Effects of topological defects on the CMB

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Inflationary cosmology & topological defects

- Simplest model of the early Universe: Inflation[a]
- Some models of inflation ("hybrid") end by producing topological defects[b]
- Defects may also be formed in subsequent thermal transitions[c]
- String/M-theory: defects from $(D\bar{D})$-brane collisions[d]
- Defects have gravitational fields & contribute to perturbations[e]

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[a] Starobinsky (1980); Sato (1981); Guth (1981); Mukhanov & Chibisov (1981); Linde (1982); Hawking & Moss (1982); Albrecht & Steinhardt (1982); Guth & Pi (1982); Hawking(1982); Hawking & Moss (1983); Bardeen, Steinhardt, Turner, (1983)

[b] Yokoyama (1989); Copeland et al (1994); Kofman, Linde, Starobinski (1996); Garcia-Bellido et al (2010)

[c] Kibble (1976); Zurek (1996); Rajantie (2002)

[d] Jones, Stoica, Tye (2002); Dvali & Vilenkin (2003); Copeland, Myers, Polchinski (2003)

[e] Kibble (1976); Zel'dovich (1980); Vilenkin (1981)

Topological defects - cosmic strings

- Cosmic strings[a] are linear distributions of mass-energy in the universe.
- Mass per unit length μ, tension T. Normally $\mu = T/c^2$
- Dynamics: acceleration $\propto$ curvature: wave equation
- In theories of high energy physics they may be
  - Elementary (string theory): zero width
  - Solitonic (field theory): non-zero width
- Made in the early universe?[b] $t \sim 10^{-36}$ s, $\mu \sim 10^{32}$ GeV^2, $w \sim 10^{-30}$ m
- If formed, still here: O(1) "infinite" string, unknown distribution of closed loops

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[a] Hindmarsh & Kibble (1994); Vilenkin & Shellard (1994); Kibble (2004)

[b] Kibble (1976); Zurek (1996); Rajantie (2002); Yokoyama (1989); Kofman, Linde, Starobinski (1996); Jones, Stoica, Tye (2002); Sarangi & Tye (2003); Copeland, Myers, Polchinski (2003); Dvali & Vilenkin (2003)

Other defects: global monopoles, textures and semilocal strings

Global monopoles and textures[a]

- Self-ordering scalar fields (Goldstone modes) from global symmetry-breaking.
- Global monopoles: point-like, with attractive force proportional to distance.
- Symmetry-breaking scale $v \sim 10^{16}$ GeV: observable perturbations.

Semilocal strings[b]

- Self-ordering scalar and vector fields from “semilocal” symmetry-breaking
- Semilocal: non-trivial combination of local and global symmetries
- Symmetry-breaking scale $v \sim 10^{16}$ GeV: observable perturbations.

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[a] Turok 1989; Spergel et al 1991; Pen, Spergel, Turok 1995; Durrer, Kunz, Melchiorri 1999,2002.

[b] Vachaspati, Achucarro 1991; Hindmarsh 1992,1993; Urrestilla et al. 2008.

Observational signals from defects

Predictions under good theoretical control:

- Cosmic Microwave Background (power spectrum, bi/tri-spectrum) [a]

Some theoretical uncertainties:

- Density perturbation (power spectrum); [b] [strings: Gravitational lensing[c]]

Large theoretical uncertainties:

- Gravitational radiation[d] [Global strings: axion radiation]; [e] [strings: cosmic rays[f]]

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[a] Gott (1984); Kaiser & Stebbins (1984); Bouchet et al. (1989); Allen et al (1996,7); Landriau & Shellard (2004,10); Wyman et al (2005); Pogosian, [Tye], Wyman (2008,9); Bevis et al (2006,7,8,10); Hindmarsh, Ringeval, Suyama (2009,10); Regan, Shellard (2010).

[b] Zel'dovich (1980); Vilenkin (1981); Avelino, Shellard, Wu, Allen (1998); . . .

[c] Vilenkin (1984); Hindmarsh (1989); de Laix & Vachaspati (1996,1997); Mack, Wesley, King (2007); . . .

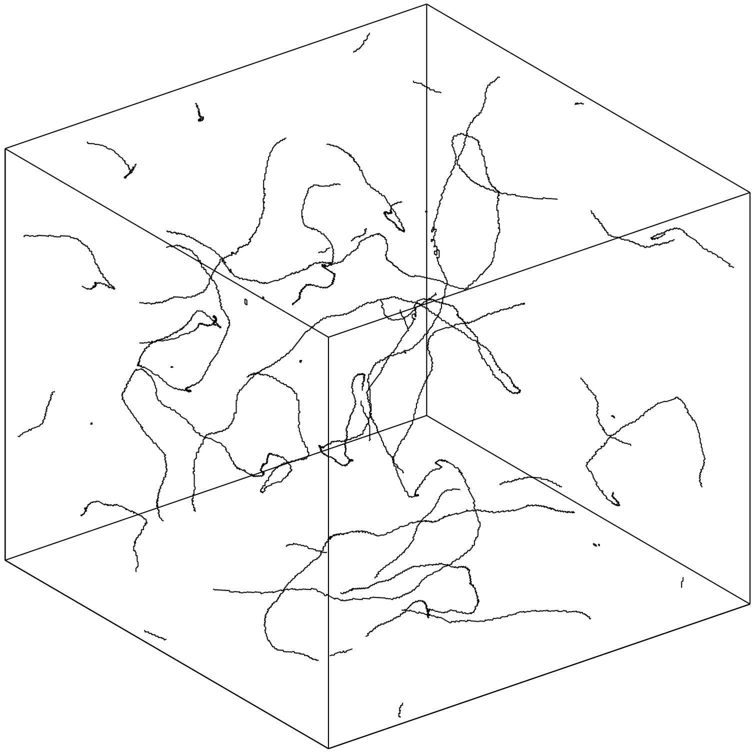
[d] Vachaspati & Vilenkin (1985); Hindmarsh (1990); Allen & Shellard (1992); Damour & Vilenkin (2000,2001,2005);. . .

[e] Battye & Shellard (1994-9); Yamaguchi, Kawasaki, Yokoyama (1998); Wantz & Shellard (2009).

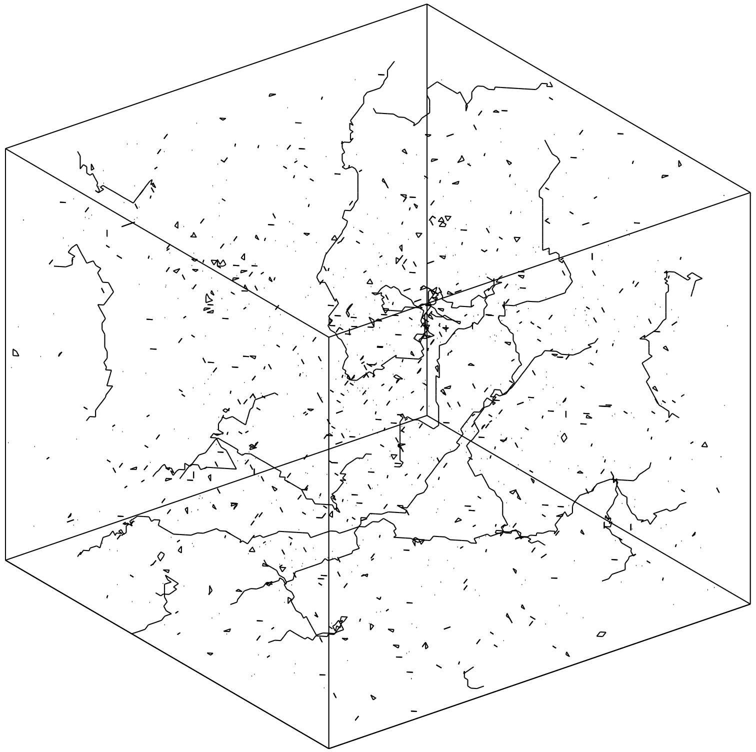
[f] Bhattarcharjee (1990); Sigl (1996); Protheroe (1996); Berezhinksi (1997); Vincent, M.H., Sakellariadou (1998); Wichowski, MacGibbon, Brandenberger (1998)

A problem for strings: small scales and loops

Classical Abelian Higgs model

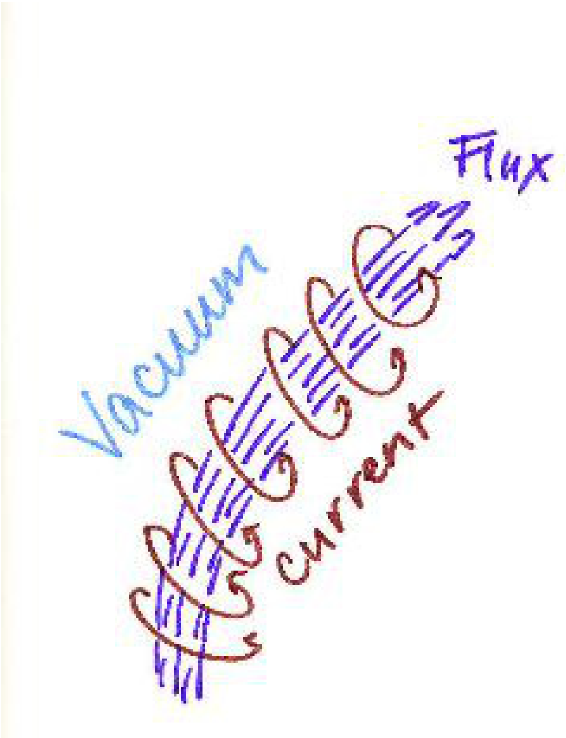


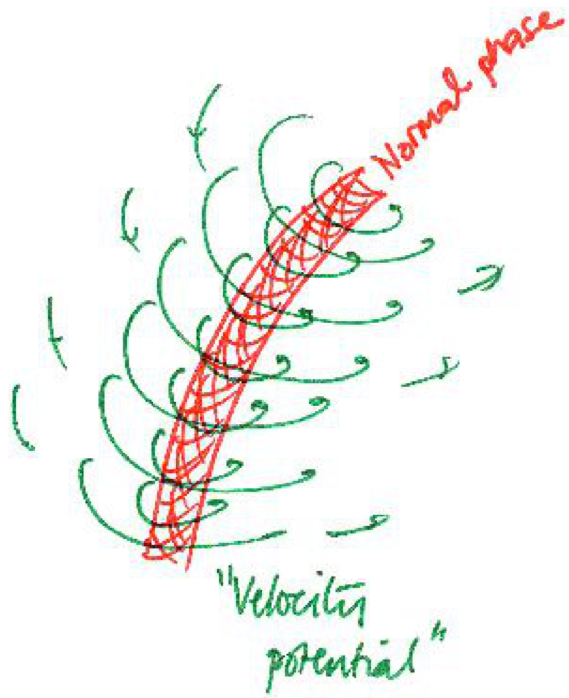
Nambu-Goto strings



How much string? How many loops?

Defect zoo: strings from field theory





Gauge/local string

Nearest living relative:

Type II superconductor flux tube

Global string

Nearest living relative:

superfluid vortex

String solutions in the Abelian Higgs model

Complex scalar field $\phi(\mathbf{x},t)$, vector field $A_\mu(\mathbf{x},t)$

$\mathcal{L} = (D\phi)^\dagger \cdot (D\phi) - V(|\phi|) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$

Static finite energy (2D) cylindrically symmetric:

$$\phi = vf(r)e^{i\theta}, \quad A_i = \frac{1}{er}A_\theta(r)\hat{\theta}_i$$

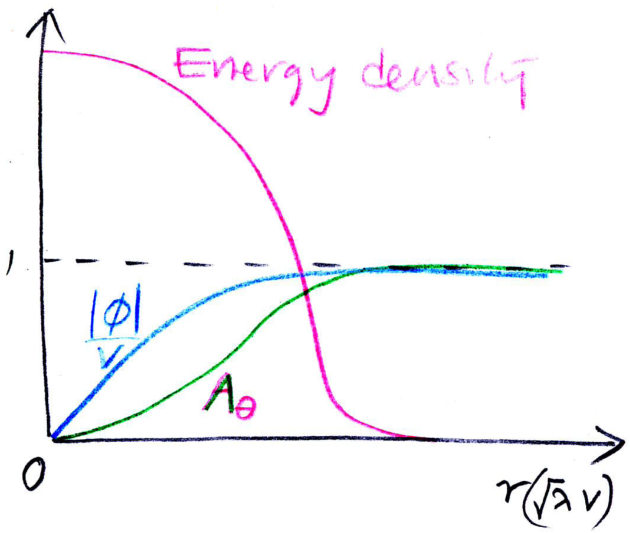
Energy density:

$$\rho = |D_i\phi|^2 + V_\lambda(\phi) + \frac{1}{2}B^2$$

ρ confined to region $r < \max(1/\sqrt{\lambda}v, 1/ev)$

String mass per unit length: $\mu = 2\pi v^2 E(\lambda/e^2)$

[E - slow function, $E(1) = 1$]



Textures

- N scalar fields $\phi^A(\mathbf{x}, t)$
- O(N) symmetry: $\mathcal{L} = \frac{1}{2}\partial\phi^A \cdot \partial\phi^A - V(|\phi|)$
- Potential: $V(|\phi|) = \frac{1}{4}\lambda(\phi^A\phi^A - v^2)^2$
- Energy density:

$$\rho = \frac{1}{2}\dot{\phi}^2 + (\nabla\phi)^2 + V$$

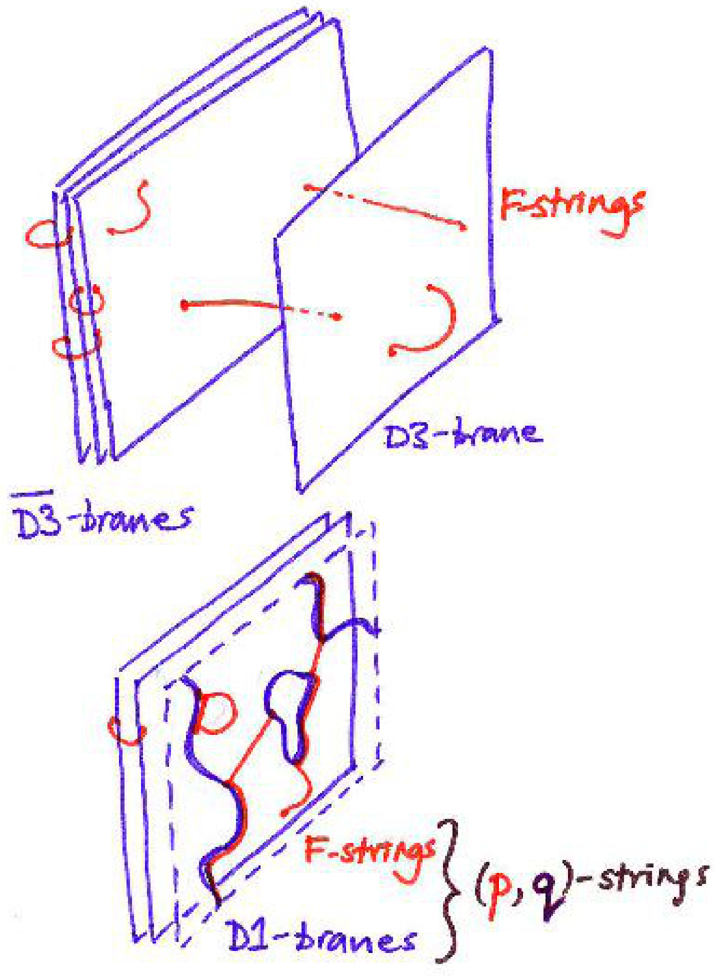
- At low energy density, $|\phi| \simeq v$ [global symmetry is broken to O($N-1$).
- Cosmic texture: field configuration with wavelength $\xi \sim t$
- Evolution timescale t.
- Energy density is $\rho \sim v^2/t^2$
- Textures exist in any kind of non-Abelian global symmetry-breaking.

Semilocal strings

- N complex scalar fields $\phi^A(\mathbf{x}, t)$
- U(N) symmetry: $\mathcal{L} = (D\phi^A)^\dagger \cdot (D\phi^A) - V(|\phi|) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$
- U(N) $=$ SU(N)$_{\rm global}\times$U(1)$_{\rm local}$.
- Potential: $V(\phi^A\phi^A) = \frac{1}{2}\lambda((\phi^A)^\dagger\phi^A - v^2)^2$
- At low energy density, $|\phi| \simeq v$ [symmetry is broken to SU($N-1$)$_{\rm global}$.
- For gauge coupling $\gg$ scalar coupling: like Abelian Higgs model
- For gauge coupling $\ll$ scalar coupling: Semilocal string
- A semilocal string is like texture, a field configuration with wavelength $\xi \sim t$
- Evolution timescale t.
- Energy density is $\rho \sim v^2/t^2$

Cosmic strings from string theory

- Fundamental strings F-strings
- Extended objects D-branes
- F-strings end on D-branes, (D1 = D-string)
- Bound states, junctions: (p, q)-strings[a]
- String tension $\mu_{p,q} = \frac{V_{6,\mathrm{eff}}}{2\pi\alpha'}\sqrt{p^2 + \frac{q^2}{g_s^2}}$ [b]
- Formation: collisions of D3-$\overline{\text{D3}}$ branes[c]



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[a] Copeland, Myers, Polchinski (2004); Firouzjahi, Leblond, Tye (2006); Dasgupta, Firouzjahi, Gwyn (2007)

[b] Copeland, Myers, Polchinski (2004); Firouzjahi, Tye (2005)

[c] Sarangi & Tye (2002); Dvali & Vilenkin (2004); Barnaby, Berndsen, Cline, Stoica (2005)

Defect scaling hypothesis

- Defects have a characteristic scale ξ (e.g. string curvature radius $\sqrt{V/L}$)
- Scaling hypothesis: $\boxed{\xi = x_* t}$ (x_* constant O(1))
- Energy density: $\rho_s \simeq v^2/\xi^2$
- Total energy density: $\rho_t \sim 1/Gt^2$:
- Defect density fraction: $\boxed{\Omega_d \sim Gv^2/x_*^2}$ - is constant.
- Grand Unification: $Gv^2 \sim 10^{-6}$

Scaling: extrapolate from $t_i \sim 10^{-36}$ s to $t_0 \sim 3 \times 10^{17}$ s today

A supersymmetric model of (hybrid) inflation ...

νMSSM + ϕ + $\bar{\phi}$ + s + W_{mix} [extra U(1)' global, gauged][a]

- For example: $W = W_{\nu\text{MSSM}} + \kappa s(\phi\bar{\phi} - M^2)$
- $V = V_D + V_{\text{soft}} + \kappa^2|s|^2(|\phi|^2 + |\bar{\phi}|^2) + \kappa^2|\phi\bar{\phi} - M^2|^2 + V_{\text{1-loop}}$
- $V_D = \frac{e'^2}{2}\left(\xi_{\text{FI}} - |\phi|^2 + |\bar{\phi}|^2\right)^2$
- Inflation for large $|s|$, ϕ = $\bar{\phi}$ = 0. F-term[b] or D-term[c] can dominate.
- Inflation ends at $|s| = |s_c(M, \xi_{\text{FI}})|$, ϕ, $\bar{\phi}$ get vevs: U(1)$'$ symmetry breaks.

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[a] e.g. Garbrecht, Pallis, Pilaftsis (2006)

[b] Copeland et al (1994)

[c] Binétruy & Dvali (1996), Halyo (1996)

... is a model with cosmic strings

- U(1)$'$ symmetry is broken after inflation: $\langle|\phi|^2\rangle \simeq \phi_0^2(M, \xi_{\rm FI})$
- Gauged U(1)': Abelian Higgs cosmic strings with tension $\mu = E(\kappa^2/{e'}^2)2\pi\phi_0^2$
- Global U(1)': Global cosmic strings with tension $\mu \sim \phi_0^2 \log(t\phi_0)$
- Inflation + strings - a natural paradigm
- Tight constraints from CMB[a]

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[a] Battye, Garbrecht, Moss (2008, 2010); Battye, Moss (2010)

Formation and evolution: Abelian Higgs in FRW background

$$S = -\int d^4x\,\sqrt{-g}\left(g^{\mu\nu}D_\mu\phi^* D_\nu\phi + V(\phi) + \frac{1}{4e^2}g^{\mu\rho}g^{\nu\sigma}F_{\mu\nu}F_{\rho\sigma}\right),$$

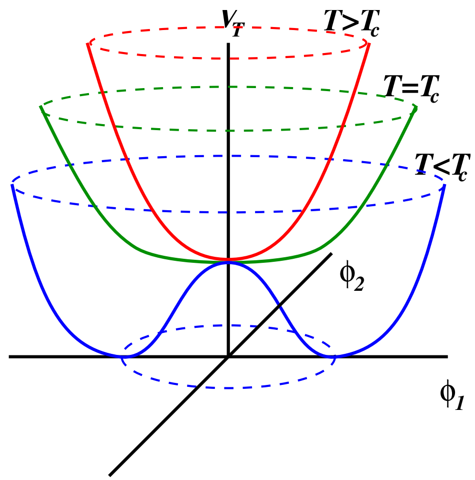
Complex scalar field $\phi(\mathbf{x},t)$, vector field $A_\mu(\mathbf{x},t)$

Covariant derivative $D_\mu = \partial_\mu - iA_\mu$.

Potential $V(\phi) = \frac{1}{2}\lambda(|\phi|^2 - v^2)^2$.

Metric $ds^2 = a^2(\tau)(-d\tau^2 + d\mathbf{x}^2)$

τ: conformal time, $\propto t, t^{\frac{1}{2}}$



Temporal gauge ($A_0 = 0$) field equations (index raised with Minkowski metric).

$$\begin{aligned}\ddot{\phi} + 2\frac{\dot{a}}{a}\dot{\phi} - D^2\phi + \lambda a^2(|\phi|^2 - v^2)\phi &= 0,\\ \partial^\mu\left(\frac{1}{e^2}F_{\mu\nu}\right) - ia^2(\phi^* D_\nu\phi - D_\nu\phi^*\phi) &= 0,\end{aligned}$$

Visualising Abelian Higgs string simulations

Isosurfaces of constant energy density. Size: 256^3, lattice spacing $0.5m^{-1}$

Initial conditions: Gaussuan random field ϕ

Boundary conditions: toroidal

Abelian Higgs model simulations: string length scale

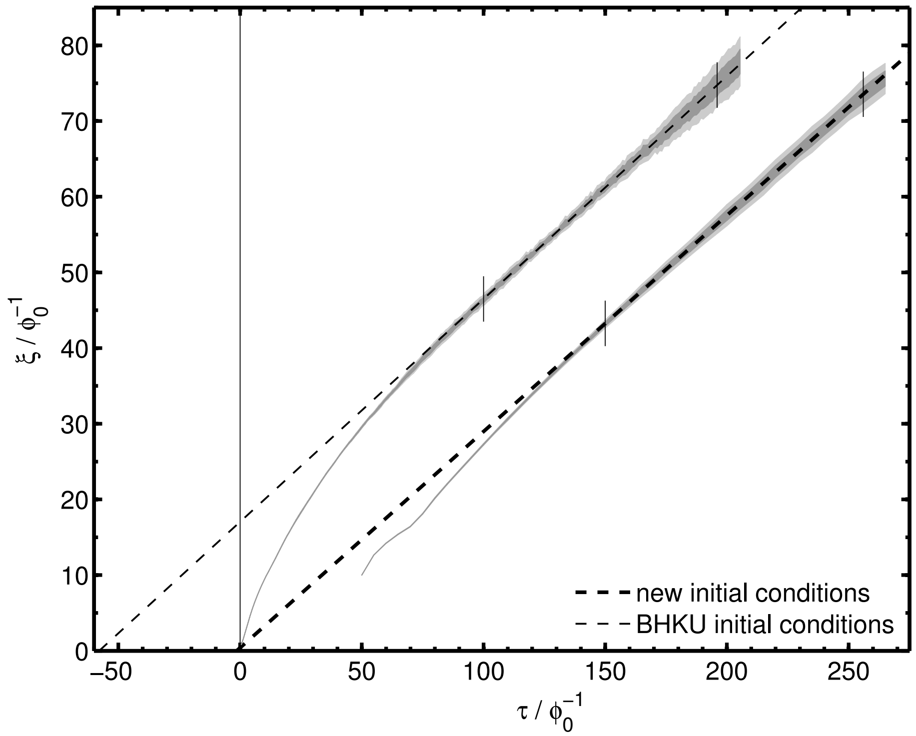
Scaling: $L/V \propto \tau^{-2}$

Network scale: $\xi = \sqrt{(V/L)}$

Hence $\xi \propto \tau$

Couplings:	$\lambda = 2, e = 1$
Masses:	$m_s = m_v = 1$
Lattice spacing:	$\Delta x \simeq 0.5$
Time step:	$\Delta t = 0.1$
Volume:	768^3,1024^3

Matter era, different initial conditions



Semilocal strings simulations: length scale

Scaling: Energy density $T_{00} \propto \tau^{-2}$

Couplings: $\lambda = 2, e = 1$

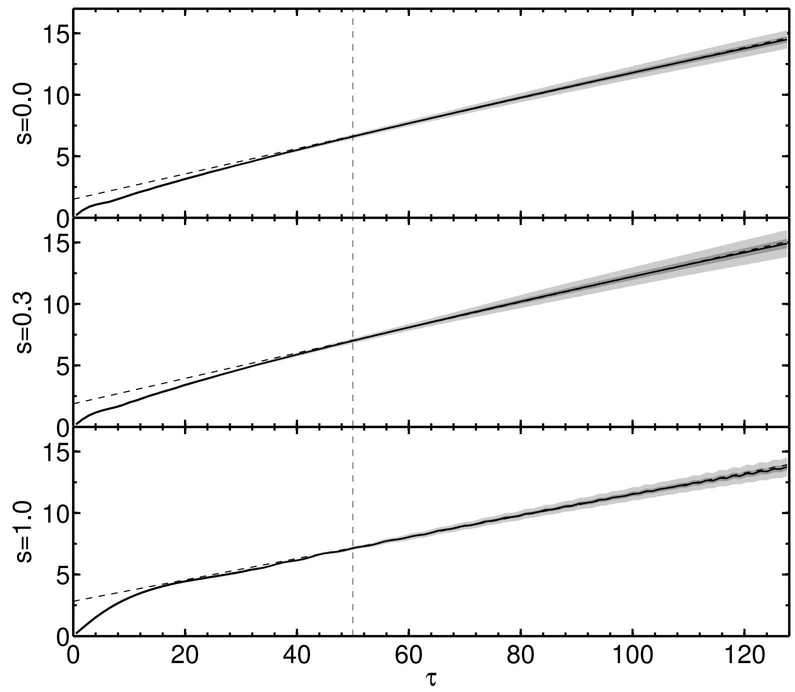
Masses: $m_s = m_v = 1$

Lattice spacing: $\Delta x \simeq 0.5$

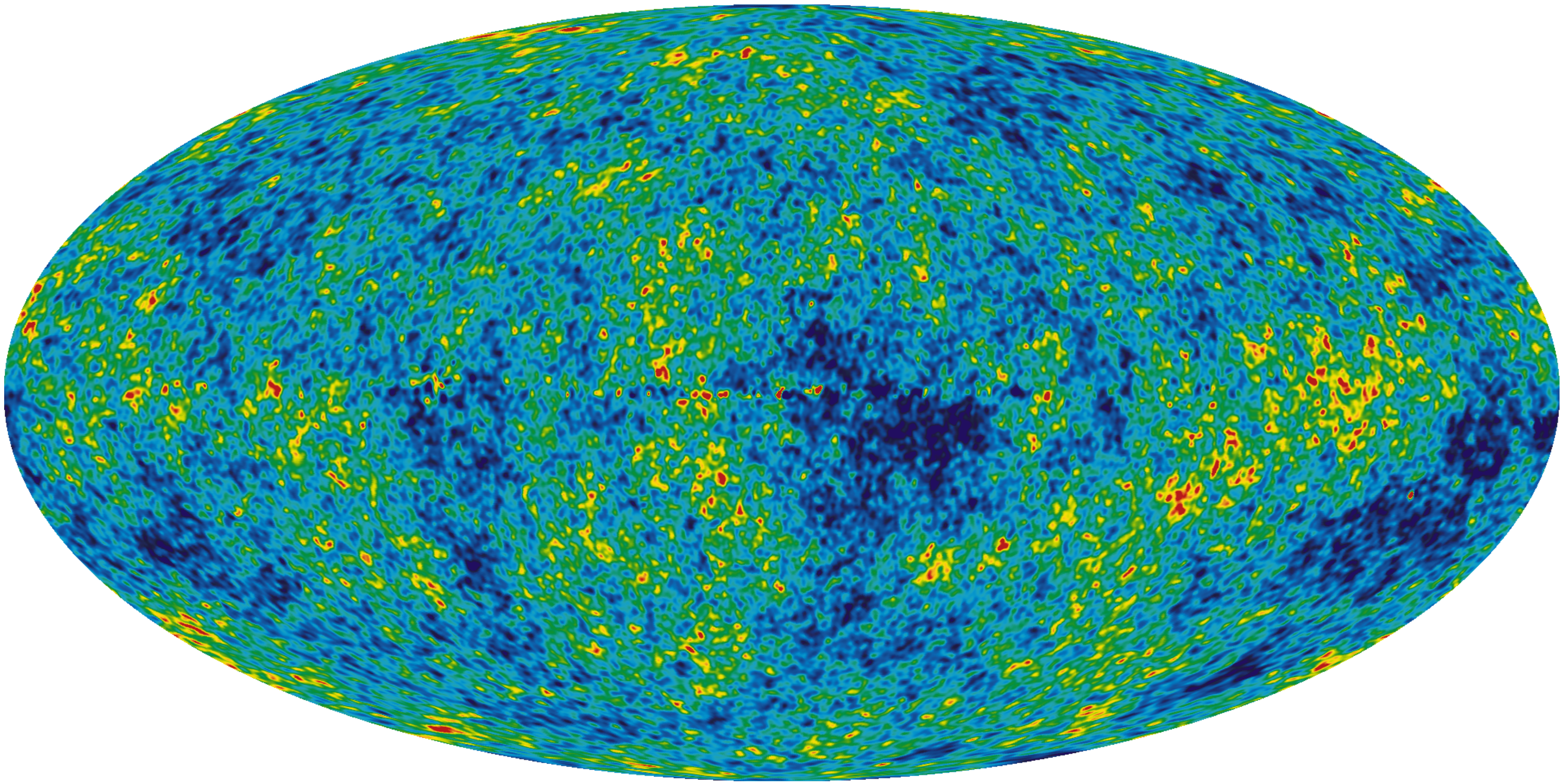
Time step: $\Delta t = 0.1$

Volume: 512^3

Radiation era, different algorithms



Cosmic Microwave Background constraints



$-200 < \Delta T < 200$ μK

Gott-Kaiser-Stebbins (GKS) effect for cosmic strings

Discontinuity[a] $\Delta T \sim 8\pi(G\mu)vT_{\rm CMB}$

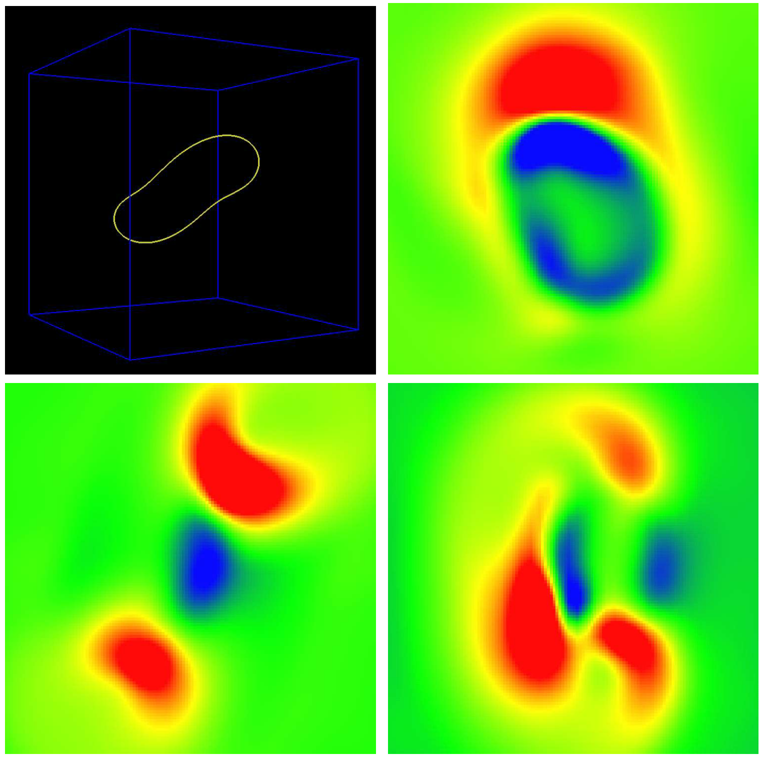
Need high resolution & high sensitivity

- OVRO + string correlators:[b]

  $(G\mu) < x_* 11 \times 10^{-6}$
- WMAP + random edge model:[c]

  $G\mu < 1.07 \times 10^{-5}$
- WMAP3 + random edge model:[d]

  $G\mu < 3.7 \times 10^{-6}$



Landriau and Shellard (2002)

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[a] Kaiser, Stebbins (1984)
[b] Hindmarsh (1993)
[c] Lo & Wright (2005)
[d] Jeong & Smoot (2006)

Calculating CMB power spectra from defects: UETC method

$h_\alpha(\tau, k)$:	linear perturbation (metric, matter, temperature . . .)
$S_\alpha(\tau, k)$:	source (string energy-momentum, separately conserved)
$D_{\alpha\beta}(\tau, k)$:	time dependent differential operator

Perturbation equation: $\mathcal{D}_{\alpha\beta}(\tau, k)h_\beta(\tau, k) = S_\alpha(\tau.k)$

Power spectrum:[a] $\left\langle |h_\alpha(\tau_0, k)|^2 \right\rangle = \int\int \mathcal{D}^{-1}\mathcal{D}^{-1} \left\langle S_\alpha(\tau, k) S^*_\alpha(\tau', k) \right\rangle$

Need unequal-time correlators (UETCs) of source (energy-momentum tensor)

$$C_{\mu\nu\rho\lambda}(k, \tau, \tau') = \left\langle T_{\mu\nu}(k, \tau) T^*_{\rho\lambda}(k, \tau') \right\rangle$$

5 independent UETCS [3 scalar, 1 vector, 1 tensor] from numerical simulations

$\mathcal{D}^{-1}$ is CMBEASY, applied to eigenvectors of UETCs

Scaling: small times, lengths $\rightarrow$ large times, lengths

[a] Pen, Seljak, Turok (1997); Dürrer, Kunz, Melchiorri (1998,2002)

Fitting CMB with inflation & cosmic defects

- Two sources of perturbations: incoherent - add in quadrature
- Cosmological model with 1 more parameter: μ
- Gauge and semilocal strings: $\mu = 2\pi v^2 E(\lambda/e^2)$
  [λ - scalar coupling, e - gauge coupling, E - slow function, $E(1) = 1$]
- Textures: $\mu = 2\pi v^2$
- Use $f_{10} = C_{10}^{\text{defect}}/C_{10}^{\text{total}}$. Proportional to $(G\mu)^2$.
- Modify COSMOMC and perform Monte Carlos.

CMB from gauge strings, textures, and semilocal strings

Multipole moments:

$$a_{lm} = \int d\Omega \Delta T(\mathbf{n}) Y^*_{lm}(\mathbf{n})$$

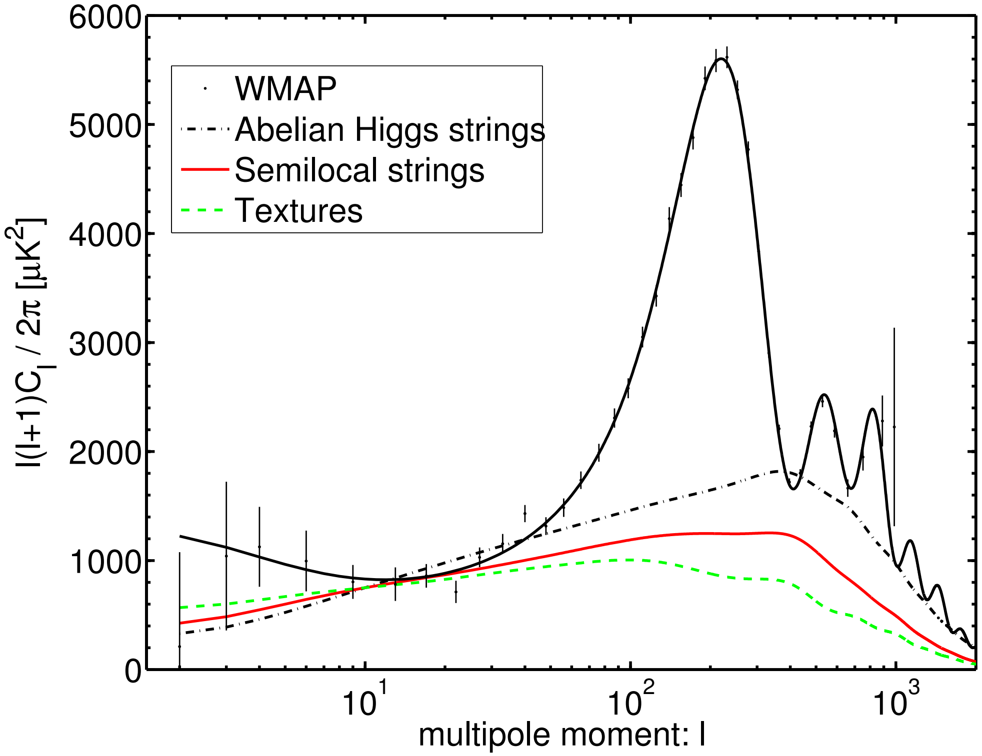
Angular power spectrum:

$$C_l = \sum_{m=-l}^{l} |a_{lm}|^2$$

Anisotropy power:

$$l(l+1)C_l/(2\pi)$$

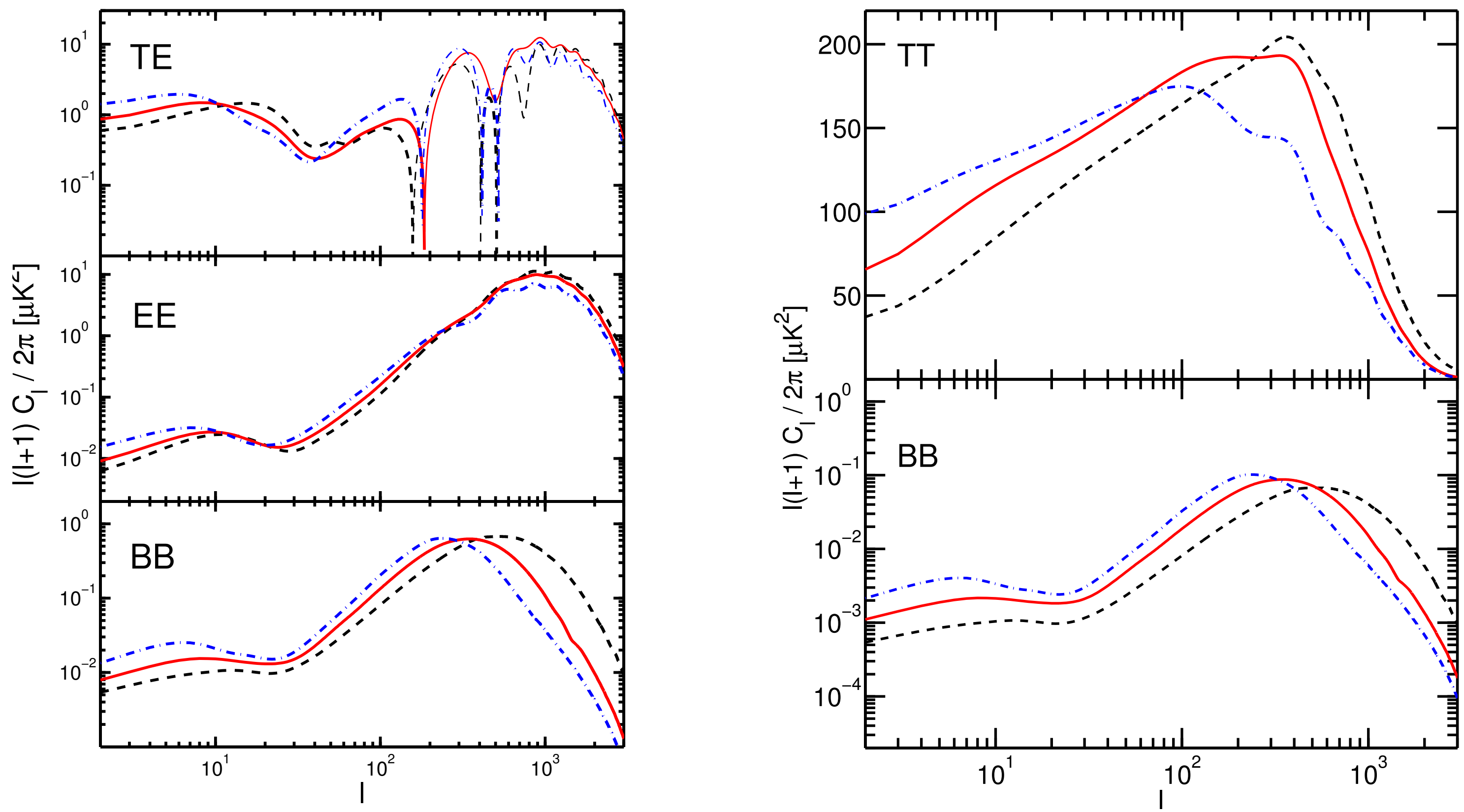
	$G\mu_{10}$
Abelian Higgs string	2.7×10^{-6}
Semilocal string	5.3×10^{-6}
Textures	4.5×10^{-6}



Defects normalised to WMAP3 ($\ell = 10$)[a]

[a] Urrestilla, Bevis, Hindmarsh, Kunz, Liddle (2008)

Polarisation from gauge strings, textures, and semilocal strings



Defects normalised to WMAP3 ($\ell = 10$) Defects normalised to 95% c.l. upper bound

Results: WMAP3, ACBAR, BOOMERANG, CBI and VSA

SL: Semilocal strings

TX: textures

AH: Abelian Higgs strings

HKP: Gaussian prior on H_0 from Hubble Key Project

BBN: Gaussian prior on $\Omega_b h^2$ from Big Bang Nucleosyntheis

95% upper bounds on f_{10}, $G\mu$ and n_s for standard 6-parameter cosmology + X

	CMB only			CMB+BBN+HKP		
Model	f_{10}	$G\mu$	n_s	f_{10}	$G\mu$	n_s
SL	0.25	2.6×10^{-6}	1.09	0.14	2.0×10^{-6}	1.01
TX	0.33	2.5×10^{-6}	1.14	0.16	1.8×10^{-6}	1.02
AH	0.17	1.1×10^{-6}	1.06	0.10	0.9×10^{-6}	1.00

Results preview

8 parameter model (Standard Cosmology + $G\mu$ + Sunyaev-Zeldovich)

MCMC fit to CMB data (WMAP7+QUAD+ACBAR)

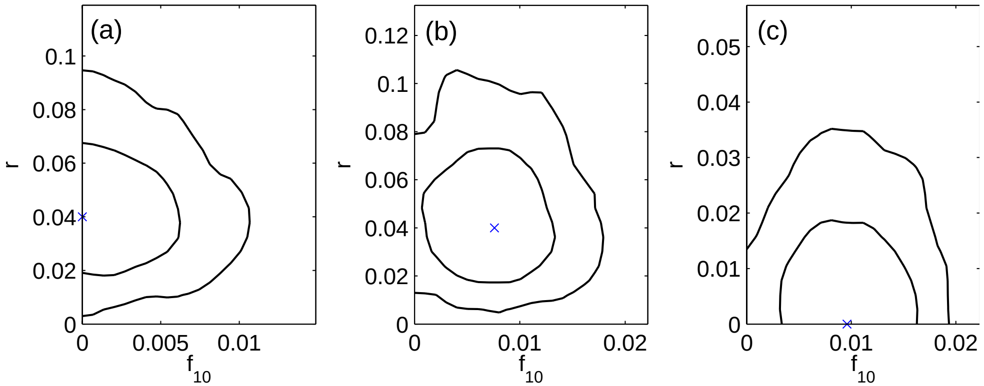
Cosmic string perturbations from Classical Abelian Higgs model[a]

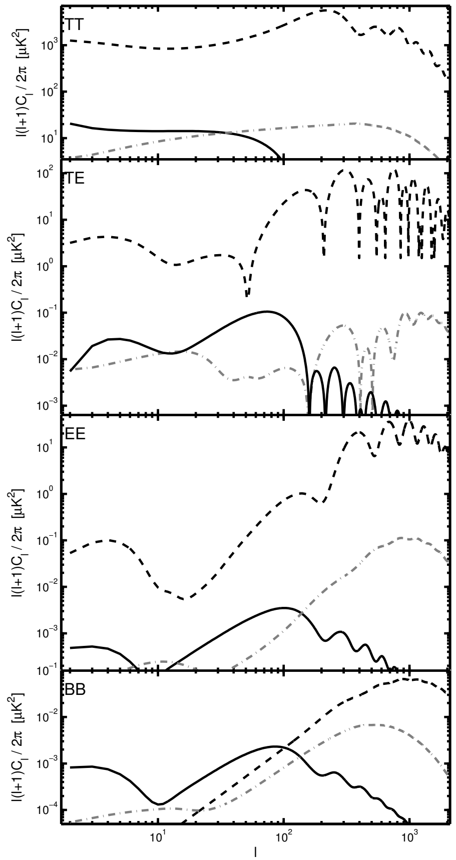
$f_{10} < 0.056$ (95%) $G\mu < 0.58 \times 10^{-6}$ (95%)

[a] Bevis et al. (2010); Bevis et al (in preparation)

Planck: distinguishing defects & tensors

- Inflation - B-modes from gravitational waves
- Defects - B-modes from vector modes
- Parameters: $r = \frac{|A_{\text{tensor}}|^2}{|A_{\text{scalar}}|^2}$, $f_{10}^{\text{string}} = \frac{C_{10}^{\text{TT,string}}}{C_{10}^{\text{TT,total}}}$
- Planck can distinguish, $f_{10} \gtrsim 0.02$[a]





scalar, black-dashed

$r = 0.04$, solid

$f_{10} = 0.01$, grey dot-dash

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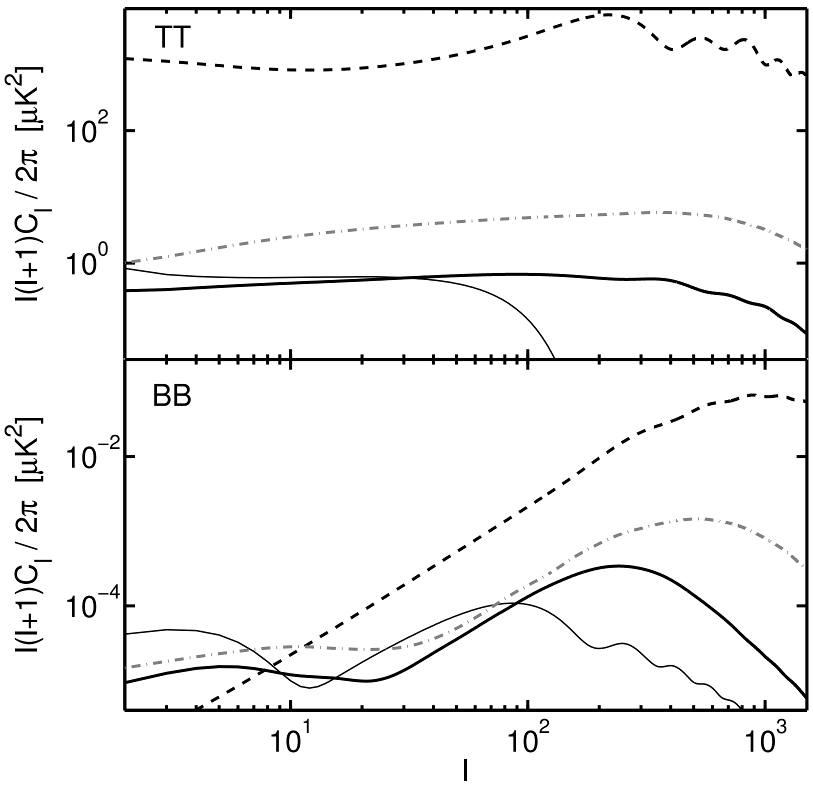
[a] Urrestilla et al. 2008

Future B-mode satellite: detecting strings/textures/tensors

- Threshold detection, assuming true model known (3σ):[a] [for CMBpol. COrE similar]
  - $f_{10}^{\text{string}} \gtrsim 0.0012$ ($G\mu = 7 \times 10^{-8}$)
  - $f_{10}^{\text{texture}} \gtrsim 0.0005$ ($G\mu = 1.0 \times 10^{-7}$)
  - $r \gtrsim 0.0018$
- No foreground, iterative delensing:
  - $f_{10}^{\text{string}} \gtrsim 10^{-4}$ ($G\mu \simeq 8 \times 10^{-9}$)[b]



scalar (black dashed),
tensor (black thin), $r = 0.0018$
cosmic strings (grey dot-dashed), $f_{10}^{\text{string}} = 0.0012$
textures (black thick) $f_{10}^{\text{texture}} = 0.0005$

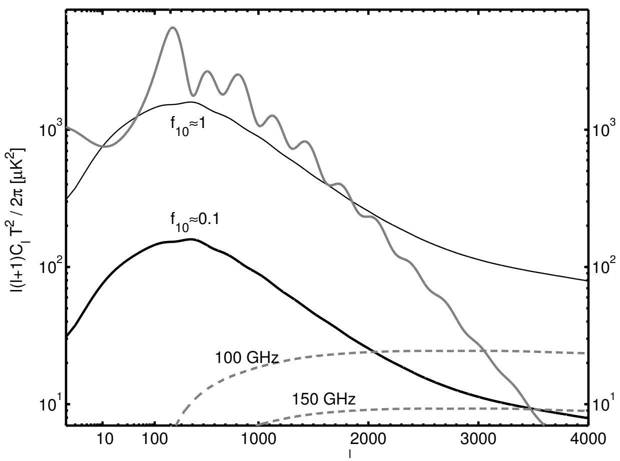
[a] Mukherjee et al (2010)

[b] Seljak, Slosar (2006); Garcia-Bellido et al (2010)

Cosmic string CMB power spectrum at small angular scales

- Inflationary scalar fluctuations damped at high ℓ
- Strings have ℓ^{-1} behaviour for $\ell > 3000$[a]
- Sunyaev-Zeldovich dominates both at high ℓ
- SZ may be removable - frequency dependence

[a] Hindmarsh (1994); Fraisse et al (2007); Pogosian, Tye, Wyman (2009); Bevis et al (2010); Yamauchi et al (2010)



Bevis et al (2010)

Cosmic string CMB bi & trispectrum at small angular scales

Temperature fluctuation $\Theta_{\mathbf{k}}$, flat sky, 2D wave vector $\mathbf{k}$: small angle approx

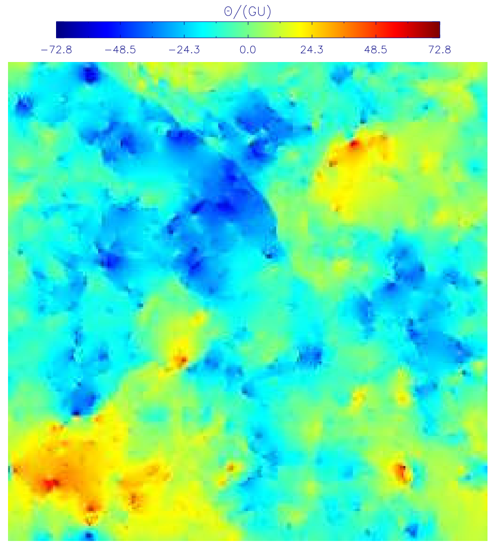
$$\langle \Theta_{\mathbf{k}_1} \Theta_{\mathbf{k}_2} \Theta_{\mathbf{k}_3} \rangle = b_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3} (2\pi)^2 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3).$$

$$\langle \Theta_{\mathbf{k}_1} \Theta_{\mathbf{k}_2} \Theta_{\mathbf{k}_3} \Theta_{\mathbf{k}_4} \rangle = T_{\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3,\mathbf{k}_4} (2\pi)^2 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 + \mathbf{k}_4).$$

Bi & trispectrum from Gott-Kaiser-Stebbins effect only[a]

- $\left[\frac{\ell(\ell+1)}{2\pi})\right]^{3/2} b_{\ell\ell\ell} \simeq (-0.5)\,(G\mu)^3 (\frac{1000}{\ell})^{-3}$,
- GKS bispectrum negative[b], $O(C_\ell^{3/2})$
- $T_{\ell\ell\ell\ell} \sim (G\mu)^4 \ell^{-\rho}$, with $6 < \rho(v_{\rm rms}) < 7$
- GKS trispectrum $O(C_\ell^2)$
- Must include fluid for $\ell \lesssim 3000$

Fraisse et al (2007);



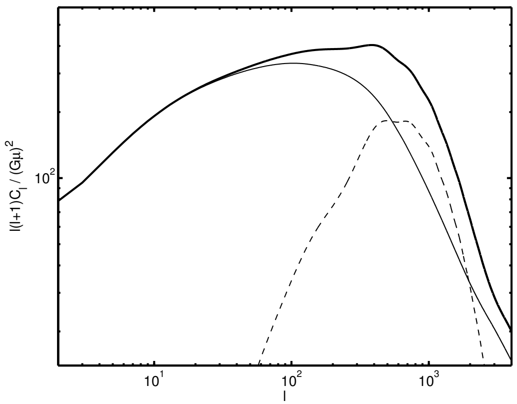
Size 7.2°, resol'n $0.41'$

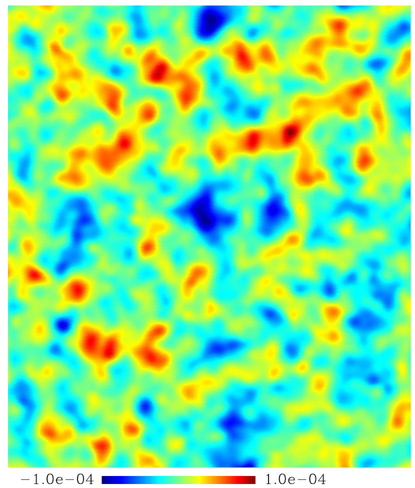
[a] Hindmarsh, Ringeval, Suyama (2009,10); Regan, Shellard (2009)

[b] Negative skewness Fraisse et al (2007); Yamauchi at al (2010)

Cosmic string CMB non-Gaussianity $100 < \ell < 3000$

- $C_\ell s$ dominated by last scattering (right).[a]
- GKS tricks don't work for fluid
- String CMB maps needed[b]
- e.g. 3° (right)
- GKS "edges" smeared by fluid response
- Skewness positive





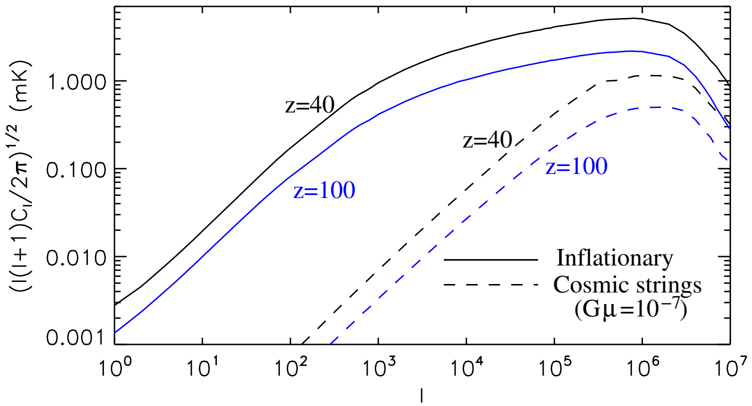
[a] Bevis et al (2010)

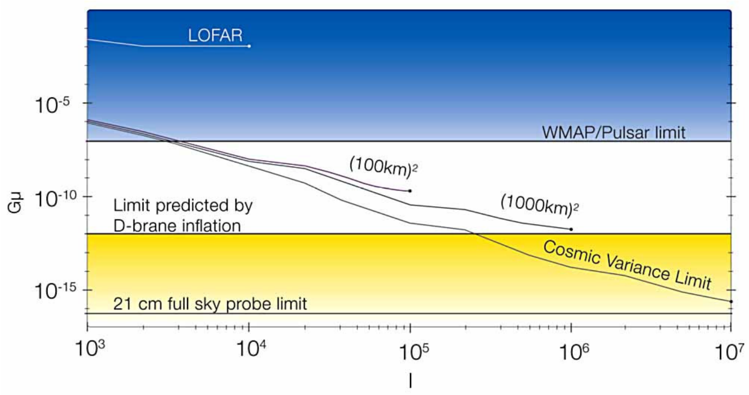
[b] Landriau & Shellard 2010

Strings and 21cm radiation background

- Strings add to 21cm power spectrum[a]
  - Need huge collecting area: $\gg 1$ km^2
- Weakly cross correlated with CMB[b]
  - X-correlation also needs huge collecting area to see $G\mu \sim 5 \times 10^{-7}$
- Cosmic string wakes observable at $z \sim 30$ with $G\mu \simeq 6 \times 10^{-7}$[c]

Khatri & Wandelt 2008:





[a] Khatri, Wandelt (2008)
[b] Berndsen, Pogosian, Wyman (2010)
[c] Brandenberger et al (2010)

Conclusions

- Well-motivated inflation models produce defects
- (Inflation + defect) cosmological models have one extra parameter, $G\mu$
- CMB indicates that $G\mu \lesssim (5 \times 10^{-7}) - 10^{-6}$
- Future:

  CMB: Planck, CMBpol/COrE/ground-based B-modes

  Gravitational radiation (strings): AdvLIGO, LISA

  Cosmic rays (strings): Fermi

  Lensing (strings): SKA (Compact Radio Sources)

  21cm ?(strings): SKA