Non-Gaussianities from Features in the Primordial Spectrum

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Review: X.C., 1002.1416
+ 1008.2485

In a broader context

Classification of Large Non-Gaussianities from Inflation Models

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Why do we need to classify non-Gaussianities?

Motivations from

Both Experimental Data Analyses and Theoretical Model Building

Data analyses

Signal to noise ratio not high enough to detect non-G model-independently

Theoretical template → Construct estimator for example f_{NL}^{local} → Fit data to constrain for example f_{NL}^{local}

(WMAP)

Or decompose into set of base functions, and constrain each separable base.

(Fergusson, Liguori, Shellard, 09,10)

- Need theoretical non-G profiles to start with
- Possible signals could be missed if we are not using the right model or set of basis
- Even if a signal were detected using one template, it does not mean we have found the right form
- Even if CMB were perfectly Gaussian, we still need do the same work to reach the conclusion

Theoretical Model Building

- Divide models into categories, so that all models in the same class share the same non-G feature
- If any such non-G were detected, we know what we learned concretely in terms of fundamental phyiscs
- Not only classify known non-G, but also discovery new ones with good model building motivations

A No-go theorem

Simplest inflation models predict unobservable non-G.

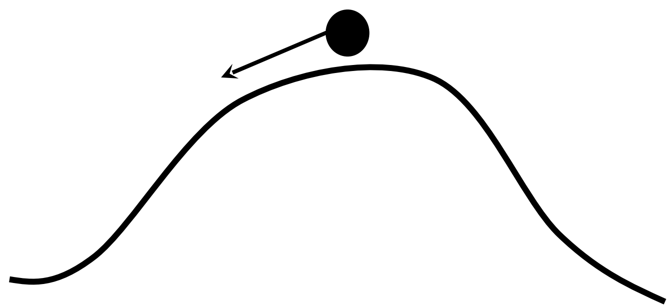
(Maldacena, 02; Acquaviva, Bartolo
Matarrese, Riotto, 02)

- Single field
- Canonical kinetic term
- Always slow-roll
- Bunch-Davies vacuum
- Einstein gravity

$$\Longrightarrow \quad f_{NL} \sim \mathcal{O}(\epsilon) \lesssim \mathcal{O}(0.01)$$

Experimentally: $f_{NL} \gtrsim \mathcal{O}(1)$

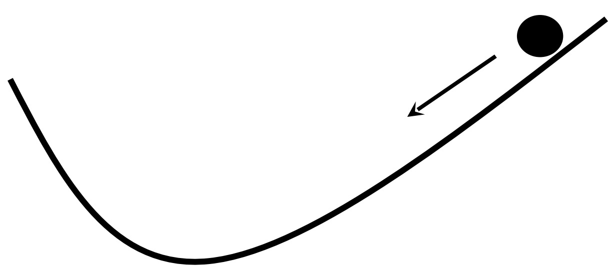
Examples of Simplest Slow-Roll Potentials



Small field:

$$V = V_0 - \frac{1}{2}m^2\phi^2$$

$$m \ll H$$



Large field:

$$V = \lambda\phi^\alpha$$

$$\phi \gg M_{\rm Planck}$$

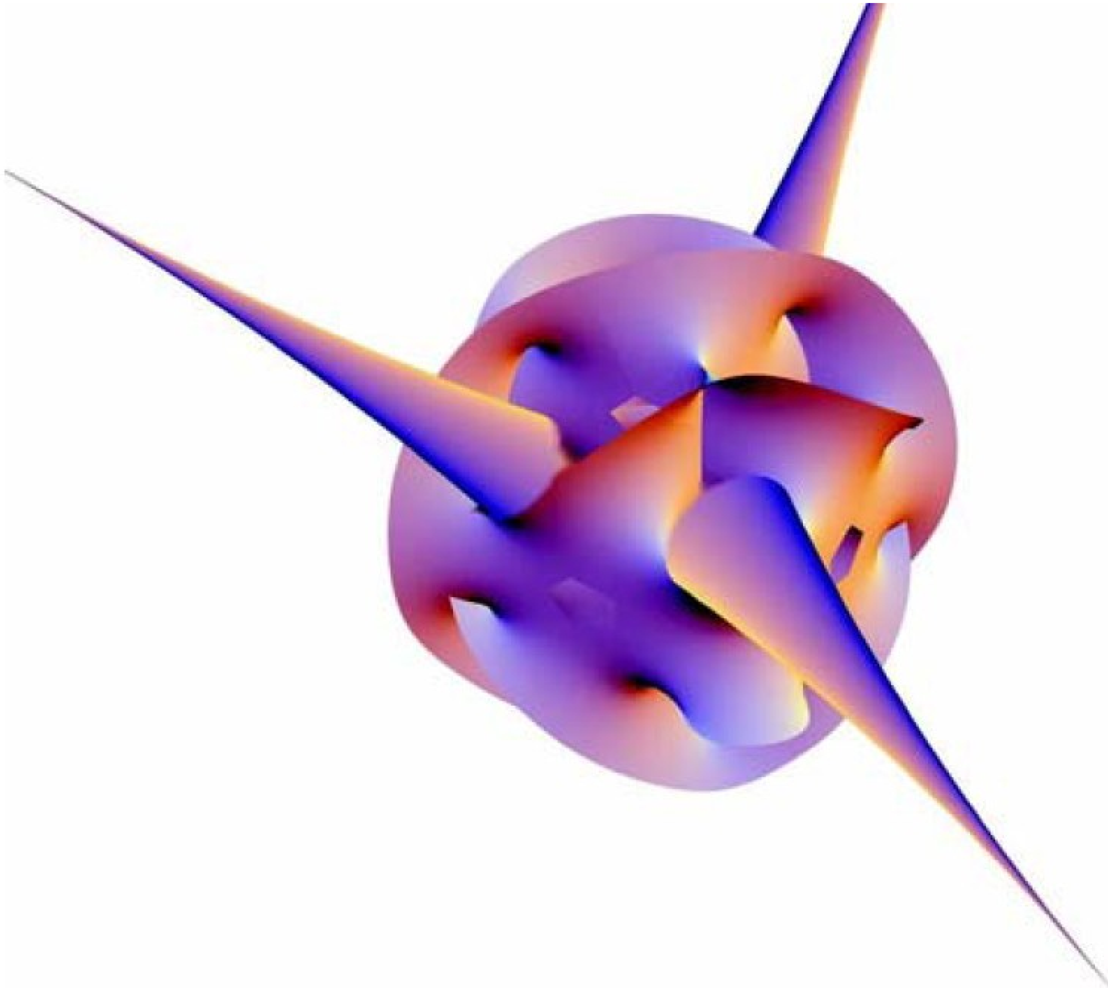
The other conditions in the no-go theorem also needs to be satisfied.

Much more complicated in realistic model building ……

Inflation Model Building

A landscape of potentials

Inflation Model Building



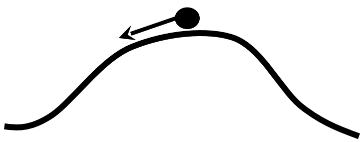
Warped Calabi-Yau

Inflation Model Building

- η -Problem in slow-roll inflation: (Dine, Fischer, Nemeschansky, 84; Copeland, Liddle, Lyth, Stewart, Wands, 04)

Backreaction from inflationary background

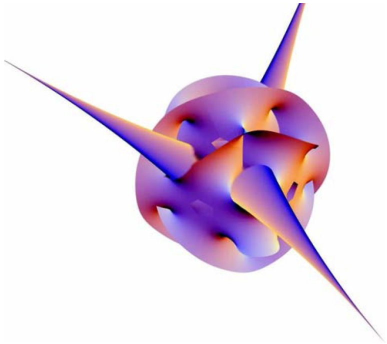
$$m \sim H, \quad \eta \sim 1 \quad \xrightarrow{?} \quad m \ll H, \quad \eta \ll 1$$



Other model-dependent tuning sources or symmetries are needed to tune a flatter potential.

Inflation Model Building

➢ h-Problem in DBI inflation: (X.C., 08)



$$ds^2 = h(r)^2 ds_4^2 + h(r)^{-2} dr^2$$

DBI inflation requires small warping: $h \ll HR$

Backreaction from inflationary background

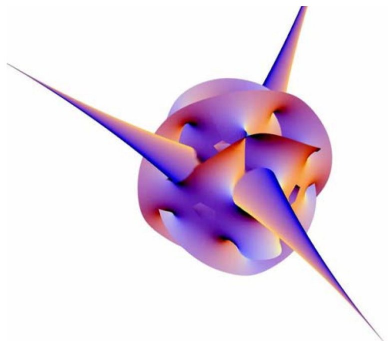
$$h \sim HR \xrightarrow{?} h \ll HR$$

Other model dependent tuning sources: moduli deformation, susy breaking etc.

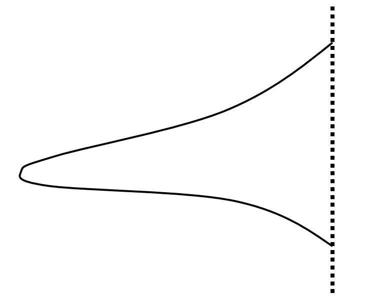
(e.g. Frey, Mazumdar, Myer, 05; McAllister, Silverstein, 07)

Inflation Model Building

- Field range bound: (X.C., Sarangi, Tye, Xu, 06; Baumann, McAllsiter, 06)

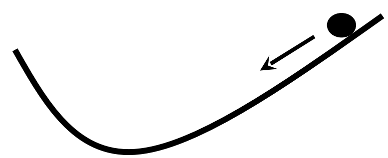


$$\phi \lesssim \mathcal{O}(0.01)\frac{M_{\rm Planck}}{\sqrt{N}}$$



$$\phi \lesssim \mathcal{O}(1)\frac{M_{\rm Planck}}{\sqrt{N}}$$

$$\phi \lesssim \frac{M_{\rm Planck}}{\sqrt{N}} \xrightarrow{?}$$



$$\phi \gg M_{\rm Planck}$$

Inflation Model Building

- Variation of potential: (Lyth, 97)

$$V(\phi) = \sum_{n=0}^{\infty} \lambda_n m_{\rm fund}^{4-n} \phi^n$$

$m_{\rm fund}$: eg. higher dim Planck mass, string mass, warped scales etc.

$$m_{\rm fund} \ll M_{\rm P}$$

Introduce shift symmetry: (e.g. Silverstein, Westphal, McAllsiter, 08)

Algebraic simplicity may not mean simplicity in nature.

Ingredients introduced to solve these problems
often make models step beyond the simplest one.

Beyond the No-Go

- Canonical kinetic term

  Non-canonical kinetic terms: DBI inflation, k-inflation, etc

- Always slow-roll

  Features in potentials or Lagrangians: sharp, periodic, etc

- Bunch-Davies vacuum

  Non-Bunch-Davies vacuum
  due to features, boundary condions, low new physics scales, etc

- Single field

  Multi-field: turning trajectories, curvatons, inhomogeous reheating surface, etc

  Quasi-single field: massive isocurvatons

In-In Formalism

(Schwinger, 61; Weinberg, 05)

Quantum fluctuations $\Longrightarrow$ Evolution inside, cross, outside horizon $\Longrightarrow$

Convert to curvature perturbation at reheating $\Longrightarrow$

Observed as correlations of fluctuations in CMB and LSS

Expectation value of an operator in a time-dependent background:

$$\langle Q(t)\rangle \equiv \langle 0| \left[\bar{T}\exp\left(i\int_{t_0}^{t} H_I(t)dt\right)\right] \; Q^I(t) \; \left[T\exp\left(-i\int_{t_0}^{t} H_I(t)dt\right)\right]|0\rangle$$

$$= \; i^n \int_{t_0}^{t} dt_1 \int_{t_0}^{t_1} dt_2 \cdots \int_{t_0}^{t_{n-1}} dt_n \, \langle [H_I(t_n), [H_I(t_{n-1}), \cdots, [H_I(t_1), Q_I(t)] \cdots]] \rangle$$

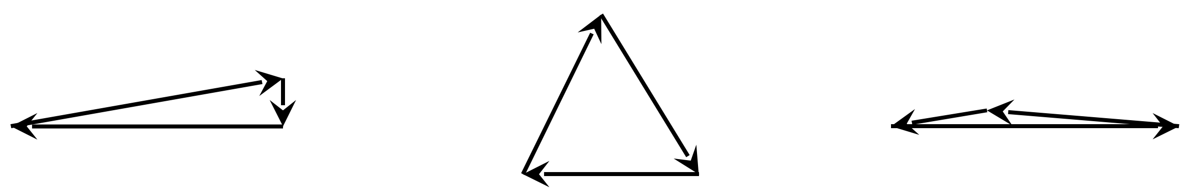
Shape and Running of Bispectra (3pt)

Bispectrum is a function, with magnitude f_{NL}, of three momenta: k_1, k_2, k_3

$$\langle\zeta^3\rangle \equiv S(k_1,k_2,k_3)\frac{1}{(k_1k_2k_3)^2}\tilde{P}_\zeta^2(2\pi)^7\delta^3(\sum_{i=1}^{3}\mathbf{k}_i)$$

- Shape dependence: (Shape of non-G)

Fix $K = k_1 + k_2 + k_3$, vary k_2/k_1, k_3/k_1.



Squeezed Equilateral Folded

- Scale dependence: (Running of non-G)

Fix k_2/k_1, k_3/k_1, vary $K = k_1 + k_2 + k_3$.

Non-Canonical Kinetic Terms

General Single Field Inflation

Consider the general single field inflation (Lorentz invariant, first derivative)

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm P}}{2} R + P(X, \phi) \right] \qquad X \equiv -\tfrac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

(Garriga, Mukhanov, 99)

- Define parameters: $c_s \quad \lambda/\Sigma$

(Seery, Lidsey, 05)

$$c_s^2 = \frac{P_{,X}}{P_{,X} + 2XP_{,XX}}$$

$$\Sigma = XP_{,X} + 2X^2 P_{,XX}$$

$$\lambda = X^2 P_{,XX} + \frac{2}{3} X^3 P_{,XXX}$$

General Single Field Inflation

Consider the general single field inflation (Lorentz invariant, first derivative)

$$S = \int d^4x \sqrt{-g} \left[\frac{M_{\rm P}}{2} R + P(X, \phi) \right] \qquad X \equiv -\tfrac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi$$

(Garriga, Mukhanov, 99)

- Define parameters: c_s λ/Σ

- Slow-variation parameters:

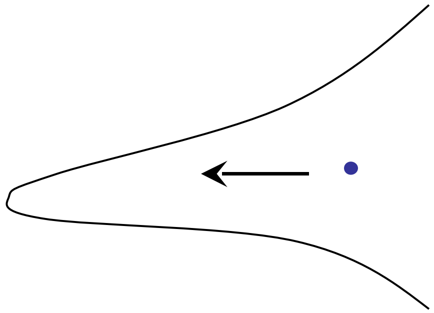
$$\epsilon = -\frac{\dot{H}}{H^2} , \quad \eta = \frac{\dot{\epsilon}}{\epsilon H} , \quad s = \frac{\dot{c}_s}{c_s H}$$

Require them to be small, independent
of mechanisms that realize the inflation

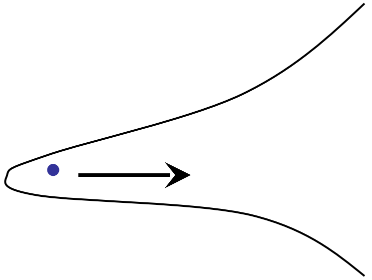
Example: DBI Inflation (Silverstein, Tong, Alishahiha, 03,04; X.C., 04)

$$ds^2 = h(r)^2 ds_4^2 + h(r)^{-2} dr^2$$

Speed limit: $\dot{r} \leq h^2$



$$V(\phi) \approx \frac{1}{2} m^2 \phi^2$$
$$m \gg M_{\rm Pl}/\sqrt{\lambda}$$



$$V(\phi) \approx V_0 - \frac{1}{2} m^2 \phi^2$$
$$m \sim H$$

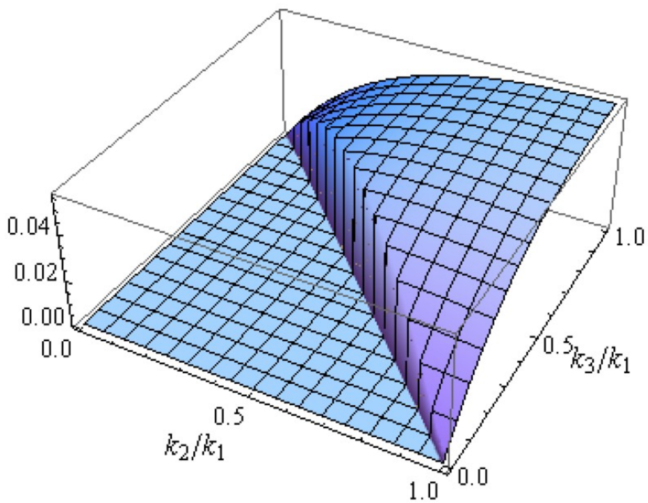
- Action: $$-\int d^4x \sqrt{-g} \left[f(\phi)^{-1} \sqrt{1 + f(\phi) g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi} - f(\phi)^{-1} + V(\phi) \right]$$

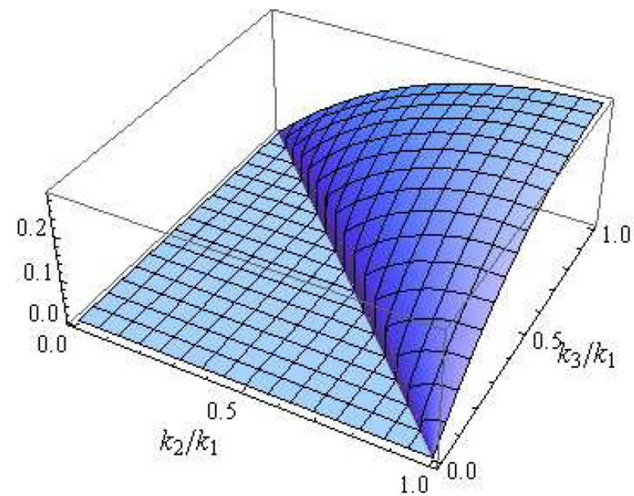
$$P = -f^{-1}\sqrt{1 - 2fX} + f^{-1} - V$$

Non-Slow-roll Mechanism

Full Bispectrum (X.C., Huang, Kachru, Shiu, 06)

- 11 shapes, speficied by 5 parameters, 3 different orders
- Leading bispectra:





$$S_\lambda : \; f_{NL}^{\lambda} = \frac{5}{81}\left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right)$$

$$S_c: \; f_{NL}^{c} = -\frac{35}{108}\left(\frac{1}{c_s^2} - 1\right)$$

Equilateral bispectra are large if $c_s \ll 1$ or $\lambda/\Sigma \gg 1$

$$-214 < f_{NL}^{\rm eq} < 266$$ (WMAP7, 10)

Physics of Large Equilateral Shape

- Generated by interacting modes during their horizon exit

Quantum fluctuations $\Rightarrow$ Interacting and exiting horizon $\Rightarrow$ Frozen

$$k_1 \sim k_2 \sim k_3$$

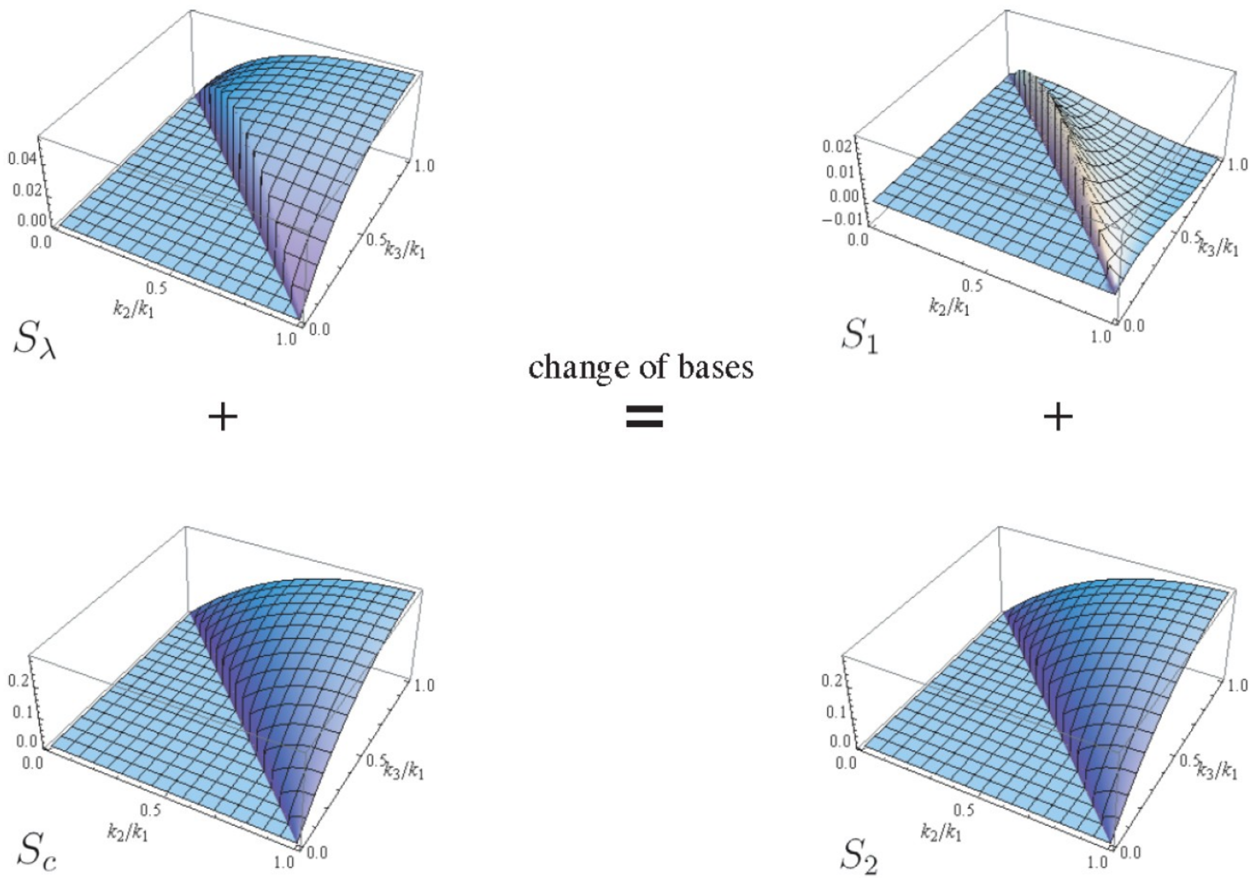
For single field, small correlation if $k_3 \ll k_1$

Higher derivative terms provide such interactions.

So, the shapes peak in the equilateral limit.

Orthogonalization

$$c_\lambda S_\lambda + c_c S_c = c_1 S_1 + c_2 S_2$$



S_1 : Orthogonal shape

(Senatore, Smith, Zaldarriaga,09)

$$-410 < f_{NL}^{\mathrm{orth}} < 6$$

(WMAP7, 10)

Features:

- Sharp feature $\Longrightarrow$ Sinusoidal running
- Periodic features $\Longrightarrow$ Resonant running
- Non-Bunch-Davies vacuum $\Longrightarrow$ Folded shape + Sinusoida/Resonant running

Sharp Features

(Kofman, 91;
Wang, Kamionkowski, 99;
Komatsu et al, 03)

(X.C., Easther, Lim, 06, 08;
Bean, X.C., Hailu, Tye, Xu, 08;
Hotchkiss, Sarkar, 09)

Sharp Features in Potentials (X.C., Easther, Lim, 06,08)

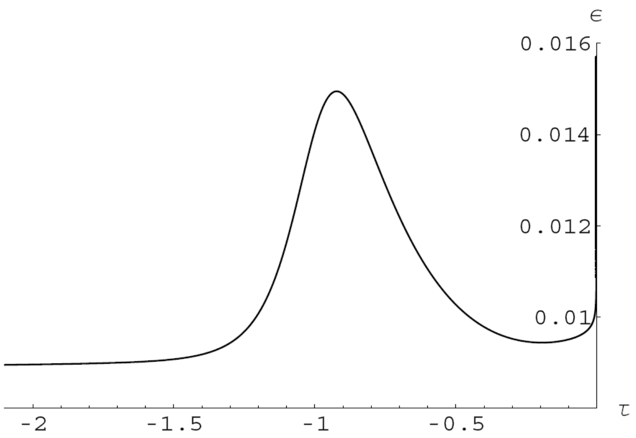
- At sharp feature, ignoring background expansion:

$$L = \frac{1}{2}\dot{\phi}^2 - V(\phi) \ , \qquad \epsilon = \frac{\dot{\phi}^2}{2M_p^2 H^2}$$

- Energy conservation:

$$\frac{1}{2}\dot{\phi}^2 + V = \text{const.}$$

$$\frac{\Delta V}{V} \ \text{small} \rightarrow \Delta\epsilon \ \text{small}$$



Sharp Features in Warp Factors (Bean, X.C., Hailu, Tye, Xu, 08)

- At sharp feature, ignoring background expansion:

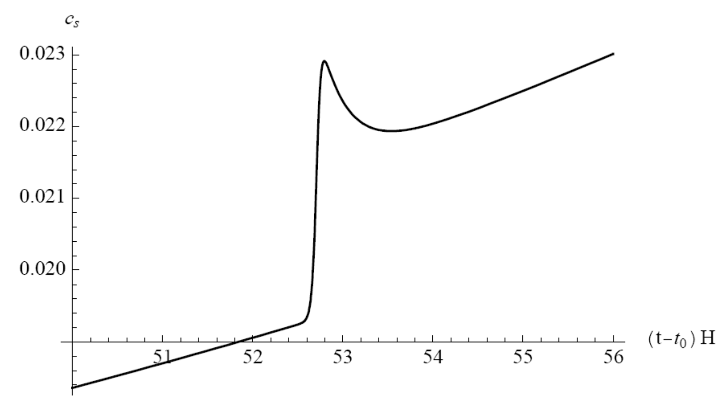
$$L = -T(\phi)\sqrt{1-\dot{\phi}^2/T(\phi)}\ , \qquad c_s = \sqrt{1-\dot{\phi}^2/T}$$

$T(\phi)$: warped brane tension

- Energy conservation:

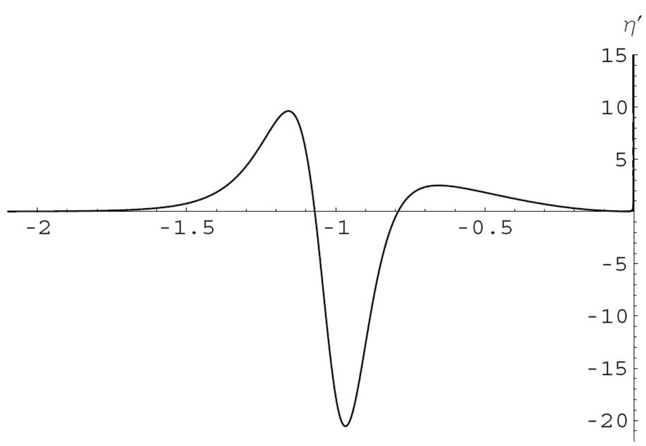
$$T/c_s = \text{const.}$$

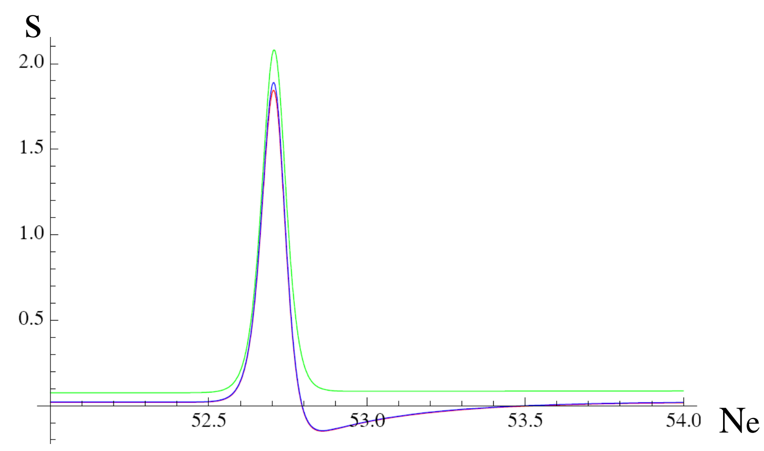
$$\frac{\Delta T}{T}\ \text{small} \rightarrow \frac{\Delta c_s}{c_s}\ \text{small}$$



- ϵ or c_s do not change much
  due to approximate energy conservation relations

- They change in a very short period of time,
  boosting $\dot{\epsilon} \to \eta$ or $\dot{c}_s \to s$ by several orders of magnitude





The Cubic Part

- The exact cubic action for scalar perturbation ζ

$$S_3 = \int dt d^3x \{ -a^3 (\Sigma(1-\frac{1}{c_s^2}) + 2\lambda) \frac{\dot{\zeta}^3}{H^3} + \frac{a^3\epsilon}{c_s^4}(\epsilon - 3 + 3c_s^2)\zeta\dot{\zeta}^2$$
$$+ \frac{a\epsilon}{c_s^2}(\epsilon - \underline{2s} + 1 - c_s^2)\zeta(\partial\zeta)^2 - 2a\frac{\epsilon}{c_s^2}\dot{\zeta}(\partial\zeta)(\partial\chi)$$
$$+ \underline{\frac{a^3\epsilon}{2c_s^2}\frac{d}{dt}(\frac{\eta}{c_s^2})\zeta^2\dot{\zeta}} + \frac{\epsilon}{2a}(\partial\zeta)(\partial\chi)\partial^2\chi + \frac{\epsilon}{4a}(\partial^2\zeta)(\partial\chi)^2 + 2f(\zeta)\frac{\delta L}{\delta\zeta}|_1 \} ,$$

$\frac{\Delta s}{c_s}$, $\Delta\left(\frac{\eta}{c_s^2}\right)$ Important in the presence of sharp features

Sinusoidal Running (X.C., Easther, Lim, 06,08)

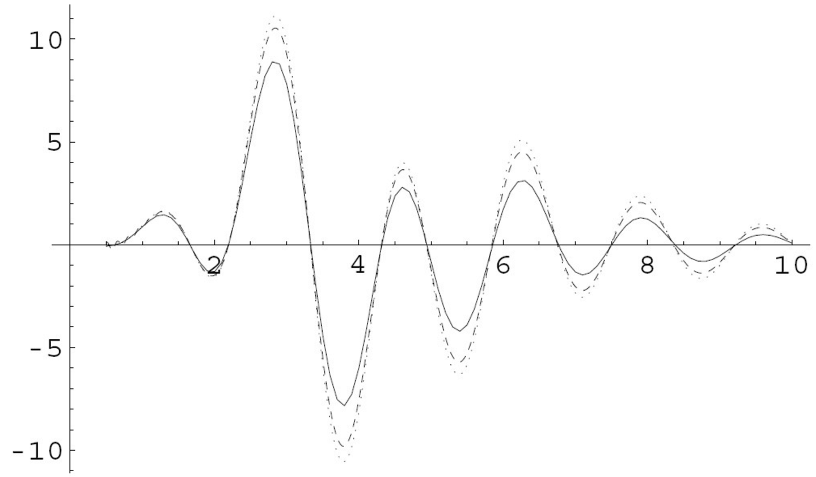
$$\int_{-\infty}^{\tau_{end}} d\tau a^2 \frac{\epsilon}{c_s^2} \frac{d}{d\tau} \left(\frac{\eta}{c_s^2} \right) \left(u_{k_1}^*(\tau) u_{k_2}^*(\tau) \frac{d}{d\tau} u_{k_3}^*(\tau) + \text{perm} \right)$$

Sharp feature at τ_*

$$e^{iK\tau}$$

Roughly, $S \sim f_{NL}^{\text{feat}} \sin\left(\frac{K}{k_*} + \phi_0\right)$, $k_* = -1/\tau_*$

- Shape is trivial (away from squeezed limit)
- Running is oscillatory and localized



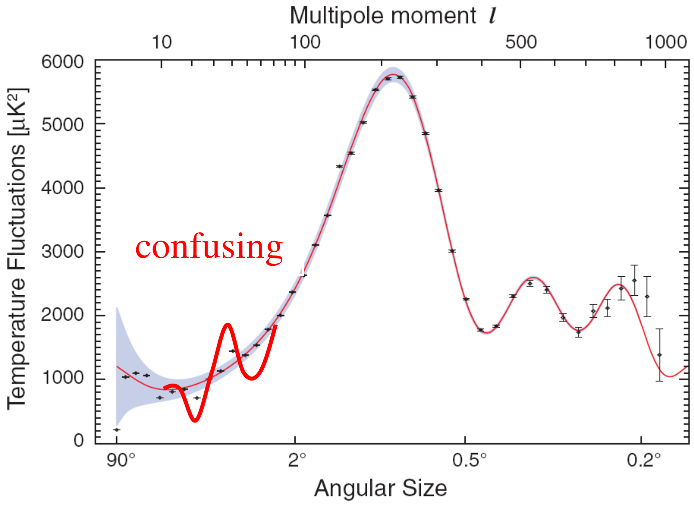
Sharp Features and Statistical Fluctuations

Power spectrum: oscillatory corrections due to the negative energy component

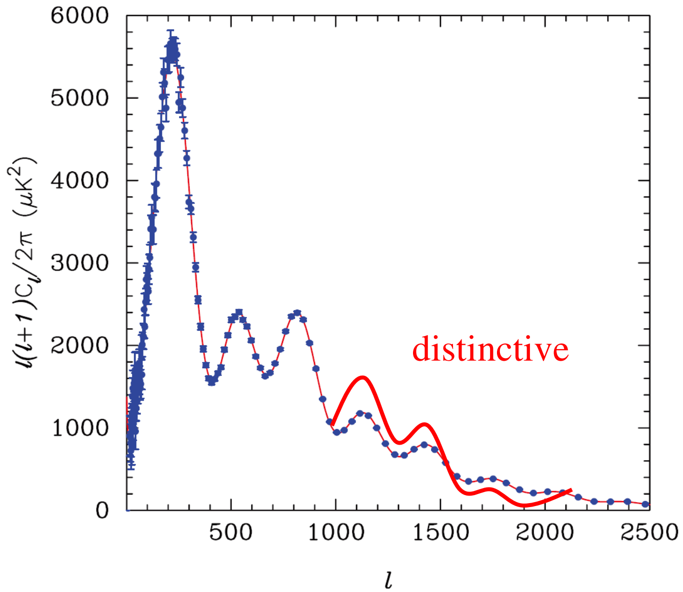
$$\Delta P_\zeta \propto \sin\left(\frac{k}{k_*/2} + \psi_0\right)$$

Osillatory frequency is given by location of feature

$$S \sim f_{NL}^{\rm feat} \sin\left(\frac{K}{k_*} + \phi_0\right)$$



WMAP



Planck

Model Building Realizations

- Small field inflation has very small ε ,
  so very sensitive to tiny features in potential or internal space.

E.g. Brane inflation. (Bean, X.C., Hailu, Tye, Xu, 08)

- Aftermath effect of sudden interactions or particle creation.

E.g. Particle creation $\Longrightarrow$ Equilateral + Sinusoidal running

(Barnaby, 10)

Periodic (Frequent) Features

(X.C., Easther, Lim, 08;
Flauger, Pajer, 10;
X.C., 10; Leblond, Pajer, 10)

Resonant Non-Gaussianity

(X.C., Easther, Lim, 08)

- Consider small periodic features in potential or Lagrangian

⇒ Small oscillatory component to slow-roll parameters with frequency ω

- Mode functions are also oscillatory, and the frequency scans

Highly oscillatory quantum fluctuations ⇒ Stretched by inflation

⇒ Frequency is continuously reduced to H ⇒ Exit horizon and Frozen

As long as $\omega > H$,

modes will resonant with the background, creating large non-G.

The Bispectrum

$$S_3 = \int dt d^3x \left[\cdots + \frac{a^3\epsilon}{2}\dot{\eta}\zeta^2\dot{\zeta} + \cdots \right]$$

Without features, $\dot{\eta} \sim \mathcal{O}(\epsilon^2)$; but here much larger.

$$\begin{aligned} \langle \zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle \;\; &\approx \;\; i\frac{H^4}{64\epsilon^3 \prod_i k_i^3}(2\pi)^3\delta^3(\sum_i \mathbf{k}_i) \\ &\times \int_{-\infty}^{0} \frac{d\tau}{\tau}\epsilon\eta' \left(1 - i(k_1+k_2)\tau - k_1k_2\tau^2\right) k_3^2 e^{iK\tau} \\ &+ \text{two perm.} + \text{c.c.}\,. \end{aligned}$$

Freq: ω

Freq: $M_{\rm P} \to H$

- Running behavior

The phase of bkgd repeats after $\Delta N_e = 2\pi H/\omega$ e-fold.

$K = k_1 + k_2 + k_3$ changes by $\Delta K = K \Delta N_e = 2\pi K H/\omega$

- Amplitude

Each *K*-mode resonants only once when it sweeps through ω .

Integrand starts to cancel after $\Delta t_1 \sim \pi/\Delta\omega$

Change from ω to $\omega - \Delta\omega$ takes $\Delta t_2 \sim \Delta\omega/(\omega H)$

Resonance duration: $\Delta t \sim \sqrt{\frac{\pi}{\omega H}}$

Number of resonance oscillation: $\frac{\omega \Delta t}{2\pi} \sim \sqrt{\frac{\omega}{4\pi H}}$

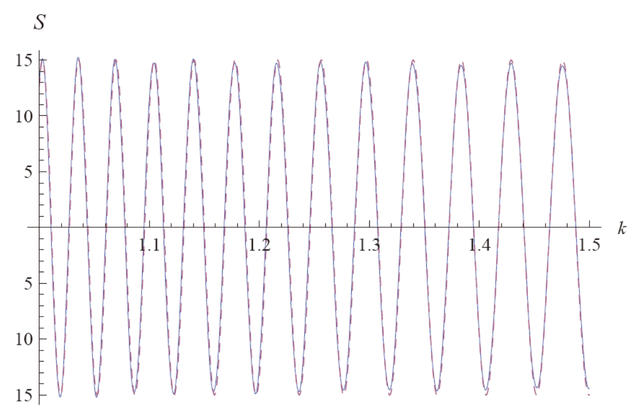
- Leading resonant bisepctrum (X.C., Easther, Lim, 08)

$$S^{\rm res}_{\rm ansatz} = f^{\rm res}_{NL} \sin\left(C \ln(K/k_*)\right)$$

$$C = 2\pi K/\Delta K = \omega/H \qquad f^{\rm res}_{NL} \sim \frac{\sqrt{\pi}}{8\sqrt{2}} \frac{\omega^{1/2}\dot{\eta}_A}{H^{3/2}}$$

Periodic-scale-invariance:
Rescale all momenta by a discrete efold: $2\pi n H/\omega$

- Characteristic running behavior



Orthogonal to all three shapes constrained in WMAP

An example of how we could miss the detection

Model Building Realizations

• Periodic features from duality cascade in brane inflation

(Hailu, Tye, 06; Bean, X.C., Hailu, Tye, Xu, 08)

Brane moving in throat ⟹ Field theory duality cascade ⟹

Dual periodic features in potential or warp factor ⟹ Sharp or resonant effect

• Periodic features from instantons in monodromy inflation

(Silverstein, Westphal, 08; Flauger, Mcallister, Pajer, Westphal, Xu, 09
Hannestad, Haugboelle, Jarnhus, Sloth, 09)

Tensor mode ⟹ Large field ⟹ Shift symmetry (string axions) ⟹

Broken by non-pert effect ⟹ Small periodic modulations ⟹ Resonant non-G

Non-Bunch-Davies Vacuum

First, very qualitatively ……

Folded Shape (X.C., Huang, Kachru, Shiu, 06; Holman, Tolly,08; Meerburg, van de Schaar, Corasaniti, Jackson, 09)

- Mode function:

$$u_k = u(\tau, \mathbf{k}) = \frac{iH}{\sqrt{4\epsilon c_s k^3}}\left(C_+(1 + ikc_s\tau)e^{-ikc_s\tau} + C_-(1 - ikc_s\tau)e^{ikc_s\tau}\right)$$

- The Bunch-Davis vacuum: $C_+ = 1$ and $C_- = 0$

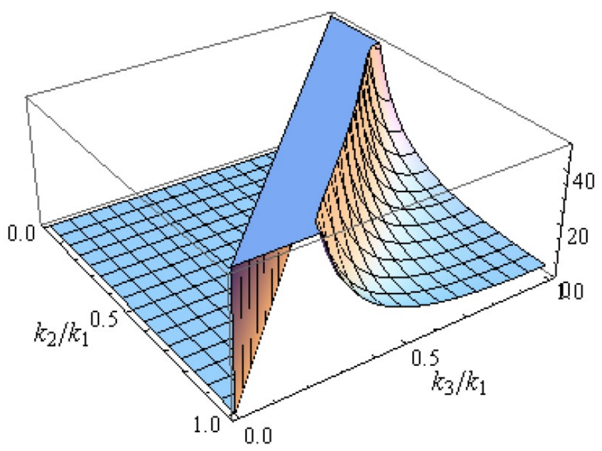
- Non-Bunch-Davis vacuum: For example, a small C_- (Martin, Brandenberger, 00)

- Bispectrum

In 3pt: $\frac{1}{k_1 + k_2 + k_3} \longrightarrow \frac{1}{k_1 + k_2 - k_3} + \text{perm.}$

Peaks at folded triangle limit

$k_1 + k_2 - k_3 = 0$ and cyclic



- **Problems:**
  Infinite peak at folded limit is unphysical;
  Non-BD vacuum put in by hand. E.g. the transition from BD to non-BD does not solve the equation of motion.

  Need a self-consistent prototypical example of non-BD case.

- Feature models: Non-BD vacuum can be easily generated:

  - General solution: linear superposition of positive and negative energy mode
  - Positive energy mode: asymptotes to BD vacuum
  - Any disturbance from features will introduce the other component.

  Analytical examples:

  Sharp feature: (Bean, X.C., Hailu, Tye, Xu, 08)

  Periodic features: (Flauger, McAllister, Pajer, Westphal, Xu, 09)

But, amplitude of the non-BD component is small, so neglected in previous examples.

A Small By-Product Playing a Large Role (X.C., 10)

- A small non-BD component in the resonant model:

$$c_- \approx \sqrt{2\pi}\delta_A \left(\frac{H}{\omega}\right)^{3/2} \exp\left[i\left(\frac{\omega}{H}\ln k + \varphi_-\right)\right] \cdot \frac{1}{2}\left[\mathrm{Erf}\left(kc_s\sqrt{\frac{2H}{\omega}}(\tau + \frac{\omega}{2kc_sH})\right) + 1\right]$$

small amplitude

resonant point

- Enhancement

General single field $\Longrightarrow$ Interaction terms with least slow-roll suppression

$$S_3 \supset \int dt d^3x \left[\frac{a^3\epsilon}{Hc_s^2}\left(\frac{1}{c_s^2} - 1 - \frac{2\lambda}{\Sigma}\right)\dot{\zeta}^3 - \frac{3a^3\epsilon}{c_s^2}\left(\frac{1}{c_s^2} - 1\right)\zeta\dot{\zeta}^2 + a\epsilon\left(\frac{1}{c_s^2} - 1\right)\zeta\left(\partial\zeta\right)^2\right]$$

Deeper inside horizon $\Longrightarrow$ factors of $\omega/H \gg 1$

Folded Resonant Non-Gaussianity (X.C., 10)

- Three effects synthesized:

  Resonant mechanism, non-BD vacuum, non-canonical kinetic terms

- Amplitude:

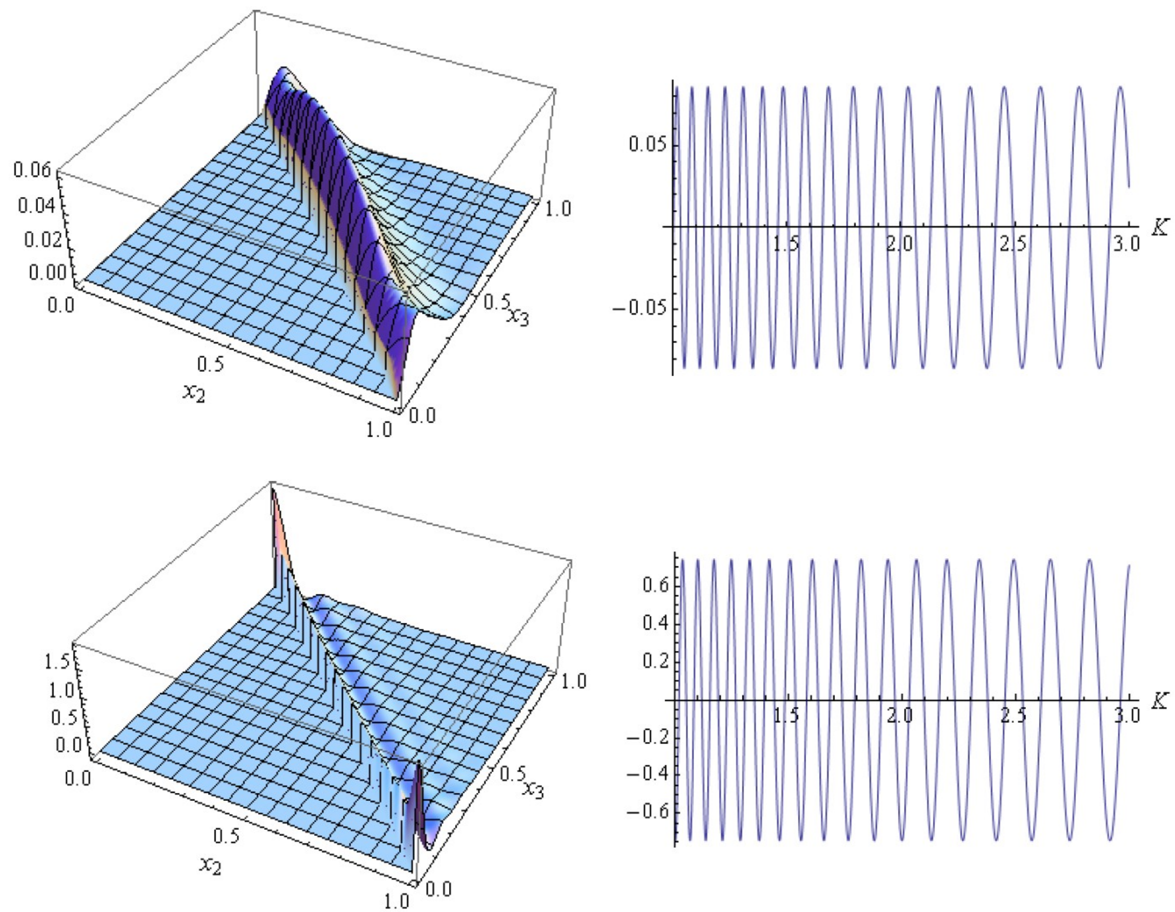
$$f_{NL}^{\rm res} \sim \frac{\Delta\epsilon}{\epsilon}\frac{1}{c_s^2}\left(\frac{\omega}{H}\right)^{5/2} \quad \text{or} \quad \frac{\Delta c_s}{c_s}\frac{1}{c_s^2}\left(\frac{\omega}{H}\right)^{5/2} ,$$

$$f_{NL}^{\rm fold\text{-}res} \sim \frac{\Delta\epsilon}{\epsilon}\frac{1}{c_s^2}\left(\frac{\omega}{H}\right)^{7/2 \ {\rm or}\ 5/2} \quad \text{or} \quad \frac{\Delta c_s}{c_s}\frac{1}{c_s^2}\left(\frac{\omega}{H}\right)^{7/2 \ {\rm or}\ 5/2}$$

Non-BD induced non-G can become the leading effect

$1/c_s^2 - 1$ and λ/Σ does not have to be much larger than one

Leading Bispectra Shapes and Running



$$
\begin{aligned}
S_{\text{ansatz}} \quad &= \quad f_{NL}^{\text{fold-res}} \ \exp\left[-\frac{C^{3/5}}{2}\left(-1+\frac{k_2}{k_1}+\frac{k_3}{k_1}\right)\right] \sin\left[\frac{C}{2}\left(-1+\frac{k_2}{k_1}+\frac{k_3}{k_1}+2\ln k_1\right)+\varphi\right] \\
&+ \quad 2 \text{ perm.}
\end{aligned}
$$

Multiple Fields:

Massive Isocurvatons

Quasi-Single Field Inflation (X.C., Wang, 09,10)

- Fields generally have mass of order *H*
- One of them has to be tuned to be << *H*

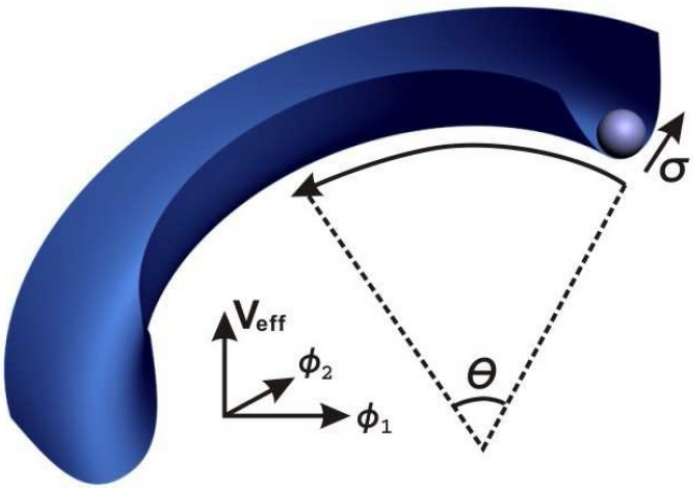
Quasi-Single field models:

One inflaton $m^2 \sim \mathcal{O}(0.01)H^2$, others have $m^2 \sim H^2$

Decoupled: same prediction as single field models

Coupled: can have novel implications on density perturbations

An Example: Turning Trajectory



In polar coordinates:

$$S_m = \int d^4x\sqrt{-g}\left[\frac{1}{2}(R+\sigma)^2 g^{\mu\nu}\partial_\mu\theta\partial_\nu\theta + \frac{1}{2}g^{\mu\nu}\partial_\mu\sigma\partial_\nu\sigma - V_{sr}(\theta) - V(\sigma)\right]$$

Slow-roll potential

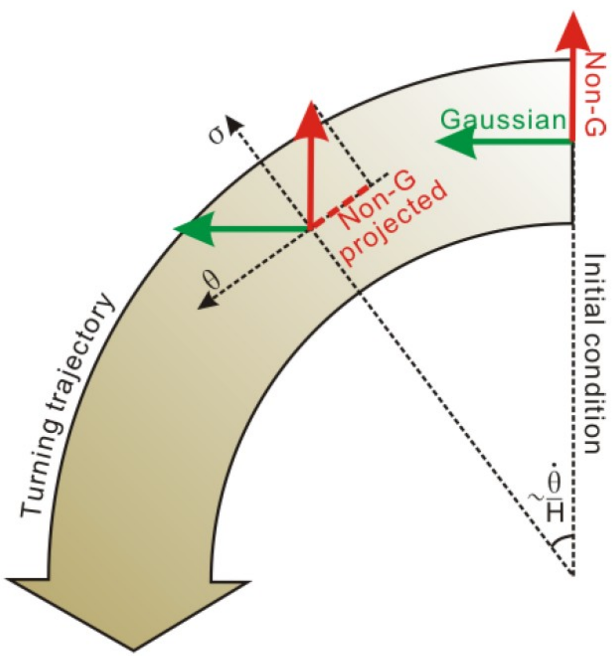
Potential for massive fields

Difference Between $V_{sr}(\theta)$ and $V(\sigma)$

$V''_{sr} \sim \mathcal{O}(\epsilon) H^2$ but $V'' \sim H^2$

$V'''_{sr} \sim \mathcal{O}(\epsilon^2) P_\zeta^{1/2} H$ but V''' can be H

V'''' can be 1 , etc



$V(\sigma)$ is the main source of large non-Gaussianities.

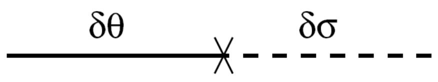
It is scale-invariant for constant turn case.

Interaction Part

- Transfer vertex

$$\mathcal{H}_2^I = -c_2 a^3 \delta\sigma_I \delta\dot{\theta}_I$$

$$c_2 = 2R\dot{\theta}_0$$

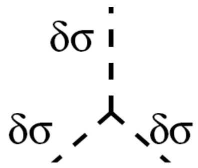


We use this transfer-vertex to compute the isocurvature-curvature conversion

- Interaction vertex

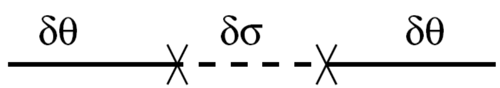
$$\mathcal{H}_3^I = c_3 a^3 \delta\sigma_I^3$$

$$c_3 = \frac{1}{6}V'''$$

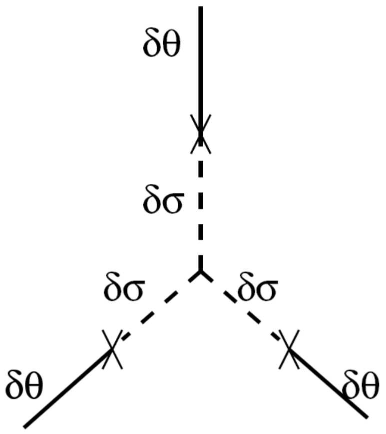


Source of the large non-Gaussianities

Perturbation Method and Feynman Diagrams



Correction to 2pt



3pt

- To use the perturbation theory, we need $\left(\frac{\dot{\theta}}{H}\right)^2 \ll 1 \,, \quad \frac{V'''}{H} \ll 1 \,.$

  These conditions are not necessary for the model building, but non-perturbative method remains a challenge.

- Choose appropriate representation of in-in formalism to cancel 9 different orders spurious IR divergences for light isocurvaton $0 < m < \sqrt{2}H$

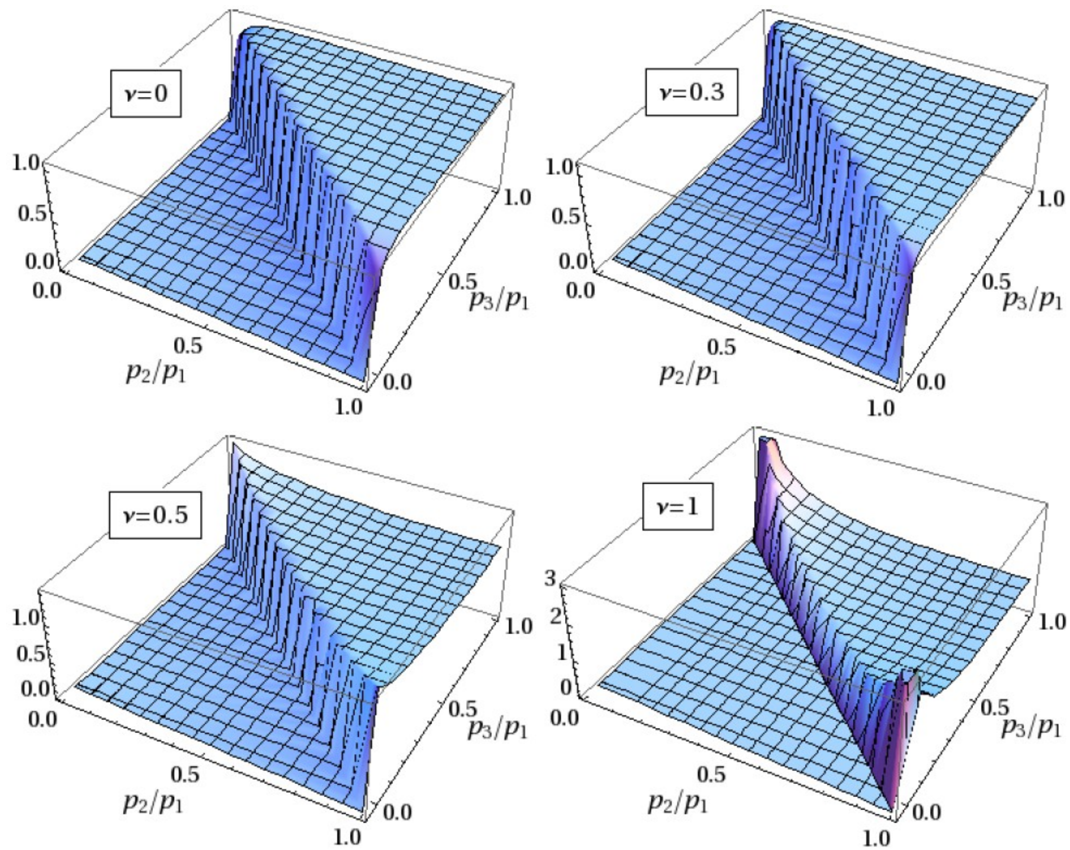
Intermediate Bispectra (X.C., Wang, 09,10)

$$S \sim \left(\frac{p_3}{p_1}\right)^{\frac{1}{2}-\nu}$$

$$0 \le \nu < 3/2$$

$$(3H/2 \ge m > 0)$$

One parameter family of shapes lying between equilateral and local

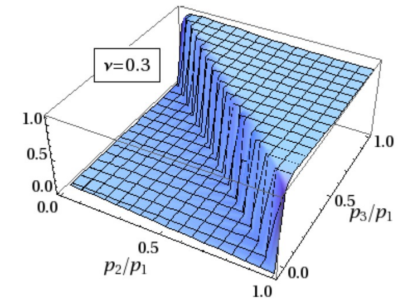


$$S^{\rm int}_{\rm ansatz} = \frac{3^{\frac{9}{2}-3\nu}}{10} \frac{f_{NL}^{\rm int}(p_1^2+p_2^2+p_3^2)(p_1p_2p_3)^{\frac{1}{2}-\nu}}{(p_1+p_2+p_3)^{\frac{7}{2}-3\nu}}$$

Physics of Large Intermediate Shapes

- Quasi-equilateral: for heavier isocurvaton $m > \sqrt{2}H$, i.e. $\nu < 1/2$

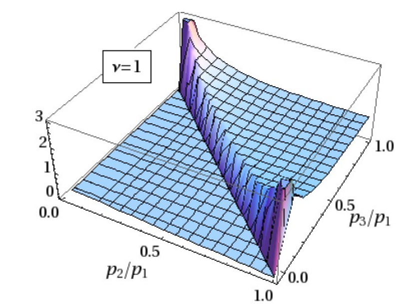
  Fluctuations decay faster after horizon-exit,
  so large interactions happen during the horizon-exit.

  Modes have comparable wavelengths:
  Closer to equilateral shape.

- Quasi-local: for lighter isocurvaton $m < \sqrt{2}H$, i.e. $3/2 > \nu > 1/2$

  Fluctuations decay slower after horizon-exit,
  so non-G gets generated and transferred
  more at super-horizon scales.

  Closer to local shape (explained in next page).

In $m \to 0$, i.e. $\nu \to 3/2$ limit, recover the local shape behavior.

Multiple Fields:

Massless Isocurvatons

Physics of Large Local Shape

- Generated by modes after horizon exit, in multifield inflation

  ➢ Isocurvature modes $\longrightarrow$ curvature mode

  ➢ Patches that are separated by horizon evolve independently (locally)

$$\delta N = \delta N_g + f_{NL}(\delta N_g)^2$$

(Starobinsky, 85; Sasaki, Stewart,95; Lyth, Rodriguez, 05)

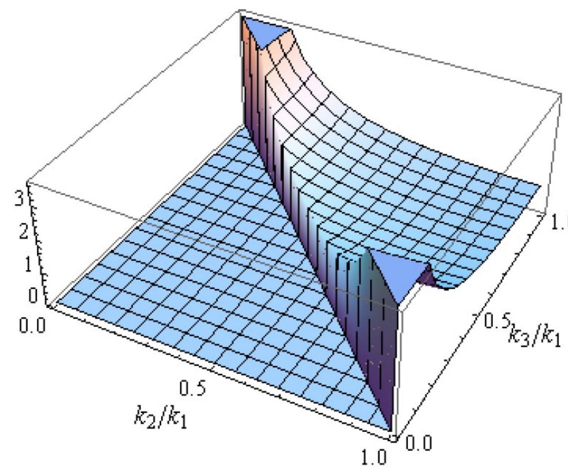
Local in position space $\longrightarrow$ non-local in momentum space

So, the shape peaks at squeezed limit.

- For example, in curvaton models;
  (Lyth, Ungarelli, Wands, 02)

  modulated reheating;
  (Dvali, Gruzinov, Zaldarriaga,03; Kofman,03)

  turning trajectory.
  (Vernizzi, Wands, 06; Rigopoulos, Shellard, van Tent, 06)



$$-10 < f_{NL}^{\rm local} < 74$$

(WMAP7, 10)

Conclusions

Using primordial non-Gaussianities to probe early universe

Different inflationary dynamics can imprint distinctive signatures in non-G;

No matter whether the primordial nonG will turn out to be observable or not, detecting/constraining them requires a complete classification of their profiles.

Conclusions

Using primordial non-Gaussianities to probe early universe

Classification:

- Non-canonical kinetic term: Equilateral shape
- Sharp feature: Sinusoidal running
- Periodic features: Resonant running
- Negative energy non-BD vacuum: Folded shape + running
- Massive isocurvatons: Intermediate shapes
- Massless isocurvaton: Local shape