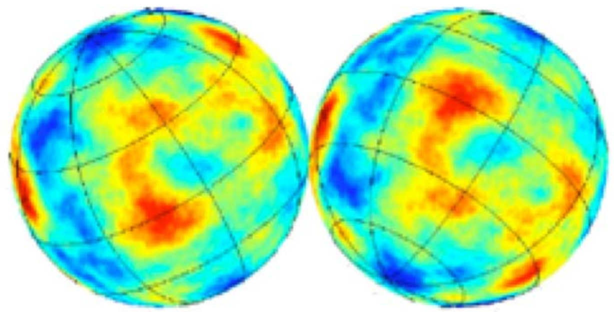

WMAP Anomalies

and Statistical Isotropy

CITA ICAT
Canadian Institute for Theoretical Astrophysics
L'institut canadien d'astrophysique theorique

Amir Hajian

http://www.cita.utoronto.ca/~ahajian

Amazing agreement between the theory and observations!

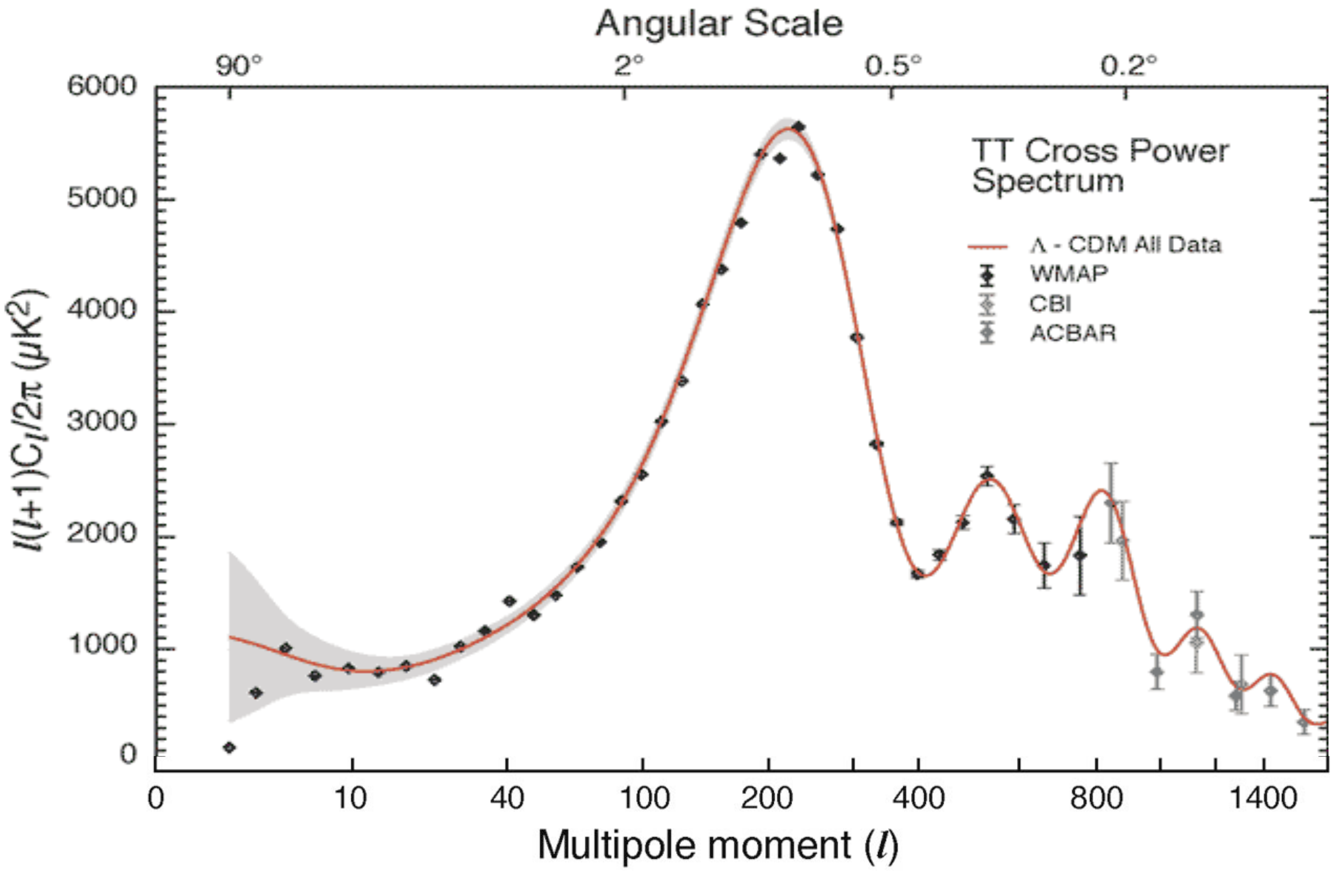

Figure: WMAP

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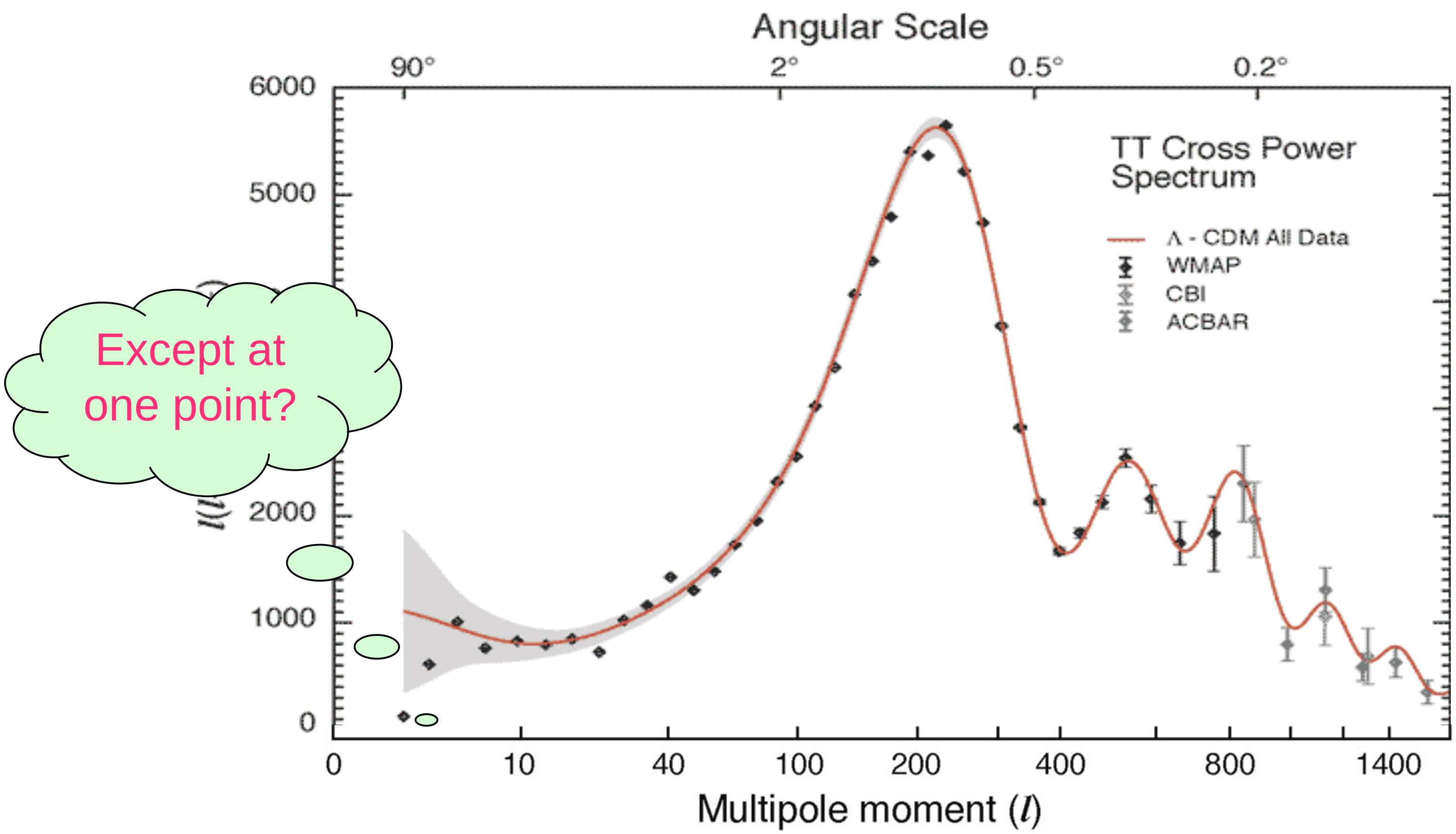
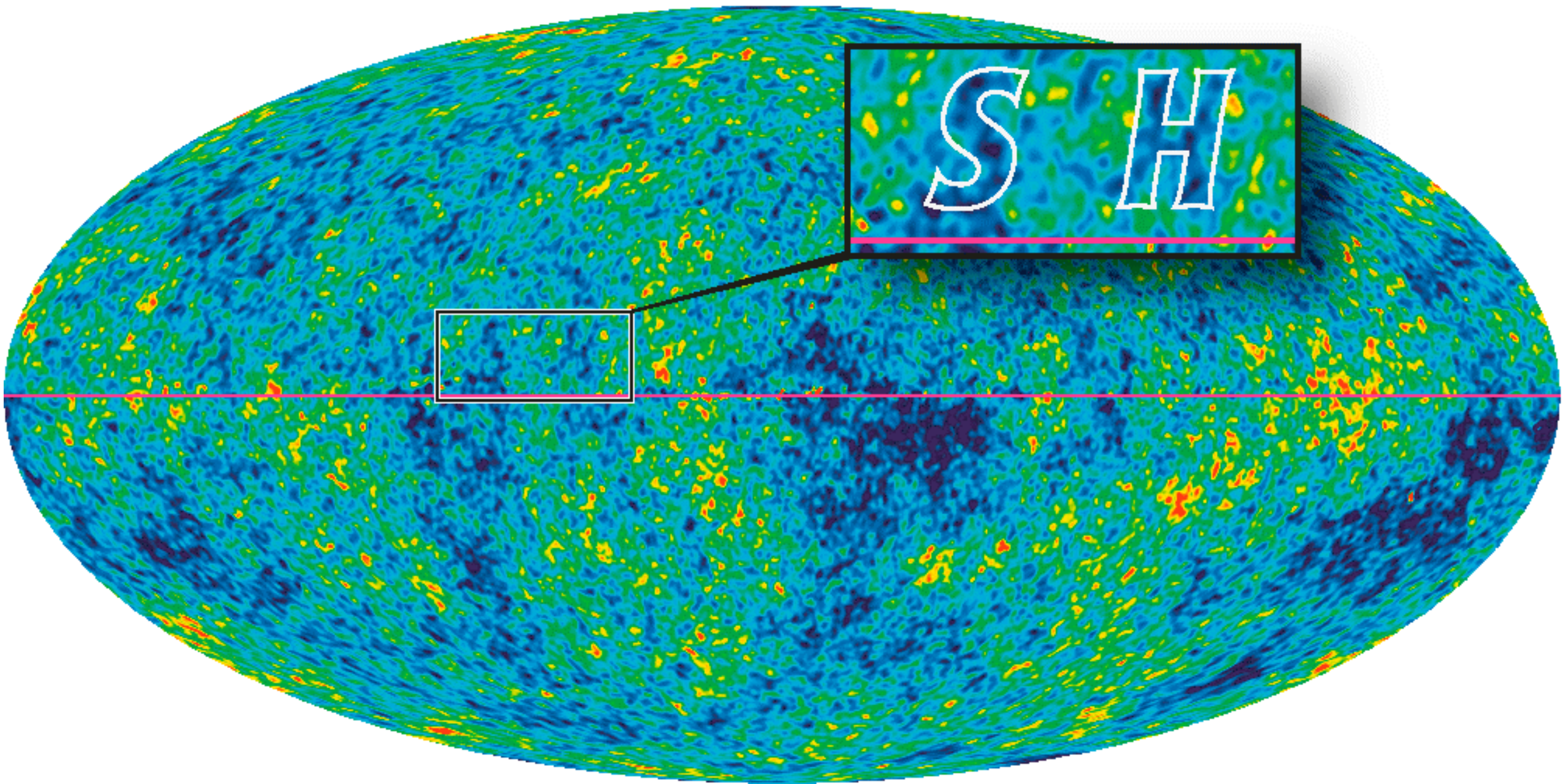
Amazing agreement between the theory and observations!



First Evidence



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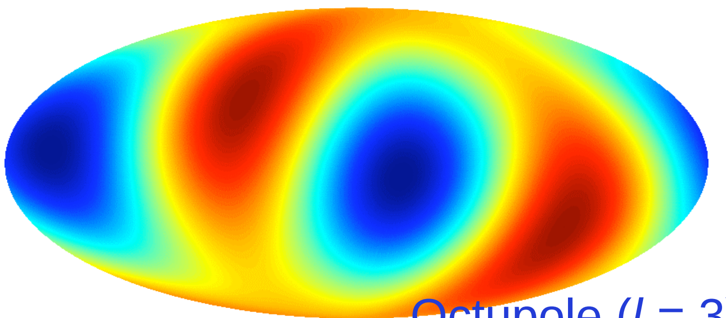
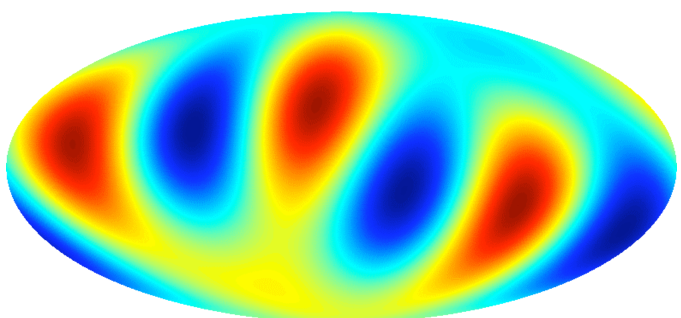
‘Anomalies’ in the CMB sky?

Quadrupole (l = 2)



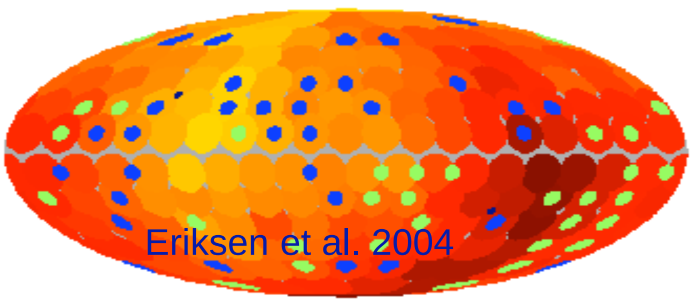
Octupole (l = 3)



- Special directions
  - Ralston & Jain 2004
  - de Oliveira-Costa, et al. 2004
  - Copi et al. 2004 – 2006
  - Land & Magueijo 2004 – 2006
  - Prunet et al., 2004 – no detection
  - Hajian, Souradeep & Cornish 2004 – no detection

de Oliveira-Costa, et al., Phys. Rev. D **69**, 063516 (2004)

‘Anomalies’ in the CMB sky?



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- N-S assymetries
  - Hansen et al. 2003
  - Eriksen et al. 2004

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'Anomalies' in the CMB sky?

- Special directions
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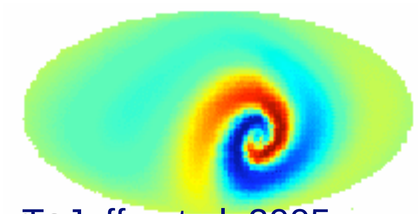
- Template patterns
  - T. Jaffe et al. 2005
  - Land & Magueijo 2005
  - Ghosh, Hajian & Souradeep 2006



T. Jaffe et al. 2005

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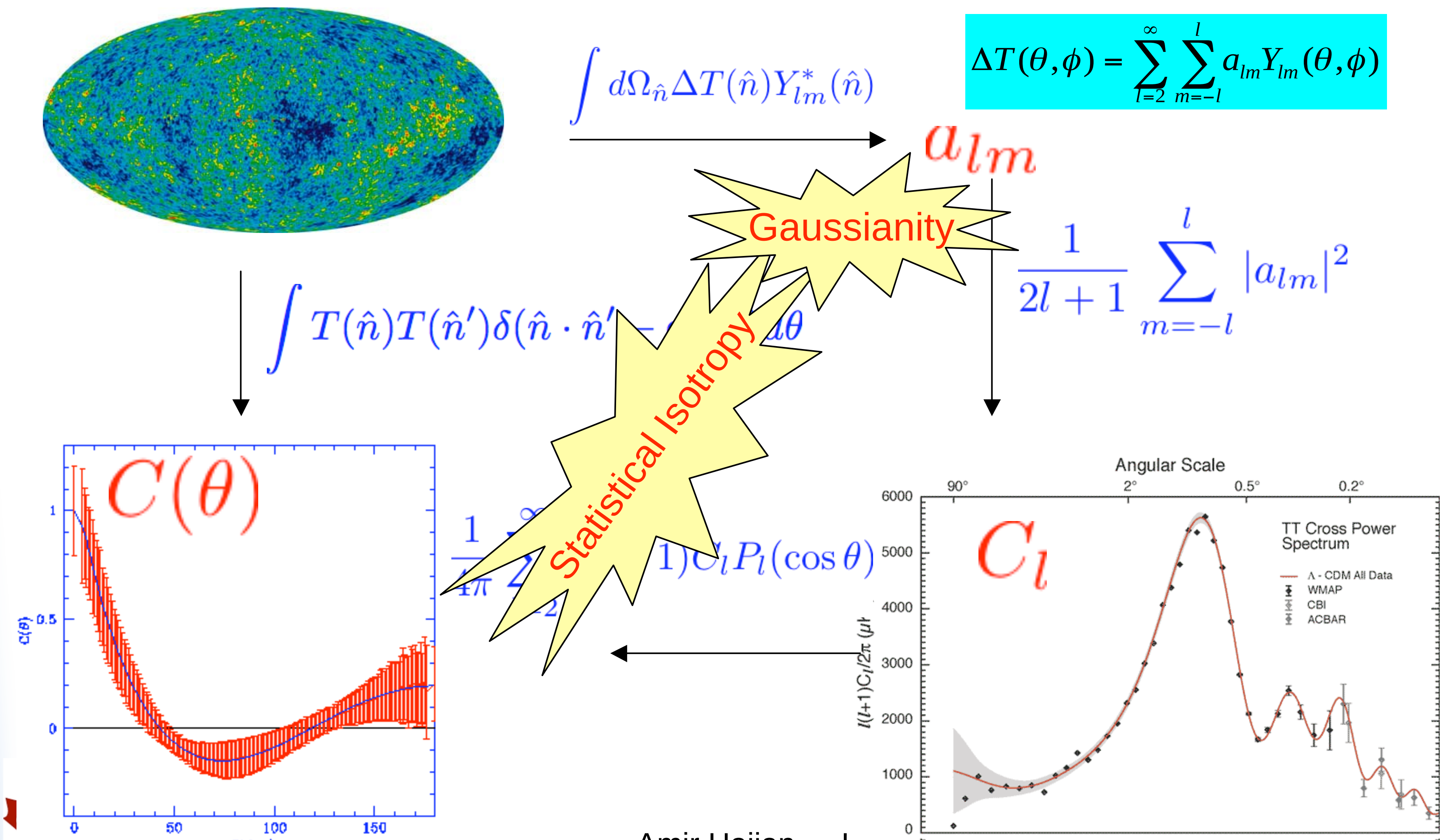
Quantifying Statistical Isotropy

Definitions



Gaussianity v.s. Isotropy

- Gaussianity guarantees that all information of the field is in its two point correlation

- Statistical Isotropy states that correlation function is invariant under rotations

We can have

Correlation Statistics
Instead of Correlation Function

- Statistically Isotropic
- Statistically Isotropic … Gaussian Models
- Statistically An-isotropic Gaussian Models
- Statistically An-isotropic non-Gaussian Models

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Most General Two Point Correlation

$$C(\hat{n}_1, \hat{n}_2) : S^2 \times S^2 \rightarrow \Re$$

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Expanding Two Point Correlation

$$C(\hat{n}_1,\hat{n}_2) = \sum_i Coefficient_i \times \underbrace{Basis_i}$$

$$\sum_{m_1 m_2} C^{LM}_{l_1 l_2 m_1 m_2} Y_{l_1 m_1}(\hat{n}_1) Y_{l_2 m_2}(\hat{n}_2)$$

Bipolar Spherical Harmonics

Natural Basis on $S^2 \times S^2$

Clebsch-Gordan Coefficients

$$|l_1 - L| \le l_2 \le |l_1 + L|$$

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Expanding Two Point Correlation

$$C(\hat{n}_1,\hat{n}_2) = \sum_{l_1 l_2 LM} A^{LM}_{l_1 l_2} \{Y_{l_1}(\hat{n}_1)\otimes Y_{l_2}(\hat{n}_2)\}_{LM}$$

$$A^{LM}_{ll'} = \sum_{mm'} \langle a_{lm} a_{lm'} \rangle \; C^{LM}_{lml'm'}$$: Measure cross correlation in a_{lm}

Statistical Isotropy => $\boxed{\langle a_{lm}\, a^{*}_{l'm'} \rangle = C_l \delta_{ll'} \delta_{mm'} = A^{00}_{ll}}$

Too many indices,
One can define:

$$A_{LM} = \sum_{l=0}^{\infty} \sum_{l'=|\ell-l|}^{\ell+l} A^{LM}_{ll'}$$

Expanding Two Point Correlation

$$C(\hat{n}_1, \hat{n}_2) = \sum_{l_1 l_2 LM} \underbrace{A^{LM}_{l_1 l_2}} \{Y_{l_1}(\hat{n}_1) \otimes Y_{l_2}(\hat{n}_2)\}_{LM}$$

$$\kappa^L \equiv \sum_{M, l_1, l_2} | A^{LM}_{l_1 l_2} |^2 \geq 0$$

Bipolar Power Spectrum

Statistical isotropy : $\kappa^L = \kappa^0 \delta_{L0}$

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Measuring Statistical Isotropy

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The Estimator

$$\boxed{\tilde{A}_{ll'}^{LM} = \sum_{mm'} \tilde{a}_{lm} \tilde{a}_{l'm'} C_{lml'm'}^{LM}}$$

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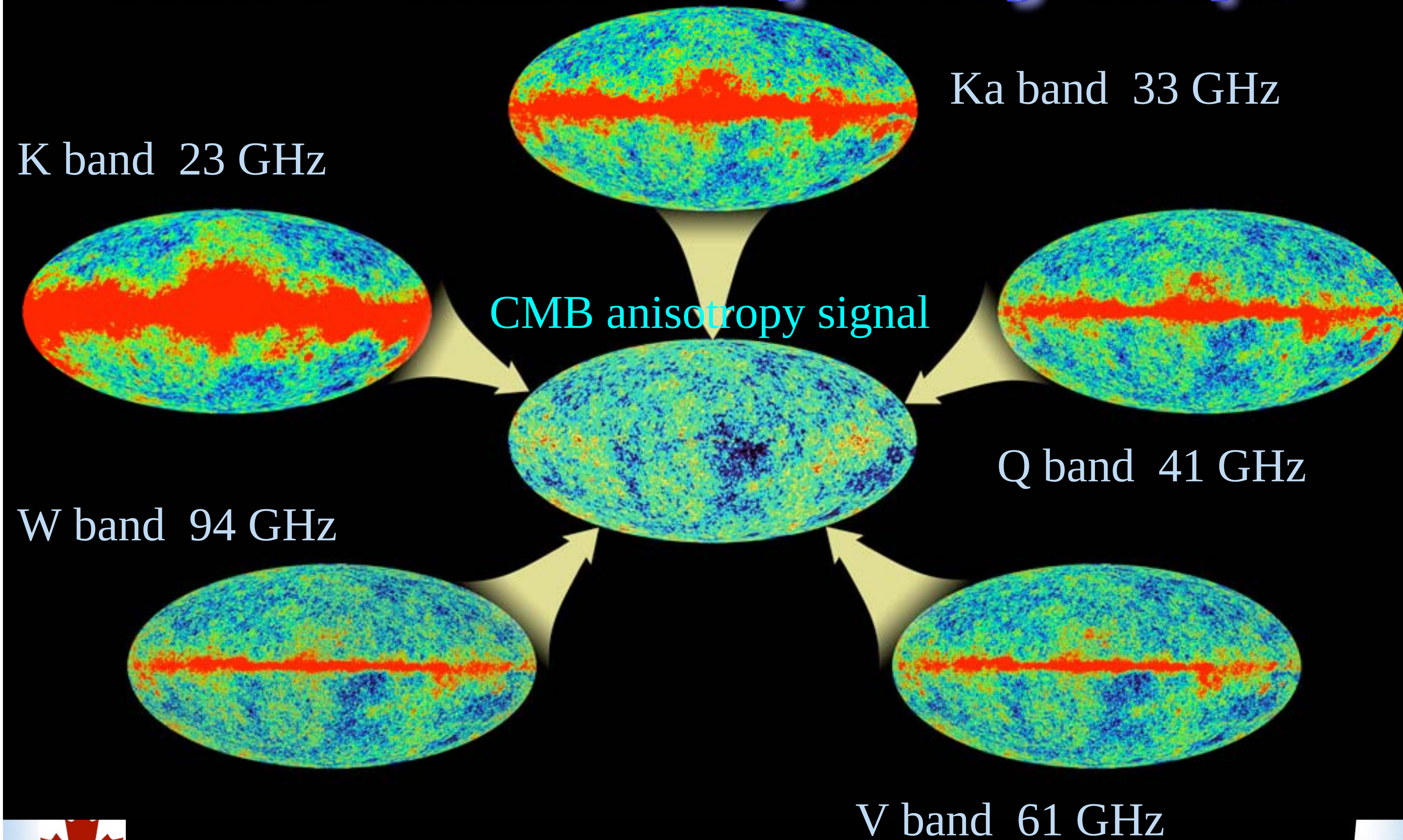
WMAP multi-frequency maps



NASA/WMAP science team

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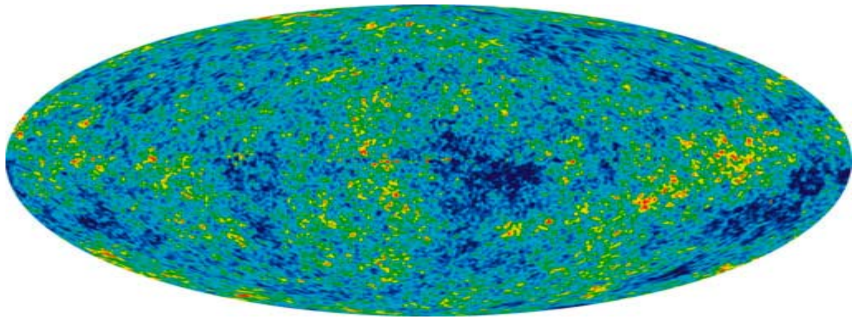
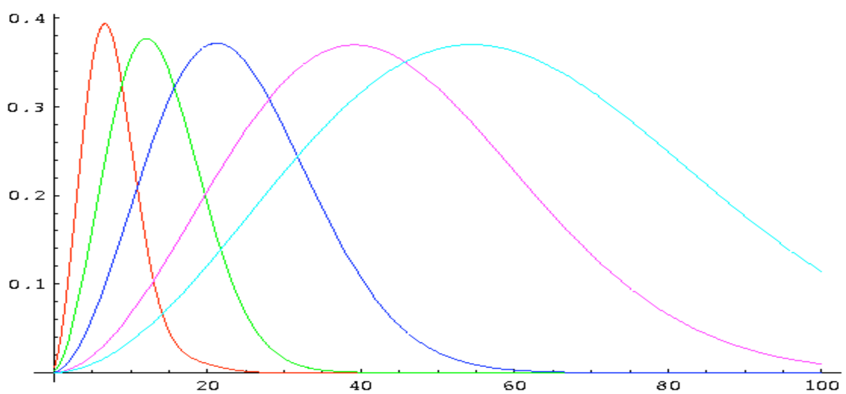
Statistical Isotropy of WMAP data





- ILC map on large scales
  - Full sky
  - Residuals from Galactic removal errors in ILC-3yr map are estimated to be less than 5 μK on angular scales greater than 10 deg
  - On large scales, the three-year ILC map is believed to provide a reliable estimate of the CMB signal, with negligible instrument noise, over the full sky
  - Exciting scenarios happen on large scales

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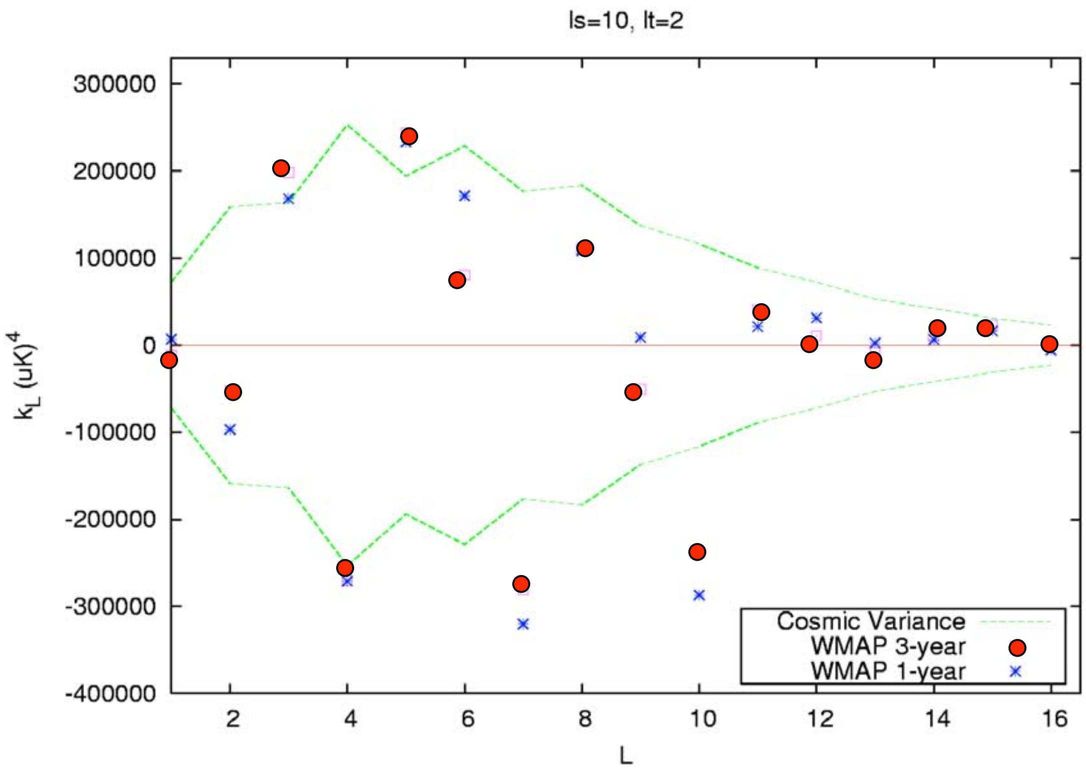
Bipolar Power Spectrum of WMAP data



Hajian & Souradeep, PRD (2006)

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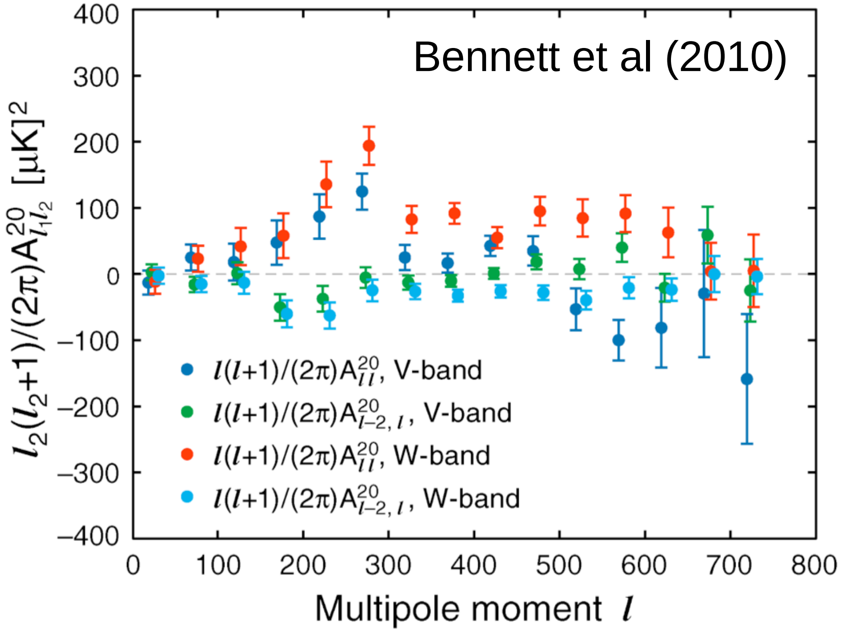
WMAP7: a BiPS study

- Used a minimum variance estimator for BiPS
- Detected non-zero BiPS at L=2, all other components are consistent with zero
- The signal is caused by M=0 in ecliptic coordinates
- The effect is larger in W and V band.
- The effects peaks at ell~200 (similar to the CMB power spectrum)
- It is equally strong in cross correlations and in auto correlations.



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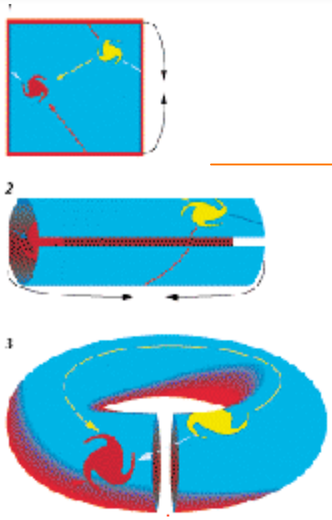
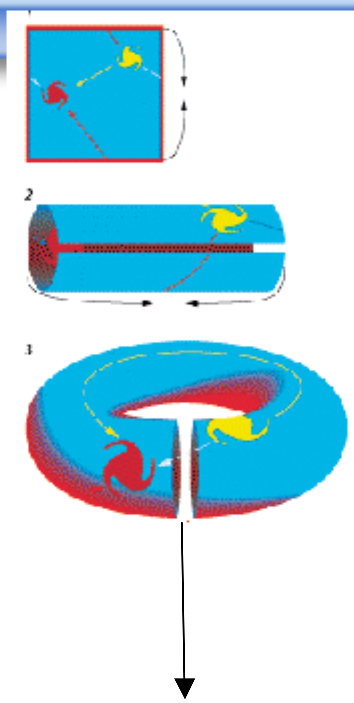
What Would A Detection Look Like?

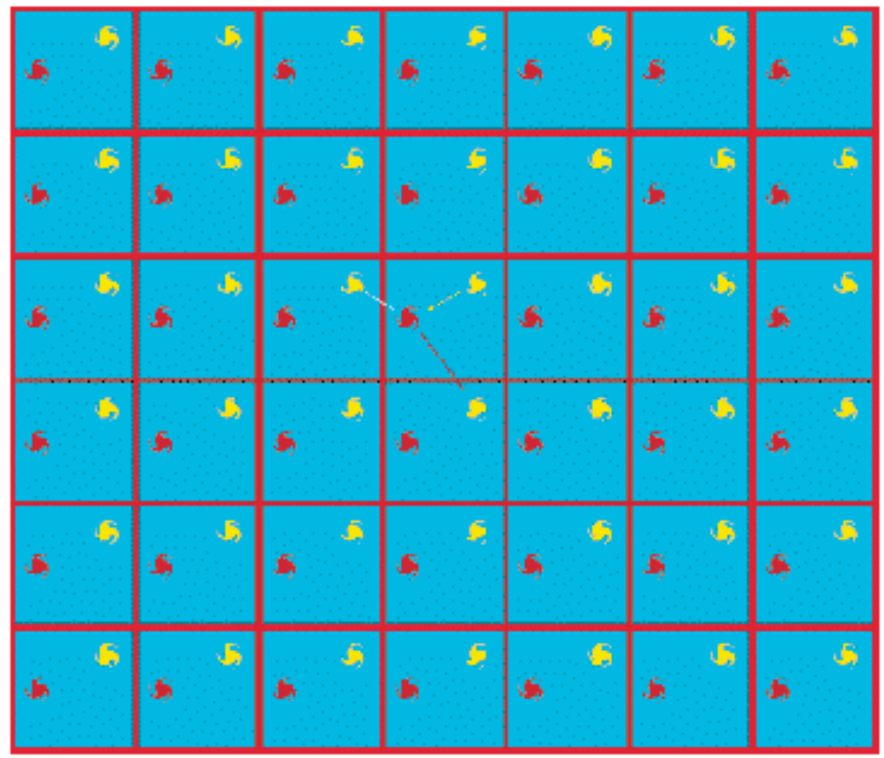
Example: A Toroidal Universe



The Euclidean 2-torus is a flat square whose opposite sides are connected.

Example: A Toroidal Universe





Pictures: Starkman et. al. 1999

Light from the yellow galaxy can reach them along several different paths. So they can see more one image of it.

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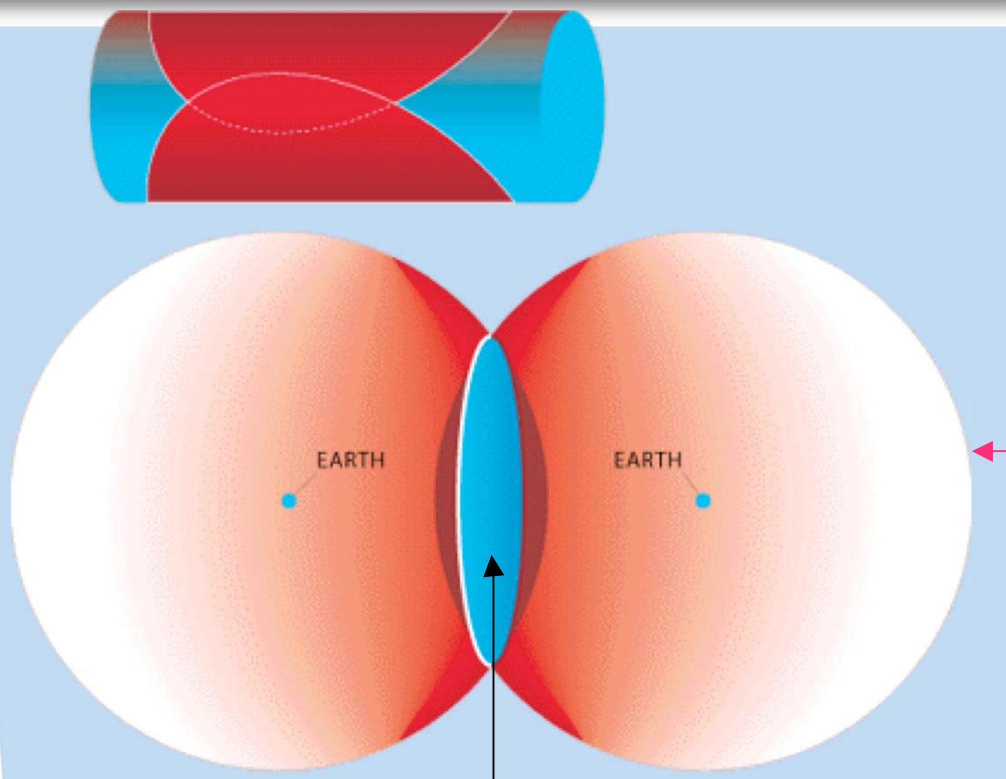
Signature of topology : correlated circles in the sky

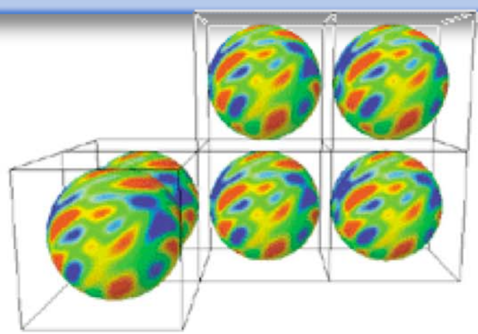


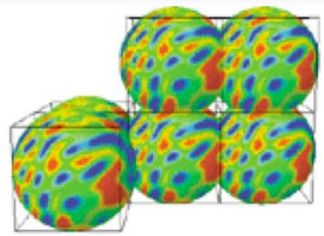
All the light received from a specific time represents a sphere.

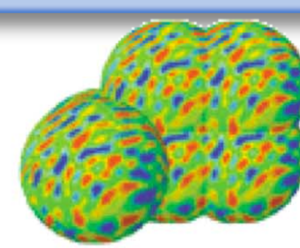
If this sphere is larger than the universe, it will intersect itself

THREE POSSIBILITIES

(Size of last scattering surface relative to the size of the compact space)



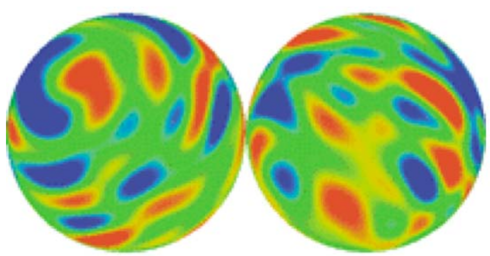


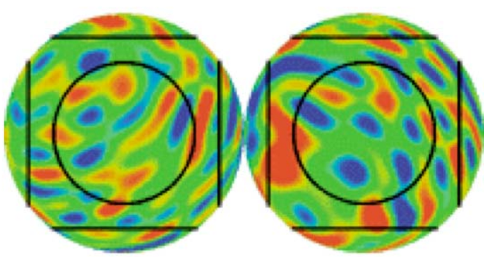


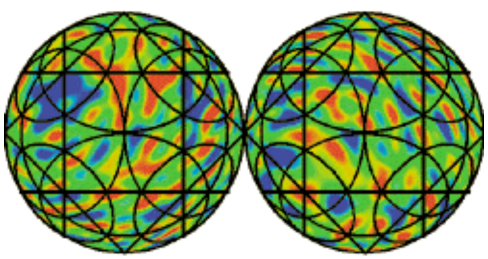
Large

Medium

Small







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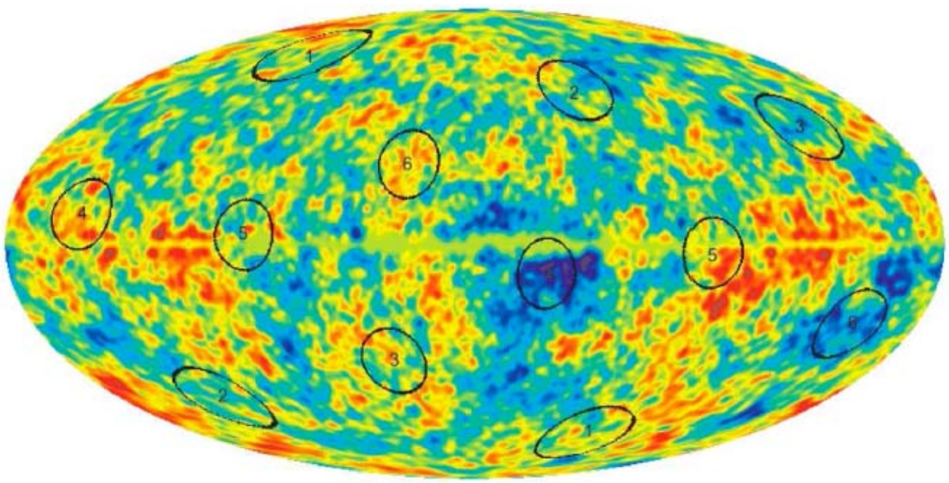
Looking for Circles in the Sky: Poincare Dodecahedral Space

- Six sets of circle pairs with a 36 degree twist expected.

- None was found in the data (Shapiro Key et al (2007))



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For bigger universes, circles may not exist. But correlations are still there.

Using Full Covariance Matrix

- For a known co… …esian analysis can be done:
    - For a gi… …f the noise Cov. Matri…

$$\frac{1}{\sqrt{(2\pi)^{N_p}\det(\ldots}} \ldots \Delta T_j$$

Fast and orientation independent methods needed

- Previous work:
    - COBE data: Bond, Pogosyan & S…
    - WMAP data: Phillips & Kogut 2004, …

Null results !

Not surprising!! You need to **guess/divine** the **true topology** (+ correct orientation) !!!

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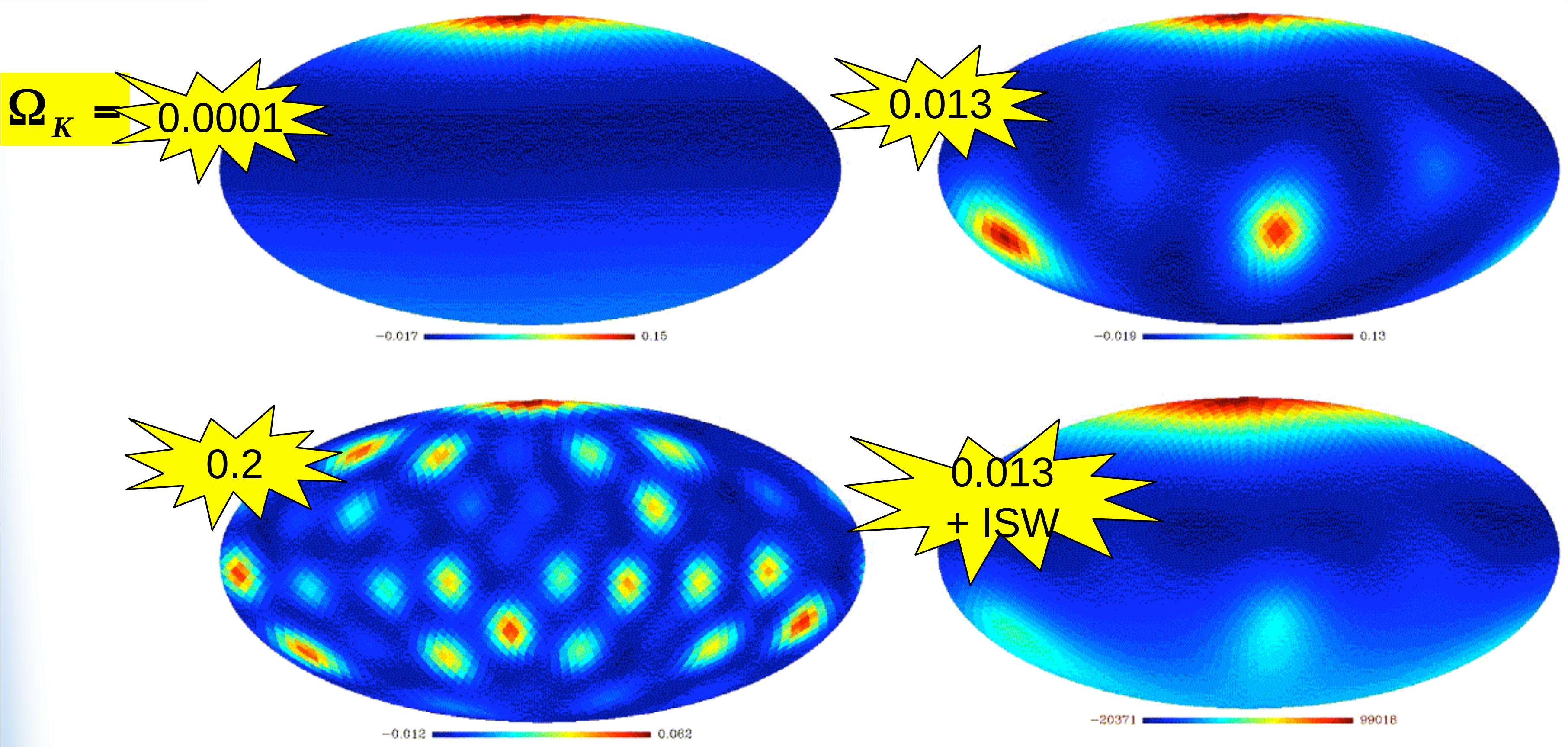
Correlation Patterns in Dodecahedral Spaces



AH, PhD Thesis

AH, Souradeep, Pogosyan, Bond, Contaldi (in progress)

- Using BiPS to look for topology is fast, and orientation independent
- Recipe:
  - For a given cosmic topology compute the correlation function
  - Find the bipolar power spectrum signature of that space (this is dictated by the symmetries of the space)
  - Compare the prediction with the bipolar power spectrum of the observed CMB data

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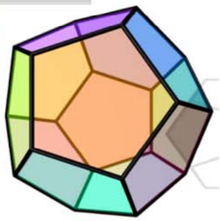
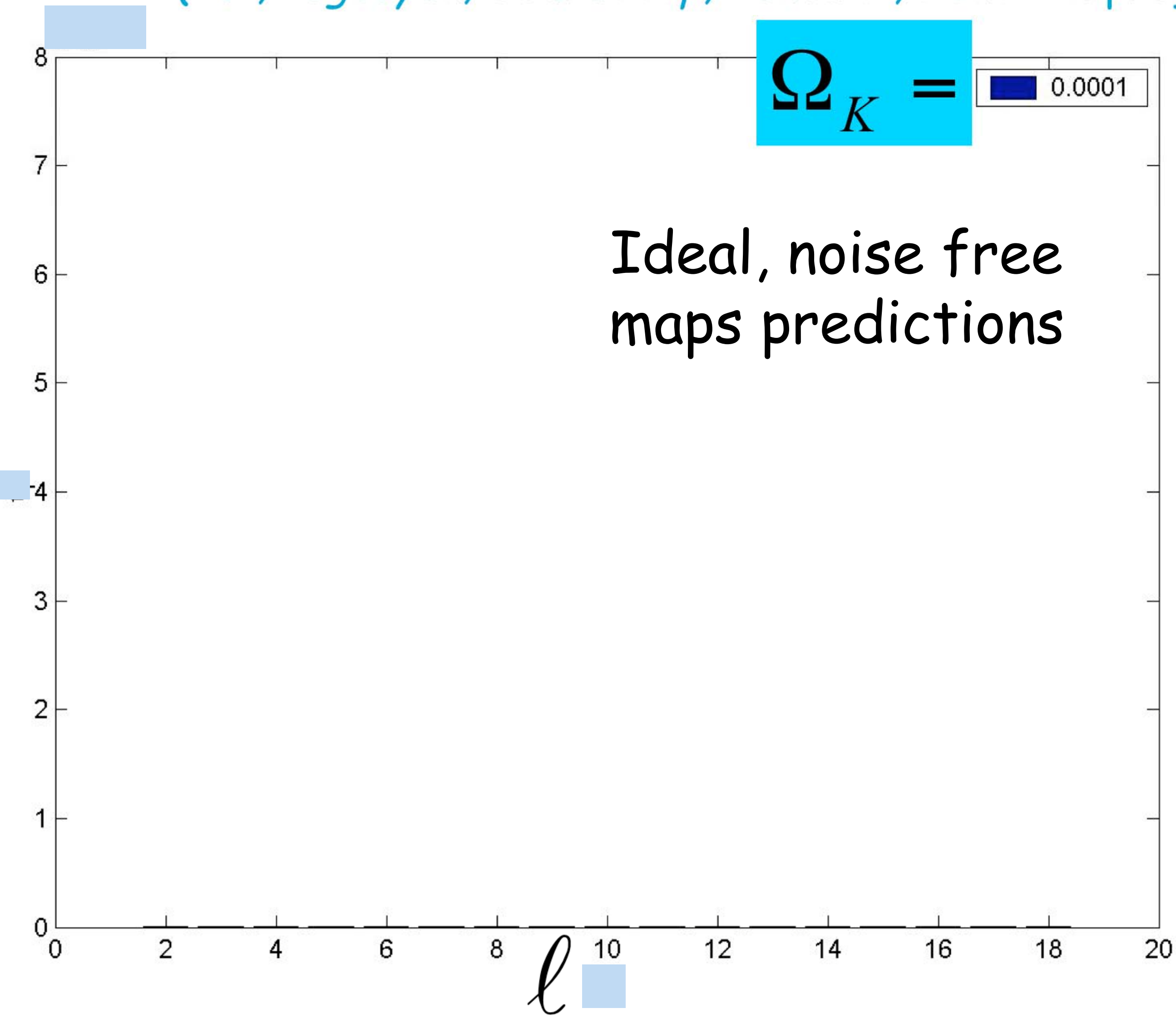
BiPS signature of a "soccer ball" universe

(AH, Pogosyan, Souradeep, Contaldi, Bond : in progress.)





Symmetries of the Correlation Function

Joshi, Jhingan, Souradeep, AH PRD(2010)

- All possible Euclidean models of compact universe exhibit reflection symmetry about xy-plane:

$$C(\theta_1,\phi_1,\theta_2,\phi_2) = C(\pi-\theta_1,\phi_1,\pi-\theta_2,\phi_2)$$

Compact topologies have even fold symmetry but emergence of this fact from symmetries of correlation function is quite instructive.

- Form of correlation function under even-fold symmetry.

$$C^{(A)}(\theta_1,\phi_1,\theta_2,\phi_2) = \sum_{m_1,m_2} f_{m_1,m_2}(\theta_1,\theta_2)\delta_{m_1+m_2,nk}\cos(m_1\phi_1+m_2\phi_2)$$

Vanishes for odd L

- Bipolar spherical harmonic coefficients:

$$A^{LM}_{l_1l_2} = [1+(-1)^{l_1+l_2-L}]\sum_{m_1m_2}\sqrt{\frac{(2l_1+1)(2l_2+1)(l_1-m_1)!(l_2-m_2)!}{(l_1+m_1)!(l_2+m_2)!}}C^{LM}_{l_1m_1l_2m_2}\delta_{m_1+m_2,nk}$$
$$\int_0^{2\pi}\int_0^{2\pi}\int_0^{\pi}\int_0^{\pi} C(\theta_1,\phi_1,\theta_2,\phi_2)\cos(m_1\phi_1+m_2\phi_2)P^{m_1}_{l_1}(\cos\theta_1)P^{m_2}_{l_2}(\cos\theta_2)d\phi_1 d\phi_2 d(\cos\theta_1)d(\cos\theta_2)$$

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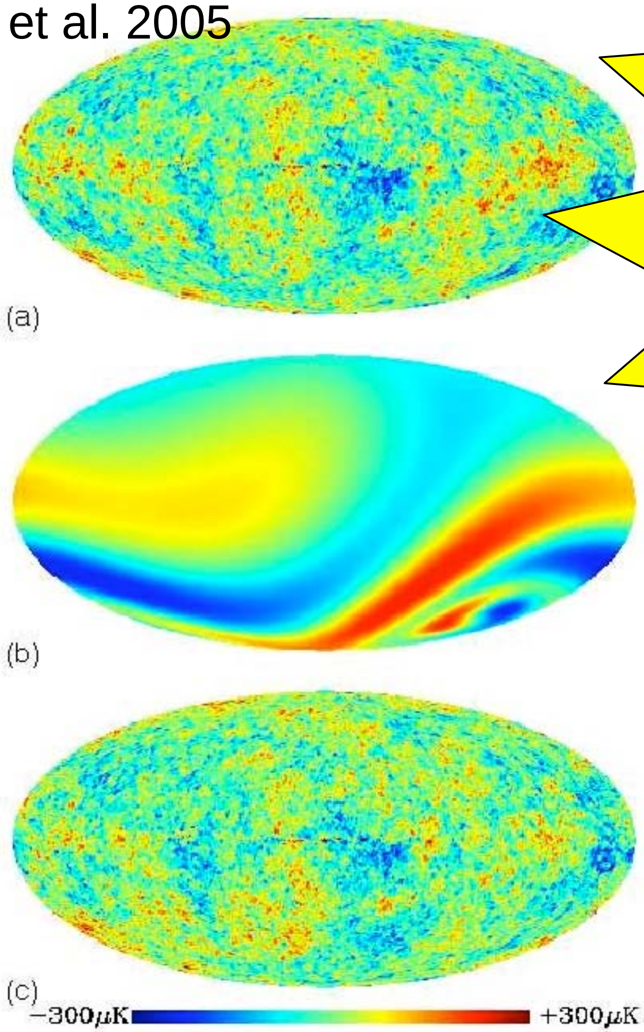
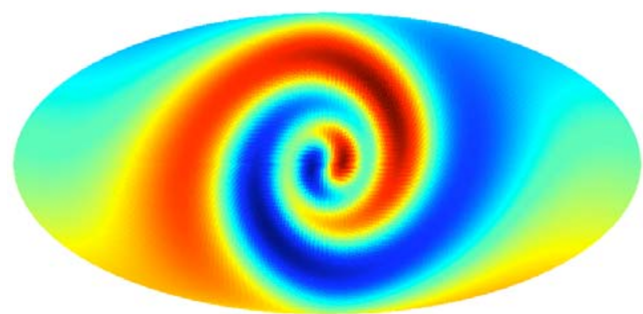
A Rotating Universe After All?!

Jaffe et al. 2005



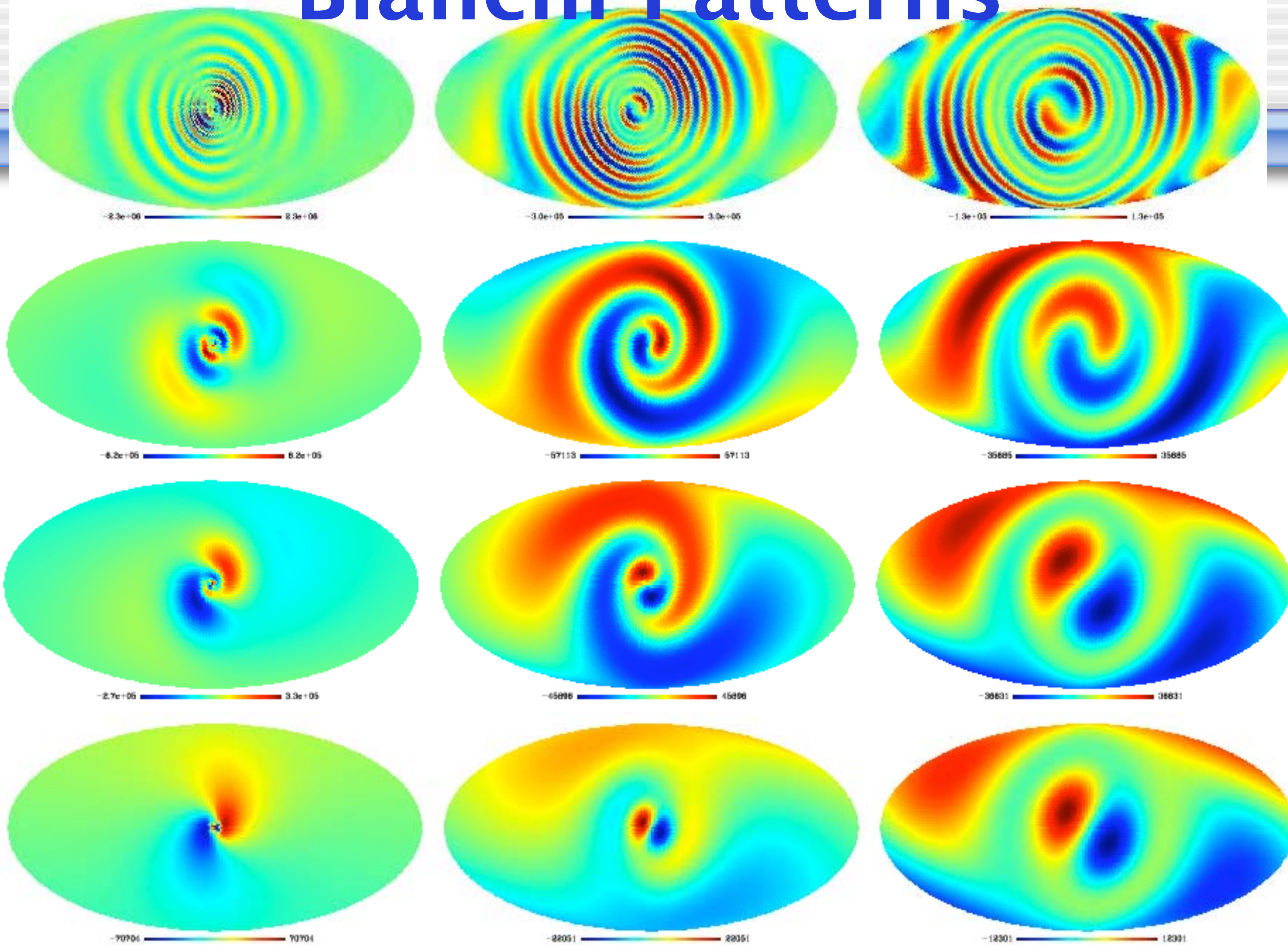
Violation of Statistical Isotropy (Ferreira et al 1998)



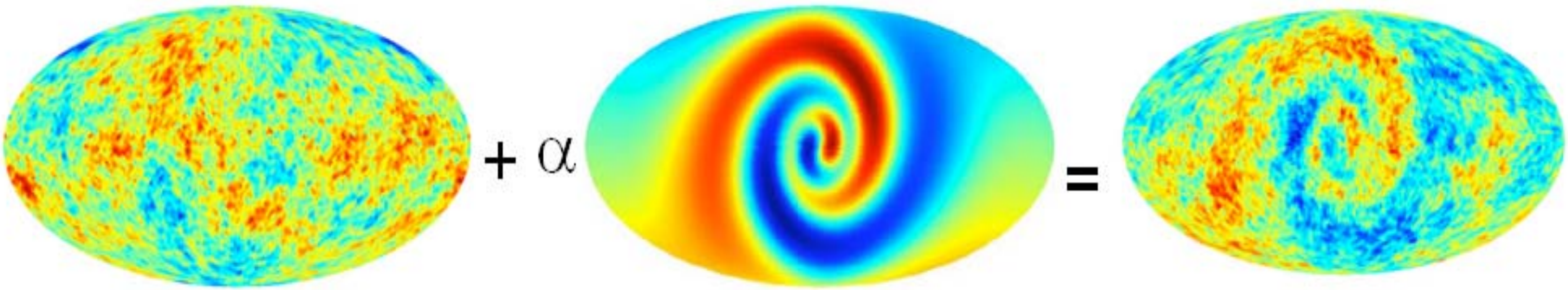
A rotating universe!

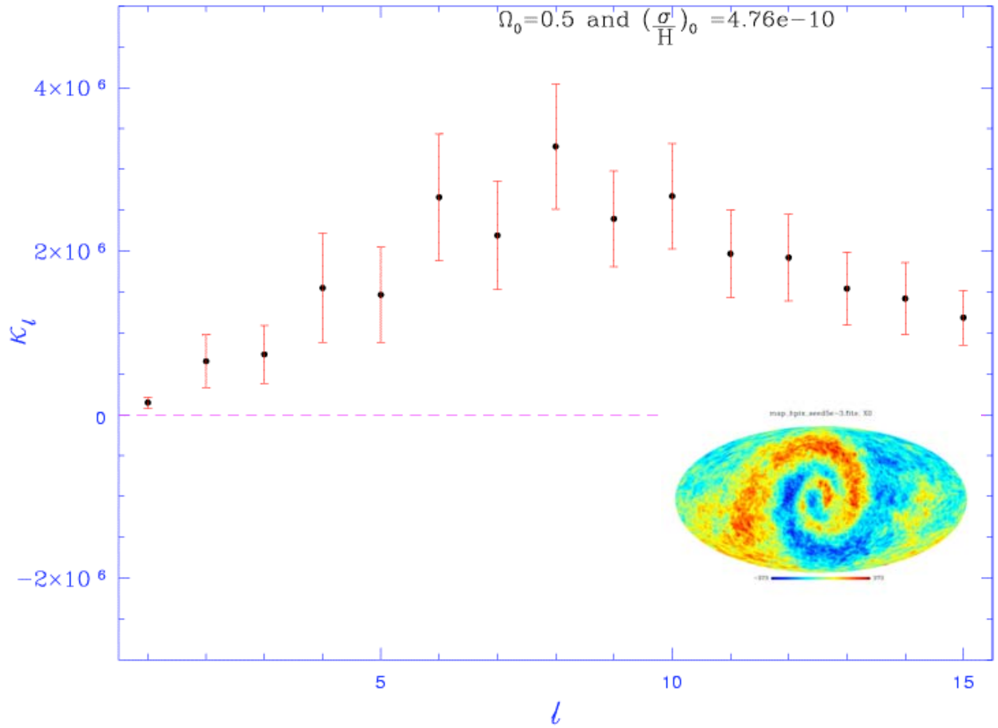
Biancho Patterns

AH, PhD Thesis



Hiding a Bianchi Template in a CMB Anisotropy Map





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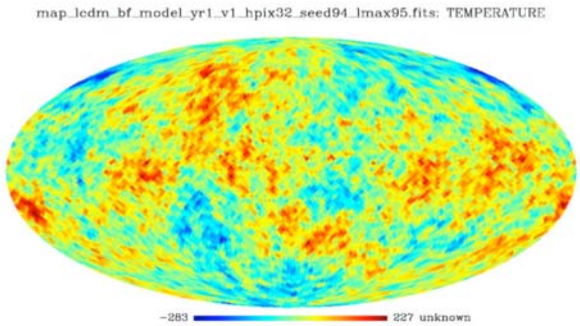
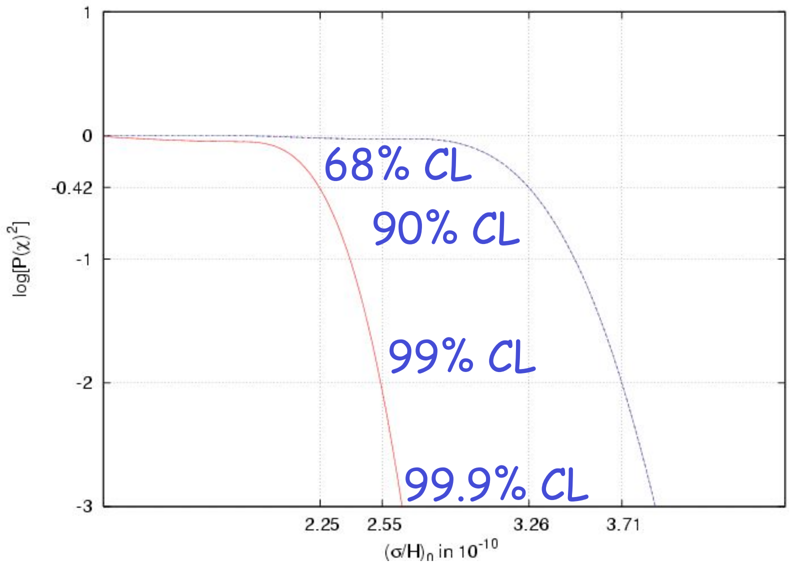
Limits from observed BiPS

Ghosh, AH, Souradeep, PRD (2006)





$$\frac{\sigma}{H_0} \leq 2.55\times10^{-10} \qquad \text{at } 99.9\% CL$$

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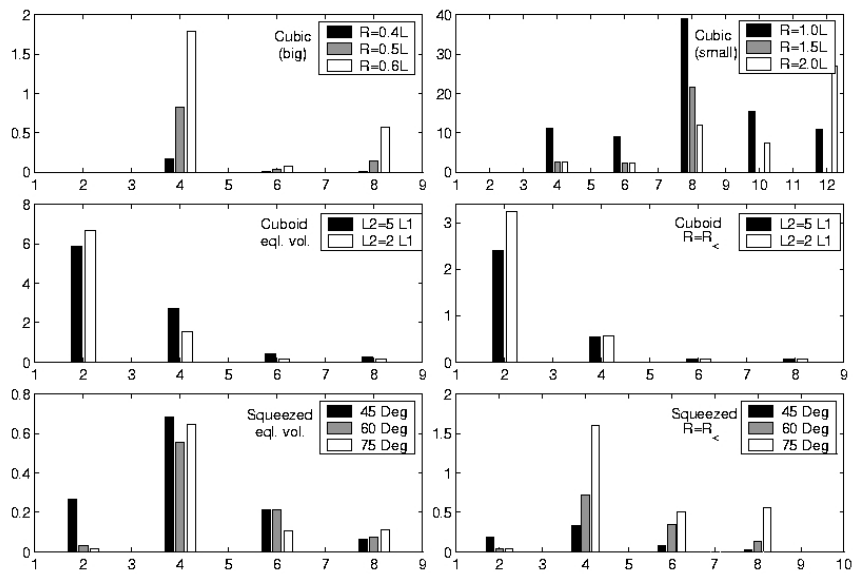
More Theoretical Motivations

Topology: multiply connected universe



Joshi, Jinghan, Souradeep, Hajian PRD(2010)

- Statistically isotropic

$$A^{LM}_{\ell_1 \ell_2} = C_{\ell_1} \delta_{\ell_1 \ell_2} \delta_{L0} \delta_{M0}$$

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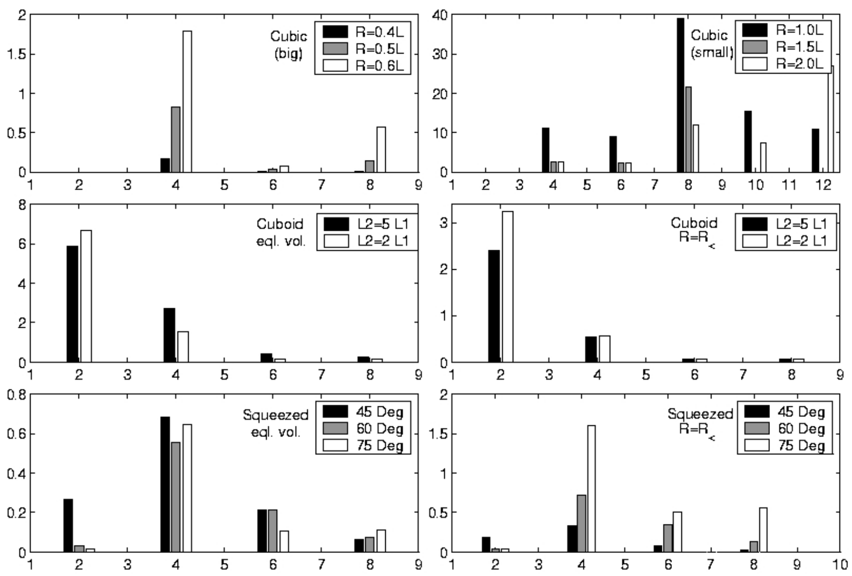
More Theoretical Motivations

Topology: multiply connected universe



Joshi, Jinghan, Souradeep, Hajian PRD(2010)

- Dipolar sky modulation

$$T(\hat{\mathbf{n}}) = \left(1 + \sum_{M=-1}^{1} w_{1M} Y_{1M}(\hat{\mathbf{n}})\right) T(\hat{\mathbf{n}})_{\rm iso}$$

$$A^{00}_{\ell\ell} = C_\ell$$

$$A^{1M}_{\ell-1,\ell} = A^{1M}_{\ell,\ell-1} = \frac{w_{1M}(C_{\ell-1} + C_\ell)}{(4\pi)^{1/2}}$$

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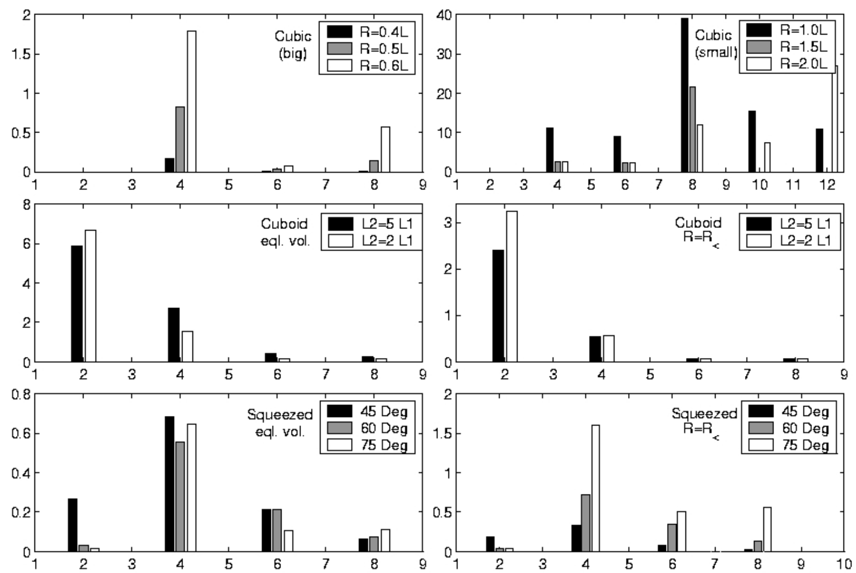
More Theoretical Motivations

Topology: multiply connected universe



Joshi, Jinghan, Souradeep, Hajian PRD(2010)

- Quadrupolar sky modulation

$$T(\hat{\mathbf{n}}) = \left(1 + \sum_{M=-2}^{2} w_{2M} Y_{2M}(\hat{\mathbf{n}})\right) T(\hat{\mathbf{n}})_{\rm iso}$$

$$A^{00}_{\ell\ell} = C_\ell$$

$$A^{2M}_{\ell\ell} = \frac{w_{2M} C_\ell}{\pi^{1/2}}$$

$$A^{2M}_{\ell-2,\ell} = A^{2M}_{\ell,\ell-2} = \frac{w_{2M}(C_{\ell-2} + C_\ell)}{(4\pi)^{1/2}}$$

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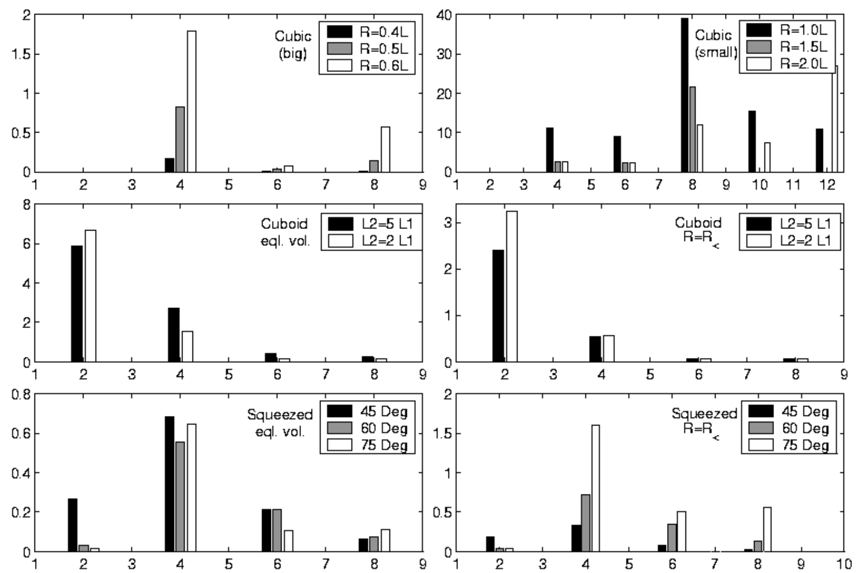
More Theoretical Motivations

Topology: multiply connected universe



Joshi, Jinghan, Souradeep, Hajian PRD(2010)

- Anisotropic early universes
  - Ackerman et al (2007)

$$\zeta(\mathbf{k}) = \left[1 + \sum_{M=-2}^{2} w_{2M} Y_{2M}(\hat{\mathbf{k}})\right] \zeta(\mathbf{k})_{\rm iso}$$

$$A^{2M}_{\ell_1\ell_2} = \frac{i^{\ell_1-\ell_2}}{(4\pi)^{1/2}} w_{2M} \int \frac{2k^2\,dk}{\pi} \Delta_{\ell_1}(k)\Delta_{\ell_2}(k)P(k)$$

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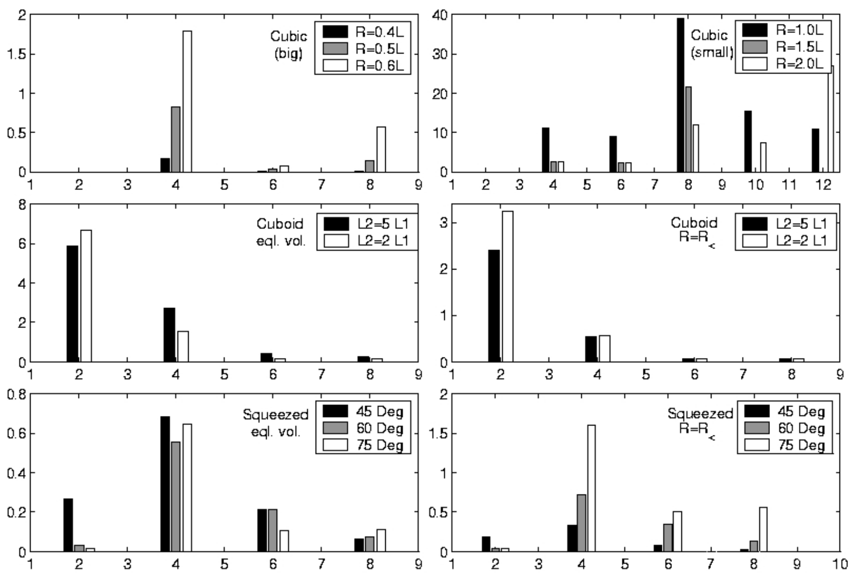
More Theoretical Motivations

Topology: multiply connected universe



Joshi, Jinghan, Souradeep, Hajian PRD(2010)

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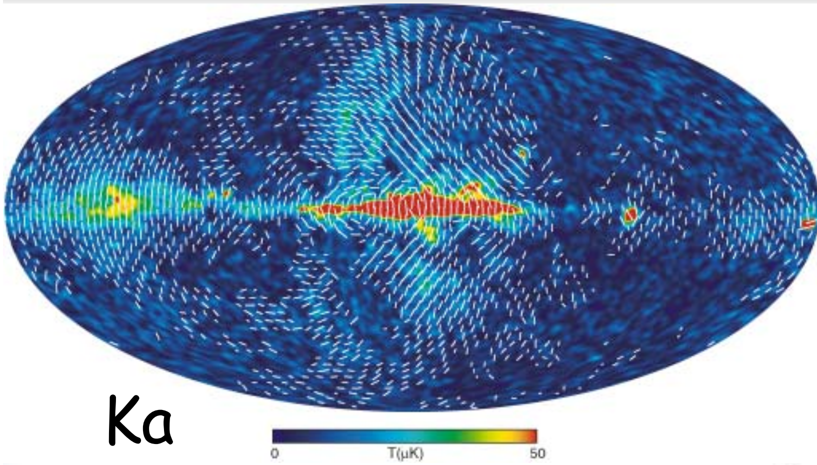
  - Non-commutative inflation (Karwan 2010)

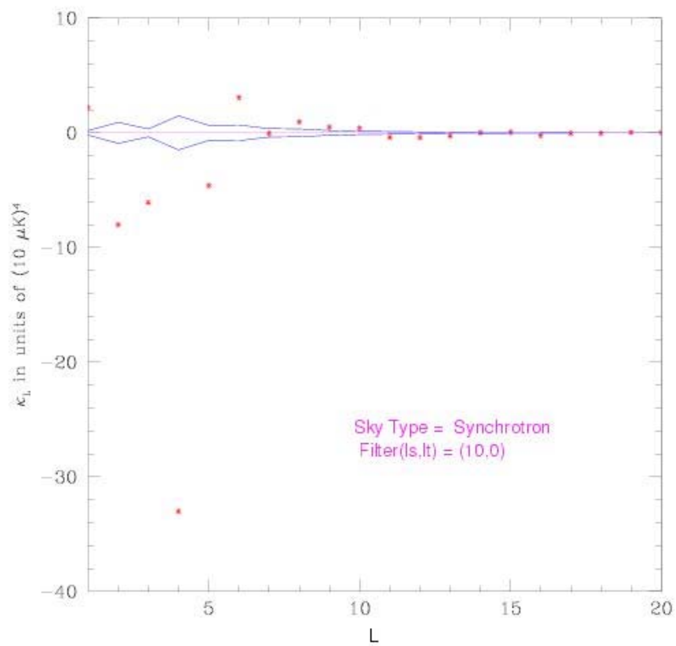
$$\mathcal{P}^{\zeta}_{\mathbf{k}} = \frac{k^3|\zeta_{\mathbf{k}}|^2}{2\pi^2} = \frac{k^3\,|v_{\mathbf{k}}/z|^2}{2\pi^2} \simeq \frac{H^2}{\pi m_p^2 c_s \epsilon}\left(1 - \frac{3}{2}\kappa^2 \sin^2(\theta)\right)$$

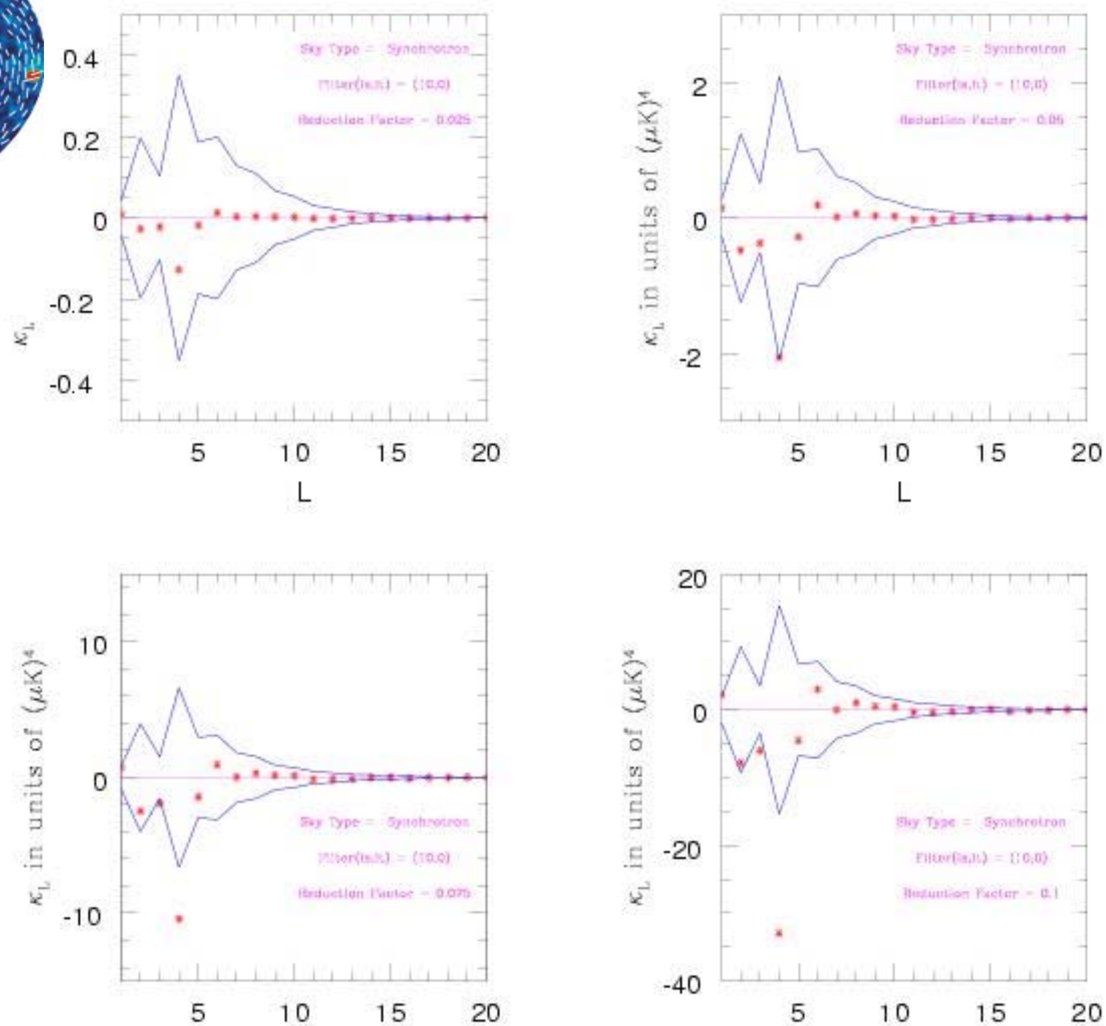
$$\begin{aligned}
\langle A_{00}\rangle &= \sum_l (-1)^l C_l \sqrt{2l+1}\left(1 - \frac{2}{3}A_s^2\Sigma\right),\\
\langle A_{20}\rangle &= -2A_s^2\Sigma \sum_l (-1)^l C_l \sqrt{\frac{l(l+1)(2l+1)}{45(2l-1)(2l+3)}}\\
&\quad -2A_s^2\Sigma \sum_{l\geq 4} (-1)^l C_l \sqrt{\frac{2l(l-1)}{15(2l-1)}}.
\end{aligned}$$

BiPS of Polarization maps



(Basak ,AH, Souradeep: Phys.Rev. D74 (2006) 021301)





Used Simulated 30 GHz maps from Planck sky Model

Circles in Penrose's Conformal Cyclic Cosmology

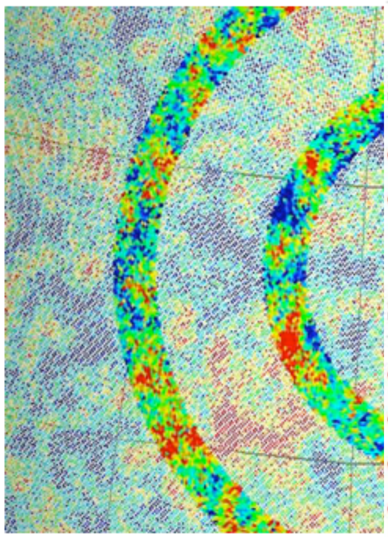
The Economist

Cosmology

Going round

In contradiction to m
evidence that the un

Dec 2nd 2010 | from PRINT



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- Cyclic universe
- Cosmic microwave background
- Inflation
- Universe
- Black holes

Published online 10 December 2010 | Nature | doi:10.1038/news.2010.665

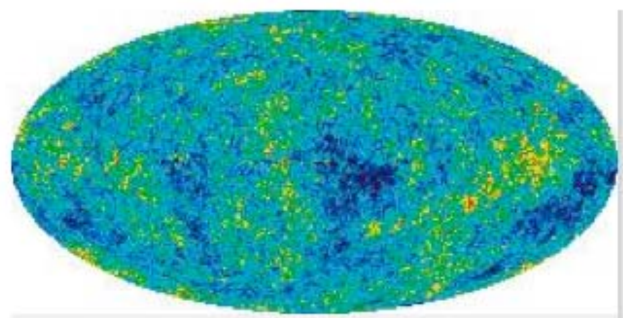
News

No evidence of time before Big Bang

Latest research deflates the idea that the Universe cycles for eternity.

Edwin Cartlidge

Our view of the early Universe may be full of mysterious circles — and even triangles — but that doesn't mean we're seeing evidence of events that took place before the Big Bang. So says a trio of papers taking aim at a recent claim that concentric rings of uniform



Circular ripples in the cosmic microwave background have been making waves with theoreticians.

NASA

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Conformal Cyclic Cosmology

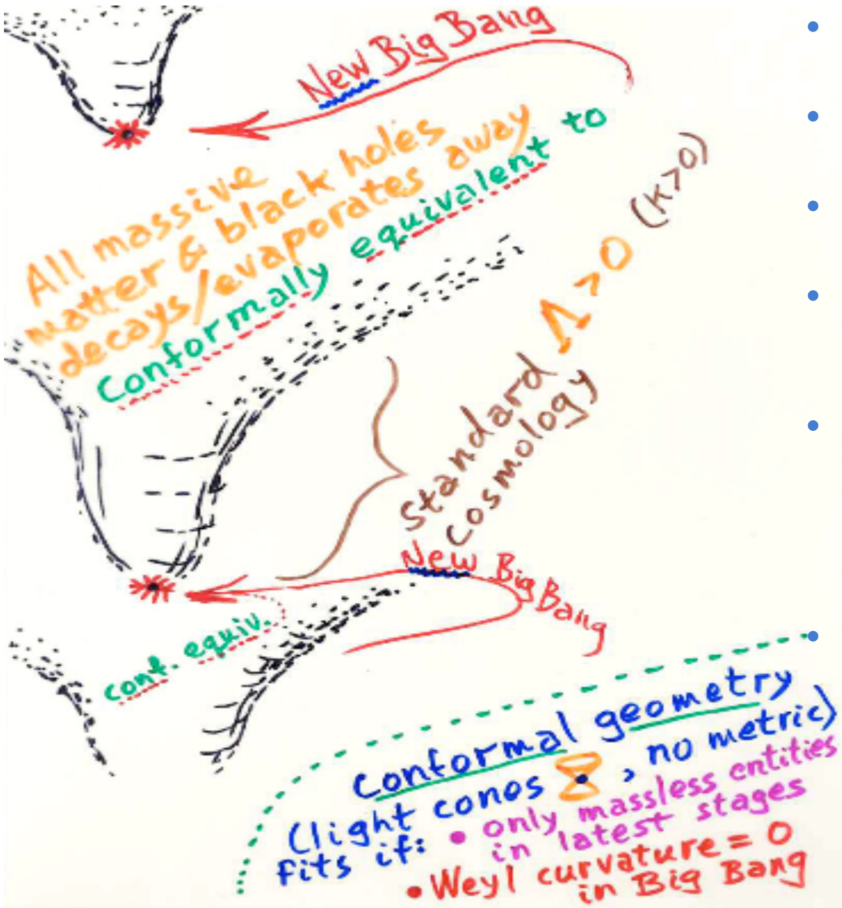
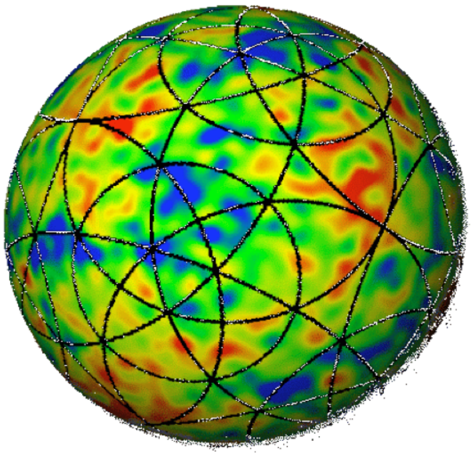


Figure courtesy of Roger Penrose

- At high energies (early times), all interactions are conformally invariant
- Conformal geometry, not metric geometry of spacetime, is relevant
- Conformal spacetime geometry extends smoothly to before big-bang
- Weyl curvature vanishes at the conformal hypersurface which represents the Big Bang -> no infinite gravity at Big Bang.
- **Fundamental Postulate**: what lies beyond the future boundary hypersurface is the big bang of a ``new universe'', what lay before our BB hypersurface was the future infinity of a ``previous universe''.
- Physical material of the universe is only sensitive to the conformal structure of the universe and is blind to the component that provides the *scale* of the metric.

… like ripples on a pond



- Hawking evaporation of black holes -> supplies the radiation content
- Spatial variations in the density after BB caused by *gravitational wave bursts* from close encounters between black holes
- Density fluctuations: *scale-invariant* (because of the exponential expansion BEFORE the Big Bang)
- CMB is a superposition of circular patterns on the surface of last scatter.

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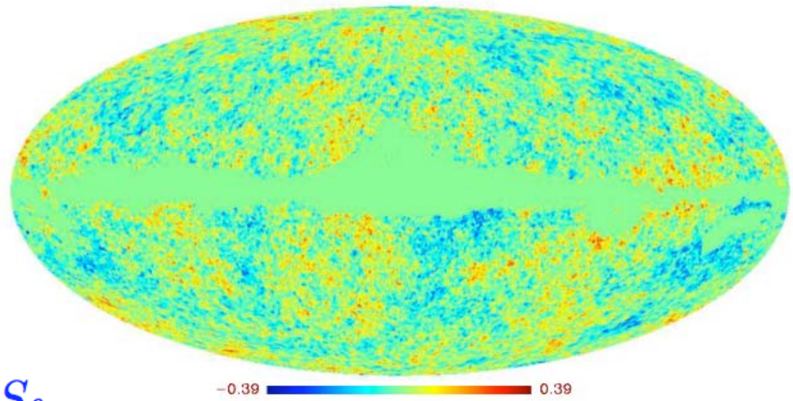
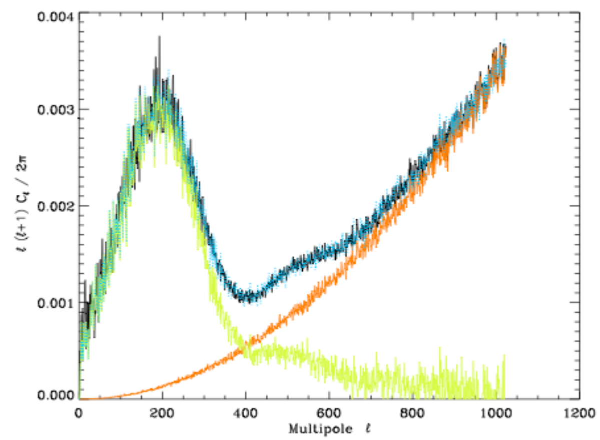
Looking for circles in CMB sky

- Using foreground cleaned W-band map (Galaxy and point sources masked)
- Looking for circularly correlated patterns with the statistic

$$\mathcal{C}_\theta(\hat{n}_i) = \frac{1}{S_\theta}\int \Delta T(\hat{n}_j)\delta(\hat{n}_i \cdot \hat{n}_j - \cos\theta)dS_\theta$$

- Simulations of the LCDM CMB maps with realistic noise are used to assess the statistical significance of the results





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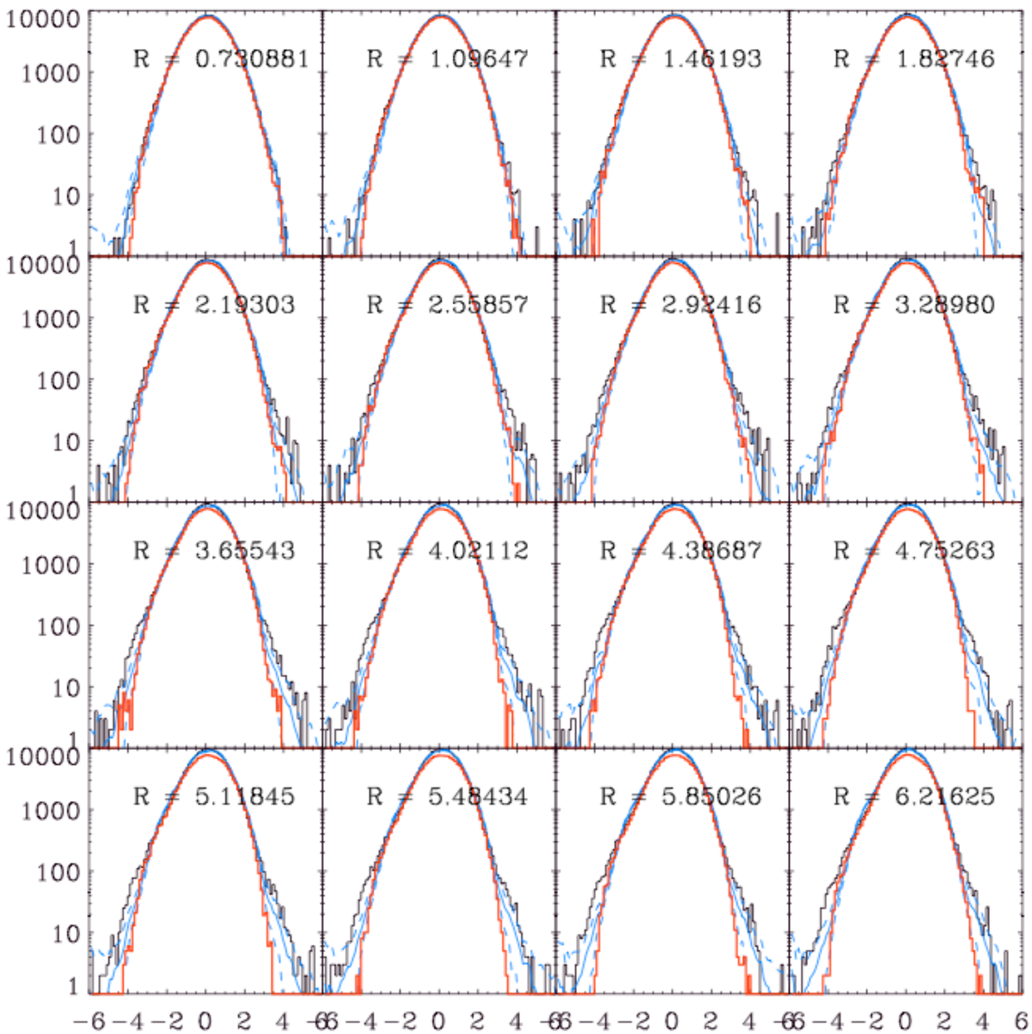
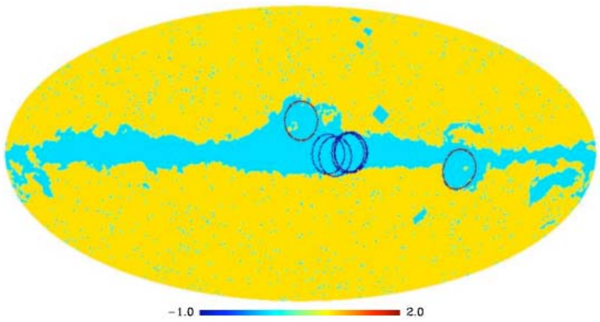
Distribution of Average Temperatures Along Circles at Different Radii





- Detected circles are all at the edges -> suffer from small number statistics
- Excluding circles with small number of pixels in them removes large deviations from Gaussian distributions.
- No circles in WMAP5 data

.0

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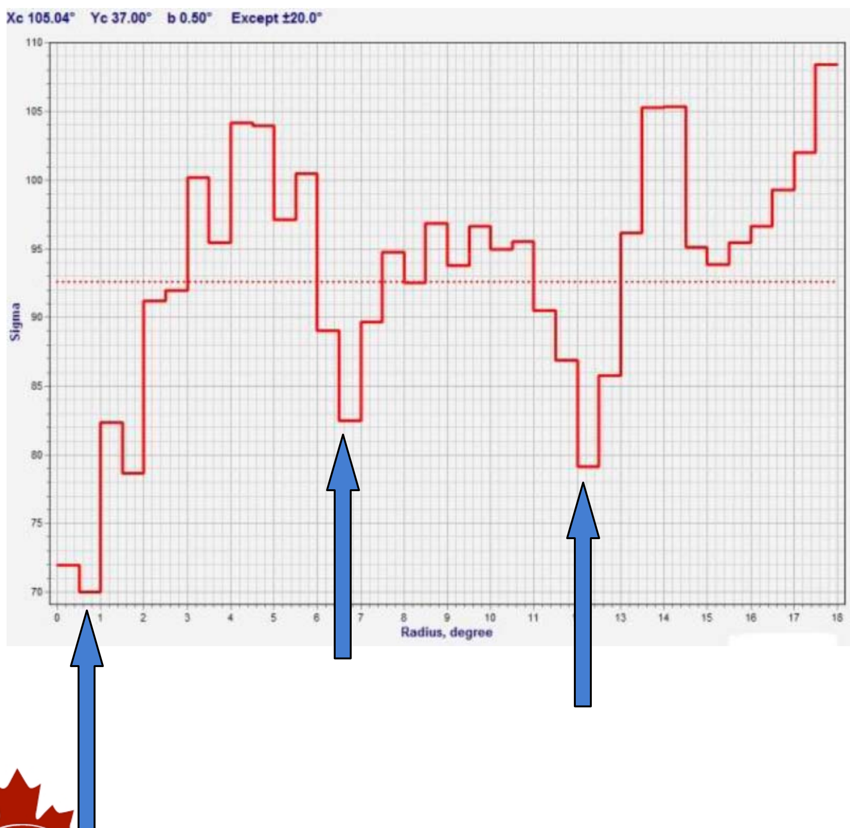
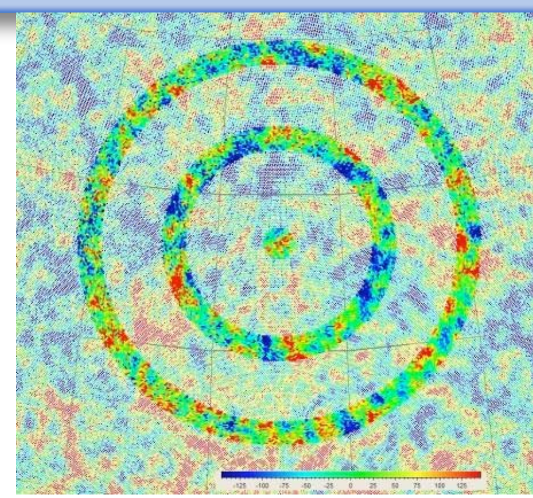
Concentric circles in WMAP7 data?





- Gurzadyan & Penrose (2010) claim to detect three families of "anomalous low-variance concentric circles" in WMAP7 data
- 3 critical papers show the patterns are not anomalous:
  - Wehus, Eriksen (arXiv:1012.1268)
  - Moss, Scott, Zibin (arXiv:1012.1305)
  - A.H. (arXiv:1012:1656)

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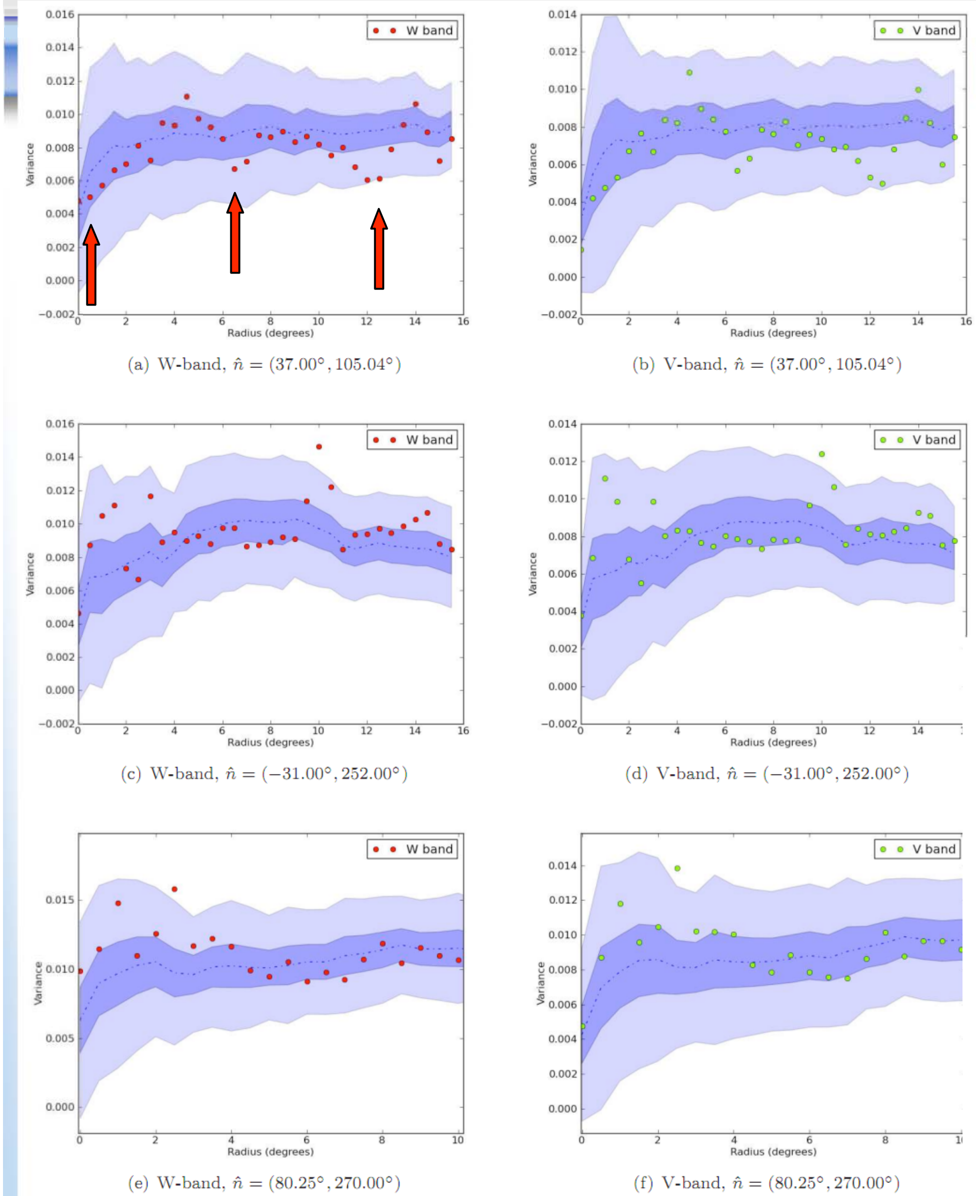
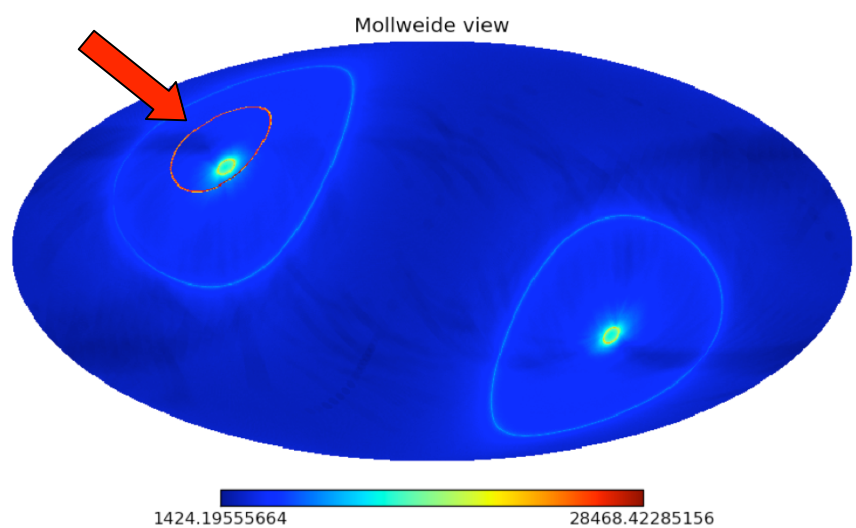
Reassessing statistical significance of the G&P patterns

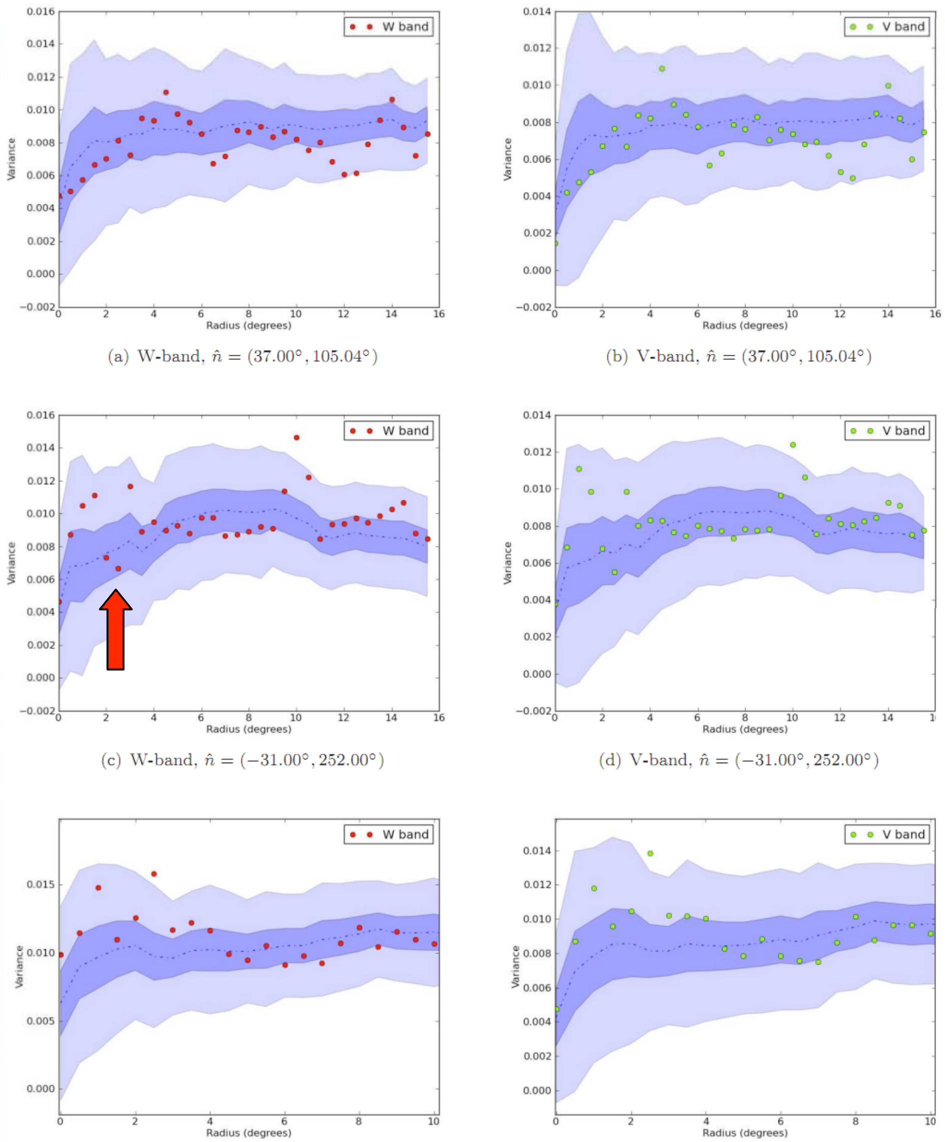


(a) W-band, $\hat{n} = (37.00°, 105.04°)$
(b) V-band, $\hat{n} = (37.00°, 105.04°)$
(c) W-band, $\hat{n} = (-31.00°, 252.00°)$
(d) V-band, $\hat{n} = (-31.00°, 252.00°)$
(e) W-band, $\hat{n} = (80.25°, 270.00°)$
(f) V-band, $\hat{n} = (80.25°, 270.00°)$

- Repeating G&P analysis at three centers we get similar patters as G&P
- 200 simulations of the sky + WMAP noise to determine statistical significance of the dips



Reassessing statistical significance of the G&P patterns



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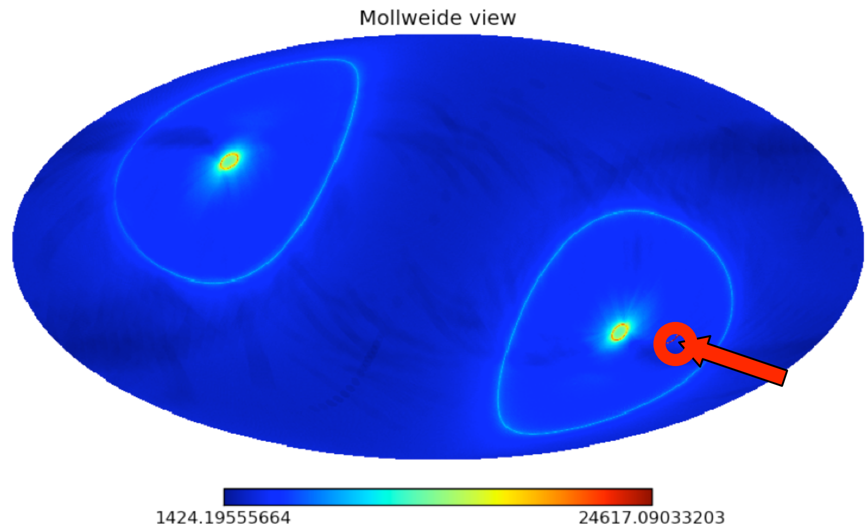

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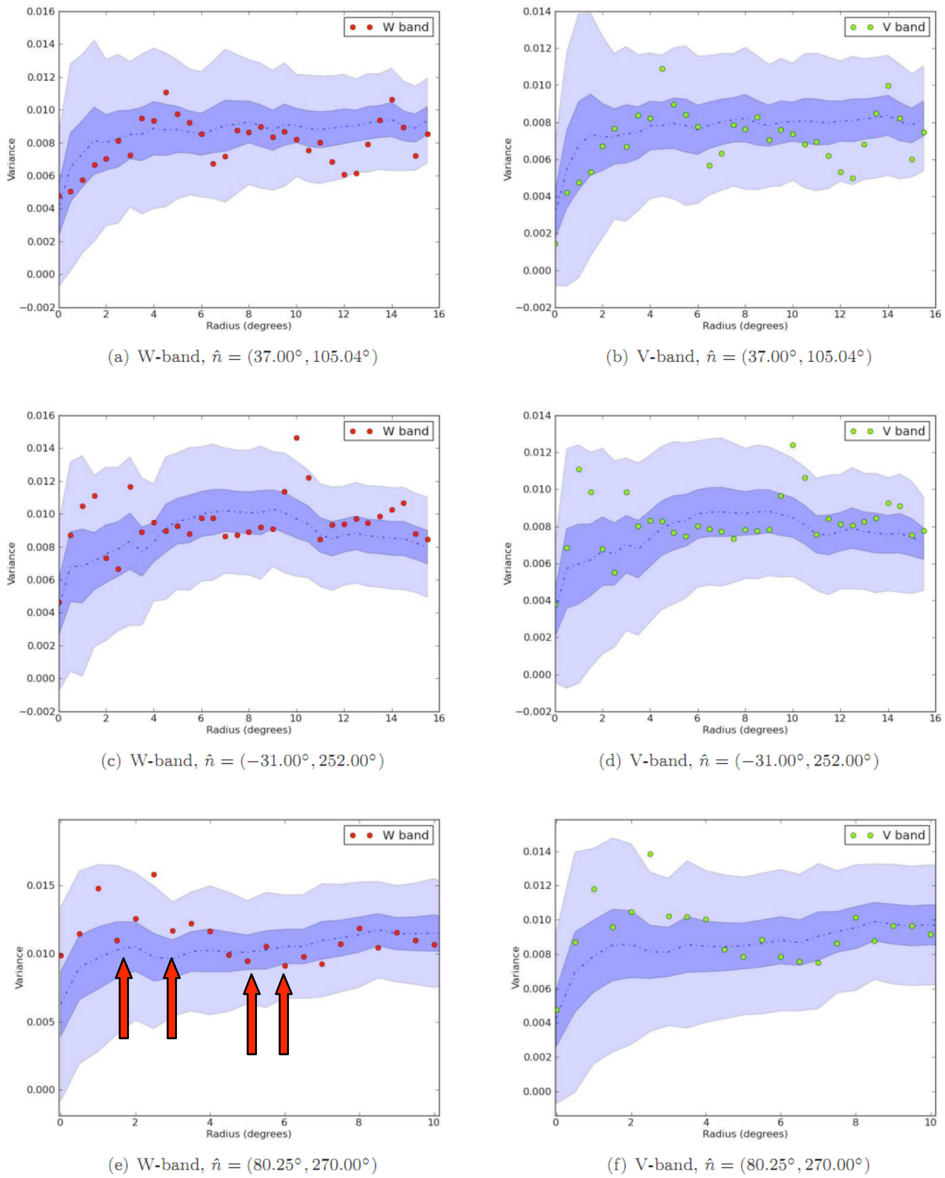

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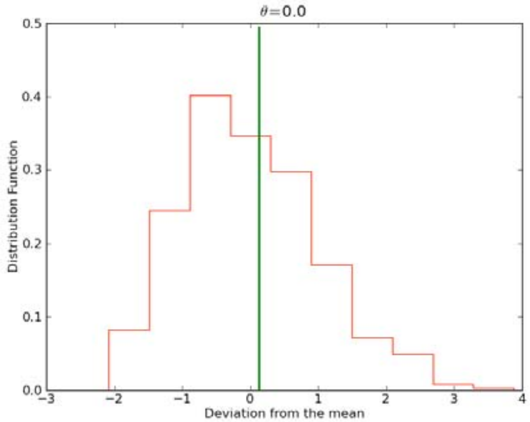
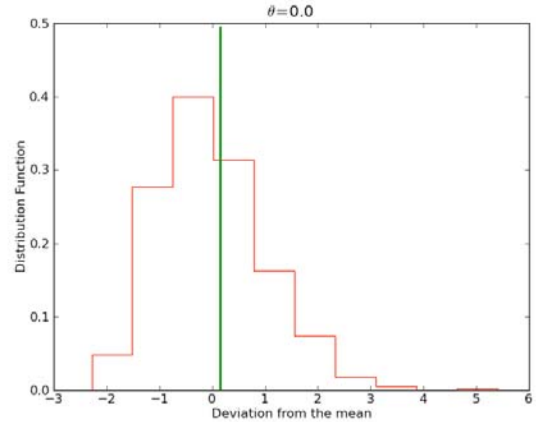
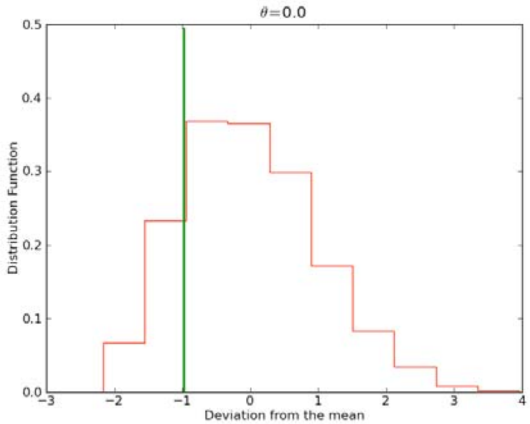
- Repeating G&P analysis at three centers we get similar patters as G&P
- 200 simulations of the sky + WMAP noise to determine statistical significance of the dips
- None of the variances are significantly low!

More details:
http://www.cita.utoronto.ca/~ahajian/pBB.html

Reassessing statistical significance of the G&P patterns







Measured variances along the G&P circles in WMAP data compared with the PDF of the variances of the same circles in 1000 simulations.

More details:
http://www.cita.utoronto.ca/~ahajian/pBB.html

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Thank you!