Scale dependence of non-Gaussianity

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Specific motivations

- We don't know how many fields drove inflation, what form their potential took, or which one gave rise to the observed perturbations
- Apart from the many inflation models, also many mechanisms after inflation could cause the perturbations
  - Curvaton – late decaying scalar field
  - Modulated reheating – time of reheating depends on position
  - Modulated preheating – efficiency of preheating varies
  - Inhomogeneous end of inflation – time inflation ends varies

Need many observables to discriminate between scenarios
Planck could provide a high significance detection
So how much could we learn, and what should we look for in the forthcoming data?
Predictions should come first!

The bispectrum

- Simplest definition, motivated but not exact

$$\zeta(\mathbf{x}) = \zeta_G(\mathbf{x}) + \frac{3}{5} f_{NL} \zeta_G^2(\mathbf{x})$$

- Can picture the bispectrum as a triangle, with wave-numbers k denoting the side lengths

$$\frac{6}{5} f_{NL}(k_1, k_2, k_3) \equiv \frac{B_\zeta(k_1, k_2, k_3)}{P_\zeta(k_1) P_\zeta(k_2) + 2 \text{ perms}}$$

- Usually reduced to an amplitude times scale-independent shape function
- Focus on local shape

  Other shapes: Equilateral, folded, orthogonal

Many recent reviews: Chen; Komatsu; Ligouri et al; ...

Some scale dependence is expected!

- Analogous to the power spectrum, f_{NL} (local) should have a mild scale dependence
- Also true for other bispectral shapes, e.g. equilateral
- Reflects evolution/dynamics during inflation (e.g. it ends)
- Observational and theoretical interest
- Breaks degeneracy between early universe models
  - As well as the trispectrum
- Can distinguish between different non-Gaussian scenarios, not just between Gaussian and non-Gaussian models
- The amplitude of f_{NL} can be tuned in most non-Gaussian models, so a precise measurement of f_{NL} wont do this
- The sign of f_{NL} can distinguish between some models
- Predictions should come first
- Avoid posterior detections (hard to quantify the significance)

Questions?

- How large is the scale dependence?
  - How to calculate it for a given model?
- How does it arise?
  - Multiple fields
  - Self interactions
- Are observations sensitive to it?
- What can we learn from it?
- How to generalise the local ansatz?

Definition of scale dependent f_{NL}

For the equilateral triangle (one k)

$$n_{f_{NL}} \equiv \frac{\partial \log |f_{NL}|}{\partial \log k}$$

- In general f_{NL} trivariate function, so definition needs care
- However $n_{f_{NL}}$ is independent of the shape provided one scales the triangle preserving the shape
  - Hence the above is a useful definition of a new observable
    Byrnes, Nurmi, Tasinato and Wands, '09
  - Not much change if the shape and size of triangle are changed together

Observable parameters, bispectrum and trispectrum

We define 3 non-linearity parameters

$$B_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3) = \frac{6}{5} f_{NL}\left[P_\zeta(k_1)P_\zeta(k_2)+P_\zeta(k_2)P_\zeta(k_3)+P_\zeta(k_3)P_\zeta(k_1)\right]$$

$$T_\zeta(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4) = \tau_{NL}\left[P_\zeta(|\mathbf{k}_1+\mathbf{k}_3|)P_\zeta(k_3)P_\zeta(k_4)+(11\text{ perms})\right] + \frac{54}{25} g_{NL}\left[P_\zeta(k_2)P_\zeta(k_3)P_\zeta(k_4)+(3\text{ perms})\right]$$

Byrnes, Sasaki & Wands '06; Seery & Lidsey '06

Note that τ_{NL} and g_{NL} both appear at leading order in the trispectrum
The coefficients have a different k dependence, $P_\zeta \propto k^{-3}$

Constraints $|f_{NL}| \lesssim 100, \ \ |\tau_{NL}| \lesssim 10^5, \ \ |g_{NL}| \lesssim 10^6$

LSS: Desjacques & Seljak '09; WMAP7; Smidt et al '10 a)

Planck forecasts $|f_{NL}| \lesssim 10, \ \ |\tau_{NL}| \lesssim 10^3, \ \ |g_{NL}| \lesssim 10^5$

Smidt et al '10 b)

Trispectrum: simplest case

$$\zeta = \zeta_G + \frac{3}{5} f_{NL} \zeta_G^2 + \frac{9}{25} g_{NL} \zeta_G^3 + \cdots$$

Valid if only one field generates the curvature perturbation; could be the curvaton or modulaton

- Consistency relation $\tau_{NL} = \left(\frac{6}{5} f_{NL} \right)^2$
- Multifield models – becomes inequality

$$\tau_{NL} > \left(\frac{6}{5} f_{NL} \right)^2$$

Suyama & Takahashi '08

- The trispectrum becomes more competitive
- g_{NL} typically depends on strength of self interactions
- Often small, due to calculational preference for quadratic potentials

Large tauNL & small fNL: Ichikawa et al '08; Byrnes et al '09; Langlois & Sorbo '09
Major overview: Suyama, Takahashi, Yamaguchi, Yokoyama '10

Simple extension of local fNL

- The multivariate local model

$$\zeta(x) = \zeta_{G,\phi}(x) + \zeta_{G,\chi}(x) + f_\chi \zeta^2_{G,\chi}(x) + g_\chi \zeta^3_{G,\chi}(x)$$

phi is the Gaussian inflaton field,
chi generates non-Gaussianity (uncorrelated to phi)

applies to mixed inflaton and curvaton/modulated reheating scenarios, provided f_χ is a constant

Bispectrum has the usual local shape – not changed

$$P_\zeta(k) = P_{\zeta_\phi}(k) + P_{\zeta_\chi}(k), \quad P_\zeta \propto k^{n-4}, \; P_{\zeta_\chi} \propto k^{n_\chi - 4}$$

$$f_{NL}(k) = \frac{5}{3}\frac{B_{\zeta_\chi}}{3P_\zeta^2} = \frac{5}{3}\frac{P_{\zeta_\chi}(k)^2}{P_\zeta(k)^2} f_\chi \propto \left(\frac{k}{k_p}\right)^{2(n_\chi - n)}$$

- So a scale dependence of f_{NL} is simple and natural
- Trispectrum $n_{\tau_{NL}} = n_{g_{NL}} = \frac{3}{2} n_{f_{NL}} = 3(n_\chi - n)$

Mixed inflaton-curvaton scenario

- The inflaton phi has Gaussian perturbations,
  the curvaton field chi (quadratic V) is non-Gaussian
  assume a small field model of inflation $\epsilon \ll \eta_{\phi\phi}$

$$n - 1 = 2(1 - w_\chi)\eta_{\phi\phi}$$

$$f_{NL}(k) = w_\chi^2(k) f_\chi$$

$$n_{f_{NL}} = 2(n_\chi - n) = -4(1 - w_\chi)\eta_{\phi\phi}$$

where $w_\chi(k) = P_{\zeta_\chi}(k)/P_\zeta(k)$

- New consistency relation $n_{f_{NL}} = -2(n-1) \simeq 0.1$

- Trispectrum $\tau_{NL} = \left(\frac{6}{5} f_{NL}\right)^2 \frac{1}{w_\chi}$ $n_{\tau_{NL}} = -3(n-1)$

Two-component hybrid inflation

$$W = W_0 \left(1 + \frac{1}{2}\eta_{\varphi\varphi}\frac{\varphi^2}{M_P^2} + \frac{1}{2}\eta_{\chi\chi}\frac{\chi^2}{M_P^2}\right)$$

If we choose initial conditions to maximise f_{NL} then

$$f_{NL} = \frac{5}{24}\eta_{\chi\chi}e^{2N(\eta_{\varphi\varphi}-\eta_{\chi\chi})}, \quad n_\zeta - 1 = \eta_{\varphi\varphi} + \eta_{\chi\chi}$$

N is the number of e-foldings from horizon crossing till the end of inflation; Scales which exit earlier are more non-Gaussian

$$n_{f_{NL}} \equiv \frac{\partial \log |f_{NL}|}{\partial \log k} = -2(\eta_{\varphi\varphi} - \eta_{\chi\chi})$$

$\eta_{\varphi\varphi}$	$\eta_{\chi\chi}$	φ_*	χ_*	f_{NL}	$n_{f_{NL}}$	$n_\zeta - 1$	r
0.04	-0.04	1	6.8×10^{-5}	-123	-0.16	0	0.006
0.08	0.01	1	0.0018	9.27	-0.14	0.09	0.026
-0.01	-0.09	1	3×10^{-6}	-132	-0.276	-0.04	0.0007

First to calculate scale dependence of local model: Byrnes, Choi & Hall '08 ii)

Observational prospects

- Planck could reach a tight constraint
- Predicted to reach $\Delta n_{f_{NL}} = 0.1$ for $f_{NL} = 50$
- CMBPol (COrE) has double this sensitivity
- Galaxy clusters will provide the best constraints

  Error bar is inversely proportional to the fiducial value of f_{NL}
- It is possible that Planck will provide the first detection of non-Gaussianity, and simultaneously detect its scale dependence!
- We have a separable ansatz for the bispectrum

CMB: Sefusatti, Ligouri, Yadav, Jackson, Pajer; '09
LSS: Becker, Huterer, Kadota '10
First LSS simulations: Shandera, Dalal & Huterer '10

General Single-field I

- Models where any single field generates the perturbations
  - Not assumed to be the inflaton
- Could be the field which modulates time of reheating or efficiency of preheating
- Arises from the non-linearity of the field evolution just after horizon exit
- Only exception is a free test field (quadratic potential)
  - has a linear equation of motion
- The assumption that f_{NL} is scale independent is only valid in the simplest toy models!
- Example is the simplest curvaton scenario
- Including the inflaton field fluctuations or self interactions will generate a scale dependence

Simplest case: Inflaton field

- Pure academic interest
- Analytic results
- Neglecting the non-Gaussianity of the fields at horizon exit (here not accurate), i.e. taking only the local part

$$\frac{6}{5} f_{NL} = 2\epsilon - \eta$$

$$n_{f_{NL}} = \frac{6\epsilon\eta - 8\epsilon^2 - \xi^2}{\eta - 2\epsilon}$$

- Scale dependence arises from the second-order field evolution near Hubble crossing

$$\delta_2\phi(t_i) = \delta_2(\phi_*) + \frac{H(t_i - t_*)}{\sqrt{2\epsilon}}(8\epsilon^2 - 6\epsilon\eta + \xi^2)\delta_1^2\phi$$

General single field II

- In models with large non-Gaussianity the single field is isocurvature during inflation (assumed adiabatic today)

$$n_{f_{NL}} \sim \frac{\sqrt{r_T}}{f_{NL}} \frac{V'''}{3H^2} \qquad r_T = \frac{P_T}{P_\zeta}$$

$$\tau_{NL} = \left(\frac{6}{5} f_{NL}\right)^2 \Rightarrow n_{\tau_{NL}} = 2 n_{f_{NL}}$$

$$n_{g_{NL}} \sim \frac{r_T}{g_{NL}} \frac{V''''}{3H^2} \sim \frac{\mathcal{P}_\zeta^{-1}}{g_{NL}} V''''$$

- Model dependent size, could be large
- Neither spectral index nor its running probe higher derivatives of the isocurvature's field potential
- Only way to probe self-interactions?
- Easy to apply our formulas, please do!

Easy to calculate

Scale dependence of non-Gaussianity parameters depends only on:

- derivatives of N (delta N formalism) – background quantities anyway required to calculate fNL
- slow-roll parameters evaluated at horizon crossing (just derivatives of the potential)

Interacting curvaton scenario I

$$V(\chi) = \frac{1}{2}m^2\chi^2 + \lambda m^4 \left(\frac{\chi}{m}\right)^p$$

Strength of self interaction (at horizon exit, *)

$$s = 2\lambda \left(\frac{\chi_*}{m}\right)^{p-2}$$

In the limit of s=0 recover scale invariance

Energy density of curvaton is subdominant during inflation, but it grows relative to that of radiation (from the decayed inflaton) while it oscillates about the minimum of its potential

Energy density of curvaton at time of decay

$$r_{dec} \equiv \left.\frac{\rho_\chi}{4\rho_{rad} + 3\rho_\chi}\right|_{decay} \qquad f_{NL} \sim \frac{1}{r_{dec}}$$

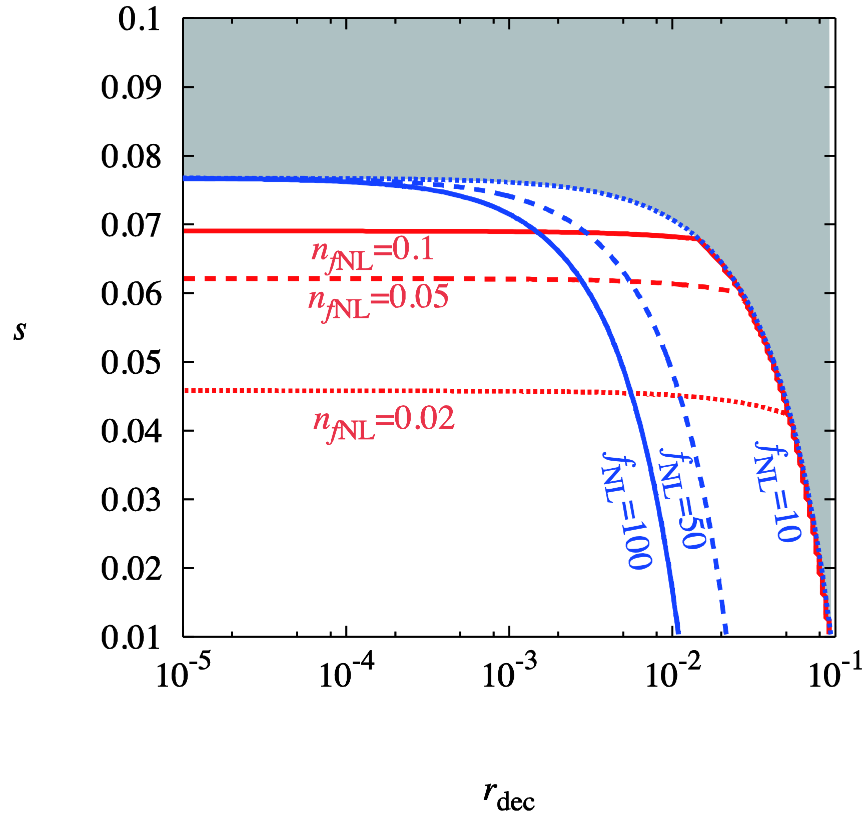
CB, Enqvist, Takahashi; 1007.5148

Interacting curvaton scenario II

$p = 6 \qquad \eta_{\chi\chi} = 0.005$

$f_{NL} < 10$ is shaded

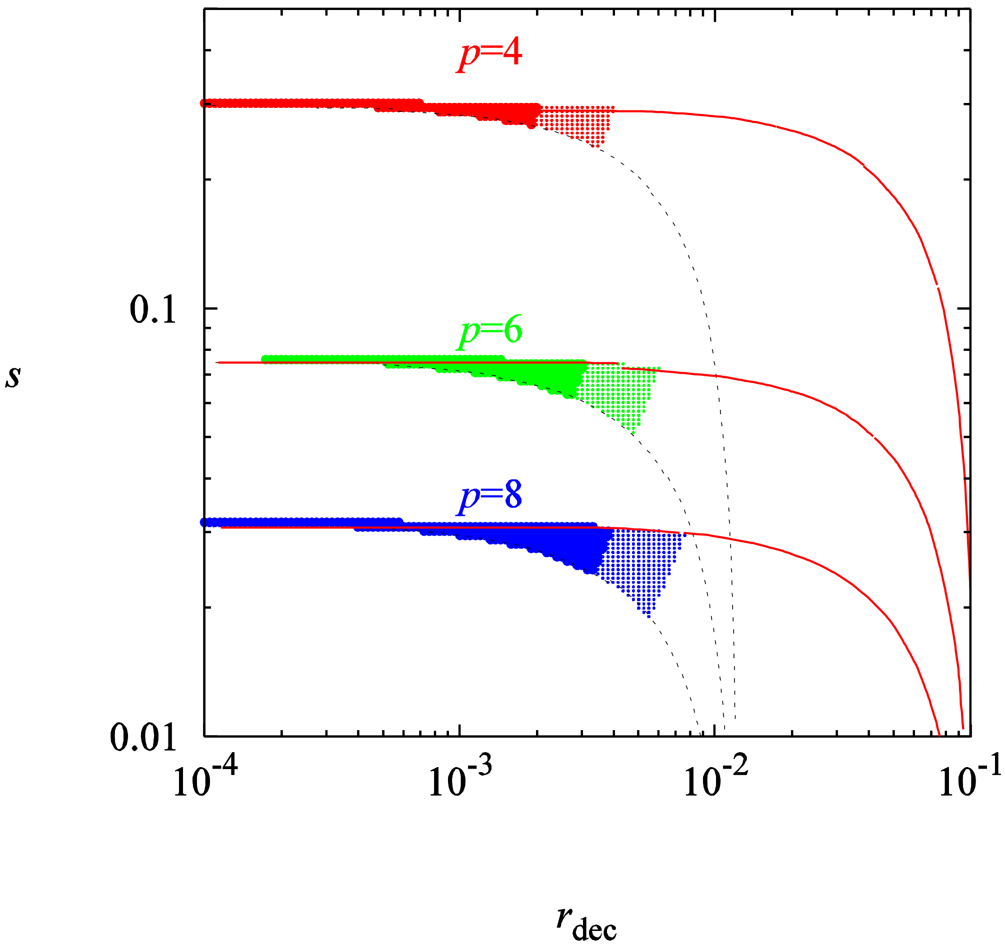
$n_{f_{NL}} > 0.1$ is possible even for small s



$$s \sim \frac{\text{self interaction}}{\text{quadratic}}$$

$$r_{dec} \sim \Omega_{\chi,decay}$$

Interacting curvaton scenario III testable region



shaded regions are testable with CMBPol at 1 and $2-\sigma$
Larger region would be testable with larger s and/or $\eta_{\sigma\sigma}$

top redline $f_{NL} = 10$
lower dashed line $f_{NL} = 100$

Interacting curvaton scenario IV Summary

- Knowledge of $f_{NL}, \;\; n_{f_{NL}}, \;\; g_{NL}$ would give us information on the curvaton parameters $m, \;\; p, \;\; s$
- Even a small self interaction significantly changes the model predictions
  - Makes all of the non-linearity parameters scale dependent
- The curvaton is required to have a quadratic minimum
  - Models which could have a pure self interaction potential (eg modulated reheating) may have larger scale dependence

CB, Gerstenlauer, Nurmi, Tasinato & Wands '10; see also Bernardeau '10

Loop corrections?

- With extreme parameter values, the bispectrum can be large through a “loop” correction

$$\zeta = \zeta_{G,\phi} + \zeta^2_{G,\chi}$$

Boubekeur & Lyth; '05
Suyama & Takahashi; '08
Preheating:
Chambers and Rajantie '08
delta N application: Byrnes et al '10
Review: Seery '10

- The bispectrum diverges in the IR
- Applying a sharp IR cut-off L

$$f_{NL} = f_{NL}^{\rm loop} \sim \frac{P^3_{\zeta_\chi}(k)}{P^2_\zeta(k)} \ln(kL)$$

- If we take L~1/H - then on CMB scales

$$n_{f_{NL}} \sim 1/\ln(kL) \sim 0.2$$

Kumar, Leblond & Rajaraman; '09

Could be distinguishable from power law scale dependence

Strong scale dependence

- Relatively small, power law scale dependence is expected
- Scale dependence could be more dramatic
- Transition of subdominant field from massive to massless can create step-function like f_{NL}
- This field generates non-Gaussianity when massless but not linear perturbations
  - Gaussian on large scales, non-Gaussian on small scales
  - Power spectrum comes from the inflaton field
  - Need to tune mass transition to be during horizon crossing of observable modes

Riotto & Sloth '10

Conclusions

- Non-Gaussianity is not given by just one amplitude
- Should include a scale-dependence (could be significant)
  - New observable
  - Unique probe of early universe models
  - Easy to calculate in many models
- Can arise due to:
  a) Multiple field effects
  b) Self interactions of the fields
- Trispectrum has more information than the bispectrum

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