Estimates of f_{NL} and g_{NL} from massive high redshift clusters

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With K. Enqvist and O. Taanilla

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HELSINKI INSTITUTE OF PHYSICS

See also: Cayon, Gordon, Silk
Hoyle, Jimenez, Verde

Measuring non-Gaussianity

We can measure/constrain non-Gaussianity by:

- Bispectrums - strictly zero for Gaussian, so very sensitive to deviations.
  - CMB $-10 < f_{\rm NL}^{\rm local} < 74$
  - LSS not yet competitive
- Correction to LSS bias. $-29 < f_{\rm NL}^{\rm loc} < +69$
- The mere existence of large mass, high-redshift clusters.
  - Very sensitive to tails of the distribution which are very sensitive to f_{nl}, g_{nl}, etc.

$$f_{\rm NL} \gtrsim 411 \,!?$$

Summary and motivation

- There exist a number of high redshift clusters that are very unlikely in LCDM.
- Non-Gaussianities can explain the existence of these clusters.

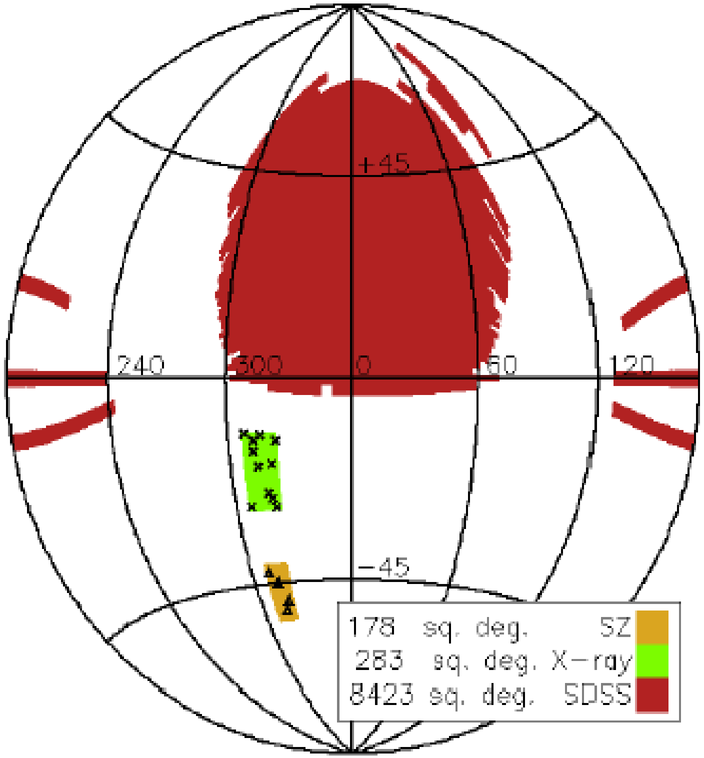
- Current f_{NL} constraints apply mass functions too far into their tails.
- g_{NL} is a higher order statistic but has attractions that distinguish it from f_{NL}

(e.g. voids, no strong scale dependence required, same number of extra parameters required)

These clusters **do** exist. They **are** "too big too early" for LCDM.

(subject only to systematic uncertainties on mass measurements by three independent methods)

getting perspective



Hoyle, Jimenez, Verde

Yellow = SZ (178 sq. deg.)

Green = Xray (283 sq. deg.)

For context:

Red = SDSS (8423 sq. deg)

White = total sky (62000 sq. deg.)

A very positive slide! (i.e. this is all very easily tested in the future)

What are these clusters?

Hoyle, Jimenez, Verde

Cluster Name	Redshift	M_{200} $10^{14}M_{\odot}$
'WARPSJ1415.1+3612' +	1.02	$3.33^{+2.83}_{-1.80}$
'SPT-CLJ2341-5119' *	1.03	$5.40^{+2.80}_{-2.80}$
'ClJ1415.1+3612' *	1.03	$3.40^{+0.60}_{-0.50}$
'XLSSJ022403.9-041328' +	1.05	$1.66^{+1.15}_{-0.38}$
→'SPT-CLJ0546-5345' *	1.06	$10.0^{+6.00}_{-4.00}$
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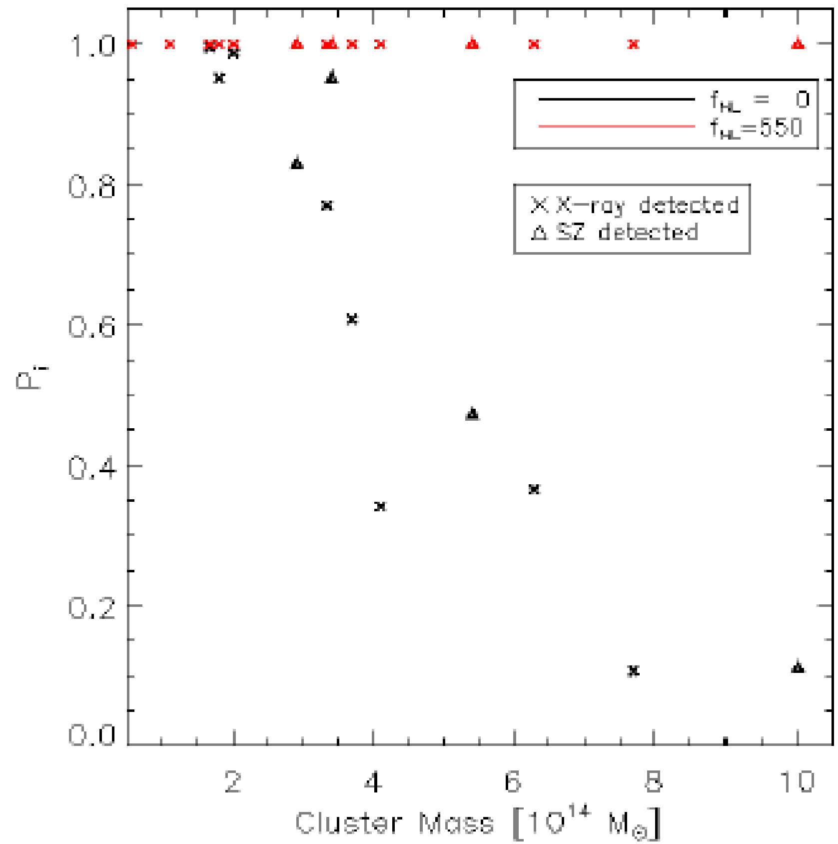
'SXDF-XCLJ0218-0510' +	1.62	$0.57^{+0.14}_{-0.14}$

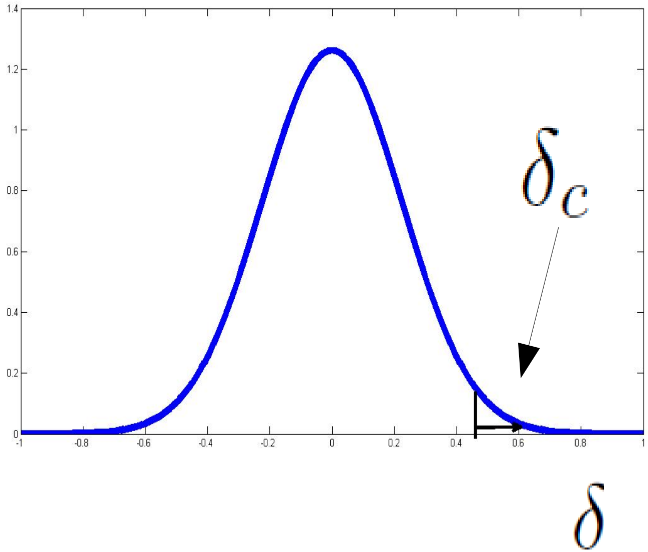
Masses in table from three different methods



Redshift $1.02 \le z \le 1.62$

Mass $0.57 \times 10^{14} M_{\odot} \le m \le 1.0 \times 10^{15} M_{\odot}$

How do we calculate Probability?



What is the probability δ is above δ_c ?

$$n(M,z) = \frac{\bar{\rho}}{M} f\left(-\frac{\mathrm{d}\ln\sigma_M}{\mathrm{d}\ln M}\right)$$

f is the interesting dimensionless quantity.

$$\sigma_R^2 = \int_0^\infty \frac{dk}{k} \alpha_R^2(k,z) \mathcal{P}(k),$$

$$\alpha_R(k,z) = \frac{2}{3\Omega_m} D(z) \left(\frac{k}{H_0}\right)^2 T(k) W_R(k)$$

$$\mathrm{Exp}(M,z,\ldots) = \int_{z_n}^{z_f} dz \int_{M_{\min}}^{M_{\max}} dM \, f_{\mathrm{sky}} \frac{dV}{dz} n_{\mathrm{NG}}$$

Now, Poisson sample from E.

How do we calculate f? (theory...)

The truth is it isn't easy.

Spherical collapse. (but what about 'cloud in cloud'?)

Excursion set – find first crossing. (but what about the choice of filter?)

Non Markovian excursion set. (what about other collapsing shapes?)

Treat δ_c stochastically to model this.

This is all before even starting to look at non-Gaussianity.

Eventually...

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Eventually... See Aseem's talk.

Status at f_{sky}=11 sq. deg.

Only one unlikely, high redshift, cluster. The wonderfully named,

XMMUJ2235.3+2557,

At redshift z=1.4, with mass = $(8.5 \pm 1.7) \times 10^{14}\ M_{\odot}$ (at the time)

$$\mathrm{Exp}(M, z, \ldots) = \int_{z_n}^{z_f} dz \int_{M_{\mathrm{min}}}^{M_{\mathrm{max}}} dM\, f_{\mathrm{sky}} \frac{dV}{dz} n_{\mathrm{NG}}$$

Probability of something existing at this redshift and mass or higher, in an 11 sq. deg. survey...

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$\sim 5 \times 10^{-3}$ Jee et al.

and now..? (with f_{sky} ~180-280)

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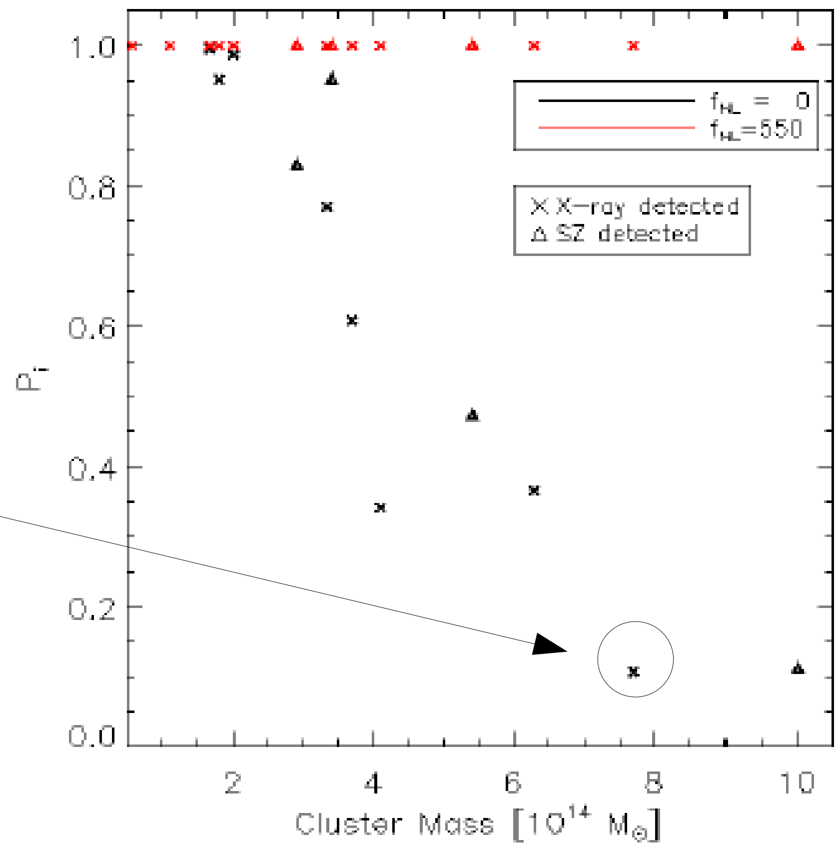

Probability is now ~0.1

However, what about all these new clusters?

and now..? (with f_{sky} ~180-280)

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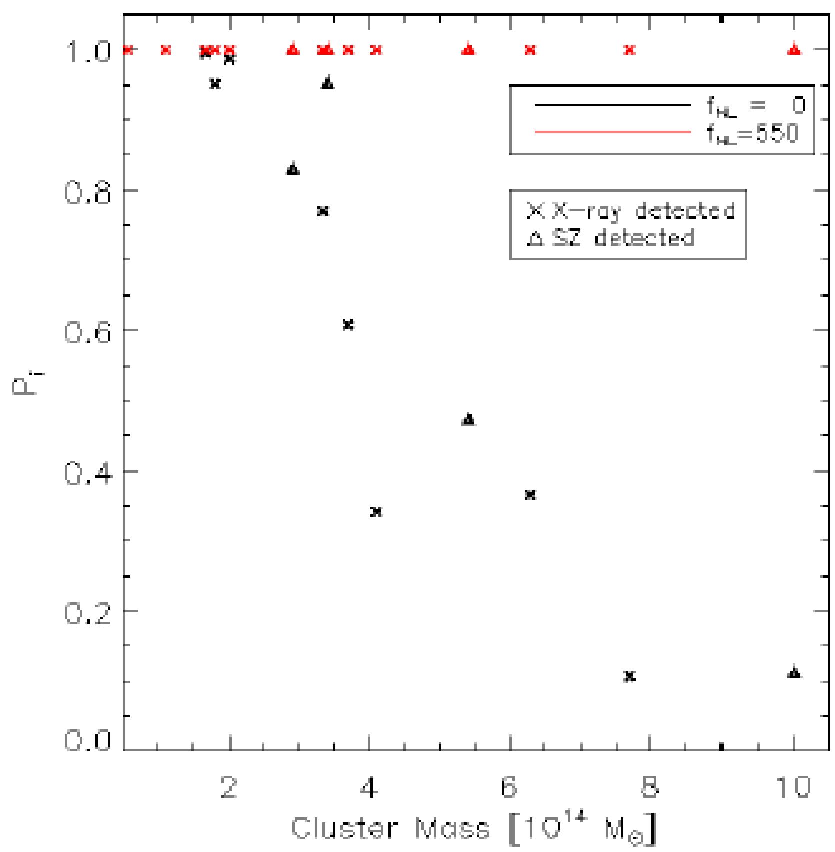
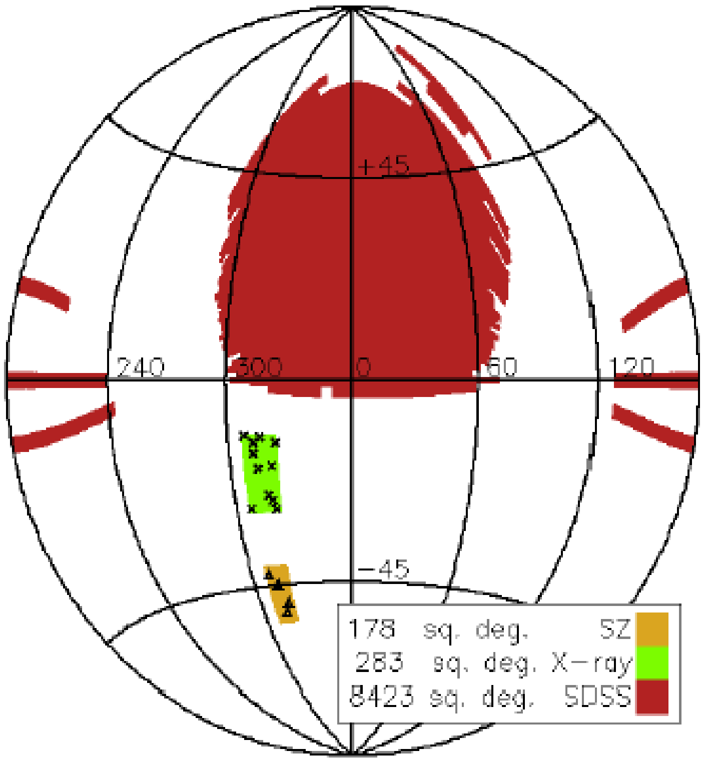
Total probability = 3×10^{-4} !



Only 15 clusters, but collectively they are very unlikely.

getting perspective



With just:

yellow = SZ (178 sq. deg.),

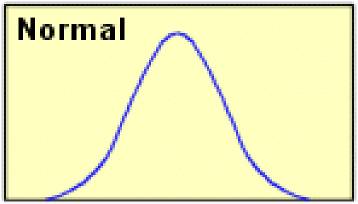
green = Xray (283 sq. deg.),

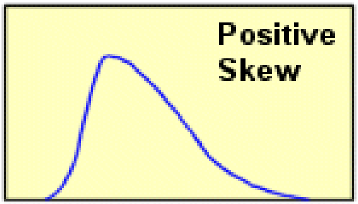
the significance is 3×10^{-4} .

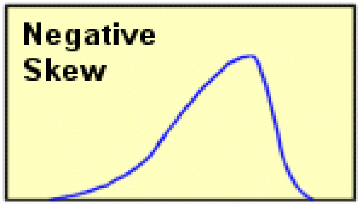
Caveats:

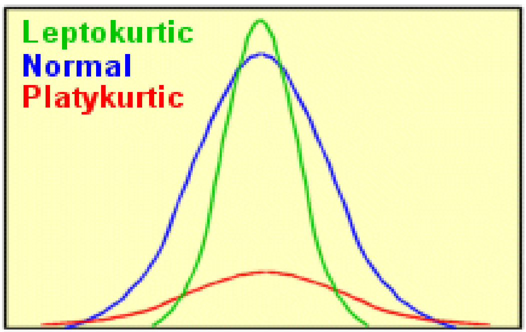
- Systematic mass errors?
- How do we pick where to look?
- 'independent' mass measurements?

Effect of NG on cluster formation









$$\Phi(\mathbf{x}) = \phi(\mathbf{x}) + f_{\rm NL}\left[\phi^2(\mathbf{x}) - \langle\phi^2\rangle\right] + g_{\rm NL}\phi^3(\mathbf{x})$$

+fnl ⟶ +skewness (S3)

+gnl ⟶ +kurtosis (S4)

+skewness ⟶ positive tail increases
negative tail decreases

+kurtosis ⟶ positive tail increases
negative tail increases

and vice versa (+fnl cannot explain voids!)

Quantitative effects of NG

- Theoretical Gaussian mass functions are getting better and better.
- Until recently weren't good enough, so typical method...

$$\mathcal{R}\left(M, z, f_{\rm NL}, g_{\rm NL}\right) = \frac{n_{\rm analytical}(M, z, f_{\rm NL}, g_{\rm NL})}{n_{\rm analytical}(M, z, f_{\rm NL}=0, g_{\rm NL}=0)}$$

$$\mathcal{R} = \exp\left(\delta_{ec}^3 \frac{S_3(M, f_{\rm NL})}{6\sigma_M^2}\right)\left\{\frac{1}{6}\frac{\delta_{ec}}{\delta_3}\frac{dS_3}{d\ln\sigma} + \delta_3\right\} \exp\left(\delta_{ec}^4 \frac{S_4(M, g_{\rm NL})}{24\sigma_M^2}\right)\left\{\frac{1}{24}\frac{\delta_{ec}^2}{\delta_4}\frac{dS_4}{d\ln\sigma} + \delta_4\right\}$$

- Sounds dodgy.... is dodgy... but tested against N-body simulations.
- In the approximate limit (large mass, high redshift) also matches best theory (see Aseem's talk).

Method used to constrain f_{NL}

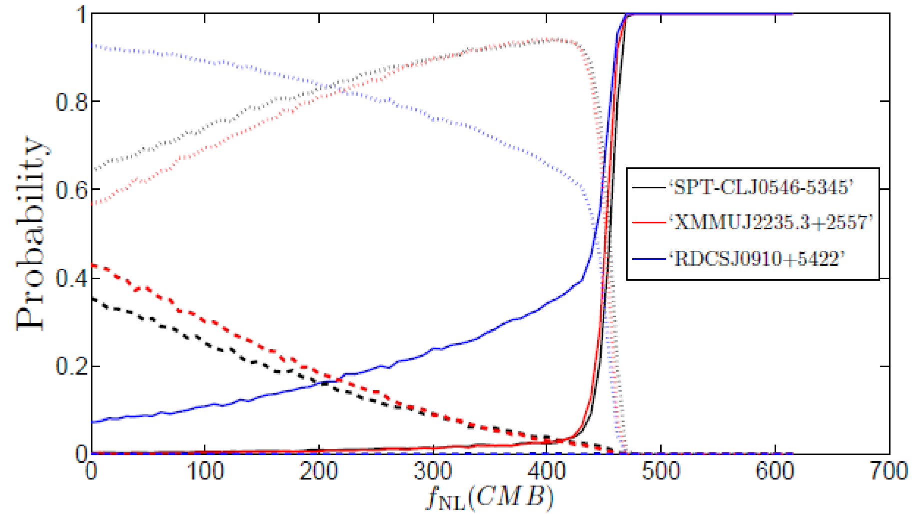
Cayon, Gordon, Silk and Hoyle, Jimenez, Verde

$$\mathrm{Exp}(M, z, \dots) = \int_{z_n}^{z_f} dz \int_{M_{\min}}^{M_{\max}} dM\, f_{\mathrm{sky}} \frac{dV}{dz} n_{\mathrm{NG}}$$

XMMUJ2235.3+2557 $\quad 7.7^{+4.4}_{-3.1} \times 10^{14} M_{\odot}$

Three mass bins

- $M < 4.6 \times 10^{14}$
- $4.6 \times 10^{14} < M < 12.1 \times 10^{14}$
- $12.1 \times 10^{14} < M$



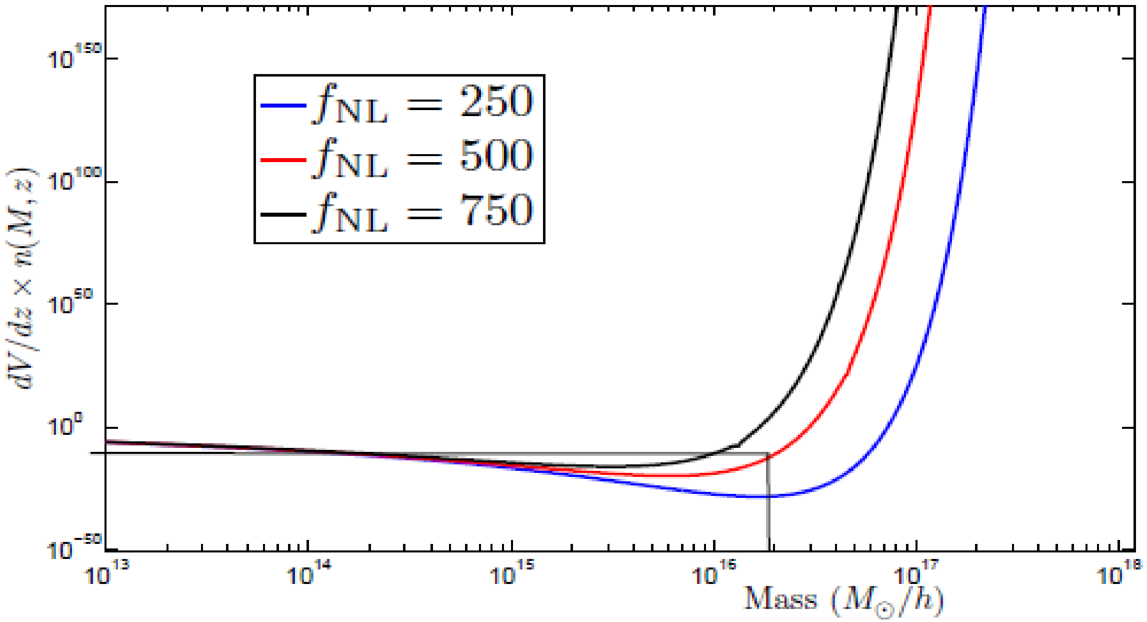
What is the probability that `most massive' cluster is in each bin?

Integrate to ∞? (be careful)

$$\mathrm{Exp}(M,z,\ldots) = \int_{z_n}^{z_f} dz \int_{M_{\min}}^{M_{\max}} dM\, f_{\mathrm{sky}} \frac{dV}{dz} n_{\mathrm{NG}}$$

$$\nu = \delta_{ec}/\sigma_M$$

$$\mathcal{R}_{NG} \Rightarrow \exp\left(\frac{\nu^3}{6}(\sigma S_3)\right)$$

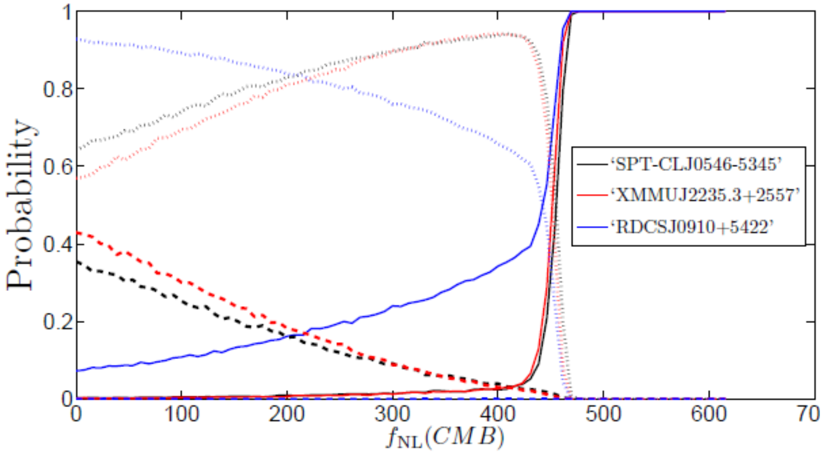


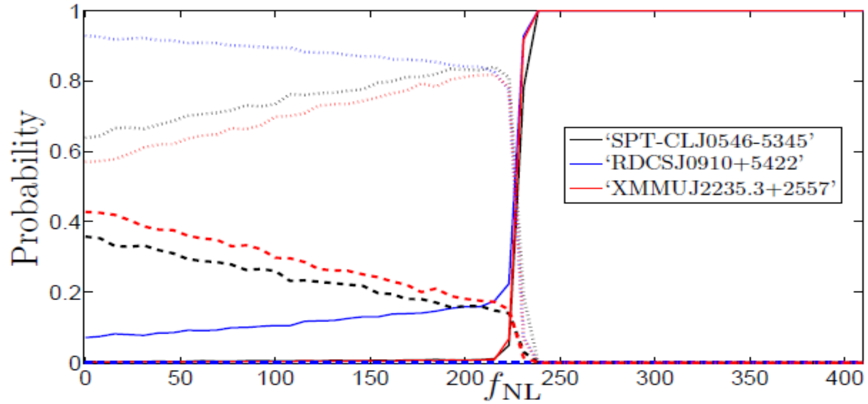
If Mmax < Mturn some of the probability is unphysical.

Once Mmax << Mturn almost all the probability is unphysical.

But, if Mmax > Mturn results can be trusted.

Choose your fnl?





Same calculation (different cutoffs)

- Looks bad, yes, but underneath a critical fnl, these are actually identical!
- In general, underestimated fnl

We know this shouldn't happen. What is going wrong? Is it actually R?

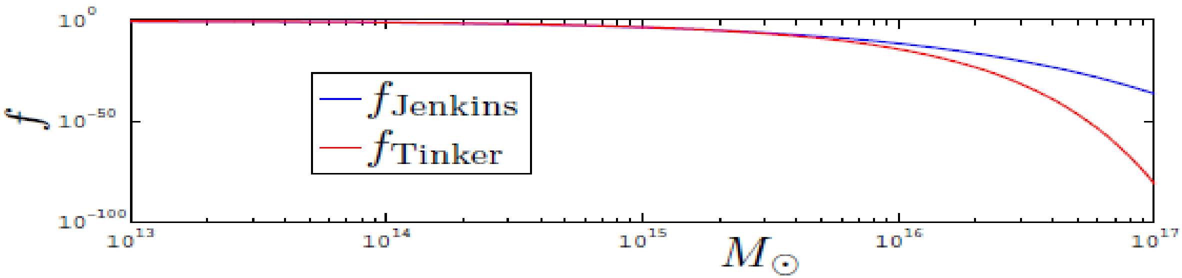
Gaussian mass function comparison

What should the asymptotic behaviour be? $f \sim \exp(-c/\sigma^2)$

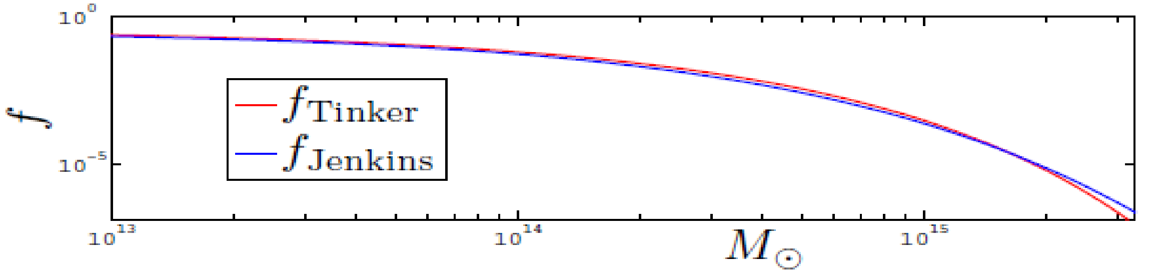
Jenkins et. al (?) $f = 0.301 \exp\left[-|\ln \sigma_M^{-1}(z) + 0.64|^{3.82}\right]$

Tinker et. al. (yes) $f(\sigma) = A\left[\left(\frac{\sigma}{b}\right)^{-a} + 1\right] e^{-c/\sigma^2}$

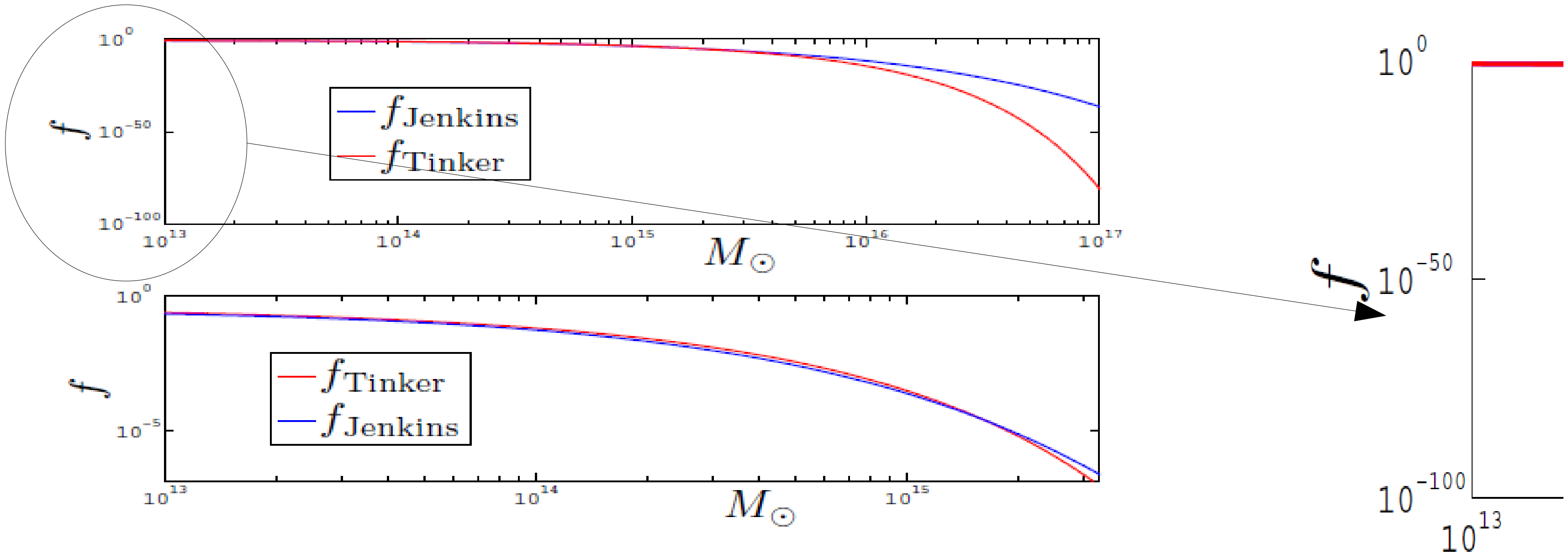
Shows how important it is to have a good understanding of expectations



Those responsible for the theoretical mass functions must enjoy the irony

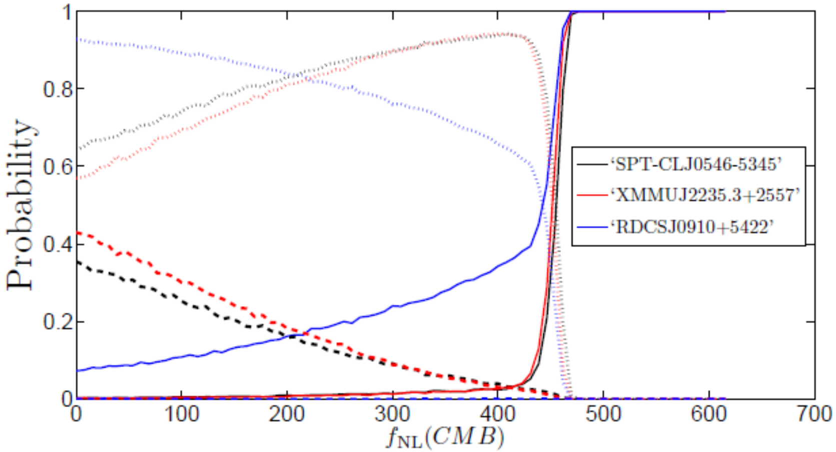


Y-axis scale

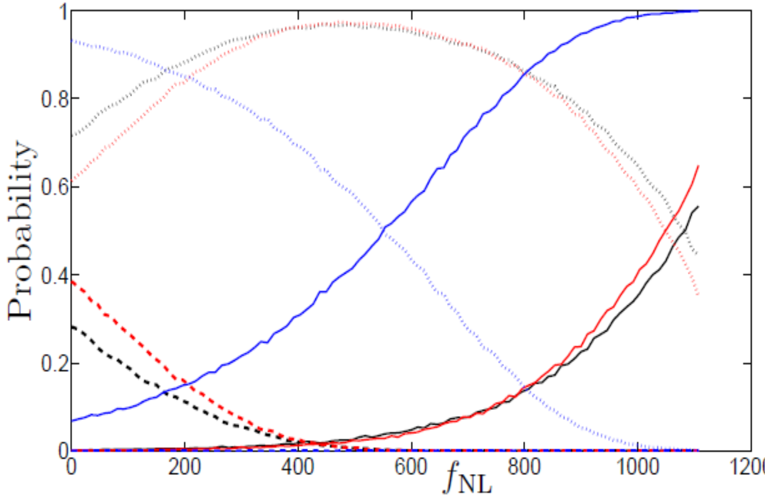


At large masses and redshifts, the difference is big

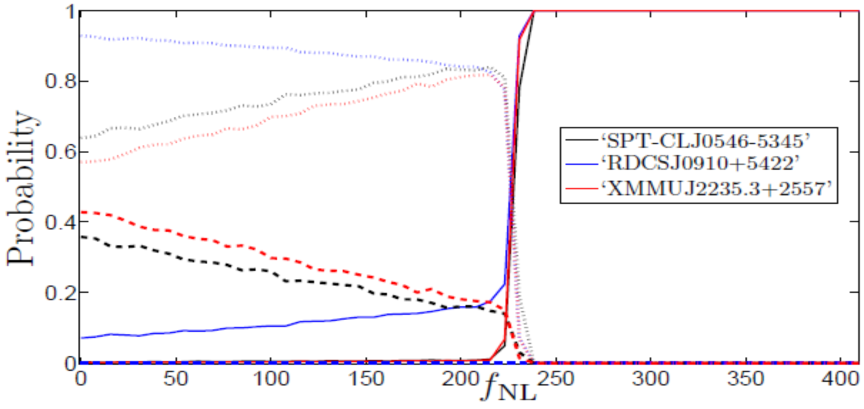
Choose your fnl?



Jenkins et. al. (different cutoffs)

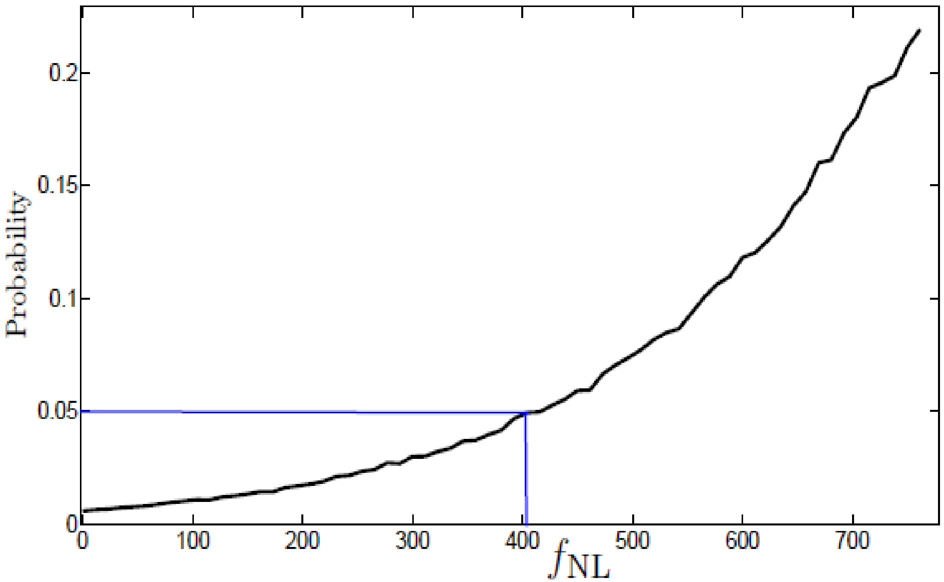


Tinker et al. (no cutoff dependence)



- Looks bad, yes, but underneath a critical fnl, these are **still** identical!
- In general, cutoff effect underestimates fnl

A lower bound for fnl



- Treat mass distribution of each cluster as log-normally distributed.
- For each cluster sample over masses and ask "what is the probability that this cluster could exist?"
- The final probability (plotted) is the product of all 15 cluster probabilities.

$$f_{NL} \gtrsim 411$$

This is similar to Hoyle, Jimenez, Verde

$f_{NL} > 478$

$$-10 < f_{NL}^{\mathrm{local}} < 74$$ → Lower bound ~ 6 times larger than upper bound

Calculating g_{NL}, (and why not h_{NL})?

What about bounds on g_{NL}?

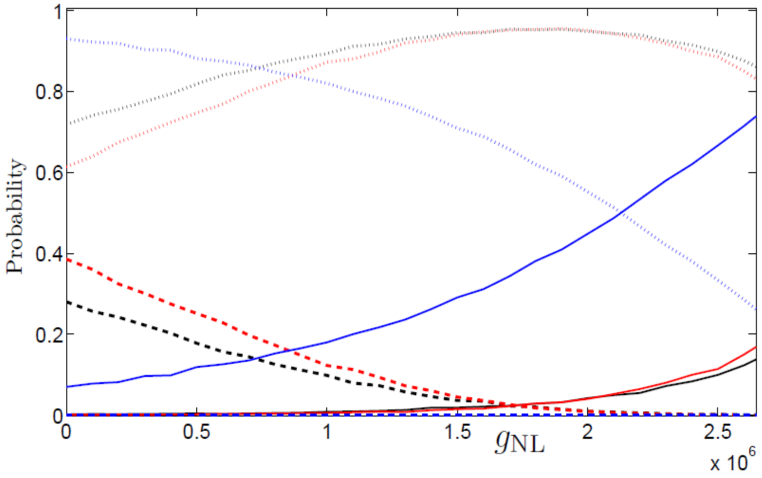
- The calculation is exactly the same as for fNL, except we use the kurtosis, S4, instead of the skewness, S3.
- We assume f_{NL} is negligible ($f_{NL} < 50$ is small enough)

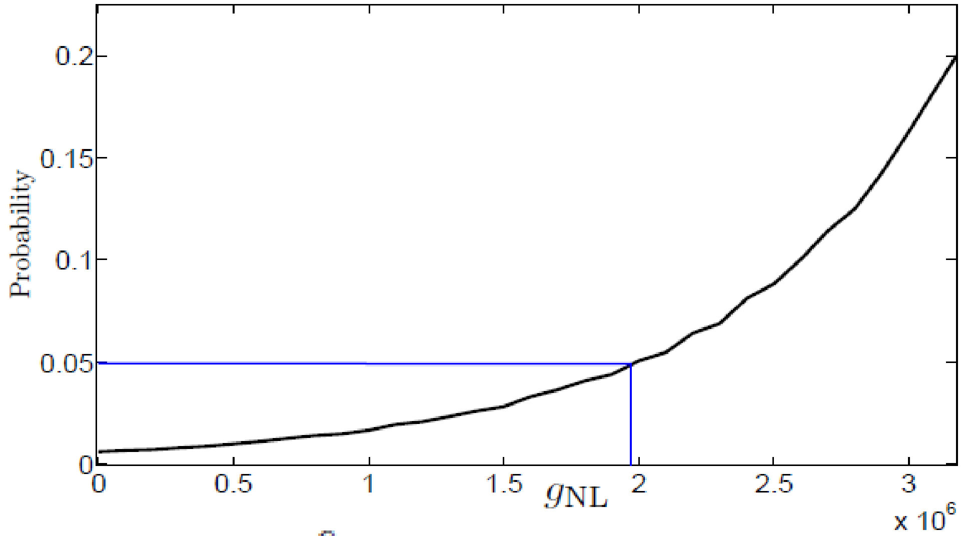
Why g_{NL}?

- **We've already introduced one extra parameter for f_{NL}, the running.**
- g_{NL}, is the next in the expansion.
- Normally, $g_{NL} \sim f_{NL}^2$ and $h_{NL} \sim f_{NL}^3$. To get around this in theory requires complexity (as does large scale dependence on f_{NL}). The higher the term in the expansion, the more complexity...
- F_{NL}, h_{NL} make voids less likely, g_{NL} makes them more likely.

g_{NL} results

For the same calculations as fNL…





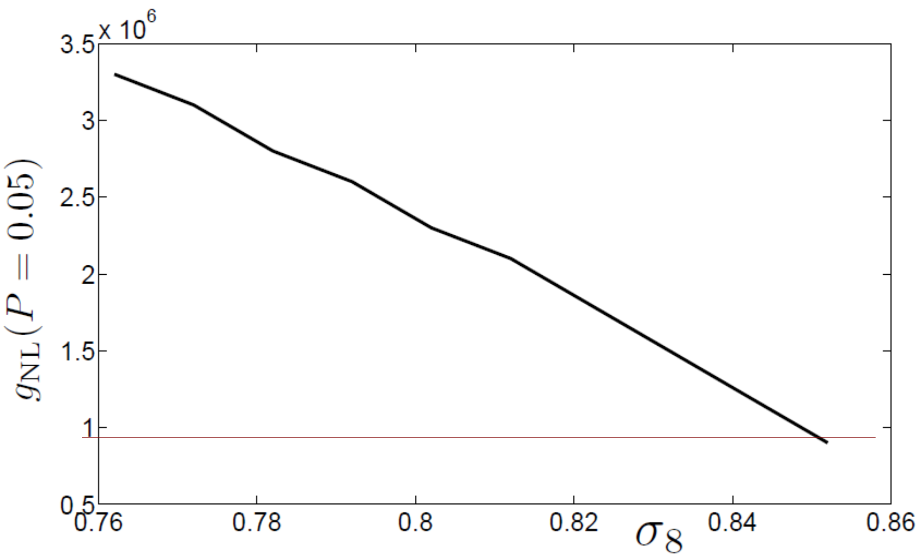
$$g_{NL}|_{P=0.05} > 2.0 \times 10^6$$

$$-3,5 \times 10^5 < g_{NL} < 8.2 \times 10^5$$

Lower bound ~ 2 times larger than upper bound.

σ_8 degeneracy

What if we change σ_8 and ask the question again?



- Looks promising. Both parameters are within WMAP bounds without running
- We need to be careful before jumping to conclusions though. WMAP bound will be similarly dependent on σ_8.

Future prospects

- Find more clusters! (or more sky without them)
- Marginalise over cosmology without cutoff contributing
- fnl and gnl bounds as function of sigma8 for other non-Gaussianity probes
- More accurate gnl
- N-body to values of mass and redshift that probe far enough into the tail
- Look at voids

- Other explanations (step in primordial spectrum, change in expansion history)
- Telling the difference between fnl, gnl, step, history...

Summary and motivation

- There exist a number of high redshift clusters that are very unlikely in LCDM.
- Non-Gaussianities can explain the existence of these clusters.

- Current f_{NL} constraints apply mass functions too far into their tails.
- g_{NL} is a higher order statistic but has attractions that distinguish it from f_{NL}

(e.g. voids, no scale dependence required, same number of extra parameters required)

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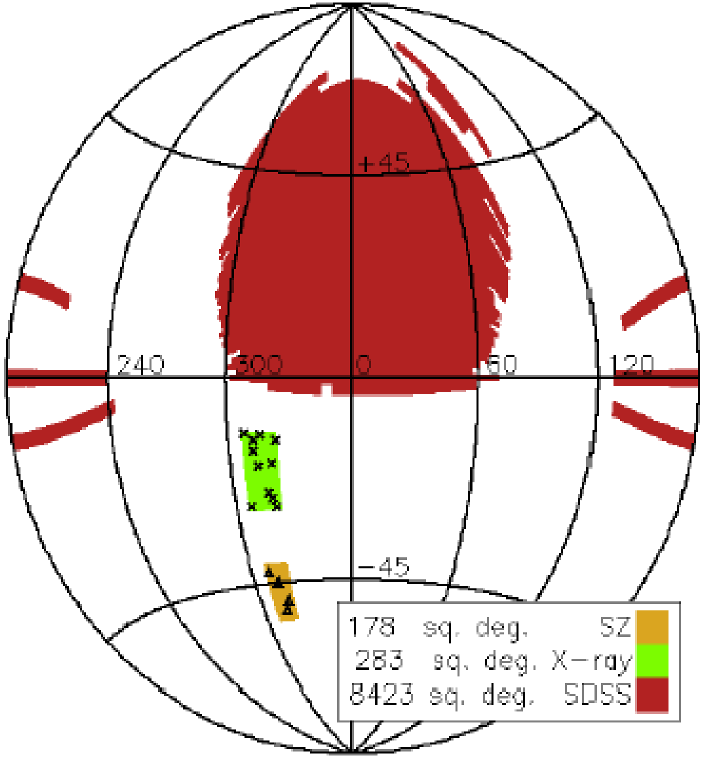
(subject only to systematic uncertainties on mass measurements by three independent methods)

End


+45
240
300
0
60
120
−45
178 sq. deg. SZ
283 sq. deg. X−ray
8423 sq. deg. SDSS