Power spectrum nulls due to non-standard inflationary evolution

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PPS features and observations

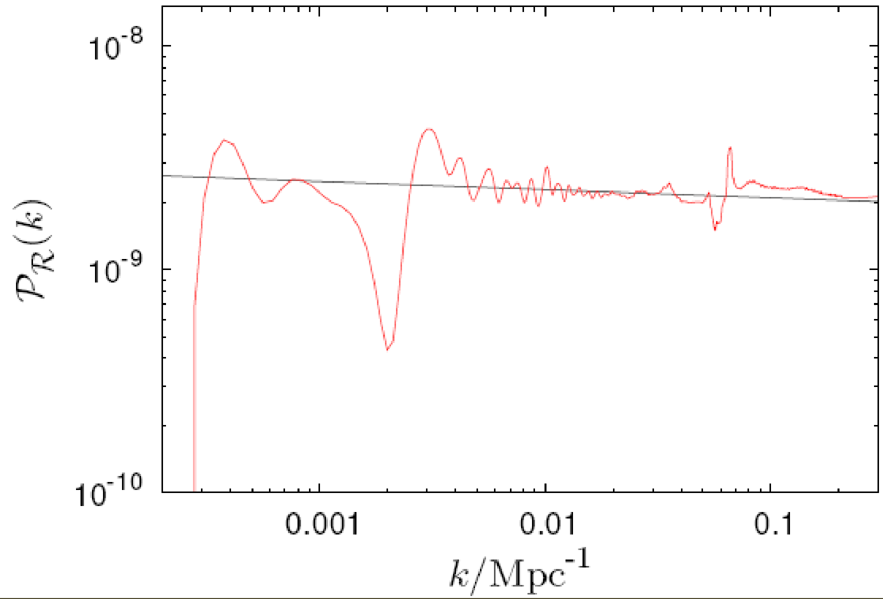
- CMB anisotropies and LSS depend on cosmological parameters and assumed form of PPS:
  - → estimates of cosmological parameters are prejudiced by the assumed PPS;
  - → important to know the PPS from observations sans theoretical bias.
  - → features in PPS: Physics of the early universe!

- A few data points (l = 2, 22 and 40) lie outside the cosmic variance for power law PPS.
  - → Oscillations in PPS » better l = 22 and l =40;

- Model independent reconstruction of PPS from CMB anisotropies done: features in PPS!
  - → Nearly scale invariant at small scales.

The results seem to indicate that PPS may have features, e.g.,

- a sharp infrared cutoff on the present Hubble scale,
- a bump (i.e. a localized excess just above the cut off),
- a ringing (i.e. a damped oscillatory feature after the infrared break).



See also: Arman Shaifloo's talk tomorrow.

Models giving PPS with features

Are there any inflationary models that produce such features?

Many!

e.g.
- Multi-field ones.
- Single field, fine-tuned initial conditions or assumptions about pre-inflationary regime.
- Single field, temporary departure from SR: introduce step in potential or slope.
- Single field, temporary departure from SR: no ad hoc feature, just fine-tuned potential.

The Punch line

- Too many models of inflation, got to distinguish between them:

✓Primordial features,

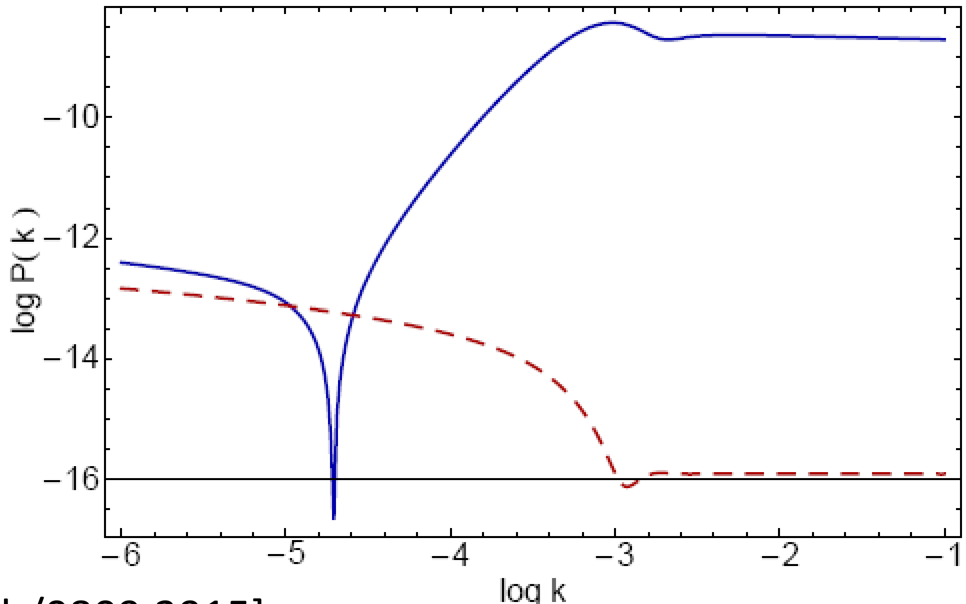
✓Non-Gaussianity

- Only the simplest ones (based on slow roll) produce nearly scale invariant primordial power spectra (PPS).

- There are many models that predict radically broken PPS.

- While it is possible (either numerically or analytically) to calculate PPS in any model, it is not clear why they take the shapes they do.

The origin of cusps PPS (with null power) can be "explained."



R.K. Jain, JCAP 09 01: 009, 2009. [arXiv:astroph/0809.3915]

Cosmological Perturbations

$$\boxed{S_2 = \frac{1}{2}\int \mathrm{d}^4x\, a^3 \frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i \mathcal{R})^2\right]}.$$

Comoving gauge

Comoving curvature perturbation R
Mukhanov-Sasaki variable v
Tensor perturbation h

$$v = R\ z \quad \text{where } z = \frac{a\ \dot{\phi}}{H}$$

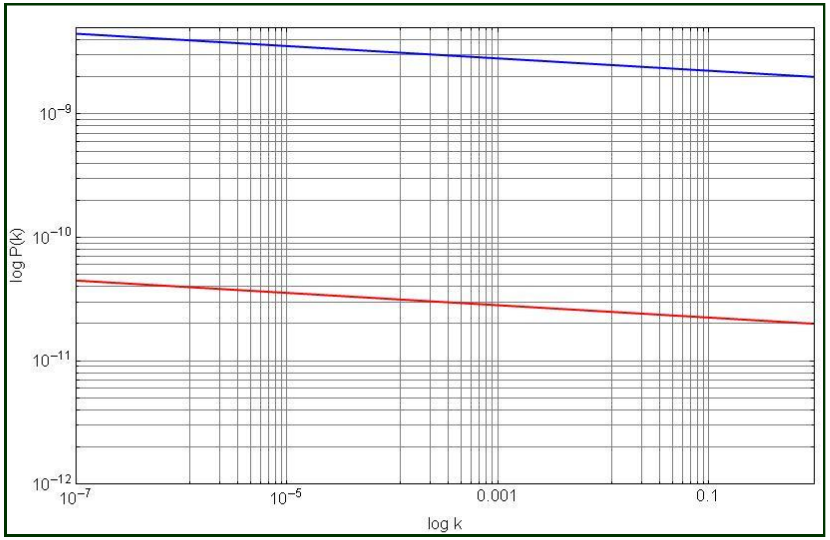
$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0$$

The B.D. initial conditions: $\lim\limits_{-k\eta\to\infty} v_k(\eta) = \frac{1}{\sqrt{2\omega_k}} \cdot e^{-ik\eta}$

$$P_S(k) = \frac{k^3}{2\pi^2}\ < |R_k|^2 > \qquad P_T(k) = 2\ \frac{k^3}{2\pi^2}\ < |h_k|^2 >$$

Slow roll inflation-

- Scalar as well as tensor PPS are featureless.
- Tensor PPS is always red.
- Scalar power is always more than tensor power so that the tensor to scalar ratio is always less than one.



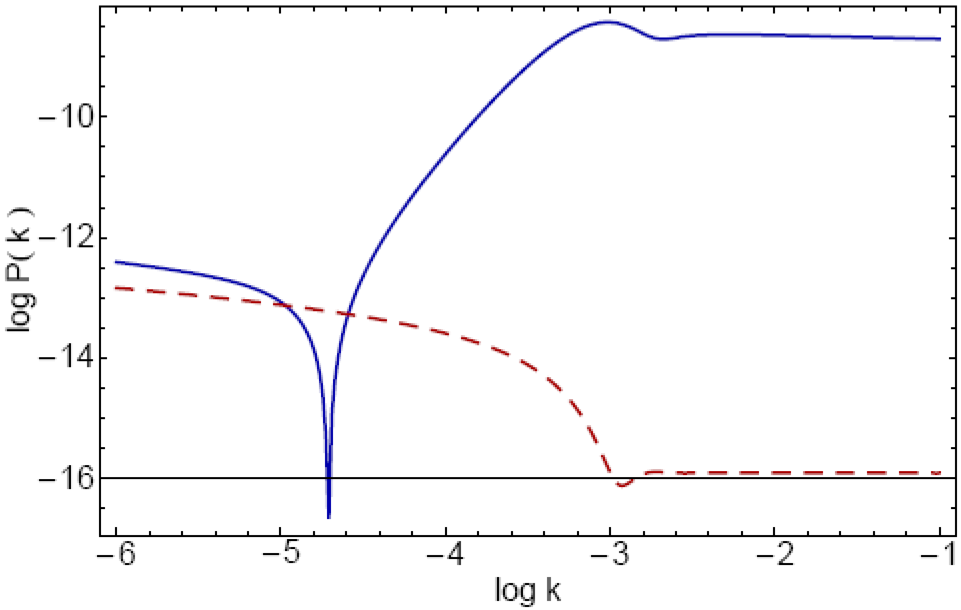
$$P_R(k) \sim A_s \; k^{n_s - 1}$$

$$P_h(k) = A_T \; k^{n_T} \sim 8 \left(\frac{H}{2\pi}\right)^2_{k=aH}$$

$$r = \frac{A_T}{A_s} \sim 16\epsilon$$

Beyond slow roll inflation-

"But, if we go beyond slow roll, all the standard results stated above get violated!"



R.K. Jain, JCAP 09 01: 009, 2009. [arXiv:astroph/0809.3915]

Evolution of modes

- There are models with various features in PSS.
- Can we have models of inflation in which scalar power is zero for some scale?
- Obvious: look at the actual evolution of the mode!
- There exist some key properties that the modes follow as they evolve.
- But a particular kind of "non standard" evolution is fairly common.

Hence, look at the evolution of Fourier mode function of comoving curvature perturbation and Mukhanov-Sasaki variable.

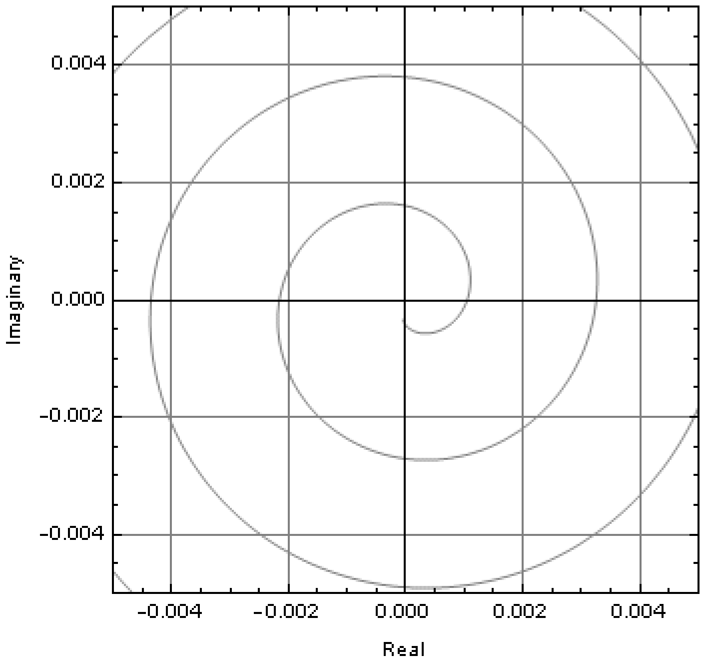
Mode functions

Scalar field in flat spacetime:

$$\phi(t,\vec{x}) = \int \frac{d^3\vec{p}}{(2\pi)^3} e^{i\vec{p}\cdot\vec{x}} \left(a(\vec{p}) \frac{e^{-iE_{\vec{p}}t}}{\sqrt{2E_{\vec{p}}}} + a^\dagger(-\vec{p}) \frac{e^{+iE_{\vec{p}}t}}{\sqrt{2E_{\vec{p}}}} \right)$$

- The mode function is a completely deterministic (complex) quantity.
- Stochasticity is due to annihilation and creation operators.

The evolution of mode function of curvature perturbation

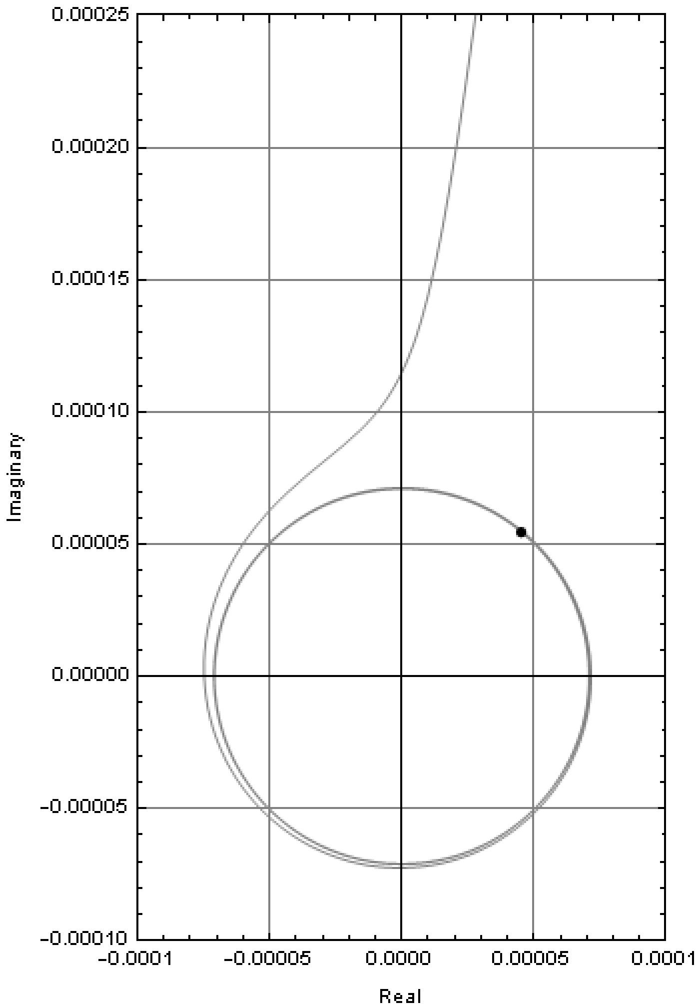


$m^2\phi^2$ case

- Extreme sub-Hubble limit: R spirals in (clockwise).
- Extreme super-Hubble limit: R just freezes.
- Prior to freezing: points radially in.

Typical of usual slow – roll

The evolution of mode function of Mukhanov-Sasaki variable



$m^2\phi^2$ case

- Extreme sub-Hubble limit: v circles (clockwise).
- Extreme super-Hubble limit: v goes radially out.

Everything can be understood from:

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0$$

And the B.D. initial conditions:

$$\lim_{-k\eta\to\infty} v_k(\eta) = \frac{1}{\sqrt{2\omega_k}} \cdot e^{-ik\eta}$$

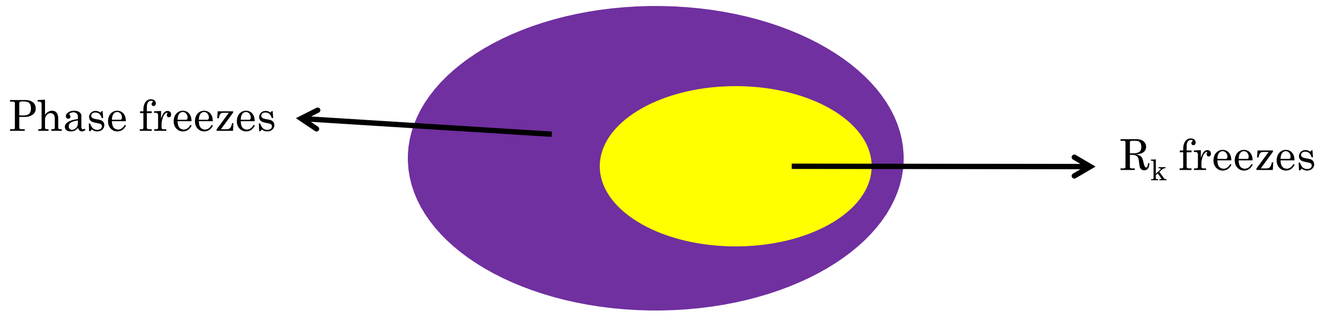
Any other initial conditions shall only turn circles into ellipses that's it, everything else shall hold good.

Origin crossing - nulling

- R freezes on super-Hubble scales: Both amplitude and phase freeze.
- If R is frozen once and is then unfrozen (by decreasing z''/z briefly), phase will remain frozen but not the amplitude.

$$v_k = re^{i\theta} \qquad\qquad \theta'' + 2\left(\frac{r'}{r}\right)\theta' = 0$$

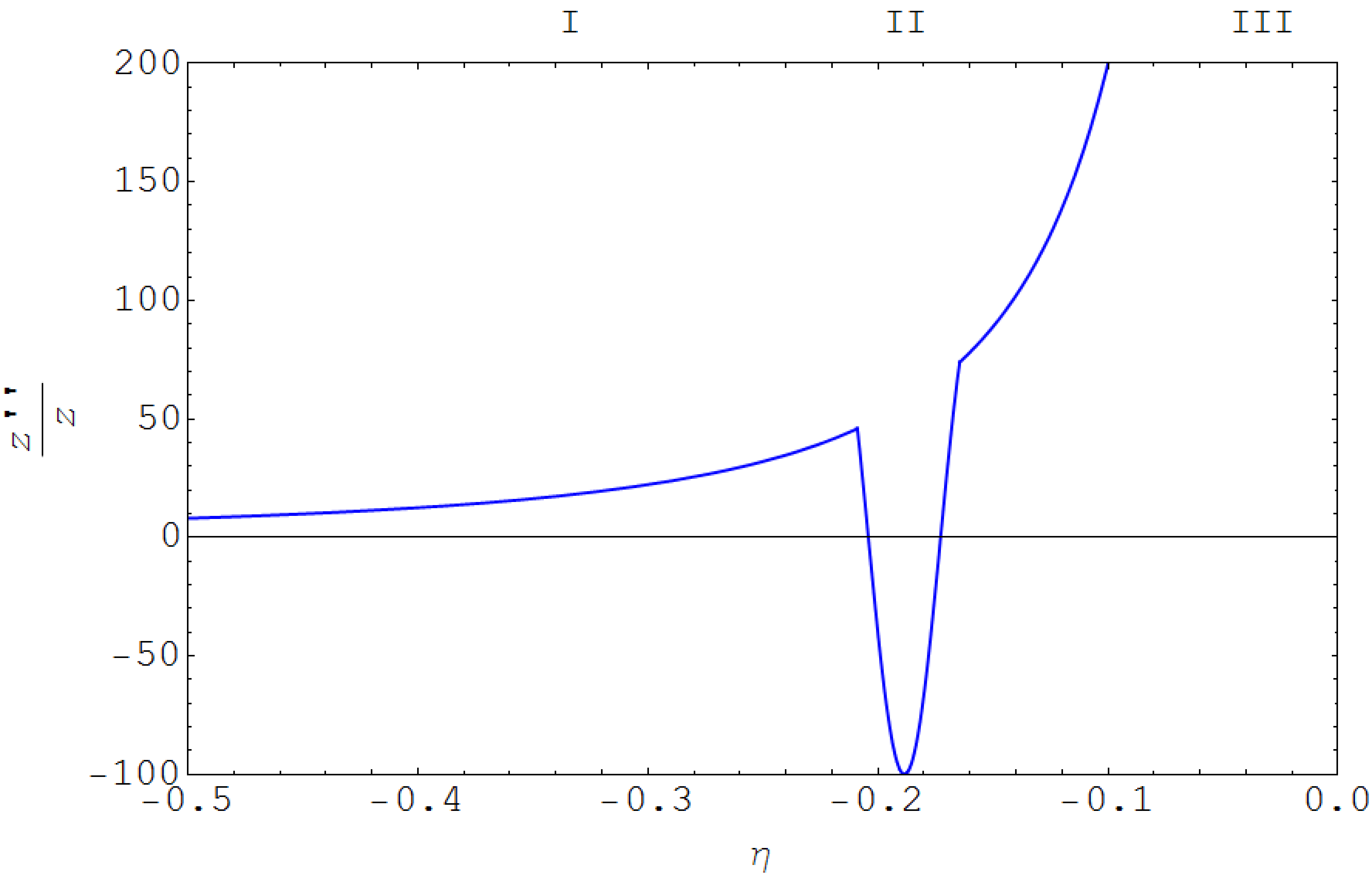
- A frozen phase can NOT unfreeze.
- Nearer we are to the epoch of phase freezing for a given mode, the less will the phase get affected by any background evolution.



"If there is a possibility of any super-Hubble evolution, it should lead to only radial trajectory in the complex plane, causing it to go towards zero."

G. Goswami et.al. [arXiv:astro-ph/1011.4914]

Sandwich models



Salient features

Super-Hubble evolution: any evolution after R_k for the mode freezes in stage I.

Radial trajectory: Super-Hubble evolution always leads to only radial trajectory in the complex plane.

Inward motion: The super-Hubble evolution involves a radially inward motion.

Amount of super-Hubble evolution: The amount of super-Hubble evolution in stage II or III is determined by the depth of the dip in z''/z in stage II.

Continuity: If a mode k1 crosses the origin in the complex plane on a radial trajectory, there should exist a mode $k*$ (with $k* < k1$) that ends up right at the origin.

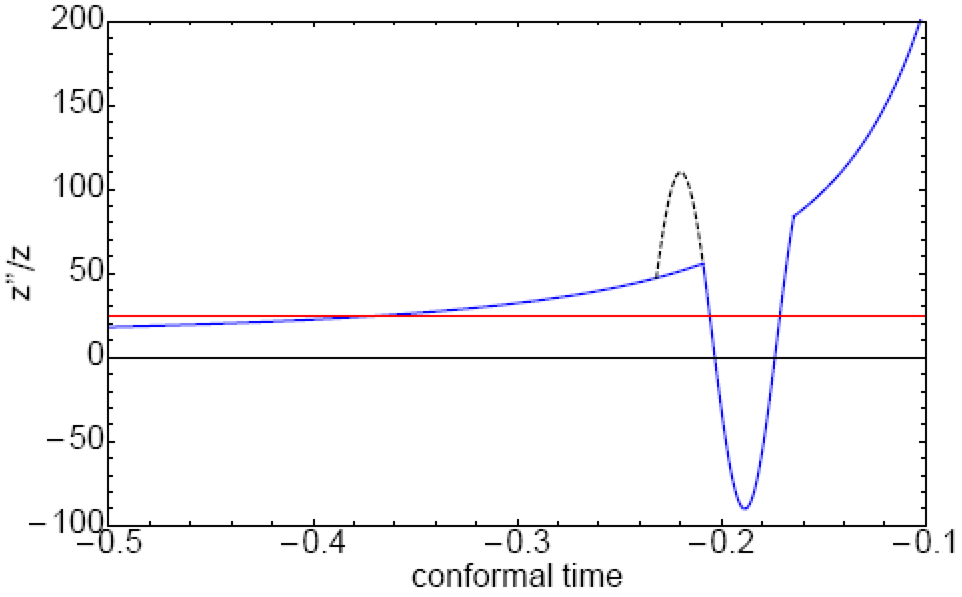
Thus, we shall have PPS having nulls.

G. Goswami et.al. [arXiv:astro-ph/1011.4914]

What if?

❑ Increasing k also unfreezes phase, won't it miss the origin?

❑ But we can always design stage I of Sandwich models such that phase shall stay frozen.



"But does all this really happen for any realistic model?"

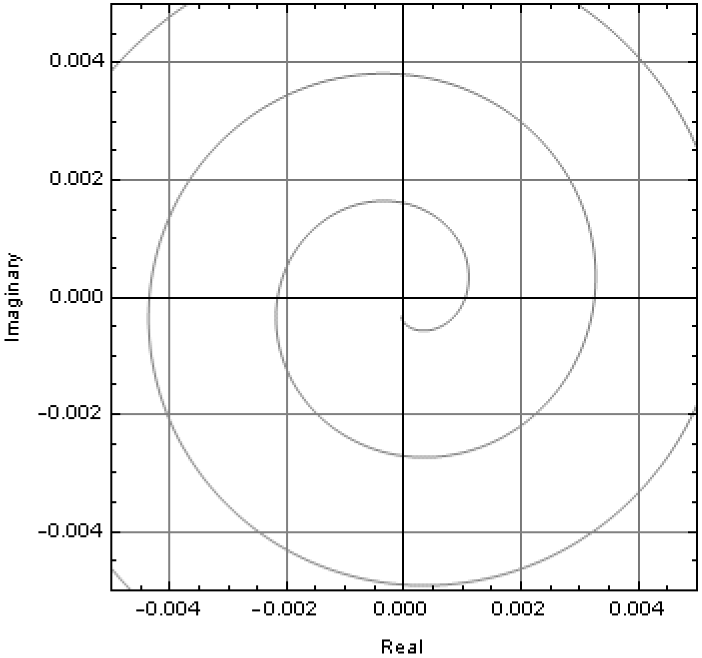
(1)

Does happen!

Illustration: Punctuated inflation (PI) model.

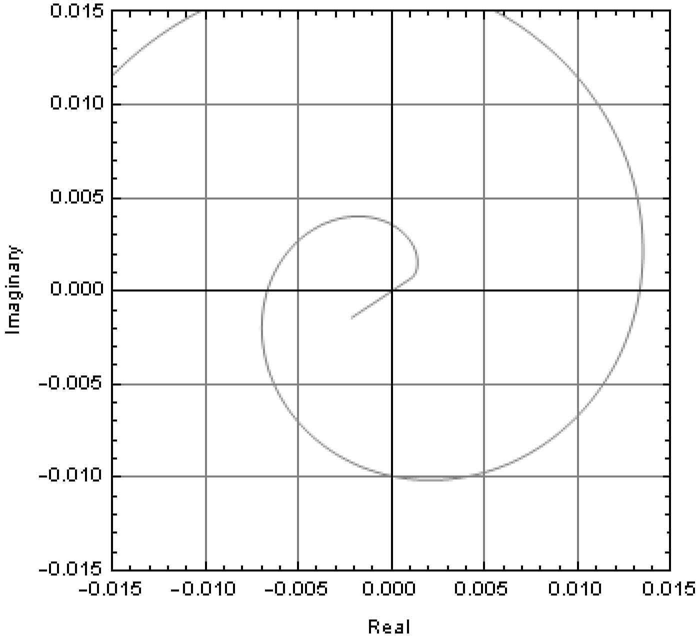
RK Jain *et al* JCAP 01 (2009) 009

The evolution of mode function of curvature perturbation



$m^2\phi^2$ case

Typical of usual slow – roll

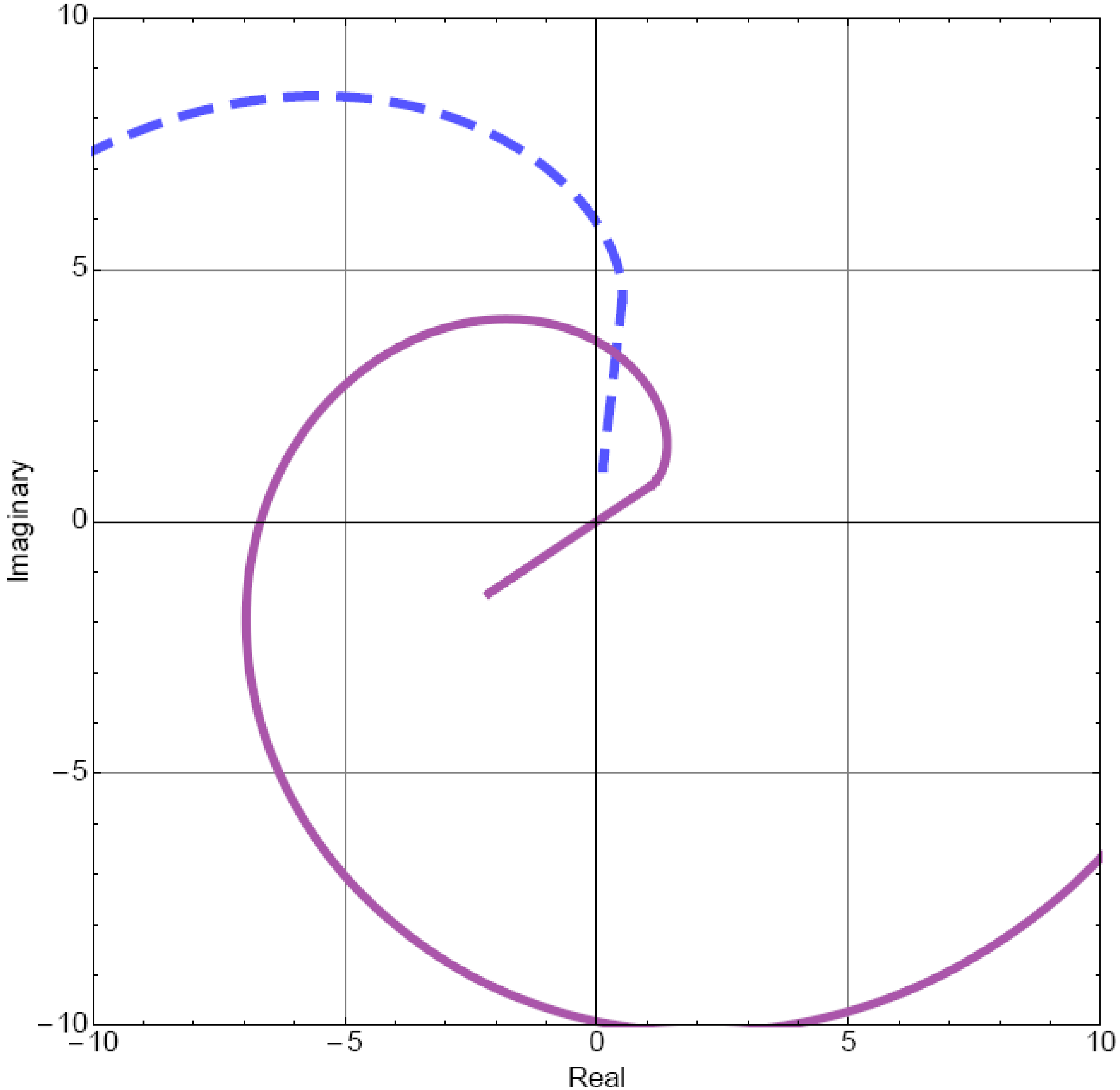


PI

Not even typical!

(2)

Suppression and/or enhancement



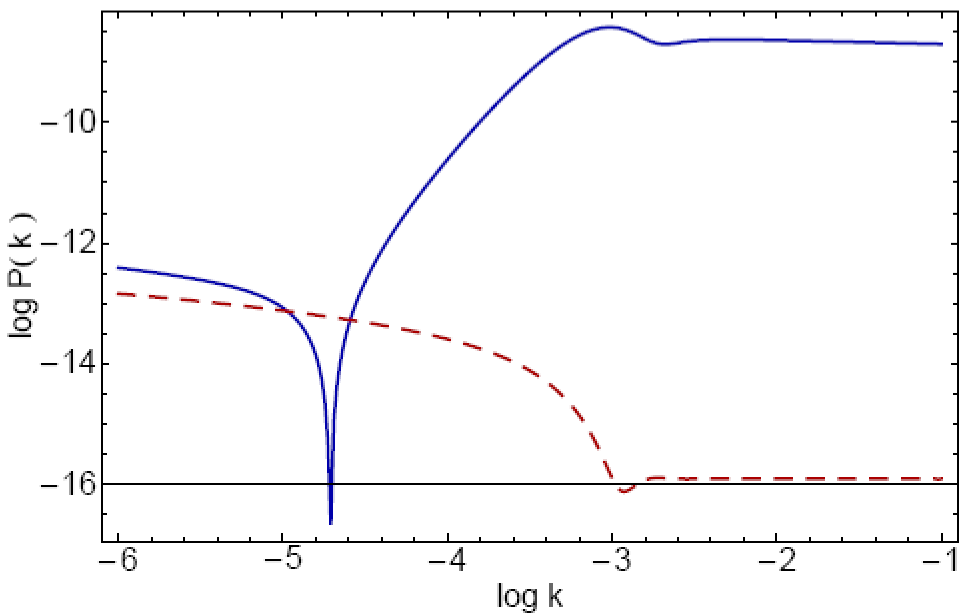
Compare: S.M. Leach et. al. Phys. Rev. D 64, 023512 (2001).

(3)

Tensor power overtaking scalar power!

•If asymptotic value of |R| becomes too small for a mode, the tensor power overtakes the scalar power and T/S becomes greater than 1. Unlike *z*, *a* can NOT decrease, so tensor modes will never suffer this fate.

•It is clear that there must be a mode for which T/S is actually infinite as R ends up at the origin in the complex plane.



R. K. Jain et. al., Phy. Rev. D 82, 023509 (2010). [arXiv:astro-ph/0904.2518]
G. Goswami et.al. [arXiv:astro-ph/1011.4914]

Exact null?

$$\boxed{S_2 = \frac{1}{2}\int \mathrm{d}^4x\, a^3 \frac{\dot{\phi}^2}{H^2}\left[\dot{\mathcal{R}}^2 - a^{-2}(\partial_i\mathcal{R})^2\right]}\,.$$

Comoving gauge

- ❑ All calculations based on only retaining quadratic terms in the action.

- ❑ Usually, the correction to PPS due to higher order perturbations too less compared to quadratic answer.

- ❑ Near the said null, the higher order corrections shall overtake the quadratic answer! Will the null survive?

- ❑ Also, there are other arguments* that prevent exact nulling.

- ❑ True quantum nature of the generation mechanism can potentially get exposed!

- ❑ Is it observable?

$\left.\begin{array}{l}\text{* D. Polarski and A. A. Starobinsky, 1996 Class. Quantum Grav. 13 377.}\\ \text{D. Polarsky and A.A. Starobinsky, Phys. Lett. B 356, 196 (1995).}\end{array}\right\}$ Quantum Positivity Principle

Conclusions

- ❑ Observations seem to favor PPS having features.
- ❑ Dips with Cusp-like features in scalar PPS seen in various models.
- ❑ The phase of v once frozen can NOT unfreeze, so any super Hubble evolution must be radial in complex plane.
- ❑ This "explains" why we see cusp-like features for small k modes in certain models in terms of origin crossing in complex plane.
- ❑ And why tensor power can overtake scalar power for a range of modes.
- ❑ Near such a (classically calculated) null, quantum effects should be important.

THANK YOU