HARISH CHANDRA RESEARCH INSTITUTE

Quantum Mechanics II

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Assignment #3

1) Consider scattering of a quantum particle of mass m off a hard sphere

$$\begin{aligned} V(\vec{r}) &= \infty; \ |\vec{r}| \le a \\ &= 0; \quad |\vec{r}| > a \end{aligned}$$

Compute the "p-wave" ($\ell = 1$) phase shift and hence the "p-wave" contribution to the total cross section. Show that this is small in comparison with the "s-wave" contribution in the low energy limit.

[10 points]

2) Let $\psi_{\text{Del}}(\vec{r})$ be a function which is vanishing outside Delhi and which satisfies

$$\int d^3\vec{r} |\psi_{\text{Del}}(\vec{r})|^2 = 1$$

Similarly, let $\psi_{\text{Che}}(\vec{r})$ be a function which is vanishing outside Chennai and which satisfies

$$\int d^3\vec{r} |\psi_{\text{Che}}(\vec{r})|^2 = 1$$

Consider two electrons with the spatial wavefunction

$$\psi(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}} \left(\psi_{\text{Del}}(\vec{r}_1)\psi_{\text{Che}}(\vec{r}_2) - \psi_{\text{Del}}(\vec{r}_2)\psi_{\text{Che}}(\vec{r}_1) \right)$$

(both the their spins are in the up state). What is the probability density of finding an electron at a point $\vec{r}_{\text{Del}}$ which is located inside Delhi? What is the general lesson that you can draw from the above?

[7 + 3 points]

4) Consider three electrons in a three dimensional harmonic oscillator potential:

$$V(\vec{r}) = \frac{1}{2} m\omega^2 |\vec{r}|^2$$

Write down the expression for a ground state wavefunction by making use of the method of Slater determinants. By carrying out an expansion of the Slater determinant, provide a final expression for the wavefunction with a clear explanation of your notation for the spatial and spin labels therein. What is the ground state degeneracy of the system? [10 + 5 points]

5) Consider two spin half fermions in an external potential $V(\vec{r})$ with no mutual interaction. Show that there is a basis in which the all energy eigenstates of the system can be written as a product of "spatial part" and a "spin part" with each part either symmetric or antisymmetric under the exchange of the particles. Does this property hold when there are any number of spin half fermions (i.e not necessarily two)? Justify your answer.

[10 + 5 points]

6) *Variational method for the* $Z = 2$ *atom*: Obtain the ground state energy of the $Z = 2$ (an atom with a nucleus of charge twice that of a proton) atom by the variational method by following the steps below:

a) Take the trial spatial wavefunction of the electrons to be

$$\psi(\vec{r}_1, \vec{r}_2) = \frac{\tilde{Z}^3}{\pi a_0^3} \exp\left(-\frac{\tilde{Z}}{a_0}(r_1 + r_2)\right).$$

with $\tilde{Z}$ as the variational parameter. Take the spin wave function to be the singlet state.

b) Compute the expectation value of the Hamiltonian of the system as a function of $\tilde{Z}$ by computing the expectation values of kinetic energy of the electrons, interaction energy between the electrons and the nucleus and the interaction energy between the two electrons.

c) Estimate the ground state energy of the system by minimising with respect to $\tilde{Z}$.

Is the value of $\tilde{Z}$ at the variational ground state wavefunction greater or lesser than 2? Try to provide an intuitive explanation of its value in comparison with 2.

[15 + 5 points]

7) Consider the scattering of two identical bosons by an interaction between them which is central in nature. What is the form of the partial wave expansion of the scattering amplitude? Justify your answer.

[10 points]