Aspects of Open Quantum Systems in Quantum Information

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Overview of Talk

- Information is represented and communicated as quantum states.
- Any physical system (process) is subjected to the effects of its environment, responsible for the phenomenon of decoherence, dissipation and it is essential to understand the functioning of the process in the presence of these.
- In this talk I will discuss the influence of decoherence and dissipation on quantum information by taking various aspects of quantum information and studying their evolution under open system evolutions.
- After motivating the need for open quantum systems, I will discuss some ingredients of open systems aimed towards quantum information.
- This will be followed by a discussion of the influence of open quantum systems on some aspects of quantum information such as: quantum correlations; geometric phase; classical capacity of a quantum channel; quantum cryptography and an environment mediated deleter.

Quantum Information: Fundamental Goals

- Identify elementary classes of static resources in quantum mechanics: example...qubit. Another example is a Bell state shared between two distant parties (entanglement).
- Identify elementary classes of dynamic resources in quantum mechanics: example...quantum information transmission between two parties and process of protecting quantum information processing against the effects of noise, a natural consequence of open systems.
- Quantify resource tradeoffs incurred performing elementary dynamical processes: example: what are minimal resources required to reliably transfer quantum information between two parties using a noisy communications channel?

Open Quantum Systems: A Brief Preview

- The theory of open quantum systems addresses the problems of damping and dephasing in quantum systems by the assertion that all real systems of interest are 'open' systems, surrounded by their environments: (W. H. Louisell:(1973)); U. Weiss: (1999); H. P. Breuer and F. Petruccione: (2007).
- The recent upsurge of interest in the problem of open quantum systems is because of the impressive progress in manipulation of quantum states of matter, encoding, transmission and processing of quantum information, for all of which understanding and control of the environmental impact are essential (Turchette *et al.*: (2000); Myatt *et al.*: (2000); O'Brien *et al.*: (2003); Chou *et al.*: (2007); Barreiro *et al.*: (2011)). This increases the relevance of open system ideas to quantum computation and quantum information.

Open Quantum Systems

- Hamiltonian of the total (closed system):

$$H = H_S + H_R + H_{SR}.$$

- S- system, R- reservoir (bath), $S - R$-interaction between them.
- System-reservoir complex evolves unitarily by:

$$\rho(t) = e^{-\frac{i}{\hbar}Ht}\rho(0)e^{\frac{i}{\hbar}Ht}.$$

- We are interested in the reduced dynamics of the system S, taking into account the influence of its environment. This is done by taking a trace over the reservoir degrees of freedom, making the reduced dynamics non-unitary.

Open Quantum Systems: continued...

- Open quantum systems can be broadly classified into two categories:
  (A). Quantum non-demolition (QND), where $[H_S, H_{SR}] = 0$ resulting in decoherence without any dissipation (Braginsky *et al.*: (1975), (1980); Caves *et al.*: (1980); SB and R. Ghosh: (2007)) and
  (B). Quantum dissipative systems, where $[H_S, H_{SR}] \neq 0$ resulting in decoherence with dissipation (Caldeira and Leggett: (1983); SB and R. Ghosh: (2003), (2007)).
- In the parlance of quantum information theory, the noise generated by a QND open system would be a "phase damping channel", while that generated by a dissipative (Lindblad) evolution would be a "(generalized) amplitude damping channel".

Quantum Non-Demolition (QND)

- Generic Hamiltonian:

$$
\begin{aligned}
H &= H_S + H_R + H_{SR} = H_S \\
&+ \sum_k \hbar\omega_k b_k^\dagger b_k + H_S \sum_k g_k (b_k + b_k^\dagger) + H_S^2 \sum_k \frac{g_k^2}{\hbar\omega_k} \quad .
\end{aligned}
$$

- S- system, R- reservoir (bath), $S-R$-interaction between them.

$$
[H_S,\ H_{SR}] = 0 \Rightarrow\ QND,
$$

- Dephasing without dissipation...
- Use made of the above Hamiltonian in the context of the influence of dephasing in quantum computation—(Unruh: (1995)), (Palma et al.: (1996)), (DiVincenzo: (1995)).
- Also used by (Turchette et al.: (2000)) in context of engineered reservoir.

Two-Level System: QND

Hamiltonian

$$H_S = \frac{\hbar\omega}{2}\sigma_z,$$

σ_z being the usual Pauli matrix.

- System eigenbasis: $|j,m\rangle$— Wigner-Dicke States

$$\begin{aligned} H_S|j,m\rangle &= \hbar\omega m|j,m\rangle \\ &= E_{j,m}|j,m\rangle, \end{aligned}$$

where $-j \leq m \leq j$.

- Initial system state:

$$|\psi(0)\rangle = \cos\left(\frac{\theta_0}{2}\right)|1\rangle + e^{i\phi_0}\sin\left(\frac{\theta_0}{2}\right)|0\rangle.$$

- Reduced Density Matrix:

$$\rho^s_{m,n}(t) = \begin{pmatrix} \cos^2(\frac{\theta_0}{2}) & \frac{1}{2}\sin(\theta_0)e^{-i(\omega t+\phi_0)}e^{-(\hbar\omega)^2\gamma(t)} \\ \frac{1}{2}\sin(\theta_0)e^{i(\omega t+\phi_0)}e^{-(\hbar\omega)^2\gamma(t)} & \sin^2(\frac{\theta_0}{2}) \end{pmatrix}.$$

Two-Level System: QND *continued ...*

- Bloch vectors:

$$\begin{aligned}
\langle \sigma_x(t) \rangle &= \sin(\theta_0)\cos(\omega t + \phi_0) e^{-(\hbar\omega)^2 \gamma(t)}, \\
\langle \sigma_y(t) \rangle &= \sin(\theta_0)\sin(\omega t + \phi_0) e^{-(\hbar\omega)^2 \gamma(t)}, \\
\langle \sigma_z(t) \rangle &= \cos(\theta_0).
\end{aligned}$$

- QND Evolution—Coplanar, fixed by the polar angle θ_0, in-spiral towards the z-axis of the Bloch sphere. This is the characteristic of a **phase-damping channel** (M. Nielsen and I. Chuang: (2000)).

Dynamics of the Reduced Density Matrix for Dissipative systems

- Two-level system interacting with a squeezed thermal bath in the weak Born-Markov, rotating wave approximation.
- $H_S \sim \sigma_z$. System interacts with bath of harmonic oscillators via the atomic dipole operator (in the interaction picture)

$$\vec{D}(t) = \vec{d}\sigma_- e^{-i\omega t} + \vec{d^*}\sigma_+ e^{i\omega t},$$

where $\vec{d}$: transition matrix elements of dipole operator.
Reduced density matrix operator (master equation) in the interaction picture can be shown

$$\begin{aligned}
\frac{d}{dt}\rho^s(t) &= \gamma_0(N+1)\left(\sigma_-\rho^s(t)\sigma_+ - \frac{1}{2}\sigma_+\sigma_-\rho^s(t)\right) \\
&+ \gamma_0 N\left(\sigma_+\rho^s(t)\sigma_- - \frac{1}{2}\sigma_-\sigma_+\rho^s(t) - \frac{1}{2}\rho^s(t)\sigma_-\sigma_+\right) \\
&- \gamma_0 M\sigma_+\rho^s(t)\sigma_+ - \gamma_0 M^*\sigma_-\rho^s(t)\sigma_-.
\end{aligned}$$

- Here γ_0 is spontaneous emission rate

$$\gamma_0 = \frac{4\omega^3|\vec{d}|^2}{3\hbar c^3},$$

and σ_+, σ_-: standard raising and lowering operators, respectively given by

$$\sigma_+ = |1\rangle\langle 0| = \frac{1}{2}(\sigma_x + i\sigma_y); \quad \sigma_- = |0\rangle\langle 1| = \frac{1}{2}(\sigma_x - i\sigma_y).$$

- The master equation may be expressed in manifestly Lindblad form

$$\frac{d}{dt}\rho^s(t) = \sum_{j=1}^{2}\left(2R_j\rho^s R_j^\dagger - R_j^\dagger R_j\rho^s - \rho^s R_j^\dagger R_j\right),$$

where $R_1 = (\gamma_0(N_{\rm th}+1)/2)^{1/2}R$, $R_2 = (\gamma_0 N_{\rm th}/2)^{1/2}R^\dagger$ and $R = \sigma_-\cosh(r) + e^{i\Phi}\sigma_+\sinh(r)$. (If $T=0$, a single Lindblad operator suffices)

$$\begin{aligned} N &= N_{th}(\cosh^2(r) + \sinh^2(r)) + \sinh^2(r), \\ M &= -\frac{1}{2}\sinh(2r)e^{i\Phi}(2N_{th}+1), \\ N_{th} &= \frac{1}{e^{\frac{\hbar\omega}{k_B T}} - 1}. \end{aligned}$$

Here N_{th}: Planck distribution giving the number of thermal photons at the frequency ω; r, Φ are squeezing parameters.

- Bloch vectors from the master equation (interaction picture)

$$
\begin{aligned}
\langle \sigma_x(t) \rangle &= \\
& \left[1 + \frac{1}{2}\left(e^{\gamma_0 a t} - 1\right)(1 + \cos(\Phi))\right] e^{-\frac{\gamma_0}{2}(2N+1+a)t} \langle \sigma_x(0) \rangle \\
&- \sin(\Phi)\sinh(\frac{\gamma_0 a t}{2}) e^{-\frac{\gamma_0}{2}(2N+1)t} \langle \sigma_y(0) \rangle, \\
\langle \sigma_y(t) \rangle &= \\
& \left[1 + \frac{1}{2}\left(e^{\gamma_0 a t} - 1\right)(1 - \cos(\Phi))\right] e^{-\frac{\gamma_0}{2}(2N+1+a)t} \langle \sigma_y(0) \rangle \\
&- \sin(\Phi)\sinh(\frac{\gamma_0 a t}{2}) e^{-\frac{\gamma_0}{2}(2N+1)t} \langle \sigma_x(0) \rangle, \\
\langle \sigma_z(t) \rangle &= \\
& e^{-\gamma_0(2N+1)t} \langle \sigma_z(0) \rangle - \frac{1}{(2N+1)}\left(1 - e^{-\gamma_0(2N+1)t}\right).
\end{aligned}
$$

where $a = \sinh(2r)(2N_{th} + 1)$.

Connection to quantum noise processes

- Interpret our results in terms of familiar noisy channels. How these environmental effects can affect quantum computing.
  In operator-sum representation, action of superoperator $\mathcal{E}$ due to environmental interaction

  $$\rho \longrightarrow \mathcal{E}(\rho) = \sum_k \langle e_k | U(\rho \otimes |f_0\rangle\langle f_0|) U^\dagger | e_k \rangle = \sum_j E_j \rho E_j^\dagger,$$

  unitary operator U represents free evolution of system, environment, as well as the interaction between the two; $|f_0\rangle$: environment's initial state; $\{|e_k\rangle\}$ a basis for the environment.
- environment-system assumed to start in a separable state.
- $E_j \equiv \langle e_k | U | f_0 \rangle$ are the Kraus operators; partition of unity: $\sum_j E_j^\dagger E_j = \mathcal{I}$. Any transformation representatable as operator-sum is a completely positive (CP) map.

Quantum non-demolition interaction

- Yields **quantum phase damping channel**: uniquely non-classical quantum mechanical noise process, describing the loss of quantum information without the loss of energy. (SB and R. Ghosh: (2007))
- Kraus operator elements

$$E_0 \equiv \begin{bmatrix} 1 & 0 \\ 0 & e^{i\beta(t)}\sqrt{1-\lambda} \end{bmatrix}; \qquad E_1 \equiv \begin{bmatrix} 0 & 0 \\ 0 & \sqrt{\lambda} \end{bmatrix},$$

where $\beta(t)$ encodes the free evolution of the system and λ the effect of the environment.

- Applying this to initial state yields

$$\rho^s(t) = \mathcal{E}(\rho^s(0)) = \begin{pmatrix} \cos^2\left(\frac{\theta_0}{2}\right) & \left(\frac{1}{2}\right) e^{-i(\beta(t)+\phi_0)} \sin(\theta_0)\sqrt{1-\lambda} \\ \left(\frac{1}{2}\right) e^{i(\beta(t)+\phi_0)} \sin(\theta_0)\sqrt{1-\lambda} & \sin^2\left(\frac{\theta_0}{2}\right) \end{pmatrix}.$$

Quantum non-demolition continued ...

- Comparing with QND interaction with bath of harmonic oscillators

$$\lambda(t) = 1 - \exp\left[-2(\hbar\omega)^2\gamma(t)\right]; \qquad \beta(t) = \omega t.$$

Dissipative Interaction with a Squeezed Thermal Bath

- This extends the generalized amplitude damping channel by allowing for finite bath squeezing and is called the **squeezed generalized amplitude damping channel**. (R. Srikanth and SB: (2007))
- It is characterized by the Kraus operators

$$
\begin{aligned}
E_0 &\equiv \sqrt{p_1}\begin{bmatrix} \sqrt{1-\alpha(t)} & 0 \\ 0 & 1 \end{bmatrix}, \quad E_1 \equiv \sqrt{p_1}\begin{bmatrix} 0 & 0 \\ \sqrt{\alpha(t)} & 0 \end{bmatrix}, \\
E_2 &\equiv \sqrt{p_2}\begin{bmatrix} \sqrt{1-\mu(t)} & 0 \\ 0 & \sqrt{1-\nu(t)} \end{bmatrix}, \\
E_3 &\equiv \sqrt{p_2}\begin{bmatrix} 0 & \sqrt{\nu(t)} \\ \sqrt{\mu(t)}e^{-i\Phi} & 0 \end{bmatrix}.
\end{aligned}
$$

Dissipative Interaction with continued ...

Here

$$
\begin{aligned}
\nu(t) &= \frac{N}{p_2(2N+1)}(1-e^{-\gamma_0(2N+1)t}), \\
\mu(t) &= \frac{2N+1}{2p_2N}\frac{\sinh^2(\gamma_0 at/2)}{\sinh(\gamma_0(2N+1)t/2)}\exp\left(-\frac{\gamma_0}{2}(2N+1)t\right), \\
\alpha(t) &= \frac{1}{p_1}\left(1-p_2[\mu(t)+\nu(t)]-e^{-\gamma_0(2N+1)t}\right),
\end{aligned}
$$

where $p_2 = 1 - p_1$, and

$$
\begin{aligned}
p_2 &= \frac{1}{(A+B-C-1)^2-4D} \\
&\times \Big[A^2B+C^2+A(B^2-C-B(1+C)-D) \\
&- (1+B)D-C(B+D-1) \\
&\pm 2\,(D(B-AB+(A-1)C+D) \\
&\times (A-AB+(B-1)C+D))^{1/2}\Big],
\end{aligned}
$$

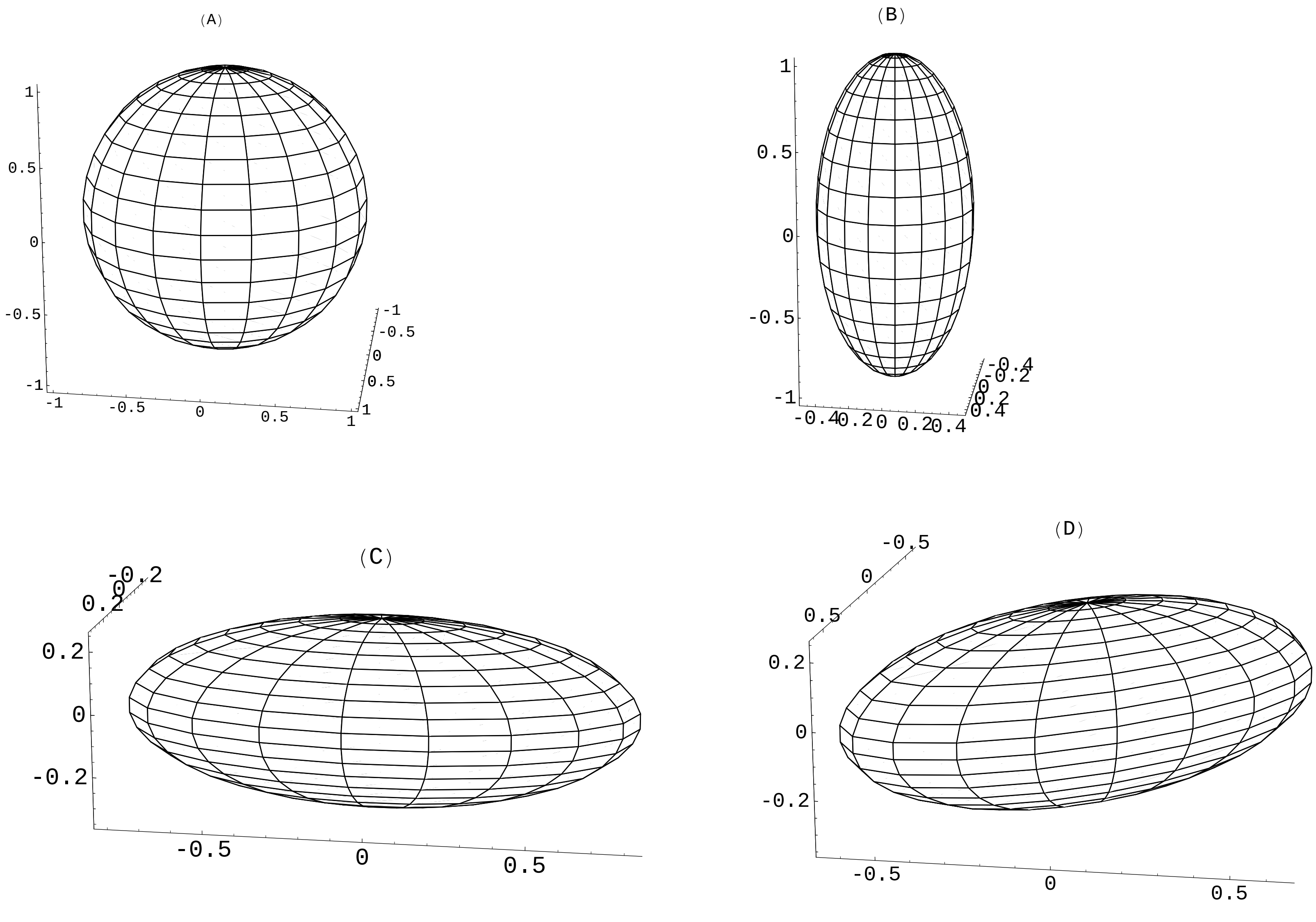
Dissipative Interaction with continued ...

with

$$
\begin{aligned}
A &= \frac{2N+1}{2N} \frac{\sinh^2(\gamma_0 a t/2)}{\sinh(\gamma_0(2N+1)t/2)} \exp\left(-\gamma_0(2N+1)t/2\right), \\
B &= \frac{N}{2N+1}(1-\exp(-\gamma_0(2N+1)t)), \\
C &= A + B + \exp(-\gamma_0(2N+1)t), \\
D &= \cosh^2(\gamma_0 a t/2)\exp(-\gamma_0(2N+1)t).
\end{aligned}
$$

- If squeezing parameter r is set to zero, the Kraus operators reduce to that of a generalized amplitude damping channel, with $\nu(t) = \alpha(t)$, $\mu(t) = 0$ and p_1 and p_2 becoming time-independent. If further $T = 0$, then $p_2 = 0$, resulting in two Kraus operators, corresponding to an amplitude damping channel.

Dissipative Interaction with continued ...



Dissipative Interaction with continued ...

Fig.1 : Effect of QND and dissipative interactions on the Bloch sphere: (A) the full Bloch sphere; (B) the Bloch sphere after time $t = 20$, with $\gamma_0 = 0.2$, $T = 0$, $\omega = 1$, $\omega_c = 40\omega$ and the environmental squeezing parameter $r = a = 0.5$, evolved under a QND interaction ; (C) and (D) the effect of the Born-Markov type of dissipative interaction with $\gamma_0 = 0.6$ and temperature $T = 5$, on the Bloch sphere – the x and y axes are interchanged to present the effect of squeezing more clearly. (C) corresponds to $r = 0.4$, $\Phi = 0$ and $t = 0.15$ while (D) corresponds to $r = 0.4$, $\Phi = 1.5$ and $t = 0.15$.

(SB and R. Ghosh: (2007))

A. Quantum Correlations

Entanglement

- What is entanglement and what is its use?
- Separability versus entanglement: that which is not separable is entangled.
- A pure state is separable if it can be expressed as a tensor product of subsystem states: $|\psi\rangle = |a\rangle \otimes |b\rangle$.
- Examples for pure states:
  (a). separable states: $|00\rangle$, $|11\rangle$
  (b). entangled states: $|\Phi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)$; $|\Psi_{\pm}\rangle = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)$: Bell states.

Entanglement continued...

- A mixed state is separable if it can be represented as a mixture of product states: $\rho = \sum_i p_i |a_i\rangle\langle a_i| \otimes |b_i\rangle\langle b_i|$. Correlations between different subsystems due to incomplete knowledge of quantum states completely characterized by classical probabilities p_i.
- Examples for mixed states:
  (a). separable state: $\rho = \frac{1}{2}(|00\rangle\langle 00| + |11\rangle\langle 11|)$
  (b). entangled state: $\rho_W = (1-p)\frac{1}{4}I + p|\Phi_+\rangle\langle\Phi_+|$, where $1/3 < p \leq 1$: Werner state.

Entanglement *continued...*

- Entanglement can be used to perform tasks not possible classically. E.g.: Using entanglement it is possible to teleport a qubit in state $|\chi\rangle = \alpha|0\rangle + \beta|1\rangle$ using a shared entangled state $|\Phi_+\rangle$.
- Thus entanglement is a resource in quantum communication and information.

Concurrence

- For a pair of qubits there exists a general formula for the entanglement of formation: E_f: based on the quantity "CONCURRENCE". (W. K. Wootters: (1998))
- Consider pure state $|\Phi\rangle$ of a pair of qubits. Concurrence

$$C(\Phi) = |\langle\Phi|\tilde{\Phi}\rangle|$$

, where $|\tilde{\Phi}\rangle = (\sigma_y \otimes \sigma_y)|\Phi^*\rangle$, σ_y is the Pauli operator, $|\Phi^*\rangle$ is the complex conjugate of $|\Phi\rangle$.

- Spin flip operation, via σ_y, when applied to a pure product state, takes the state of each qubit to the orthogonal state, i.e., state diametrically opposite on the Bloch sphere resulting in zero concurrence. A completely entangled state is left invariant by a spin flip, resulting in C taking the maximum value 1.
- Relation between entanglement and concurrence of a pure state is: $E(\Phi) = \mathcal{E}(C(\Phi))$, where $\mathcal{E}(C) = h\left(\frac{1+\sqrt{1-C^2}}{2}\right)$, $h(x) = -x\log_2(x) - (1-x)\log_2(1-x)$.

Concurrence continued...

- $\mathcal{E}(C)$ is monotonically increasing for $0 \leq C \leq 1$ implying that concurrence can be regarded as a measure of entanglement in its own right.
- Concurrence of a mixed state of two qubits is: (W. K. Wootters: (1998))
  $C(\rho) = max\Big\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\Big\}$,
  where λ_i are the square roots of the eigenvalues of $\rho\tilde{\rho}$ in descending order and $\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^*(\sigma_y \otimes \sigma_y)$.

Quantum Correlations

Discord

- Correlation between two random variables X and Y is: ‘Mutual Information’ $J(X:Y) = H(X) - H(X|Y)$.
- Here $H(X|Y)$ is the conditional entropy of X given that Y has already occured and $H(X)$ is the Shannon entropy of the random variable X.
- $H(X|Y) = H(X,Y) - H(Y)$: an alternative expression for mutual information $I(X:Y) = H(X) + H(Y) - H(X,Y)$.
- Classically: no ambiguity between these two expressions of mutual information and they are same.

Discord continued...

- Situation different in quantum regime (H. Ollivier and W. H. Zurek:(2001); L. Henderson and V. Vedral:(2001); S. Luo: (2008)).
- Consider a bipartite state ρ_{XY}: where ρ_X and ρ_Y are the states of the individual subsystems.
- Shannon entropies $H(X), H(Y)$ are replaced by von-Neumann entropies (e.g: $H(X) = S(\rho_X) = -Tr_X \rho_X Log(\rho_X)$).
- Conditional entropy $S(X|Y)$ requires a specification of the state of X given the state of Y.
- Such a statement in quantum theory is ambiguous until the to-be-measured set of states of Y are selected.
- Focus on perfect measurements of Y defined by a set of one dimensional projectors $\{\pi_j^Y\}$. The subscript j is used for indexing different outcomes of this measurement.

Discord continued...

- The state of X, after the measurement is given by

(1)
$$\rho_{X|\pi_j^Y} = \frac{\pi_j^Y \rho_{XY} \pi_j^Y}{Tr(\pi_j^Y \rho_{XY})},$$

with probability $p_j = Tr(\pi_j^Y \rho_{XY})$.

- $S(\rho_{X|\pi_j^Y})$ is the von-Neumann entropy of the system in the state ρ_X, given that projective measurement is carried out on system Y.
- The conditional entropy of X, given the complete set of measurements $\{\pi_j^Y\}$ on Y is: $S(X|\{\pi_j^Y\}) = \sum_j p_j S(\rho_{X|\pi_j^Y})$.
- The quantum analogue of $J(X:Y)$ is thus

(2)
$$J(X:Y) = S(X) - S(X|\{\pi_j^Y\}),$$

where a supremum is taken over all $\{\pi_j^Y\}$.

Discord continued...

- $I(X:Y)$ is similar to its classical counterpart

(3) $$I(X:Y) = S(X) + S(Y) - S(X,Y).$$

- It is clearly evident that these two expressions are not identical in quantum theory. Quantum discord is the difference between these two generalizations of classical mutual information,

(4) $$D(X:Y) = I(X:Y) - J(X:Y).$$

- Quantum discord aims to quantify the amount of quantum correlation that remains in the system and also points out that classicality and separability are not synonymous. In other words, it actually reveals the quantum advantage over the classical correlation.

Quantum Correlations

Bell's Inequality

- Bell's inequality: one of the first tools used to detect entanglement. (J. Bell: (1965;1971)): It is not possible for a local, realistic theory to reproduce all the statistical predictions of quantum mechanics.
- The Clauser-Horne-Shimony-Holt inequality (J. F. Clauser and A. Shimony: (1978)), derived on the premises of a local realistic theory is:

  $$E[(M1)(M3] + E[(M2)(M3] + E[(M2)(M4] - E[(M1)(M4)] \leq 2, \tag{5}$$

  where E stands for the mean value.
- Interestingly, it can be seen that in standard quantum theory, it is always possible to design experiments for which this inequality gets violated (A. Aspect, P. Grangier and G. Roger: (1981)). This shows that quantum physics can violate local realism.

Bell's Inequality continued...

- One can express the most general form of Bell-CHSH inequality for the two-qubit mixed state
  $\rho = \frac{1}{4}[I \otimes I + (r.\sigma) \otimes I + I \otimes (s.\sigma) + \sum_{n,m=1}^{3} t_{mn}(\sigma_m \otimes \sigma_n)]$
  as $M(\rho) < 1$,
  where $M(\rho) = max(u_i + u_j)$, and u_i, u_j are the eigenvalues of the matrix $T^\dagger T$ (where the elements of the correlation matrix T is given by,
  $t_{mn} = Tr[\rho(\sigma_m \otimes \sigma_n)]$) (Horodecki family: (1995)).
- Violation of Bell's inequality for a given quantum state indicates that the state is entangled. But at the same time, there are certain entangled states which do not violate Bell's inequality.

Quantum Correlations

Teleportation Fidelity

- In addition to all these measures of quantum correlation one could also attempt to quantify them in terms of an application, for e.g., fidelity of teleportation (C. H. Bennett et al.: (1993)).
- The basic idea is to use a pair of particles in a singlet state shared by sender (Alice) and receiver (Bob). Pairs in a mixed state could be still useful for (imperfect) teleportation (S. Popescu: (1994)).
- The general mixed state of a two-qubit system :

$$\begin{aligned} \rho &= \frac{1}{4}[I \otimes I + (r.\sigma) \otimes I \\ &+ I \otimes (s.\sigma) + \sum_{n,m=1}^{3} t_{mn}(\sigma_m \otimes \sigma_n)]. \end{aligned} \tag{6}$$

- The quantities $t_{nm} = Tr[\rho(\sigma_n \otimes \sigma_m)]$ are the coefficients of a real matrix denoted by T. This representation is most convenient when one talks about the inseparability of mixed states. In fact, all the parameters fall into two different classes: those that describe the local behaviour of the state, i.e., (r and s), and those responsible for correlations (T matrix).

Teleportation Fidelity continued...

- In the standard teleportation scheme a mixed state ρ acts as a quantum channel.
- One of the particles is with Bob while the other one and a third particle in an unknown state $|\phi\rangle$ are subjected to joint measurement in Alice's Hilbert space. These measurement operators are given by a family of projectors

  (7) $$P_k = |\psi_k\rangle\langle\psi_k|, k = 0, 1, 2, 3,$$

  where ψ_k constitute the so-called Bell basis.
- Using two bits Alice sends Bob the result of outcome k on basis of which he applies some unitary transformation U_k, obtaining in this way his particle in a state k.
- Fidelity of transmission of the unknown state is given by (S. Popescu: (1994); Horodecki family: (1996)),

  (8) $$F = \int_S dR(\phi) \sum_k p_k Tr(\rho_k P_\phi),$$

  where the integral is taken over all states (indexed by the angle ϕ) belonging to the Bloch sphere with uniform distribution R and $p_k = Tr[(P_k \otimes I)(P_\phi \otimes \rho)]$ denotes the probability of the k-th outcome.

Teleportation Fidelity continued...

- The task is to find those unitary transformations U_k that produce the highest fidelity (a choice of a quadruple of such U_k is what would be called a strategy).
- Maximizing F over all strategies gives (Horodecki : (1996))

$$\begin{aligned} F_{max} &= \frac{1}{2}(1+\frac{1}{3}N(\rho)) \\ &= \frac{1}{2}(1+\frac{1}{3}[\sqrt{u_1}+\sqrt{u_2}+\sqrt{u_3}]). \end{aligned} \tag{9}$$

  Here u_i and u_j are the eigenvalues of $U = T^\dagger(\rho)T(\rho)$, where $T(\rho) = [T_{ij}], T_{ij} = Tr[\rho(\sigma_i \otimes \sigma_j)]$ and $T^\dagger$ implies the Hermitian conjugate of T. The classicial fidelity of teleportation in the absence of entanglement is obtained as $\frac{2}{3}$. Thus whenever $F_{max} > \frac{2}{3}(N(\rho) > 1)$, teleportation is possible.
- At this point it is interesting to note that there is a non-trivial interplay between Bell's inequality and teleportation fidelity. This is because both $M(\rho), N(\rho)$ are dependent on the correlation matrix T. The relationship between these two quantities is the inequality $N(\rho) > M(\rho)$. Hence, it is clear that states which do violate Bell's inequality are always useful for teleportation. However, this does not rule out the possibility of existence of entangled states that do not violate Bell's inequality, but can still be useful for teleportation.

Dynamics: Model: (A).Two-qubit QND system

(SB, V. Ravishankar and R. Srikanth: (2009))

- Hamiltonian, describing the QND interaction of two qubits with the bath:

$$\begin{aligned} H &= H_S + H_R + H_{SR} \\ &= \sum_{n=1}^{L=2} \hbar \varepsilon_n J_z^n + \sum_k \hbar \omega_k b_k^\dagger b_k + \sum_{n,k} \hbar J_z^n (g_k^n b_k^\dagger + g_k^{n*} b_k). \end{aligned}$$

- H_S, H_R and H_{SR} stand for the Hamiltonians of the system, reservoir and system-reservoir interaction, respectively. $b_k^\dagger$, b_k denote the creation and annihilation operators for the reservoir oscillator of frequency ω_k, g_k^n stands for the coupling constant (assumed to be position dependent) for the interaction of the oscillator field with the qubit system and are taken to be

$$g_k^n = g_k e^{-ik.r_n},$$

  where r_n is the qubit position. Since $[H_S, H_{SR}] = 0$, the Hamiltonian (1) is of QND type.

Dynamics of the Reduced Density Matrix continued...

- The position dependence of the coupling of the qubits to the bath helps to bring out the effect of entanglement between qubits through the qubit separation: $r_{mn} \equiv r_m - r_n$. This allows for a discussion of the dynamics in two regimes:

  (a). Localized (independent) Decoherence:

  where $k.r_{mn} \sim \frac{r_{mn}}{\lambda} \geq 1$

  and

  (b). Collective Decoherence:

  where $k.r_{mn} \sim \frac{r_{mn}}{\lambda} \rightarrow 0$.

Model: (B).Two-qubit Dissipative system

(SB, V. Ravishankar and R. Srikanth: (2010))

- Hamiltonian, describing the dissipative, position dependent, interaction of two qubits with bath (modelled as a 3-D electromagnetic field (EMF)) via dipole interaction as:

$$
\begin{aligned}
H &= H_S + H_R + H_{SR} \\
&= \sum_{n=1}^{N=2} \hbar\omega_n S_n^z + \sum_{\vec{k}s} \hbar\omega_k (b^\dagger_{\vec{k}s} b_{\vec{k}s} + 1/2) - i\hbar \sum_{\vec{k}s} \sum_{n=1}^{N} [\vec{\mu}_n . \vec{g}_{\vec{k}s}(\vec{r}_n)(S_n^+ + S_n^-) b_{\vec{k}s} - h
\end{aligned}
$$

$\vec{\mu}_n$: transition dipole moments, dependent on the different atomic positions $\vec{r}_n$

$$S_n^+ = |e_n\rangle\langle g_n|, \; S_n^- = |g_n\rangle\langle e_n| :$$

dipole raising and lowering operators satisfying the usual commutation relations

$$S_n^z = \frac{1}{2}(|e_n\rangle\langle e_n| - |g_n\rangle\langle g_n|) :$$

energy operator of the nth atom

Dynamics of the Reduced Density Matrix continued...

- $b^{\dagger}_{\vec{k}s}$, $b_{\vec{k}s}$: creation and annihilation operators of the field mode (bath) $\vec{k}s$ with the wave vector $\vec{k}$, frequency ω_k and polarization index $s = 1, 2$
- System-Reservoir (S-R) coupling constant:

$$\vec{g}_{\vec{k}s}(\vec{r}_n) = (\frac{\omega_k}{2\varepsilon\hbar V})^{1/2}\vec{e}_{\vec{k}s}e^{i\vec{k}.r_n}.$$

  V: the normalization volume and $\vec{e}_{\vec{k}s}$: unit polarization vector of the field.
- S-R coupling constant: dependent on the atomic position r_n.
  This leads to a number of interesting dynamical aspects.

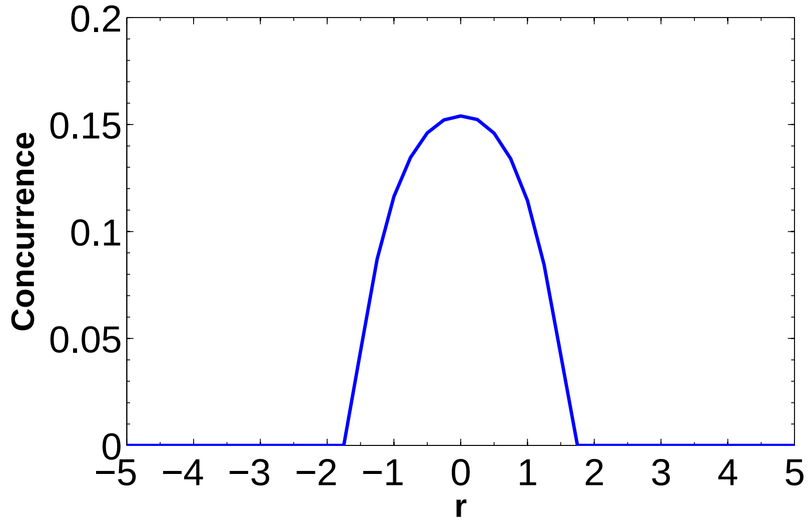
Dynamics of the Reduced Density Matrix continued...

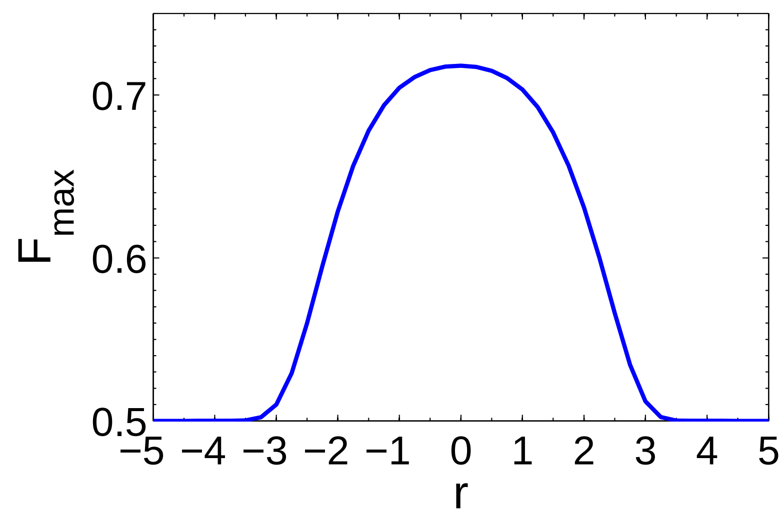
- Wavevector $k_0 = 2\pi/\lambda_0$, λ_0 being the resonant wavelength, sets up a length scale into the problem depending upon the ratio r_{ij}/λ_0. This is thus the ratio between the interatomic distance and the resonant wavelength, allowing for a discussion of the dynamics in two regimes:
  (a). localized decoherence: where $k_0.r_{ij} \sim \frac{r_{ij}}{\lambda_0} \geq 1$
  and
  (b). collective decoherence: where $k_0.r_{ij} \sim \frac{r_{ij}}{\lambda_0} \to 0$.
- Collective decoherence would arise when the qubits are close enough for them to feel the bath collectively or when the bath has a long correlation length (set by the resonant wavelength λ_0) in comparison to the interqubit separation r_{ij}.

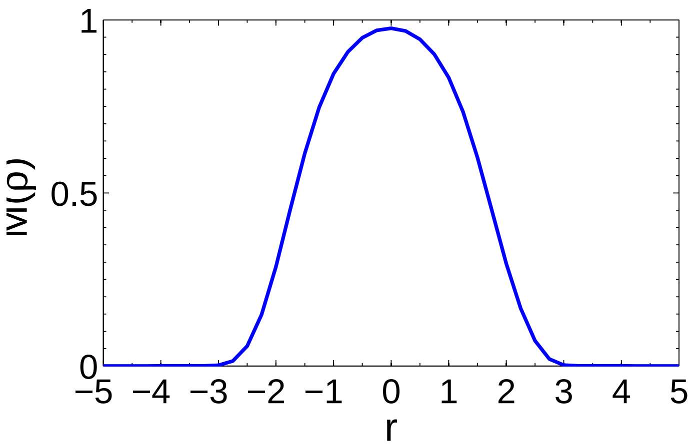
Dynamics of Quantum Correlations

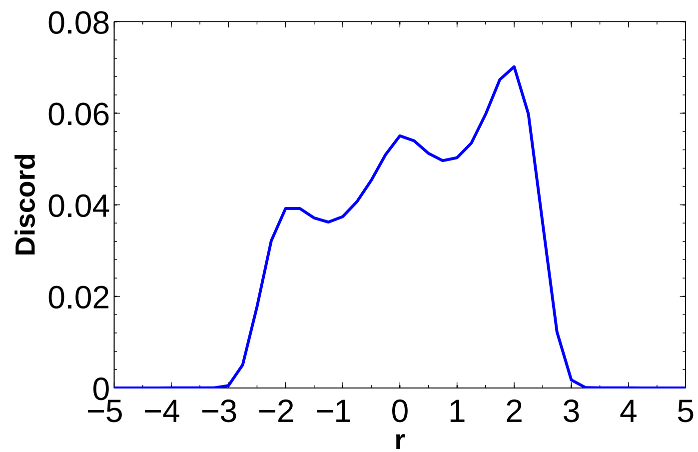
- We made a comparative study, on states generated by our model, of various features of quantum correlations like teleportation fidelity (F_{max}), violation of Bell's inequality $M(\rho)$ (violation takes place for $M(\rho) \geq 1$), concurrence $C(\rho)$ and discord with respect to various experimental parameters like, bath squeezing parameter r, inter-qubit spacing r_{12}, temperature T and time of evolution t (I. Chakrabarty, SB, N. Siddharth: (2011)).
- A basic motivation of this work is to have realistic open system models that generate entangled states which can be useful for teleportation, but at the same time, not violate Bell's inequality. We provide below some examples of such states. Interestingly, we also find examples of states with positive discord, but zero entanglement, reiterating the fact that entanglement is a subset of quantum correlations.

Dynamics of Quantum Correlations: QND





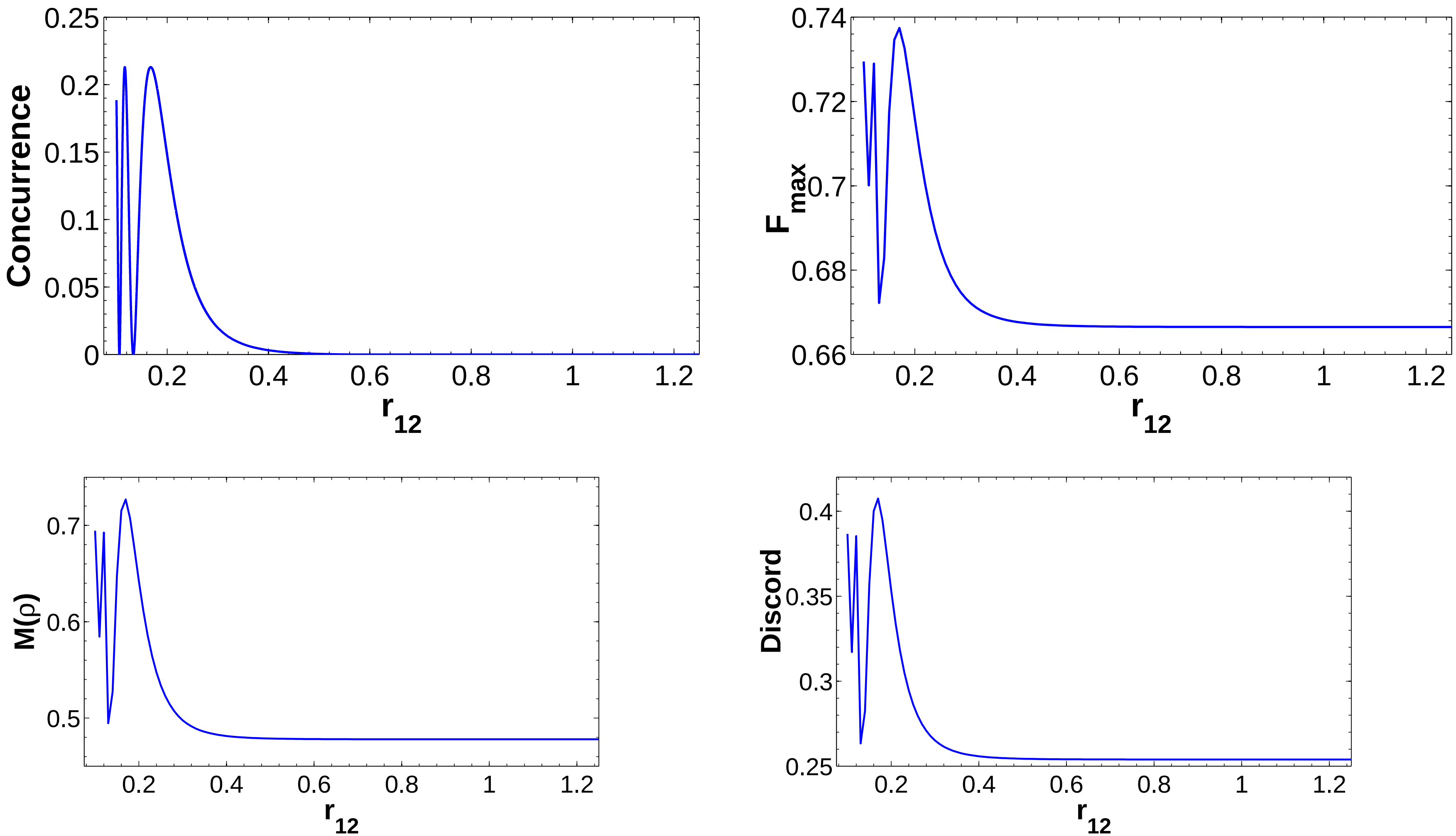




Dynamics of Quantum Correlations: QND continued...

Fig. 2 : The example depicted in Figs. (2 (a)), (b), (c), (d), study two-qubit density matrices, as a function of bath squeezing parameter r, from the localized (independent) model. In Fig. (2 (a)), concurrence is plotted with respect to bath squeezing parameter r. It is seen that states are entangled when r lies in the range $[-1.8, 1.8]$. Teleportation fidelity, as in the Fig. (2 (b)), indicates that the states are useful for teleportation for the same range of r, i.e., when they are entangled. However, from Bell's inequality, as shown in Fig. (2 (c)), in the same range, we see that the states do not violate Bell's inequality. Interestingly, from Fig. (2 (d)), we find a non zero quantum discord in the range $[-3, 3]$ and particularly, in the range $[-3, -1.8] \cup [1.8, 3]$, i.e., where entanglement as depicted in Fig. (2 (a)) is zero, discord is non-zero. This brings out the fact that the amount of entanglement present in a system is not equivalent to the the total amount of quantum correlation in it. For the parameters chosen, we see a similar pattern of the various correlation functions as in the previous figure, i.e., with the exception of discord, the quantum correlation measures fall with the increase in bath squeezing.

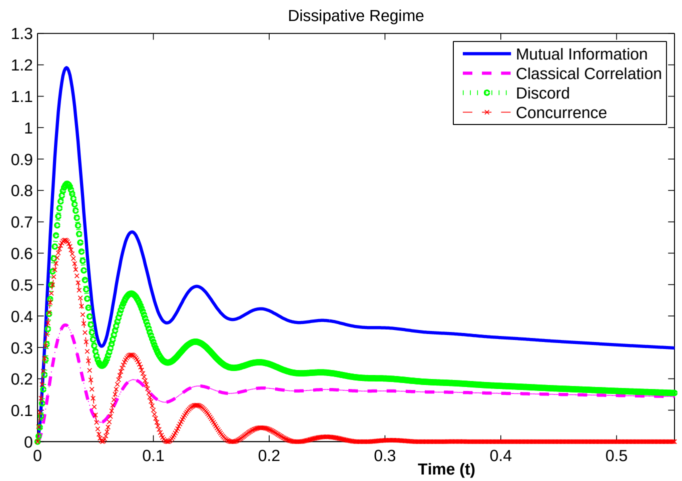
Dynamics of Quantum Correlations: Dissipative



Dynamics of Quantum Correlations: Dissipative continued...

Fig. 3 : Quantum correlations in a two-qubit system undergoing a dissipative evolution. The Figs. (a), (b), (c) and (d) represent the evolution of concurrence, maximum teleportation fidelity F_{max}, test of Bell's inequality $M(\rho)$, discord as a function of inter-qubit distance r_{12}. Here temperature $T = 300$, evolution time t is 0.1 and bath squeezing parameter $r = -1$. From Fig. (3 (a)), we find that the two qubit density matrix is entangled with a positive concurrence except at the point 0.133 (approx) and for $r_{12} \geq 0.4$. Figure (3 (b)) illustrates that $F_{max} > \frac{2}{3}$, for all values of r_{12} except where there is no entanglement. However, from Fig. (3 (c)) we find that $M(\rho) < 1$ for all values of r_{12}, clearly demonstrating that the states can be useful for teleportation despite the fact that they satisfy Bell's inequality. Moreover, from Fig. (3 (d)), a positive discord is seen for the complete range of r_{12}, even in the range where there is no entanglement. As a function of the inter-qubit distance, the various correlation measures exhibit oscillatory behavior, in the collective regime of the model, but flatten out subsequently to attain almost constant values in the independent regime of the model. This oscillatory behavior is due to the strong collective behavior exhibited by the dynamics due to the relatively close proximity of the qubits in the collective regime.

Dynamics of Quantum Correlations: Dissipative continued...



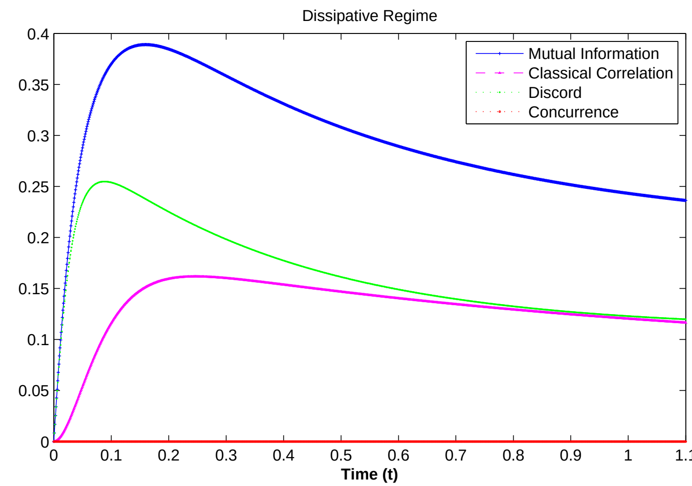


Fig. 4 & 5 : An example showing vanishing entanglement, but non vanishing discord, for a dissipative two-qubit evolution. Figures (4) and (5) represent the evolution of mutual information (blue), quantum discord (quantum correlation) (green), concurrence (red) and classical information (pink) with respect to the time of evolution t, evolving under a dissipative interaction for collective and local models, respectively. We find that in the absence of entanglement from a certain time $t > \bar{t}$, the classical correlation and the quantum discord becomes identical.

Geometric Phase (GP) in Open Quantum Systems

Brief history of GP

- Pancharatnam defined a phase characterizing the intereference of classical light in distinct states of polarization (1956).
- Berry (1984) discovered that under cyclic adiabatic evolution, system acquires extra phase over dynamical phase.
- Simon (1983) establised geometric nature of GP, linked to notion of parallel transport, depends only on area covered by motion, independent of how motion is executed.
  (consequence of the holonomy in a line bundle over parameter space)
- Generalization of GP to non-adiabatic evolution (Aharonov and Anandan 1987) to non-cyclic evolution (Samuel and Bhandari 1988)
- GP as a consequence of quantum kinematics (Mukunda & Simon 1993).
- GP defined for nondegenerate density opertors undergoing unitary evolution (Sjöqvist *et al.* 2000)
- Extended by Singh *et al.* (2003) to the case of degenerate density operators.
- Kinematic approach to define GP in mixed states undergoing nonunitary evolution Tong *et al.* (2004) – which we use.

Motivation

- The geometric nature of GP implies an inherent fault tolerance and would be useful for quantum computers (Duan, Cirac, Zoller: (2001)).
- There have been proposals to observe GP in superconducting nanocircuits (Falci, Fazio, Palma, *et al.*: (2000)). Here the effect of the environment is never negligible (Nakamura, Pashkin, Tsai: (1999)).
- The above points provide a strong motivation for studying GP in the context of Open Quantum Systems. Work along these lines was initiated by (Whitney, Gefen: (2003); Whitney, Makhlin, Shnirman, Gefen: (2005)).

A. Geometric Phase (GP) in Two-Level Open System

(SB and R. Srikanth: (2007))

- System Hamiltonian

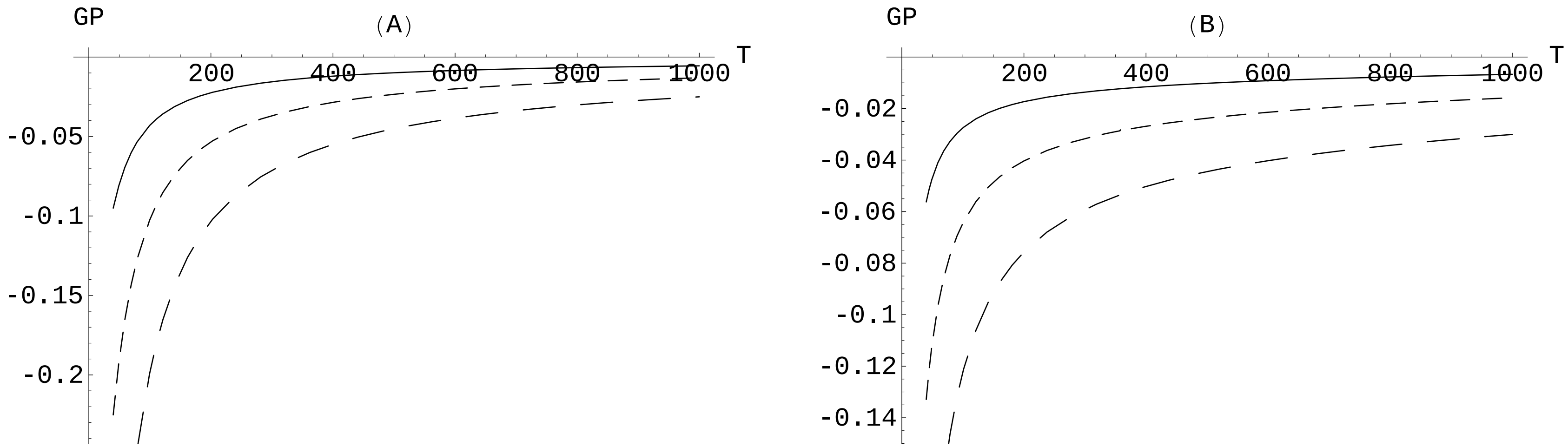
$$H_S = \frac{\hbar\omega}{2}\sigma_z.$$

- System interacts with a squeezed-thermal bath via a QND or a dissipative interaction.
- Use prescription of Tong *et al.* (2004)

$$\Phi_{\mathrm{GP}} = \arg\left(\sum_{k=1}^{N}\sqrt{\lambda_k(0)\lambda_k(\tau)}\langle\Psi_k(0)|\Psi_k(\tau)\rangle e^{-\int_0^\tau dt\langle\Psi_k(t)|\dot{\Psi}_k(t)\rangle}\right).$$

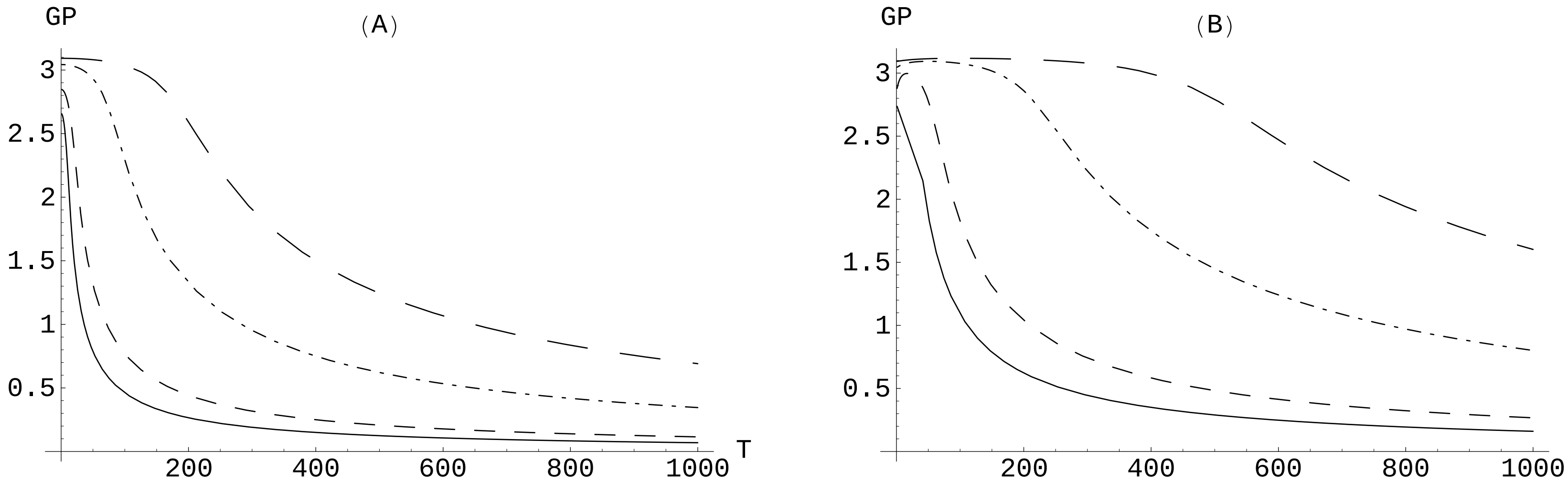
- $\lambda_k(\tau)$, $\Psi_k(\tau)$: eigenvalues, eigenvectors of reduced density matrix.

Geometric Phase (GP)... continued...



Figs. 6 & 7 : GP (in radians) as function of temperature (T) for QND interaction with a bath of harmonic oscillators. Fig. (6) $\gamma_0 = 0.005$ and vanishing squeezing. The solid, dashed and larger-dashed lines correspond to $\theta_0 = \pi/8, 3\pi/16$ and $\pi/4$. Fig. (7) same as Figure (6), except that here squeezing is non-vanishing, with $r = 0.7$ and $a = 0.1$. Increasing T or squeezing causes tip of Bloch vector to inspiral more towards the σ_z axis, leading to suppression of GP.

Geometric Phase (GP)... continued...



Figs. 8 & 9 : GP (in radians) as function of temperature (T) for dissipative interaction with a bath of harmonic oscillators. Here $\omega = 1$, $\theta_0 = \pi/2$, the large-dashed, dot-dashed, small-dashed and solid curves, represent, $\gamma_0 = 0.005, 0.01, 0.03$ and 0.05, respectively. Fig. (8) zero squeezing, Fig. (9): squeezing non–vanishing, with $r = 0.4$ and $\Phi = 0$. GP falls with T. Effect of squeezing: GP varies more slowly with T, by broadening peaks and flattening tails. Counteractive action of squeezing on influence of T on GP useful for practical implementation of GP phase gates.

B. Geometric Phase (GP) in Three-Level Quantum Optical System

(Sandhya and S. Banerjee: (2011))

- Holonomic quantum computation requires the evolution of qubits in parameter space: by the control of parameters which are physically feasible.
- Atom photon interaction provides a rich ground for exploring geometric phase with the added advantage of the existence of control parameters for manipulating the photon states.
- We study the geometric phase of the two-photon state corresponding to the two modes emitted by the two dipole transitions of three level cascade system interacting with two driving fields. The two photon state is in general a mixed state.
- The evolution in the state space is made by varying the control parameters namely, the driving field strength and detuning.

B. Geometric Phase (GP) in Three-Level... continued...

Model

- The scheme considered here corresponds to a three level cascade system interacting with two coherent fields which address the only two allowed dipole transitions $|i\rangle \leftrightarrow |i+1\rangle, i = 1, 2$ with energy separation given by ω_i.
- Two counter propagating (Doppler free geometry) driving fields of nearly equal frequencies ω_{L1} and ω_{L2} and respective strengths Ω_1 and Ω_2 are resonant with these two transitions.
- The decay constants of the energy levels $|3\rangle$ and $|2\rangle$ are indicated by Γ_3 and Γ_2, respectively. The parameters $\Delta_1, \ \Delta_2$ refer to the detunings of the driving fields.
- This scheme may be realized for example in ^{87}Rb vapor with the corresponding energy levels $5S_{1/2}, 5P_{3/2}$ and $5D_{5/2}$ and has been used by (Banacloche *et al.*: (1995)) for studying electro-magnetically induced transparency.
- We use the prescription given in (Tong *et al.* (2004)) to determine the geometric phase of the two-photon mixed states emitted by the three level system.

B. Geometric Phase (GP) in Three-Level... continued...

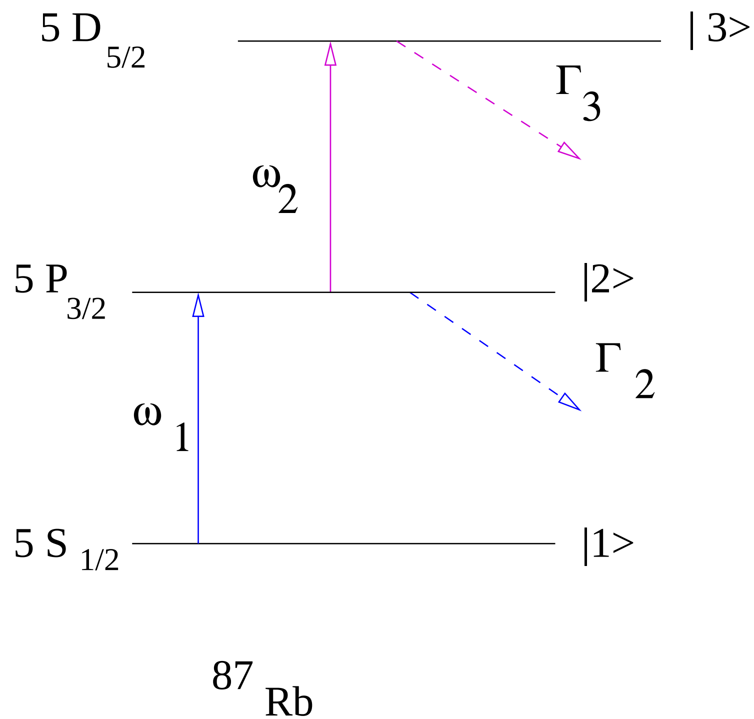


Fig. 10 : Three level cascade system corresponding to the ^{87}Rb atoms driven by two fields ω_1 and ω_2.

B. Geometric Phase (GP) in Three-Level... continued...

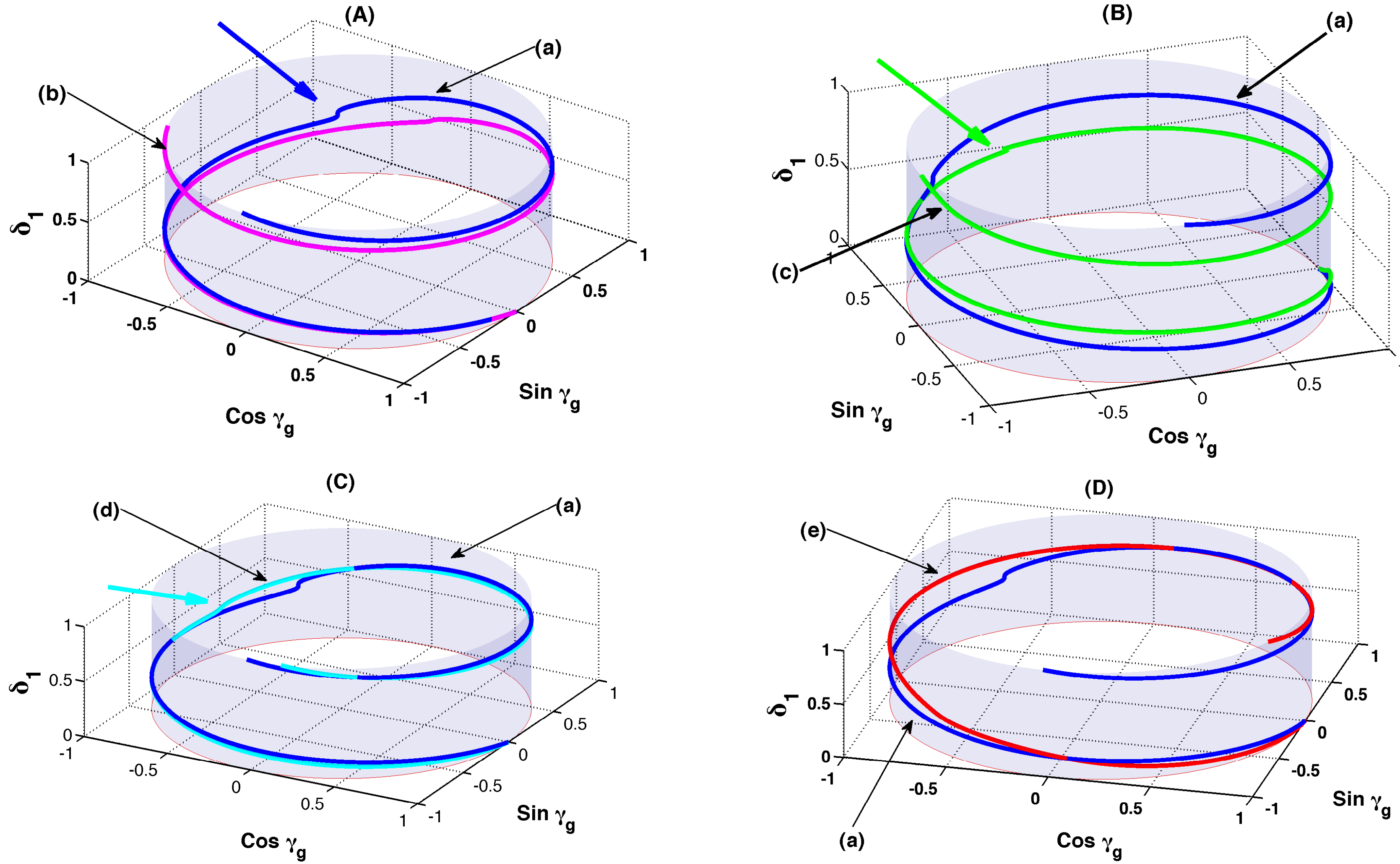


Fig. 11 : Variation of the geometric phase with the rescaled detuning parameter $\delta_1 = (\Delta_1 + 10.0)/20.0$.

B. Geometric Phase (GP) in Three-Level... continued...

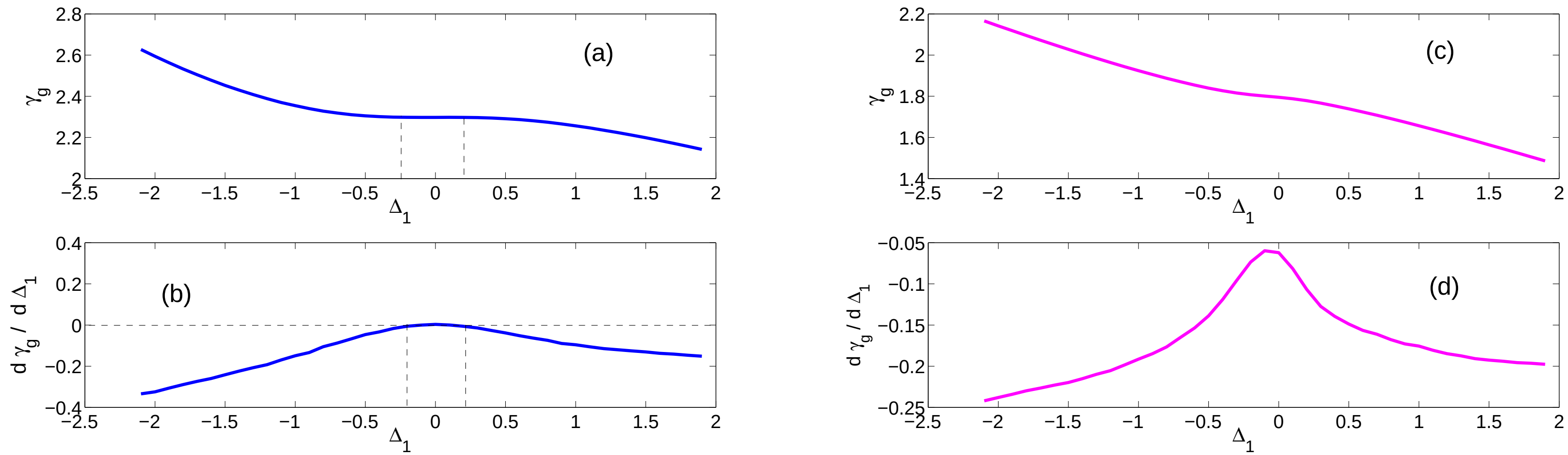


Fig. 12 : Variation of the geometric phase and its derivative with the detuning parameter Δ_1. Parameter values of figures 13 (a) and (b) are those corresponding to the curve (a) of Figs. 11; figures 12 (c) and (d) correspond to curve (b) of Figs. 11. The parameter values of the curves are: (a) $\Omega_1 = \Omega_2 = 6.0, \Delta_2 = 0$, (b) $\Omega_1 = 3.0, \Omega_2 = 6.0, \Delta_2 = 0$, (c) $\Omega_1 = 6.0, \Omega_2 = 3.0, \Delta_2 = 0$, (d) $\Omega_{1,2} = 6.0, \Delta_2 = 3.0$ and (e) $\Omega_{1,2} = 6.0, \Delta_2 = 6.0$.

B. Geometric Phase (GP) in Three-Level... continued...

Model

- In Figs. 11, for the case of curve (a) γ_g does not change in the neighborhood of $\delta_1 = 0.5$ which results in a smaller sweep while the variation of the angle is uniformly slow in the case of curve (e).
- In Figs. 12, the details of the variation of the curves (a) and (b) in the neighborhood of $\Delta_1 = 0$ are presented.
- Figs. 12 (a), (b) show the variation of γ_g and the rate of change of γ_g, respectively with Δ_1 near $\Delta_1 = 0$ of the curve (a) of Figs. 11. γ_g is constant in the region $-0.25 < \Delta_1 < 0.25$. This is substantiated by the vanishing of the derivative in this region thus indicating *that the geometric phase in this case is stable under small perturbations in the neighborhood of* $\Delta_1 = 0$.
- In contrast, in the case of Figs. 12 (c), (d) γ_g is never a constant even though *the rate of change of* γ_g *slows down and shows an extremum at two photon resonance*.

Classical Capacity of Squeezed Generalized Amplitude Damping Channel

(SB and R. Srikanth: (2008))

- A quantum communication channel can be used to perform a number of tasks: transmitting classical or quantum information.
- How information communicated over squeezed generalized amplitude damping channel is degraded...
- Consider the following situation: there is a sender A and receiver B; A has a classical information source producing symbols $X = 0, \cdots, n$ with probabilities $p_0, \cdots, p_n$ which are encoded as quantum states ρ_j $(0 \leq j \leq n)$ and communicated to B, whose optimal measurement strategy maximizes the accessible information, which is bounded above by the Holevo quantity

$$\chi = S(\rho) - \sum_j p_j S(\rho_j),$$

where $\rho = \sum_j p_j \rho_j$, and ρ_j are various initial states and S(ρ) is the von Neumann entropy.

Classical Capacity of Squeezed Generalized continued ...

- Here assume A encodes binary symbols of 0 and 1 in terms of pure, orthogonal states of the form $|\psi(0)\rangle = \cos(\frac{\theta_0}{2})|1\rangle + e^{i\phi_0}\sin(\frac{\theta_0}{2})|0\rangle$, and transmits them over the squeezed generalized amplitude damping channel ($\mathcal{E}$).
- Further assume that A transmits messages as product states, i.e., without entangling them across multiple channel use. Then, the (product state) classical capacity C of the quantum channel is defined as the maximum of $\chi(\mathcal{E})$ over all ensembles $\{p_j, \rho_j\}$ of possible input states ρ_j .

Classical Capacity of Squeezed Generalized continued...

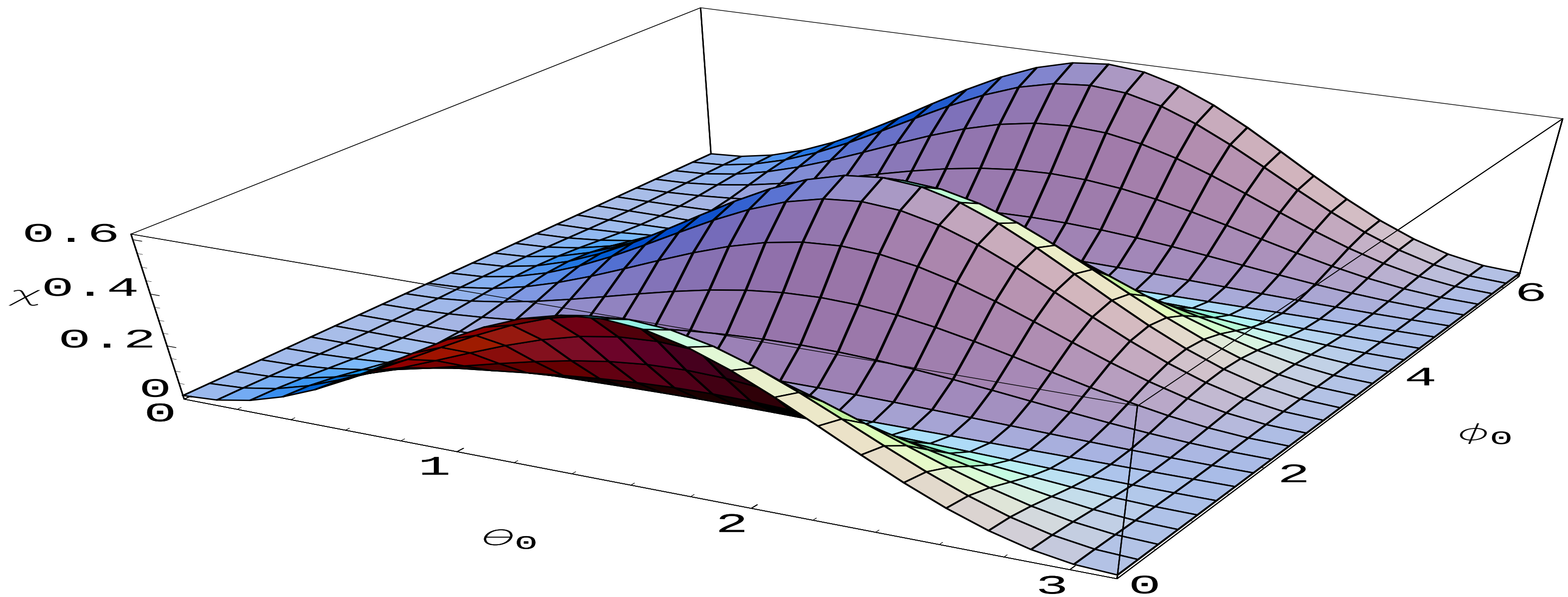
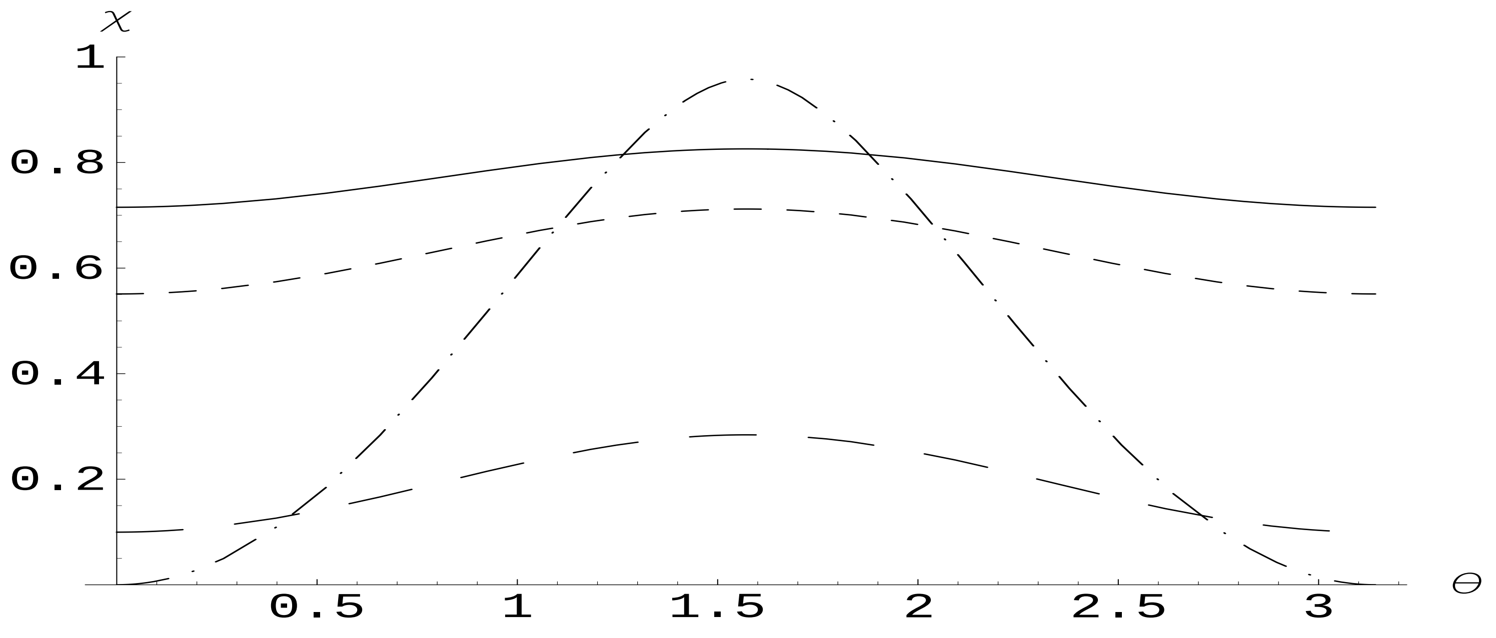


Fig. 13 : Holevo bound χ for a squeezed generalized amplitude damping channel with $\Phi = 0$, over the set $\{\theta_0, \phi_0\}$, which parametrizes the ensemble of input states $\{(\theta_0, \phi_0),\ (\theta_0 + \pi, \phi_0)\}$, corresponding to the symbols 0 and 1, respectively, with probability of the input symbol 0 being $f = 0.5$. Here temperature $T = 5$, $\gamma_0 = 0.05$, time $t = 5.0$ and bath squeezing parameter $r = 1$. The channel capacity C is seen to correspond to the optimal value of $\theta_0 = \pi/2$ (i.e., the input states $\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$ for $\phi_0 = 0$).

Classical Capacity of Squeezed Generalized continued...



Classical Capacity of Squeezed Generalized continued ...

- Fig. 14 illustrates optimal source coding for the squeezed amplitude damping channel, with χ plotted against θ_0 corresponding to the "0" symbol. Here $\Phi = 0$, $\gamma_0 = 0.05$ and $f = 0.5$. It is seen that χ is maximized for states of the form when the pair of input states are given by $(\theta_0 = \frac{\pi}{2}, \phi_0 = 0)$ and $(\theta_0 = \frac{\pi}{2} + \pi, \phi_0 = 0)$ (i.e., states $\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$). The solid and small-dashed curves represent temperature $T = 0$ and bath squeezing parameter $r = 0$, but $t = 1$ and 2, respectively. The large-dashed and dot-dashed curves represent $T = 5$ and $t = 2$, but with $r = 0$ and 2, respectively.
- A comparison of the solid and small-dashed (small-dashed and large-dashed) curves demonstrates the expected degrading effect on the accessible information, of increasing the bath exposure time t (increasing T).
- A comparison of the large-dashed and dot-dashed curves demonstrates the dramatic effect of including squeezing. In particular, whereas squeezing improves the accessible information for the pair of input states $\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$, it is detrimental for input states (θ_0, ϕ_0) given by $(0, 0)$ (i.e., $|1\rangle$) and $(\pi, 0)$ (i.e., $|0\rangle$).

Classical Capacity of Squeezed Generalized continued...

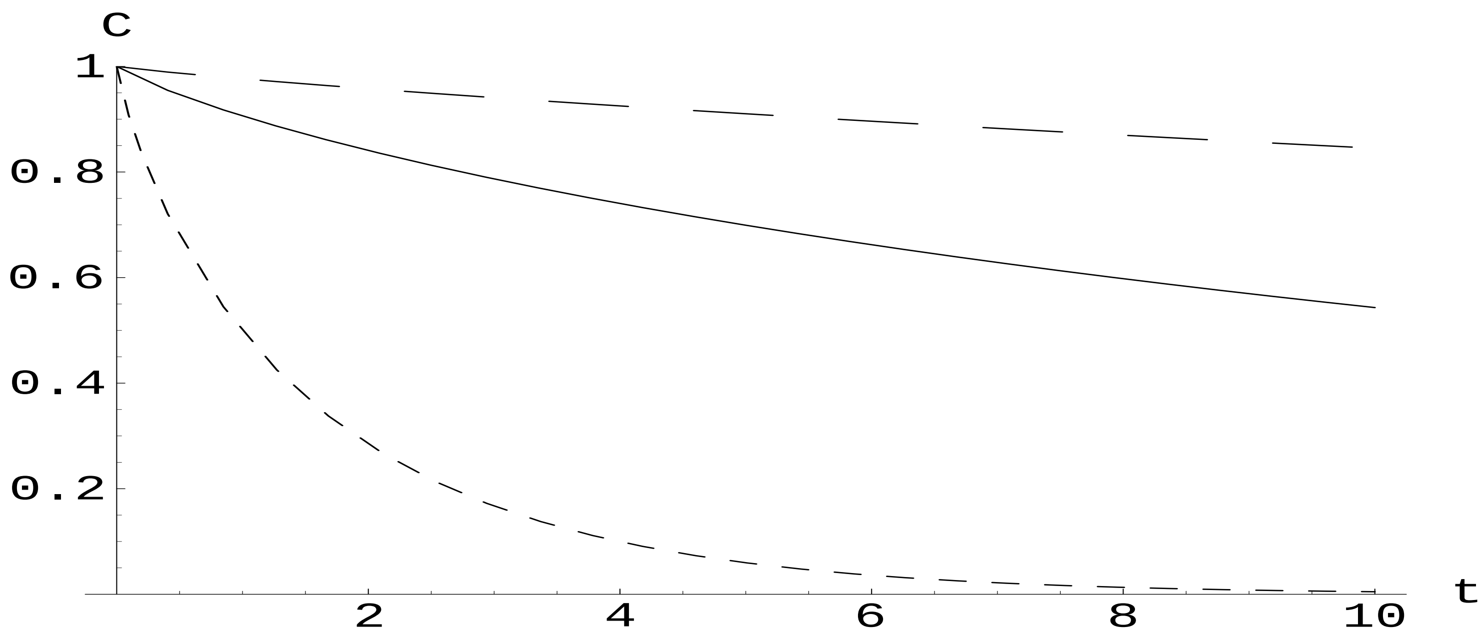


Fig. 15 : Interplay of squeezing and temperature on the classical capacity C of the squeezed amplitude damping channel (with input states $\frac{1}{\sqrt{2}}(|0\rangle \pm |1\rangle)$, and $f = 1/2$, corresponding to the optimal coding). Here $\Phi = 0$ and $\gamma_0 = 0.05$. The solid and small-dashed curves correspond to zero squeezing r, and temperature $T = 0$ and 5, respectively. The large-dashed curve corresponds to $T = 5$ and $r = 2$. A comparison between the solid and large-dashed curves shows that squeezing can improve C. This highlights the possible usefulness of squeezing to noisy quantum communication.

Quantum Cryptographic Switch

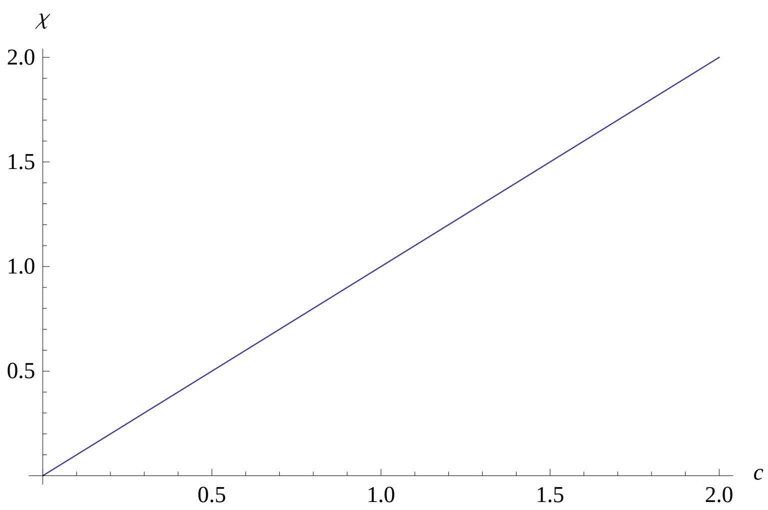
(S. Narayanaswamy, O. Srikrishna, R. Srikanth, SB and A. Pathak: (2011))

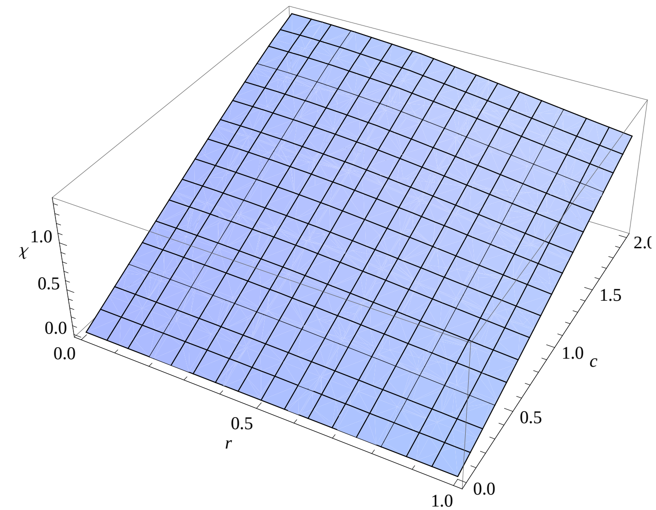
- We illustrate using a quantum system the principle of a cryptographic switch, in which a third party (Charlie) can control to a continuously varying degree the amount of information the receiver (Bob) receives, after the sender (Alice) has sent her information. Suppose Charlie transmits a Bell state to Alice and Bob. Alice uses dense coding to transmit two bits to Bob. Only if the 2-bit information corresponding to choice of Bell state is made available by Charlie to Bob can the latter recover Alice's information. By varying the information he gives, Charlie can continuously vary the information recovered by Bob.
- The performance of the protocol subjected to the squeezed generalized amplitude damping channel is considered.
- This can also be visualized as an application of *controlled dense coding*, in which the controller is Charlie, who determines how much classical information is delivered to Bob after Alice sends him all her dense coding qubits.

Quantum Cryptographic Switch continued ...

- In this protocol, all four Bell states can be used. The key information is thus 2 bits.
- A family of protocols can be considered in which the key information c varies continuously as $0 \leq c \leq c_{\max} = 2$.
- The problem can be treated as a communication situation in which Alice is signaling Bob by means of Bell states. Then the maximum information Bob can extract from the pair of qubits is the Holevo quantity. When $c = 2$, Bob can extract Alice's 2 bits of information.

Quantum Cryptographic Switch continued ...





Figs. 16 & 17 : Fig. 16 depicts information recovered by Bob, quantified by the Holevo quantity χ, as a function of the key information c communicated by Charlie, in the noiseless case. Fig. 17 shows information recovered by Bob, as a function of the squeezing parameter r, coming from the Squeezed Generalized Amplitude Damping Channel, and key information c communicated by Charlie. The time of evolution $t = 0.5$, while temperature $T = 0.1$. The Holevo quantity χ increases with c, but not as much as in the noiseless case: due to the randomness introduced by the noise.

Quantum Cryptographic Switch continued...

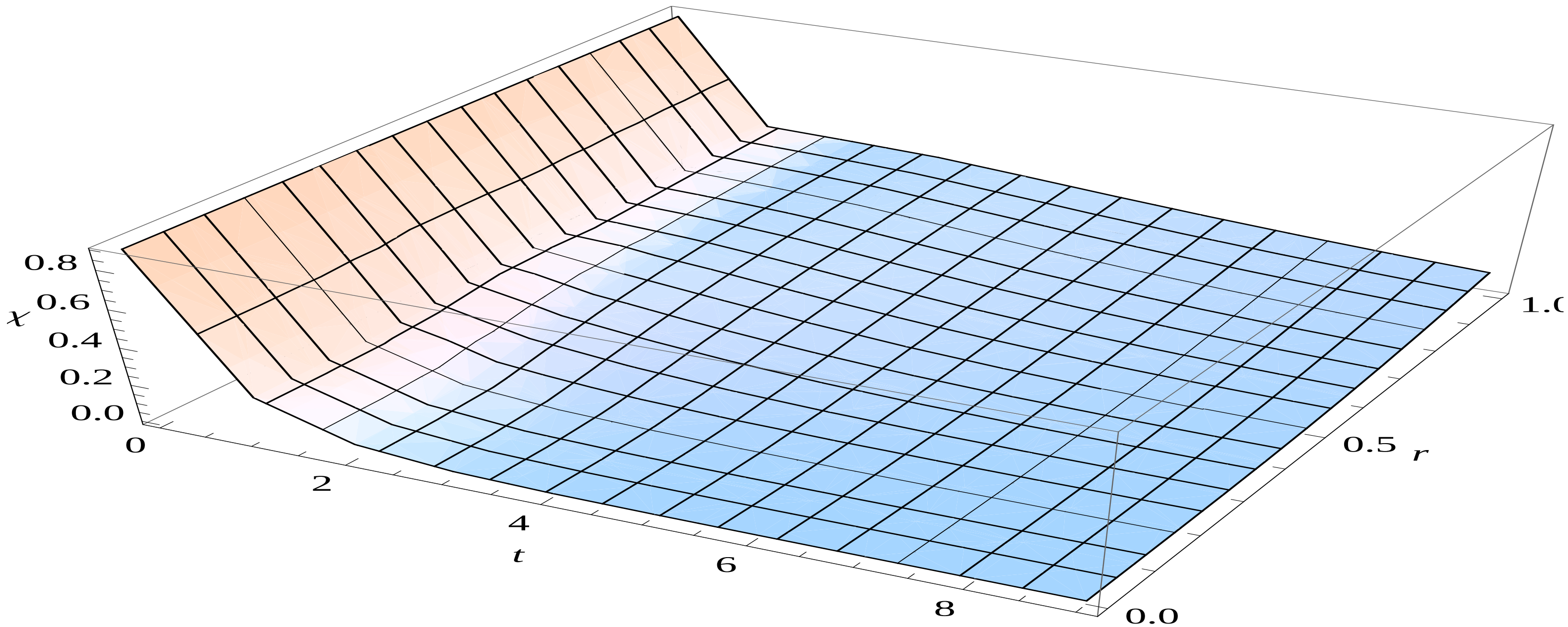


Fig. 18 : Information recovered by Bob, as a function of the squeezed generalized amplitude damping channel parameters r (squeezing) and t (time of evolution), assuming Charlie communicates one bit of information. For sufficiently early times, squeezing fights thermal effects (here $T = 0.1$) to cause an increase in the recovered information.

Environment-Mediated Quantum Deleter

(R. Srikanth and SB: (2007))

- Quantum computation is well known to solve certain types of problems more efficiently than classical computation (M. A. Nielsen and I. L. Chuang: (2000)).
- Although quantum mechanical linearity endows a quantum computer with greater-than-classical power (P. Shor: (1995); L. K. Grover: (1997)), it also imposes certain restrictions, such as the prohibition on cloning (W. K. Wooters and W. H. Zurek: (1982)) and on deleting (A. K. Pati and S. Braunstein: (2000)). The latter result means that quantum mechanics does not allow us to delete a copy of an arbitrary quantum state perfectly.

Requirement of Open Quantum System

- A quantum computational task can be broadly divided into three stages:
  - (A). Initializing the quantum computer, by putting all qubits into a standard ‘blank state’;
  - (B). Executing the unitary operation that performs the actual computation. This is the area where “decoherence” is an obstacle. A variety of techniques, including quantum error correction (S. Calderbank and P. Shor: (1996); A. Steane: (1996)), dynamic decoupling (L. Viola and S. Lloyd: (1998); D. Vitali and P. Tombesi: (2001)) , fault tolerant quantum computation (P. Shor: (1996)), decoherence-free subspaces (D. A. Lidar, I. L. Chuang and K. B. Whaley: (1998)), among others exist to combat decoherence;
  - (C). Performing measurements to read off results.
- In step (A), we must be able to erase quantum memory at the end of a computational task, in order to prepare the state of a quantum computer for a subsequent task. What is required is a quantum mechanism that with high probability allows us to prepare standard ‘blank states’. It is clear that no unitary process can achieve this, since true deletion would be irreversible, and hence non-unitary. Further, the no-deleting theorem implies that no qubit state can be erased against a copy (A. K. Pati and S. Braunstein: (2000)).

Requirement of Open... continued...

- A direct method for initializing the quantum computer would be to measure all qubits in the computational basis. This results in a statistical mixture of $|0\rangle$'s and $|1\rangle$'s, and there is no unitary way in a closed system to convert the $|1\rangle$'s while retaining the $|0\rangle$'s. However, open quantum systems, in particular a decohering environment, can effect non-unitary evolution on a sub-system of interest. We are thus led to conclude that decoherence is in fact *necessary* for step (A), since there would be no other way to delete quantum information.
- Here this insight is used to argue that decoherence can be useful to quantum computation. In particular, it is shown that a dissipative environment, the *amplitude-damping channel* in the parlance of quantum information theory, can serve as an effective deleter of quantum information.

Fidelity as a function of Temperature

- Fidelity is defined as

$$
\begin{aligned}
f(t) &= \sqrt{\langle 0|\rho^s(t)|0\rangle} = \sqrt{\frac{1-\langle\sigma_3(t)\rangle}{2}} \\
&= \frac{1}{\sqrt{2}}\left[\left(1-e^{-\Gamma t}\langle\sigma_3(0)\rangle\right) + \frac{\left(1-e^{-\Gamma t}\right)}{2N+1}\right]^{1/2},
\end{aligned}
$$

where $\Gamma \equiv \gamma_0(2N+1)$ and $\langle\sigma_3(0)\rangle$ is the expectation value of σ_3 at time $t=0$.

Fidelity as a function... continued...

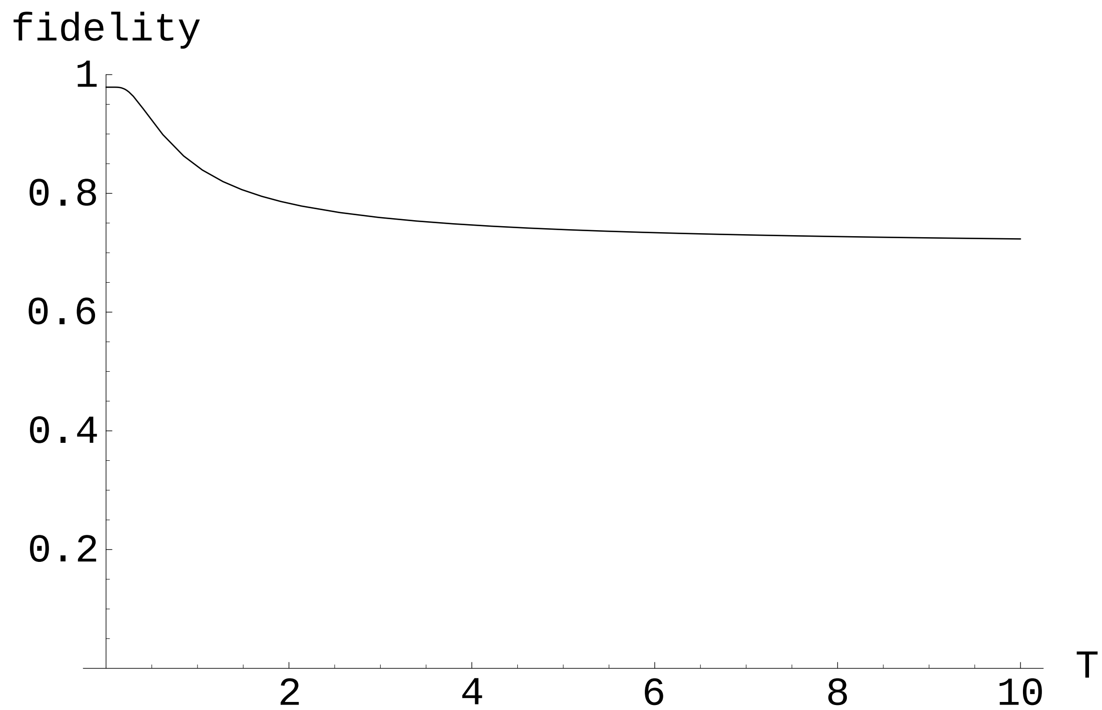


Fig. 22 : Fidelity ($f(t)$) falls as a function of temperature (T, in units where $\hbar \equiv k_B \equiv 1$) until it reaches the value $1/\sqrt{2}$ corresponding to a maximally mixed state. The case shown here corresponds to $\theta_0 = 0$, $\gamma_0 = 0.5$, $\omega = 1.0$ and time $t = 10$. Here we set the squeezing parameters r and Φ to zero.

Summary

- I discussed some basic ideas in Open Quantum Systems.
- This was followed by an application of these ideas to quantum information. In particular, the evolution of quantum correlations in the presence of open system effects was discussed followed by applications to GP, classical capacity of a quantum channel, quantum cryptography and an environment-mediated deleter.
- These studies are aimed to help us understand the role played by open systems in various aspects of quantum information.

Thank you!