Statistical Isotropy violation of the CMB brightness fluctuations
in the baryon photon fluid

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PFNG, HRI, Allahabad

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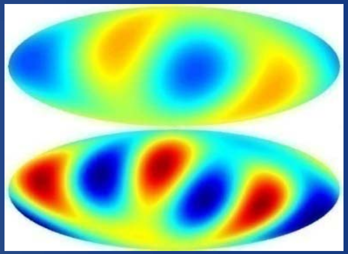
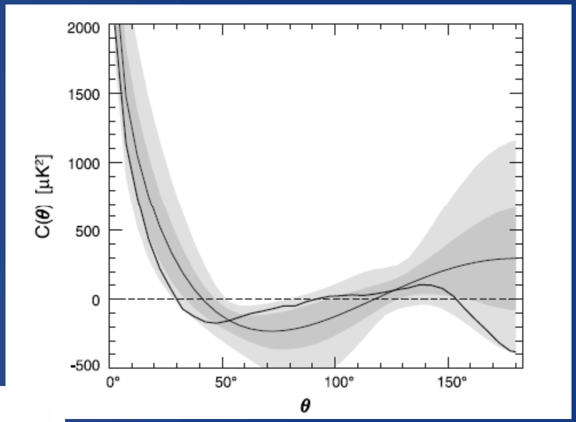
Motivation





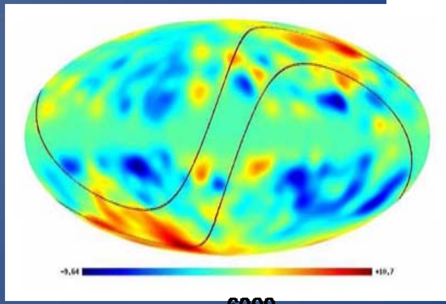
- Low multipole allignment

Tegmark et al. 2003; de Oliveira-Costa et al. 2004;

Schwarz et al. 2004; Copi et al.2006

- Lack of large scale power

Bennett et al., 2010

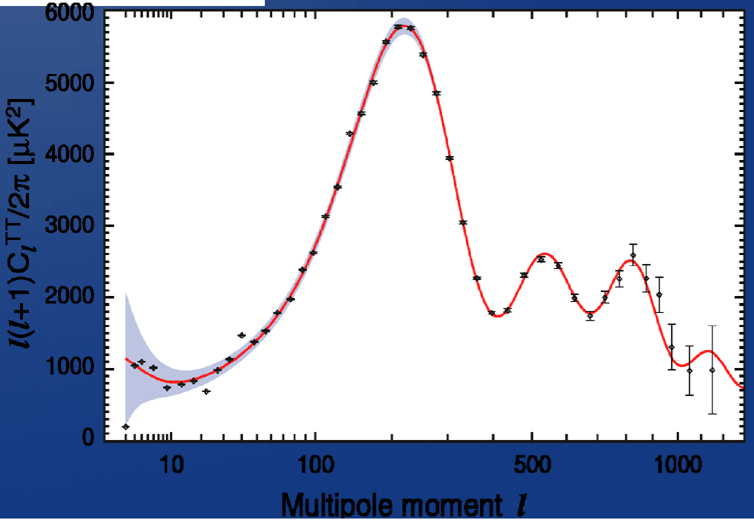


- North-South power asymmetry

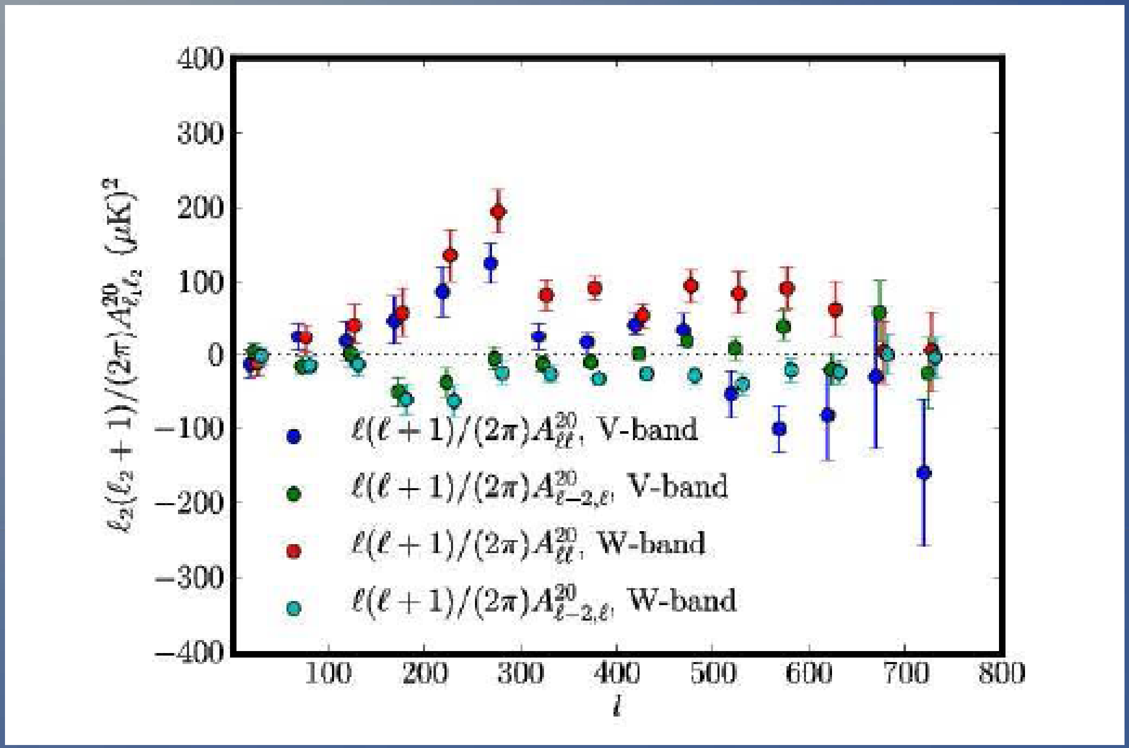
Eriksen , et al. 2004, Hansen et al. 2004, 2009
Larson & Wandelt 2004 , Park 2004

- WMAP outliers

Larson, et al., 2010, Jarosik et al. 2010



Quadrupolar bipolar power spectrum: WMAP 7



Bennett et. al, 2010

arXiv:1001.4758

A statistically significant quadrupolar effect of SI violation is observed at **l = 200** which is the scale at which the CMB power spectrum has the **baryon acoustic peak**.

It would be interesting to study **SI violations in the CMB power spectrum** which arises due to **SI violations in the baryon photon fluid**.

Statistical isotropy violation

Correlation function $\boxed{C(\hat{n}, \hat{n'}) \neq C(\hat{n}.\hat{n'})}$

Angular power spectrum $\langle a_{\ell m} a^*_{\ell' m'} \rangle \neq C_\ell \delta_{\ell\ell'} \delta_{mm'}$

SI violation in temperature anisotropy can be seeded by SI violation in

a) Primordial power spectrum $\langle \phi(\vec{k})\phi(\vec{k'}) \rangle = P(\vec{k})\delta(\vec{k} - \vec{k'})$

b) Radiative transfer function kernel / CMB brightness fluctuation

$$\tilde{\Delta}(\vec{k}, \hat{n}, \tau) \not\equiv \tilde{\Delta}(k, \hat{k} \cdot \hat{n}, \tau)$$

Relate products of these SI violating brightness fluctuations in terms of measurable Bipolar spherical harmonic (BipoSH) coefficients

$$A^{JN}_{\ell\ell'} \longrightarrow \tilde{\Delta}^{JN}_{\ell_1 \ell} \tilde{\Delta}^{JN}_{\ell_1 \ell'}$$

Bipolar Power Spectrum (BiPS) :

A Generic Measure of Statistical Anisotropy

Correlation is a *two point function* on a sphere

$$\cancel{C(\hat{n},\hat{n}') = C(\theta) = \sum_{\ell}(2\ell+1)C_\ell P_\ell(\cos\theta)}$$

$$C(\hat{n},\hat{n}') = \sum_{\ell\ell' JN} A^{JN}_{\ell\ell'}\{Y_\ell(\hat{n})\otimes Y_{\ell'}(\hat{n}')\}_{JN}$$

BipoSH coefficients

Bipolar spherical harmonics (BipoSH)

$$\{Y_\ell(\hat{n})\otimes Y_{\ell'}(\hat{n}')\}_{JN} = \sum_{mm'} \mathcal{C}^{JN}_{\ell m \ell' m'} Y_{\ell m}(\hat{n}) Y_{\ell' m'}(\hat{n}')$$

Correlation is a two point function on a sphere & can be expanded in bipolar spherical harmonics (BipoSH).

Bipolar Formalism

A.Hajian and T. Souradeep, ApJ 597 L5 (2003)

$$A^{JN}_{\ell\ell'} = \sum_{mm'} (-1)^{m'} \langle a_{\ell m} a^*_{\ell' m'} \rangle \mathcal{C}^{JN}_{\ell m \ell' -m'}$$

Linear combination of off-diagonal elements

$$A^{00}_{\ell\ell'} = (-1)^{\ell} \sqrt{2\ell+1}\, C_\ell\, \delta_{\ell\ell'}$$

Statistical isotropic term

Properties of the BipoSH coefficients

$$C(\hat{n}, \hat{n'}) = C(\hat{n}, \hat{n'})$$

$$A^{JN}_{\ell_2\ell_1} = (-1)^{\ell_1+\ell_2-J} A^{JN}_{\ell_1\ell_2}$$

$$C(\hat{n}, \hat{n'})^* = C(\hat{n}, \hat{n'})$$

$$A^{JN\,*}_{\ell_1\ell_2} = (-1)^{\ell_1+\ell_2-J+N} A^{J-N}_{\ell_1\ell_2}$$

Symmetric and antisymmetric combinations of BipoSH coefficients

$$S^{JN}_{\ell_1\ell_2} = \frac{1}{2}[A^{JN}_{\ell_1\ell_2} + A^{JN}_{\ell_2\ell_1}]$$

$$T^{JN}_{\ell_1\ell_2} = \frac{1}{2}[A^{JN}_{\ell_1\ell_2} - A^{JN}_{\ell_2\ell_1}]$$

$$S^{JN}_{\ell_1\ell_2} = S^{JN}_{\ell_2\ell_1} = A^{JN}_{\ell_1\ell_2}$$

for l1 + l2 - J = even

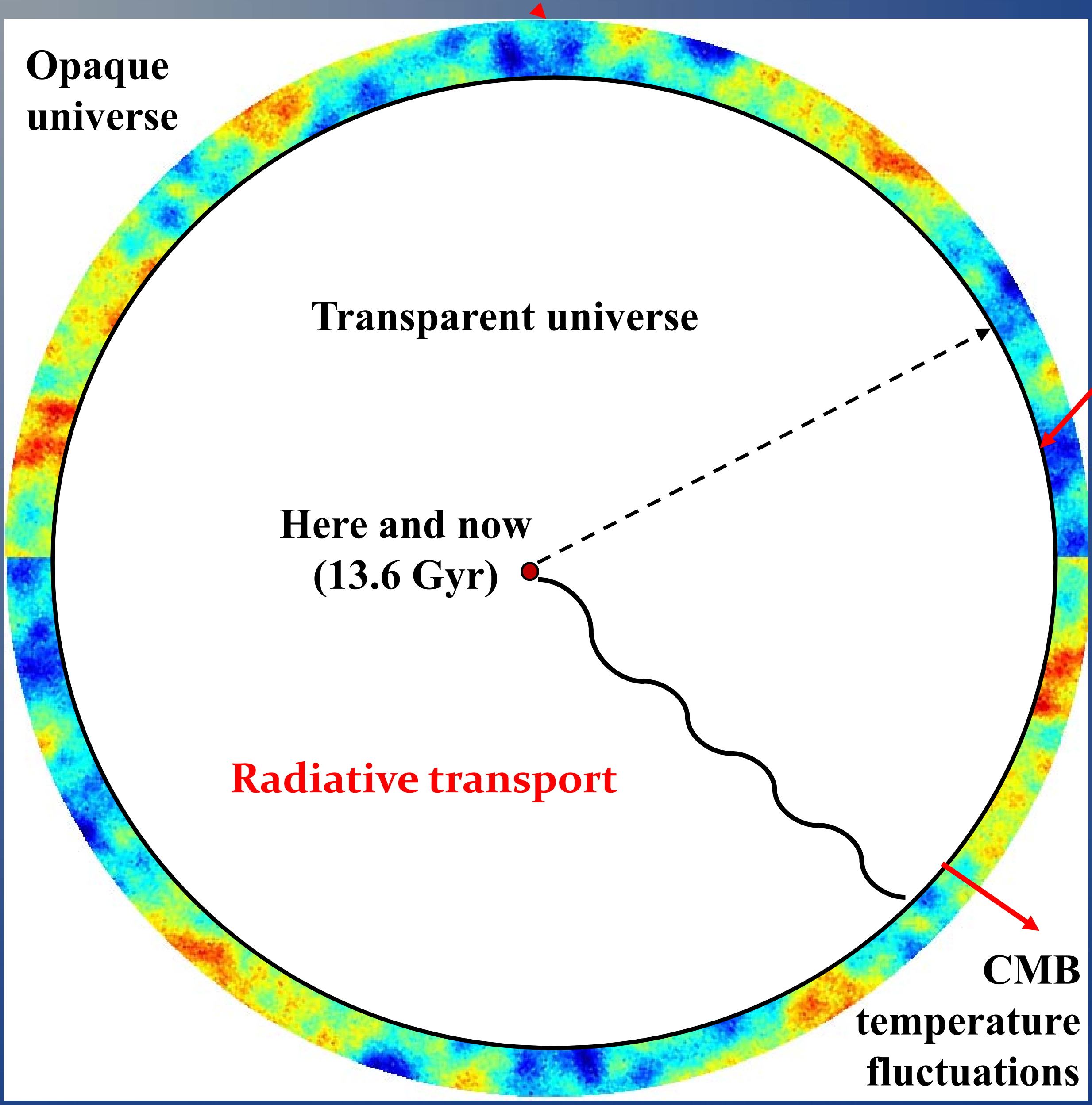
$$T^{JN}_{\ell_1\ell_2} = -T^{JN}_{\ell_2\ell_1} = A^{JN}_{\ell_1\ell_2}$$

for l1 + l2 - J = odd

Bennett et. al, 2010 arXiv:1001.4758

Kamionkowski & Souradeep, 2010

CMB Temperature anisotropy



Last scattering surface

Primordial power spectrum

CMB Temperature anisotropy

Temperature fluctuations ΔT(n) → Convolution of the Primordial power spectrum and radiative transport kernel

$$\Delta T(\hat{n}) \sim |P(\vec{k})|^{1/2} * \tilde{\Delta}_\ell(k,\tau)$$

CMB brightness fluctuations at present epoch related to baryon-photon physics at the last scattering through the evolution equation.

$$\tilde{\Delta}_\ell(k,\tau) = \sum_{\ell'} (...) j_\ell(k,\tau-\tau_s)\tilde{\Delta}_{\ell'}(k,\tau_s)$$

Monopole and dipole of the CMB brightness fluctuation at the last scattering dominate in the fluid approximation regime.

Deviations from Statistical Isotropy

Case A - Anisotropic power spectrum

$$P(\vec{k}) = P_0(k)\left[1 + \sum_{\mathfrak{l}>0}\sum_{\mathfrak{m}=-\mathfrak{l}}^{\mathfrak{l}} g_{\mathfrak{l}\mathfrak{m}}(k)\, Y_{\mathfrak{l}\mathfrak{m}}(\hat{k})\right]$$

Pullen & Kamionkowski
Phys. Rev. D 2007

$$\langle a_{\ell m} a^*_{\ell' m'}\rangle = \delta_{\ell\ell'}\delta_{mm'}C_\ell + \sum_{\mathfrak{l}\mathfrak{m}} \xi^{\mathfrak{l}\mathfrak{m}}_{\ell m \ell' m'} D^{\mathfrak{l}\mathfrak{m}}_{\ell\ell'}$$

Case B - Anisotropic CMB brightness fluctuations

$$\tilde{\Delta}(\vec{k}, \hat{n}, \tau) \not\equiv \tilde{\Delta}(k, \hat{k}\cdot\hat{n}, \tau)$$

Anisotropic CMB brightness fluctuations at last scattering free-stream to SI violating brightness fluctuations and hence angular correlations at present epoch.

Aich & Souradeep
Phys. Rev. D 81:083008

From CMB brightness fluctuation to angular power spectrum

CMB brightness fluctuations

$$\Delta(\vec{x}, \hat{n}, \tau) \equiv \frac{\delta T}{T}$$

$$\tilde{\Delta}(\vec{x} = 0, \hat{n}, \tau) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n})$$

<u>Statistical isotropy</u>

$$\tilde{\Delta}(\vec{k}, \hat{n}, \tau) = \sum_{\ell} (...) \tilde{\Delta}_{\ell}(k, \tau) P_{\ell}(\hat{k} \cdot \hat{n})$$

Angular power spectrum

$$\langle a_{\ell m} a^{*}_{\ell' m'} \rangle$$

From CMB brightness fluctuation to angular power spectrum

CMB brightness fluctuations

$$\Delta(\vec{x}, \hat{n}, \tau) \equiv \frac{\delta T}{T}$$

$$\tilde{\Delta}(\vec{x} = 0, \hat{n}, \tau) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n})$$

<u>Statistical isotropy deviation</u>

$$\tilde{\Delta}(\vec{k}, \hat{n}, \tau) = \sum (\ldots) \tilde{\Delta}^{LM}_{\ell_1 \ell_2} \, Y_{\ell_1 m_1}(\hat{k}) \, Y_{\ell_2 m_2}(\hat{n})$$

Angular power spectrum

$$\langle a_{\ell m} a^*_{\ell' m'} \rangle$$

<u>BipoSH coefficients</u>

$$A^{JN}_{\ell \ell'} = \sum_{mm'} (-1)^{m'} \langle a_{\ell m} a^*_{\ell' m'} \rangle \mathcal{C}^{JN}_{\ell m \ell' -m'}$$

Deviations from Statistical Isotropy:

CMB Photon distribution

- Start with an **anisotropic brightness distribution function**
- Brightness fluctuations expanded in **Bipolar Spherical Harmonic series**

Generalized Statistical Isotropy Violation

Primordial power spectrum

$$\begin{aligned}\langle a_{\ell m} a^*_{\ell' m'}\rangle &= (4\pi)^2 \int\int \frac{d^3k}{(2\pi)^3}\frac{d^3k'}{(2\pi)^3} \boxed{\langle \phi(\vec{k})\phi(\vec{k'})\rangle} \\ &\times \sum_{\ell_1 m_1 LM}\sum_{\ell_2 m_2 L'M'} \boxed{\tilde{\Delta}^{LM}_{\ell_1\ell}(k,\tau)\tilde{\Delta}^{L'M'}_{\ell_2\ell'}(k',\tau)^*} \\ &\times \mathcal{C}^{LM}_{\ell_1 m_1 \ell m}\mathcal{C}^{L'M'}_{\ell_2 m_2 \ell' m'} Y_{\ell_1 m_1}(\hat{k}) Y_{\ell_2 m_2}(\hat{k}')\end{aligned}$$

Transfer function kernel

For a directional dependent power spectrum

$$P(\vec{k}) = P_0(k)\left[1 + \sum_{l>0}\sum_{m=-l}^{l} g_{lm}(k)\, Y_{lm}(\hat{k})\right]$$

Generalized BipoSH coefficients with SI violating CMB brightness fluctuation

$$\begin{aligned} A^{JN}_{\ell\ell'} &= \boxed{(A^{JN}_{\ell\ell'})_{l=0}} + \int \frac{k^2 dk}{2\pi^2} P_0(k) \sum_{lm\ell_1\ell_2} g_{lm}\; \mathcal{C}^{\ell_2 0}_{\ell_1 0 l 0} \\ &\times \sum_{\ell_3 m_3} \mathcal{C}^{JN}_{\ell_3 m_3 lm} \sum_{LL'} (\dots) \boxed{\{\tilde{\Delta}^{L}_{\ell_1\ell} \otimes \tilde{\Delta}^{L'}_{\ell_2\ell'}\}_{\ell_3 m_3}} \end{aligned}$$

Tensor product of CMB brightness fluctuations in bipolar spherical harmonic space

$$\left\{\tilde{\Delta}^{L}_{\ell_1\ell} \otimes \tilde{\Delta}^{L'}_{\ell_2\ell'}\right\}_{JN} = \sum_{MM'} (-1)^{M'} \mathcal{C}^{JN}_{LML'-M'} \tilde{\Delta}^{LM}_{\ell_1\ell}(k,\tau)[\tilde{\Delta}^{L'M'}_{\ell_2\ell'}(k,\tau)]^*$$

For isotropic power spectrum

$$\langle \phi(\vec{k})\phi(\vec{k'})\rangle = P_0(k)\delta(\vec{k}-\vec{k'})$$

$$(A^{JN}_{\ell\ell'})_{l=0} = \int \frac{k^2 dk}{2\pi^2} P_0(k) \sum_{\ell_1 L L'} (\ldots) \left\{ \begin{array}{ccc} L & J & L' \\ \ell' & \ell_1 & \ell \end{array} \right\} \boxed{\left\{ \tilde{\Delta}^{L}_{\ell_1 \ell} \otimes \tilde{\Delta}^{L'}_{\ell_1 \ell'} \right\}_{JN}}$$

With J = 2, N = 0, we obtain the non-zero BipoSH coefficients for a statistically significant quadrupolar effect in the WMAP 7 yr data

$$A^{20}_{\ell\ell} \sim -2A^{20}_{\ell-2\ell} \neq 0$$

Bennett et. al, 2010 arXiv:1001.4758

Generalized Evolution Equation

SI violation observed at *current epoch* can be related to non-trivial effects at the *surface of last scattering*

***Free stream equation* for the SI violating brightness fluctuations**

$$\tilde{\Delta}^{LM}_{\ell_1\ell_2}(k,\tau) = \sum_{\ell\ell_3\ell_4}(\ldots)j_\ell(k\Delta\tau)\boxed{\tilde{\Delta}^{LM}_{\ell_3\ell_4}(k,\tau_s)} \times \mathcal{C}^{\ell_3 0}_{\ell 0 \ell_1 0}\,\mathcal{C}^{\ell_2 0}_{\ell 0 \ell_4 0}\begin{Bmatrix} \ell_1 & L & \ell_2 \\ \ell_4 & \ell & \ell_3 \end{Bmatrix}$$

Statistical Isotropy
L = 0

$$\tilde{\Delta}_\ell(k,\tau) = \sum_{\ell' L}(-i)^{\ell+\ell'-L}(2\ell'+1)j_L(k\Delta\tau)\boxed{\tilde{\Delta}_{\ell'}(k,\tau_s)}[\mathcal{C}^{L0}_{\ell 0 \ell' 0}]^2$$

Generalized Evolution Equation: Asymptotic Limit

Asymptotic Limit ***(large multipoles at present time)***
– *Fluid approximation*

$\ell_1, \ell_2 >> \ell_3, \ell_4$

Diagonal terms

$$\tilde{\Delta}^{LM}_{\ell_1\ell_1}(k,\tau) = \sum_{\ell}(\ldots) j_\ell(k\Delta\tau)\boxed{\tilde{\Delta}^{LM}_{\ell_1-\ell,\ell_1-\ell}(k,\tau_s)}$$

$$\ell, \ell_1 >> |\ell_1 - \ell|$$

Off-diagonal terms

$$\tilde{\Delta}^{LM}_{\ell_1,\ell_2}(k,\tau) = \sum_{\ell}(\ldots) j_\ell(k\Delta\tau)\boxed{\tilde{\Delta}^{LM}_{\ell_1-\ell,\ell_2-\ell}(k,\tau_s)}$$

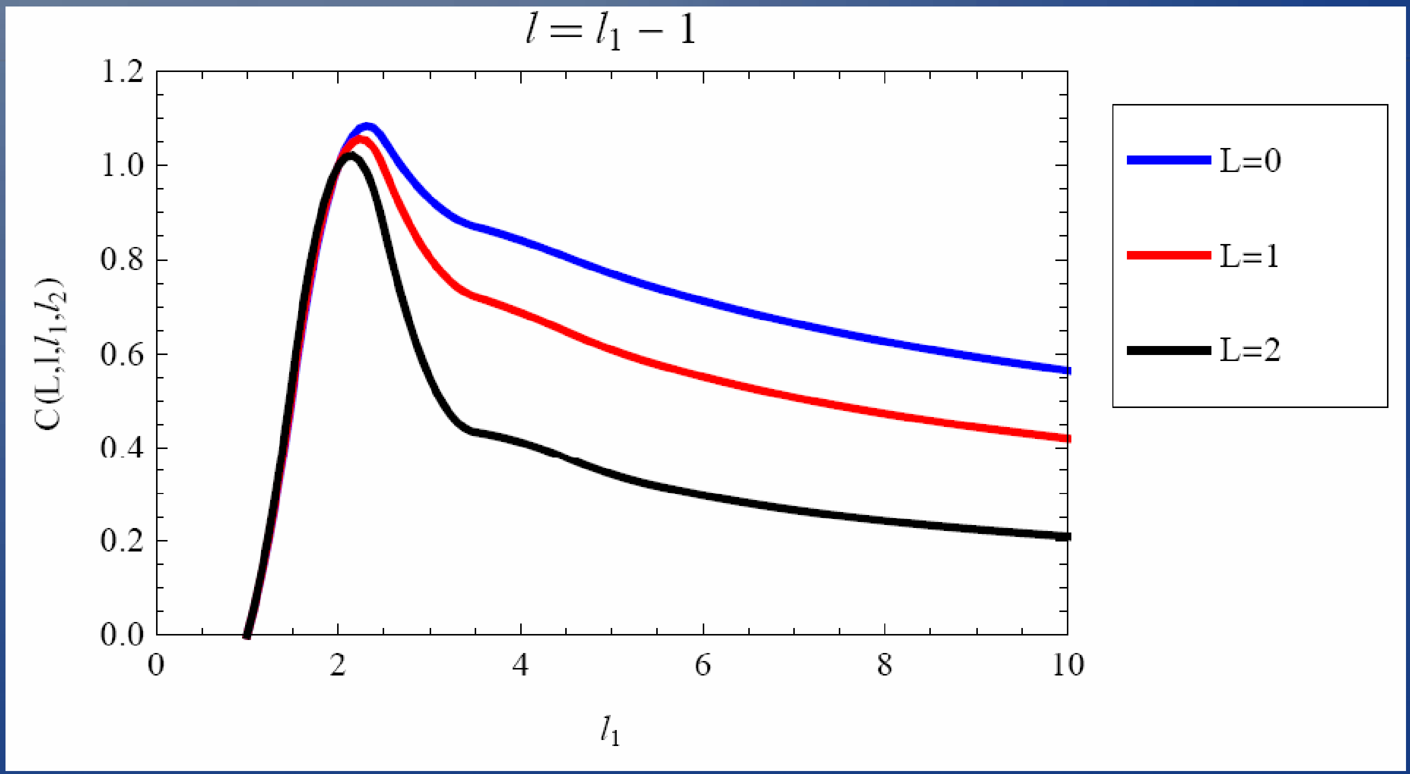
$$\ell,\ \ell_n >> |\ell_n - \ell|,\ L \text{ with } n = 1, 2$$

Smaller multipoles at last scattering, due to statistical anisotropy (i.e. L ≠ 0) feeds into the larger multipoles of the brightness function hence in the CMB anisotropy correlations today

Generalized Evolution Equation

Diagonal terms of the CMB brightness fluctuations -
Non SI Dipole at LSS
ℓ3 = ℓ4 = 1 and L = {0, 1, 2}

$$\tilde{\Delta}^{LM}_{\ell_1\ell_2}(k,\tau) = \sum_{\ell}(\ldots)j_\ell(k\Delta\tau)\ \tilde{\Delta}^{LM}_{11}(k,\tau_s)\ \boxed{\mathcal{C}^{10}_{\ell 0 \ell_1 0}\ \mathcal{C}^{\ell_2 0}_{\ell 0 1 0} \begin{Bmatrix} \ell_1 & L & \ell_2 \\ 1 & \ell & 1 \end{Bmatrix}}$$



Evolution & Parity of the Bipolar coefficients

BipoSH coefficients in terms of brightness fluctuations at the last scattering surface

$$A^{JN}_{\ell\ell'}|_\tau = \sum(...)\, j_{\ell_2}(k\Delta\tau)\, j_{\ell_5}(k\Delta\tau)\, \mathcal{C}^{JN}_{LML'-M'}\, \tilde{\Delta}^{LM}_{\ell_3\ell_4}(k,\tau_s)[\tilde{\Delta}^{L'M'}_{\ell_3\ell_6}(k,\tau_s)]^*$$

$$A^{JN}_{\ell\ell'}|_\tau =\sim \sum(...)\, j_{\ell_2}(k\Delta\tau)\, j_{\ell_5}(k\Delta\tau)\, \mathcal{A}^{JN}_{\ell_4\ell_6}|_{\tau_s},$$

Parity is conserved in the “evolution equation” of the BipoSH coefficients

$$\begin{aligned}
\ell_4+\ell_6+J &= \text{even} \longrightarrow \ell+\ell'+J=\text{even}\\
\ell_4+\ell_6+J &= \text{odd} \longrightarrow \ell+\ell'+J=\text{odd}
\end{aligned}$$

Non-SI dipolar Source Terms

Dominating SI term: L = 0

$$\tilde{\Delta}_{11}^{00}(k,\tau)\tilde{\Delta}_{11}^{00}(k,\tau) \sim \iint d_\Omega(\ldots)|\Delta|^2 \boxed{(\hat{k}\cdot\hat{n})(\hat{k}\cdot\hat{n}')}$$

Dipole term: L = 1

$$\tilde{\Delta}_{11}^{1M}(k,\tau)\tilde{\Delta}_{11}^{1M}(k,\tau) \sim \iint d_\Omega(\ldots)|\Delta|^2 \boxed{\{\hat{k}\otimes\hat{n}\}_{1M}\{\hat{k}\otimes\hat{n}'\}_{1M}}$$

Quadrupole term: L = 2

$$\tilde{\Delta}_{11}^{2M}(k,\tau)\tilde{\Delta}_{11}^{2M}(k,\tau) \sim \iint d_\Omega(\ldots)|\Delta|^2 \boxed{\{\hat{k}\otimes\hat{n}\}_{2M}\{\hat{k}\otimes\hat{n}'\}_{2M}}$$

SI violating physical effects at last scattering

CMB anisotropy in presence of a homogeneous magnetic field at last scattering

CMB brightness fluctuations
sourced by
bipolar dipole, L = 1

$$\Delta(\vec{k},\hat{n},\tau_s) = 3i\sqrt{\frac{3}{2}}\sum_M \boxed{\tilde{\Delta}_{11}^{1M}(k,\tau_s)}\ (\hat{k}\times\hat{n})_M$$

Source term comprising magnetic field, velocity of plasma etc.

$$\begin{aligned}\langle\Delta(\vec{k},\hat{n},\tau_s)\Delta(\vec{k},\hat{n}',\tau_s)\rangle &\propto \tilde{\Delta}_{11}^{1M}(k,\tau_s)\ \tilde{\Delta}_{11}^{1M'}(k,\tau_s)\\ &\times \left[(\hat{n}\cdot\hat{n}')(\hat{k}\cdot\hat{k}) - (\hat{n}\cdot\hat{k})(\hat{n}'\cdot\hat{k})\right]\end{aligned}$$

Durrer et al., PRD 1998,
Kahniashvili et al., PRD 2008

$$\langle a_{\ell m}a^*_{\ell' m'}\rangle = (\ldots)\ \delta_{\ell'\ell}\delta_{mm'} + (\ldots)\ \delta_{\ell'\ \ell\pm 2}\delta_{mm'}$$

Summary

- Motivated by anomalies in CMB maps, we incorporate SI deviation in the CMB brightness fluctuations, in addition to SI violations due to anisotropic P(k)
- We obtain expressions for angular correlations and hence the BipoSH coefficients, arising due to the violation of SI.
- Angular correlations at present epoch can further be related to CMB brightness fluctuations at last scattering by the generalized free-streaming equation.
- We are interested in the large multipole moments of the CMB brightness fluctuations today which are not dominated by the cosmic variance regime.
- Due to SI violation, lower multipoles at the last scattering free stream to higher multipoles at the present epoch.
- We have used our formalism to represent and match the well known case for SI violation in presence of a homogeneous magnetic field.
- Parity remains conserved during the evolution of the BipoSH coefficients

Thank You