Harish Chandra Research Institute

Quantum Mechanics II

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Assignment #1

1) Consider a free quantum particle in three dimensions of zero intrinsic spin. The operator $\hat{O}$ is defined to be

$$\hat{O} = \sigma(L_x - L_y) + \rho L^2$$

where L_x, L_y are the angular momentum operators in the x, y directions; and $L^2 = L_x^2 + L_y^2 + L_z^2$ (with L_z the angular momentum in the z direction) is the total angular momentum squared operator. σ, ρ are constants. A measurement of the total angular momentum of the particle is made; this yields $L^2 = 6\hbar^2$. Immediately after this, a measurement of $\hat{O}$ is made, what are the possible outcomes of this measurement as a function of σ and ρ ?

['10 points]

2) Carry out the real integral

$$\int_{-\infty}^{\infty} dx \frac{\sin^2 tx}{x^2},$$

by a suitable modification of the contour. t is a real constant.

[10 points]

3) Consider a quantum system which is initially in one of its energy eigenstates $|\ell\rangle$, it experiences a time dependent perturbation

$$\begin{aligned} \hat{V} &= 0, \quad \text{for } t < 0 \\ &= \hat{V}_0 \cos(\omega t) \quad \text{for } t > 0 \end{aligned} \tag{1}$$

If the system makes transitions to a continuum of states derive an analogue of the Golden rule.

[10 points]

4) In this problem ,we will consider the evolution of the spin of an electron in a slowly varying magnetic field. At $t = 0$, the magnetic field is $\vec{B} = B_0 \hat{x}$ and spin of the electron is in the state

$$|\psi\rangle = \frac{1}{\sqrt{2}} \left(|+\rangle + |-\rangle \right),$$

where $|+\rangle, |-\rangle$ are eigenstates of the S_z operator. The magnetic field changes with time:

$$\vec{B} = B_0 \cos(\omega t)\hat{x} + B_0 \sin(\omega t)\hat{y}$$

Assuming that the adiabatic approximation is valid, obtain the state of the spin of the electron (i.e coefficients of its expansion when it is written in terms eigenstates of the S_z operators) as a function of time. Under what condition is the adiabatic approximation valid ?

[10 points]