Harish Chandra Research Institute

Quantum Mechanics II

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Assignment #2

1) Consider a hydrogen atom in its ground state located at $\vec{r} = 0$ under the influence of a time dependent electric field of an electromagnetic wave.

$$\vec{E} = \vec{E}_0 \cos(\vec{k}.\vec{r} - \omega t)$$

Assume that the wavelength of the wave is much larger than the Bohr radius, i.e the electric field experienced by the atom can be can be taken to be

$$\vec{E} = \vec{E}_0 \cos(\omega t)$$

By making use of a Golden rule you have derived in a previous assignment, compute the rate of ionisation (that is the electron is scattered off to infinity as a result of the interaction) of the atom. You can assume that the the frequency of the light is high enough so that the wavefunction of the scattered electron is a plane wave (i.e the distortion effects due the presence of the Coulomb potential are small). Also assume that the proton is infinitely massive and hence has no dynamics. We have not discussed the magnetic field associated with electromagnetic wave in the treatment of the ionisation process. Is this justified or not (provide a reason for your answer)? We have taken the wavelength of the light to be much larger than the size of the atom, and its frequency such that deviation of the wavefunction of the ionised electron from the plane wave form is small. Show that these two criteria can be simultaneously met.

[15 + 5 + 5 points]

2) Verify by explicit computation that

$$G^{+}(r, r') = \frac{e^{ik|r-r'|}}{|r - r'|}$$

satisfies the Greens function equation

$$\left(\nabla^2 + k^2\right) G^{+}(r, r') = -4\pi\delta^3(\vec{r} - \vec{r'}).$$

Next, derive the Greens function by considering the equation in Fourier space and making a suitable choice of contour for integration involved. [4 + 6 points]

3) Consider the scattering of a particle of mass m and charge q off a ball of uniform charge density of radius r_0 and total charge Q.

(a) Compute the scattering amplitude in the Born approximation.

(b) Consider the limit of zero transfer of the Born approximation result and compare your result with that for scattering off a point like object of charge Q.

(c) Consider the Born approximation result in the regime of large transfer. What are the key differences from scattering off a point-like object in the same regime ?

For the purposes of this problem you can treat the Coulomb interaction as a limit of the Yukawa interaction. You can ignore issues associated with the validity of the Born approximation.

[4+3+3 points]

4) Show that for an object with a spherically symmetric charge density $\rho(\vec{x})$ the form factor (for Born approximation) has a low q (transfer) expansion

$$F(q) = Q\left(1 - \frac{1}{6}R_Q^2 q^2 + ...\right),$$

where Q is the total charge of the object and R_Q the charge radius

$$R_Q^2 \equiv \frac{1}{Q}\int d^3x\ (\vec{x}.\vec{x})\rho(\vec{x})$$

5) Far away from the interaction region (at large $|\vec{x}| = r$) we use the wavefunction

$$\Psi_{\vec{k}}(x) \approx N\left(e^{i\vec{k}.\vec{x}} + \frac{e^{ikr}}{r} f_{\vec{k}}(\hat{x})\right)$$

to describe the scattering process. The first term in the right hand side of the above equation is the incoming wave and the second term the scattered wave. From considerations of conservation of the probability current and taking into account the effects of interference between the incoming wave and the scattered wave derive the optical theorem.

[10 points]

6) A particle of mass m is under the influence of a spherically symmetric attractive potential well in three dimensions

$$\begin{aligned} V(r) &= -V_0, \quad r \leq a \\ &= \quad 0\ , \quad r > a, \end{aligned}$$

where V_0 is a constant. Determine the s-wave phase shift by explicitly computing the s-wave wave functions in the interior and exterior of the well. What is the total cross section at low energies ?

[5 points]