Non-Gaussianity as a signature of thermal initial condition of inflation

Phys.Rev.D80:123537,2009
arXiv:0908.2305 [astro-ph.CO]

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Outline

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Power spectrum in slow-roll single field model

Quantum fluctuations in inflaton field
$\phi(\mathbf{x}, t) \longrightarrow$ Comoving curvature perturbations in spatially flat gauge

$$\mathcal{R}(t, \mathbf{x}) = \frac{H}{\dot{\phi}} \delta\phi(t, \mathbf{x})$$

Two-point correlation
of $\mathcal{R}(\mathbf{k}, t)$ in Fourier space $\longrightarrow$ Comoving curvature power spectrum

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2} \langle \mathcal{R}(k) \mathcal{R}(k) \rangle \longleftrightarrow \left(\frac{H}{\dot{\phi}} \right)^2 \langle \delta\phi(k) \delta\phi(k) \rangle$$

Well measured through CMB anisotropy spectrum

Outline

Bispectrum in slow-roll single field model

Bispectrum $\longrightarrow$ Three point correlation function $\longrightarrow$ Non-linear parameter f_{NL}

- WMAP : $-151 < f_{NL} < 253$ (95% CL)
- PLANCK : $f_{NL} \sim 5$
- 21-cm Background Anisotropy Measurement : $f_{NL} < 0.1$

  A. Cooray, Phys. Rev. Lett. **97**, 261301 (2006)

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In a free theory

$$\langle \delta\phi(k_1)\delta\phi(k_2)\delta\phi(k_3)\rangle = 0$$

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In a free theory

$$\langle \delta\phi(k_1)\delta\phi(k_2)\delta\phi(k_3)\rangle = 0$$

But a theory with $V(\phi) = \lambda\phi^3 \longrightarrow$ Non-vanishing Bispectrum

$$\langle \delta\phi(k_1)\delta\phi(k_2)\delta\phi(k_3)\rangle \sim \lambda/H$$

$\lambda/H \longrightarrow$ Too small (QFT) $\sim \mathcal{O}(10^{-7}) \longrightarrow$ Not measurable

T. Falk, R. Rangarajan and M. Srednicki, Astrophys. J. **403**, L1 (1993)

Bispectrum in single field slow-roll model

In the non-linear limit

$$\mathcal{R}_{NL}(t,\mathbf{x}) = \frac{H}{\dot\phi}\delta\phi_L(t,\mathbf{x}) + \frac{1}{2}\frac{\partial}{\partial\phi}\left(\frac{H}{\dot\phi}\right)\delta\phi_L^2(t,\mathbf{x}) + \mathcal{O}(\delta\phi_L^3)$$

Three-point correlation exists in momentum space

$$\langle\mathcal{R}_{NL}\mathcal{R}_{NL}\mathcal{R}_{NL}\rangle \simeq \left(\frac{H}{\dot\phi}\right)^2\frac{1}{2}\frac{\partial}{\partial\phi}\left(\frac{H}{\dot\phi}\right)\langle\delta\phi\delta\phi\delta\phi^2\rangle$$

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Defining the non-linear parameter f_{NL}

$$\langle\mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3)\rangle = (2\pi)^{-\frac{3}{2}}\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\frac{6}{5}f_{NL}\left(\frac{P_{\mathcal{R}}(k_1)}{k_1^3}\frac{P_{\mathcal{R}}(k_2)}{k_2^3} + 2\ \mathrm{perms.}\right)$$

Non-linear parameter for bispectrum $f_{NL} = \frac{5}{6}(\delta-\epsilon) \rightarrow$ too small to be detected

Outline

Trispectrum in slow-roll single field model

Trispectrum $\longrightarrow$ Connected part of four point correlation function

$$\langle\mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3)\mathcal{R}(\mathbf{k}_4)\rangle_c \equiv \langle\mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3)\mathcal{R}(\mathbf{k}_4)\rangle$$
$$-\left(\langle\mathcal{R}_L(\mathbf{k}_1)\mathcal{R}_L(\mathbf{k}_2)\rangle\langle\mathcal{R}_L(\mathbf{k}_3)\mathcal{R}_L(\mathbf{k}_4)\rangle + 2\ \mathrm{perm}\right)$$

Experimental bound on Non-linear parameter τ_{NL}

- WMAP : $|\tau_{NL}| \lesssim 10^8$
- PLANCK : $|\tau_{NL}| \sim 560$
- 21-cm Background Anisotropy Measurement : $\tau_{NL} \sim 10$

A. Cooray, C. Li and A. Melchiorri, Phys. Rev. D **77**, 103506 (2008)

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In free theory $\longrightarrow$ Four-point function $\rightarrow$ Square of two-point function $\longrightarrow$ Connected four-point function vanishes $\rightarrow$ $\tau_{NL} = 0$

Trispectrum in single field slow-roll model

Expanding non-linearly upto third order

$$\mathcal{R}_{NL}(t, \mathbf{x}) \to \mathcal{O}(\delta\phi_L^3)$$

Non-vanishing connected four-point function $\longrightarrow$ Yields non-linear parameter

$$\tau_{NL} = \left(\frac{6}{5} f_{NL}\right)^2$$

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$\tau_{NL} < f_{NL}$

τ_{NL} too small to be detected

Outline

Features due to initial thermal bath

If pre-inflationary radiation era exists $\longrightarrow$ Thermal vacuum contains real particle

$$N_k|\Omega\rangle = n_k|\Omega\rangle$$

The probability of the system to be in the state $\varepsilon_k \equiv n_k k$

$$p(\varepsilon_k) \equiv \frac{e^{-\beta n_k k}}{\sum_{n_k} e^{-\beta n_k k}} = \frac{e^{-\beta n_k k}}{z}$$

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Thermal statistical average of two-point correlation

$$\begin{aligned} P^{\rm th}_{\delta\phi}(k) &= \frac{k^3}{2\pi^2}\langle\Omega|\delta\phi(k,t)\delta\phi(k,t)|\Omega\rangle_\beta \\ &= \frac{k^3}{2\pi^2}\sum_{\varepsilon_k} p(\varepsilon_k)\langle\Omega|\delta\phi(k,t)\delta\phi(k,t)|\Omega\rangle \end{aligned}$$

Features due to initial thermal bath

Due to the thermal distribution of the inflaton field

$$\begin{aligned} \langle\Omega|\delta\phi(k,t)\delta\phi(k,t)|\Omega\rangle &= |\varphi_k(t)|^2 \langle\Omega|\left(1+2N_k\right)|\Omega\rangle \\ &= |\varphi_k(t)|^2 \left(1+2n_k\right) \end{aligned}$$

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The thermal power spectrum

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P^{\rm th}_{\delta\phi}(k) &= \frac{k^3}{2\pi^2} |\varphi_k(t)|^2 \frac{1}{z} \sum_{n_k} e^{-\beta n_k k} (1+2n_k) \\
&= \frac{k^3}{2\pi^2} |\varphi_k(t)|^2 (1+2f_B(k))
\end{aligned}
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$f_B(k) \equiv \frac{1}{e^{\beta k}-1} \longrightarrow$ Bose-Einstein distribution

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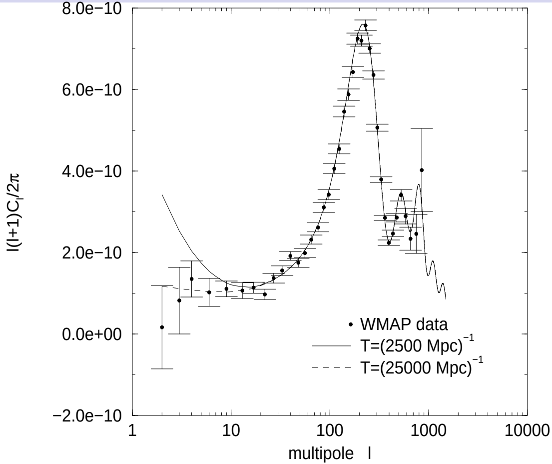
$f_B(k) \equiv \frac{1}{e^{\beta k}-1} \longrightarrow$ Bose-Einstein distribution

Power is enhanced by a factor

$$1+2f_B(k) = \coth(\beta k/2)$$

Outline

Power Spectrum



K. Bhattacharya, S. Mohanty and R. Rangarajan,
Phys. Rev. Lett. **96**, 121302 (2006)

Power spectrum
$\mathcal{P}_{\mathcal{R}}^{\rm th}(k) = \coth(\beta k/2)\mathcal{P}_{\mathcal{R}}(k)$

Enhancement in power at low ℓ due to thermal inflaton

Bound on temperature
$T < 1.0 \times 10^{-3}\ \mathrm{Mpc}^{-1}$

Outline

Non-Gaussianity from Bispectrum

The three point function too has to be thermal averaged

$$\langle \mathcal{R}_{NL}\mathcal{R}_{NL}\mathcal{R}_{NL}\rangle_\beta \simeq \left(\frac{H}{\dot{\phi}}\right)^2 \frac{1}{2}\frac{\partial}{\partial\phi}\left(\frac{H}{\dot{\phi}}\right)\langle\delta\phi\delta\phi\delta\phi^2\rangle_\beta$$

where the thermal average of four inflaton field

$$\langle\delta\phi_{k_1}\delta\phi_{k_2}\delta\phi_{k_3}\delta\phi_{k_4}\rangle_\beta = \sum_{\{n_{k_i}\}} p(k_1,k_2,k_3,k_4)\,\langle\Omega|\delta\phi_{k_1}\delta\phi_{k_2}\delta\phi_{k_3}\delta\phi_{k_4}|\Omega\rangle$$

Here the probability of occupancy of different energy states is

$$p(k_1,k_2,k_3,k_4) \equiv \frac{\prod_r e^{-\beta n_{k_r}k_r}}{\prod_r \sum_{n_k} e^{-\beta n_{k_r}k_r}} = \frac{\prod_r e^{-\beta n_{k_r}k_r}}{Z}$$

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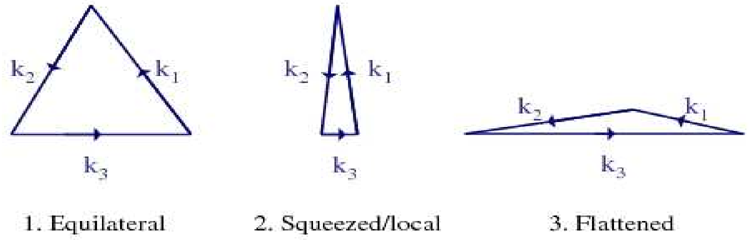
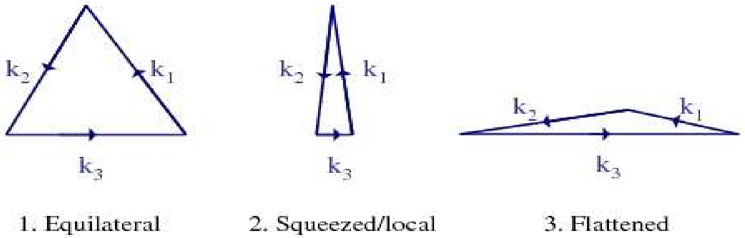
f_{NL} is enhanced due to thermal averaging

Non-Gaussianity from Bispectrum



See K. Simth's talk

Non-Gaussianity from Bispectrum



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$$f_{NL}^{\mathrm{th}} = f_{NL} \times \left(3 + \frac{5}{4\sinh^2\left(\frac{\beta k}{2}\right)}\right)$$

$$f_{NL}^{\mathrm{th}} = f_{NL} \times 2\left(1 + 3.72\coth\left(\frac{\beta k}{2}\right)\right)$$

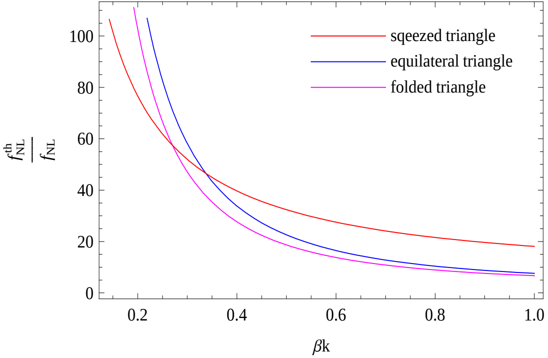
$$f_{NL}^{\mathrm{th}} = f_{NL} \times \left(3 + \frac{1}{\sinh^2\left(\frac{\beta k}{2}\right)}\right)$$

Enhancement : 90.85

Enhancement : 64.82

Enhancement : 73.28

Non-Gaussianity from Bispectrum



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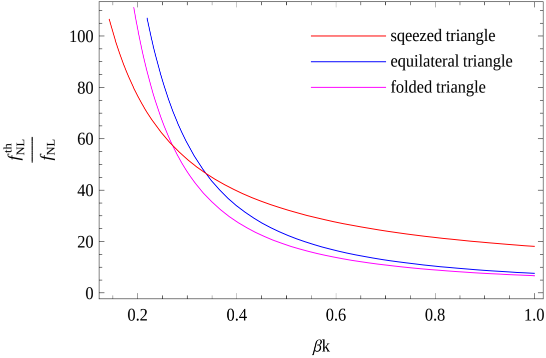

- Maximum contribution $\longrightarrow$ "Equilateral" configuration
- Non-Gaussianity $\longrightarrow$ Measurable by the 21-cm background radiation observations

Similar analysis by Agullo and Parker (arXiv:1010.5766)

- Calculated non-Gaussinities in single field inflationary models when perturbations are already present in the initial state.
- Considerd a generic statistical density operator ρ (in our case p) that includes probabilities for nonzero numbers of scalar perturbations to be present at early times during inflation.
- Only Bispectrum features are analyzed and found that the initial presence of quanta can significantly enhance non-gaussianities

$$\mathrm{Tr}[\rho\hat{\mathcal{R}}^{I}_{\vec{k}_1}(0)\hat{\mathcal{R}}^{I}_{\vec{k}_2}(0)\hat{\mathcal{R}}^{I}_{\vec{k}_3}(0)] =$$

$$\delta_{(\sum\vec{k}_i),0} P^0_{\mathcal{R}}(k_1) P^0_{\mathcal{R}}(k_2) \left[\frac{\ddot{\phi}}{\dot{\phi}H} + \frac{1}{4M_P^2}\frac{\dot{\phi}^2}{H^2}\left(1 + \frac{k_1^2 + k_2^2}{k_3^2}\right)\right] F(\rho, \vec{k}_1, \vec{k}_2) + 2\ \mathrm{perm}$$

where

$$F(\rho, \vec{k}_1, \vec{k}_2) = \mathrm{Tr}[\rho(2N_{\vec{k}_1}+1)(2N_{\vec{k}_2}+1)] - \delta_{\vec{k}_1,\vec{k}_2}\mathrm{Tr}[\rho N_{\vec{k}_1}(N_{\vec{k}_1}+1)]$$

Outline

Non-Gaussianity from Trispectrum

Due to thermal averaging

$$\begin{aligned}\langle\mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3)\mathcal{R}(\mathbf{k}_4)\rangle_c &\neq \langle\mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3)\mathcal{R}(\mathbf{k}_4)\rangle_\beta \\ &- \left(\langle\mathcal{R}_L(\mathbf{k}_1)\mathcal{R}_L(\mathbf{k}_2)\rangle_\beta \langle\mathcal{R}_L(\mathbf{k}_3)\mathcal{R}_L(\mathbf{k}_4)\rangle_\beta + 2\text{perm}\right)\end{aligned}$$

The connected part turns out to be

$$\begin{aligned}\langle\mathcal{R}(\mathbf{k}_1)\mathcal{R}(\mathbf{k}_2)\mathcal{R}(\mathbf{k}_3)\mathcal{R}(\mathbf{k}_4)\rangle_c &= \tau_{NL}\left[\frac{P_{\mathcal{R}}(k_1)}{k_1^3}\frac{P_{\mathcal{R}}(k_2)}{k_2^3}\delta^3(\mathbf{k}_1+\mathbf{k}_3)\delta^3(\mathbf{k}_2+\mathbf{k}_4)\right. \\ &\left. +2\text{ perm.}\right]\end{aligned}$$

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τ_{NL} does not depend upon the slow-roll parameters

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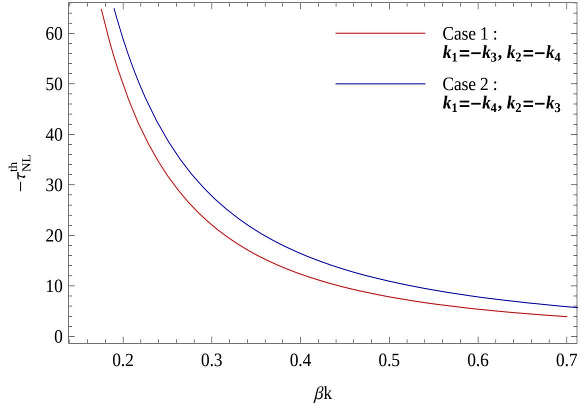

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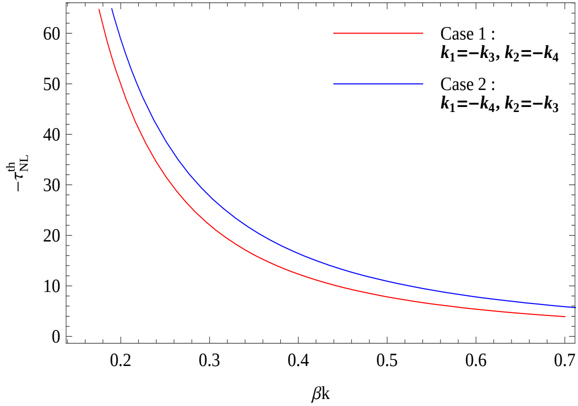

τ_{NL} can be as large as -42.58

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- In case of trisepctrum : Detectable τ_{NL} by 21 cm background measurement

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- In case of trisepctrum : Detectable τ_{NL} by 21 cm background measurement
- Signature of thermal inflaton background $\longrightarrow$ Large trispectrum compared to bispectrum