Delving the symmetries of CMB correlation function

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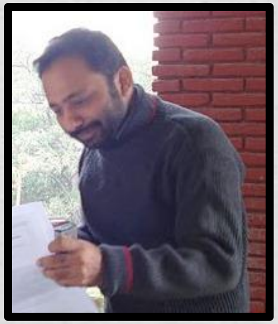

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astro-ph/0912.3217

Outline

1) Motivation- Evidence of Statistical Isotropy violation?

2) An Introduction to Bipolar formalism.

3) Symmetries and Bipolar space.

4) Bipolar coefficients- SI and non-SI models.

5) Conclusion.

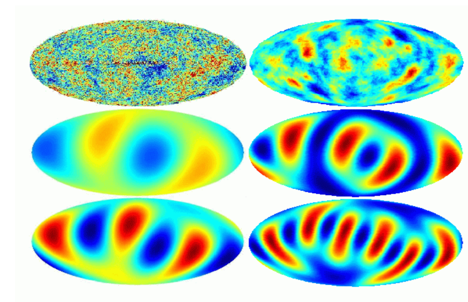
Is statistical isotropy broken?

Evidences of SI Breakdown ?

WMAP anomalies (deviations from isotropy- Inconsistent with standard cosmological model)

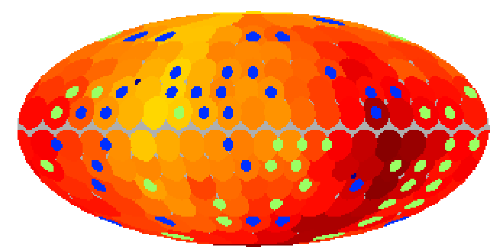
Low multipole Peculiarities

- Peculiar alignment of quadrupole & octupole along a preferred axis. (Tegmark et al. 2003; de Oliveira-Costa et al. 2004; Schwarz et al. 2004; Copi et al. 2006).
- Indications of preferred axis(Land & Magueijo 2005)
- Suppression of power in low l multipoles(Bennett et al 1992,2010);

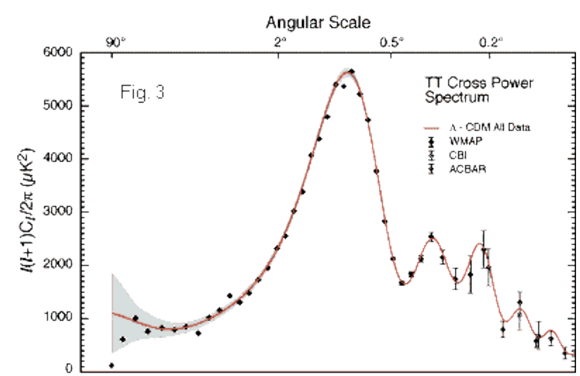


<u>North-south power asymmetry</u>

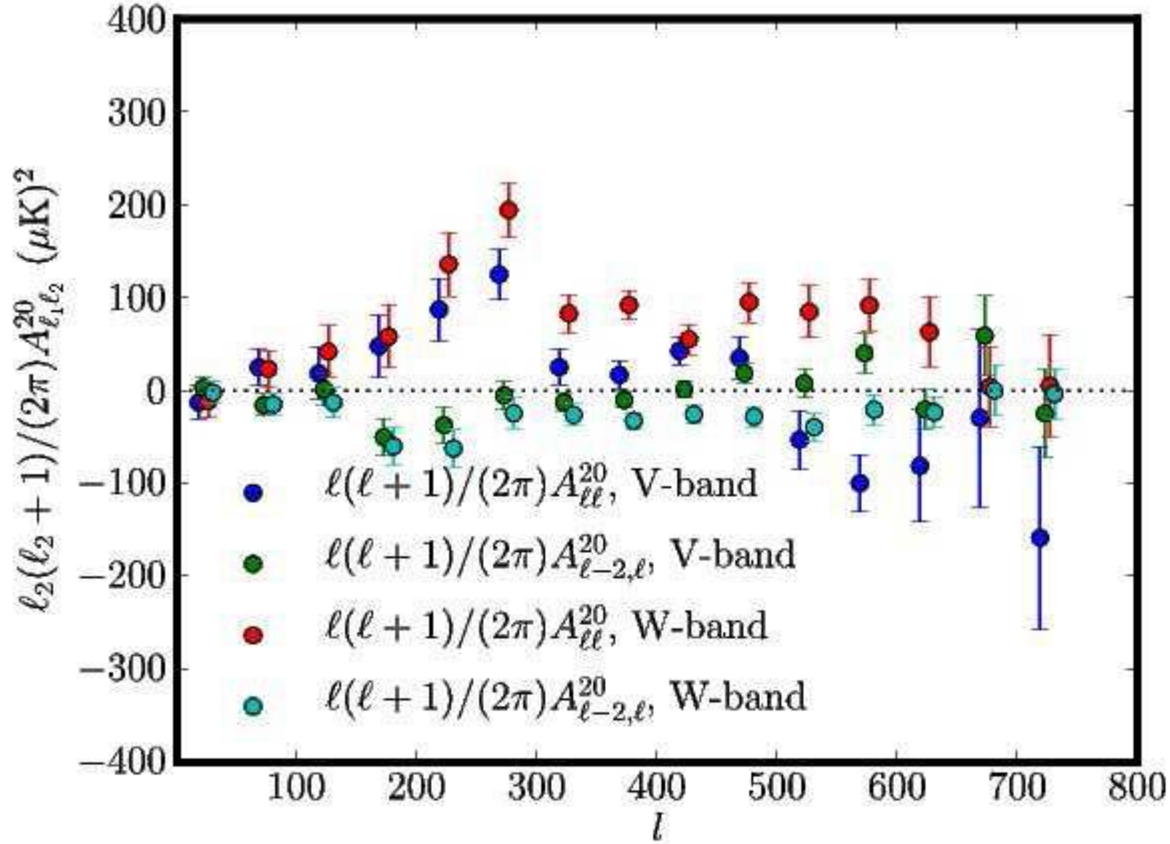
- *(*Eriksen , et al. 2004,2007, Hansen et al. 2004,2007, 2009 Larson & Wandelt 2004 , Park 2004,Hoftuft2009, Ackerman et al. 2007, Groeneboom et al. 2009).
- Axis of maximum asymmetry found to be close to ecliptic



- Outliers in wmap-seven year data(Bennett et al. 2010)



Statistical properties are <u>not</u> invariant under rotation of the sky ???



WMAP-seven year
Bennett et al. 2010

The quadrupolar bipolar power spectra, binned with l = 50, using the KQ75y7 mask.

Bipolar Spherical Harmonics

Basics of Bipolar Statistics

- Correlation is a two point function & can be expanded in bipolar spherical harmonics basis.

$$C(\hat{n}_1, \hat{n}_2) = \sum_{l_1, l_2, J, M} A^{JM}_{l_1 l_2} \{Y_{l_1}(\hat{n}_1) \otimes Y_{l_2}(\hat{n}_2)\}_{JM}$$

$A^{JM}_{l_1 l_2} \rightarrow$ Bipolar spherical harmonic coefficients(BipoSH)

$\{Y_{l_1}(\hat{n}_1) \otimes Y_{l_2}(\hat{n}_2)\}_{JM} \rightarrow$ Bipolar spherical harmonics

Convenient basis of expansion for functions depending on two vector directions

$$\{Y_{l_1}(\hat{n}_1) \otimes Y_{l_2}(\hat{n}_2)\}_{JM} = \sum_{m_1 m_2} C^{JM}_{l_1 m_1 l_2 m_2} Y_{l_1 m_1}(\hat{n}_1)\, Y_{l_2 m_2}(\hat{n}_2)$$

Recall $Y_{lm}(\hat{n})$

$|l_1 - l_2| \leq L \leq l_1 + l_2$ Triangularity conditions $M = m_1 + m_2$

Orthonormal functions for different set of l_1, l_2, J, M

1) Bipolar coefficients

Inverse transform

$$A^{JM}_{l_1 l_2} = \int d\Omega_{\hat{n}_1} \int d\Omega_{\hat{n}_2} C(\hat{n}_1, \hat{n}_2) \{Y_{l_1}(\hat{n}_1) \otimes Y_{l_2}(\hat{n}_2)\}^*_{JM}$$

$$a_{lm} = \int d\Omega_{\hat{n}} \Delta T(\hat{n}) Y_{lm}(\hat{n})$$

RECALL

$$A^{JM}_{l_1 l_2} = \sum_{m_1 m_2} (-1)^{m_2} < a_{l_1 m_1} a^*_{l_2 m_2} > C^{JM}_{l_1 m_1 l_2 - m_2}$$

Measures cross-correlation in spherical harmonic Coefficients.

Combining BipoSH in different ways to reduce Cosmic Variance.

2) Reduced Bipolar Coefficients- rBipoSH

Orientation Dependent

$$A_{LM} = \sum_{l_1=0}^{\infty} \sum_{l_2=|L-l_1|}^{L+l_1} A^{LM}_{l_1 l_2}$$

3) Bipolar Map

$$\theta(\hat{n}) = \sum_{LM} A_{LM} Y_{LM}(\hat{n})$$

4) Bipolar Power Spectrum

Recall $C_l = \frac{1}{(2l+1)} \sum_m |a_{lm}|^2$

Orientation- independent measure $\longrightarrow$ $$\kappa_L = \sum_{l_1,l_2,M} |A^{LM}_{l_1 l_2}|^2$$

$$\kappa_L = \left(\frac{2L+1}{8\pi}\right)^2 \int d\Omega_{\hat{n}_1} \int d\Omega_{\hat{n}_2} \left[\int dR\, \chi^L(R)\, C(R\hat{n}_1, R\hat{n}_2)\right]^2$$

A weighted average of the correlation function over all rotations

Characteristic Function of Rotation group $\longrightarrow$ $$\chi^L(R) = \sum_M D^L_{MM}(R)$$

Wigner rotation matrix

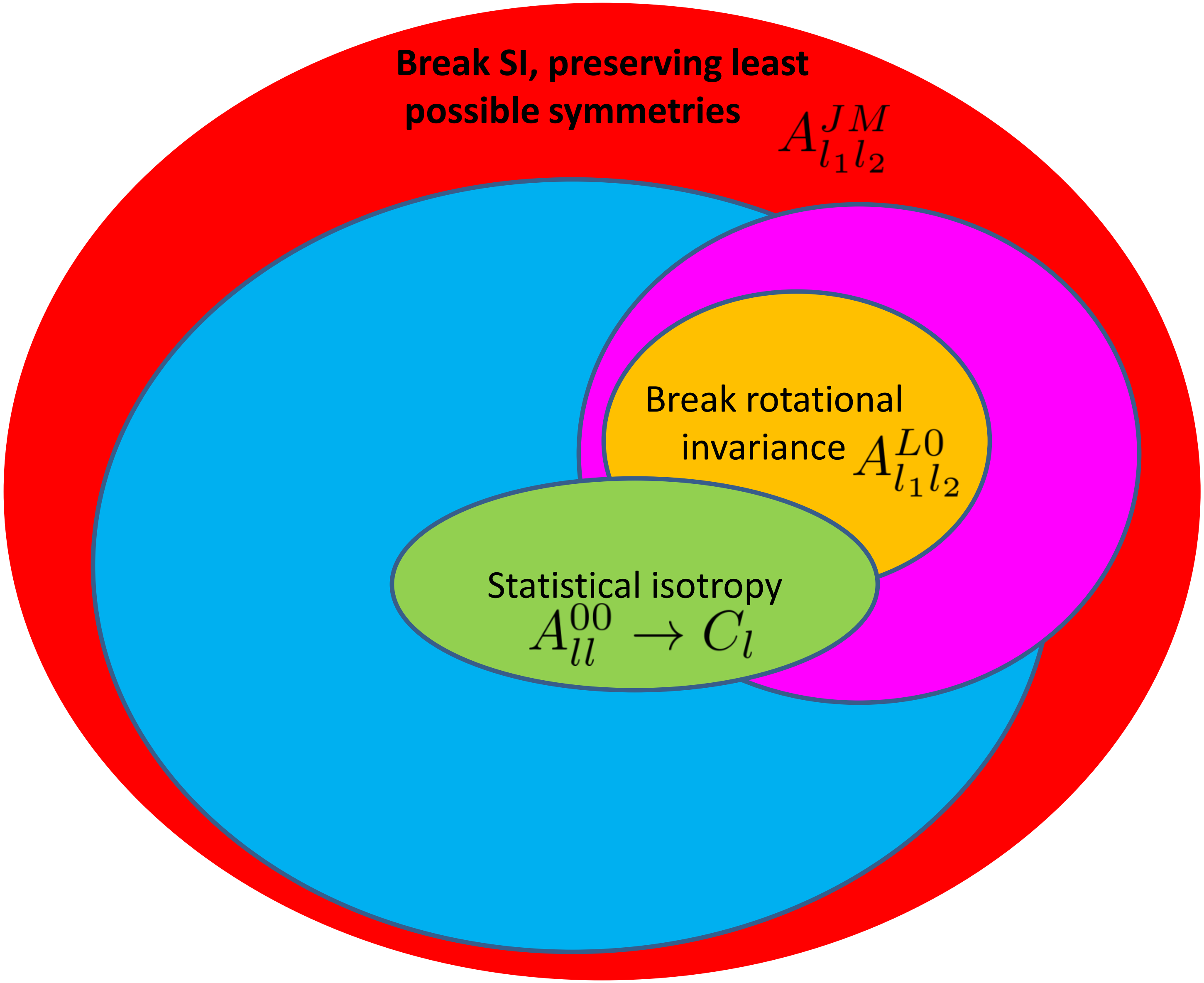
Statistical Isotropy $\longrightarrow$ $$\kappa_L = \kappa_0 \delta_{L0}$$

Breakdown of SI implies non-zero components of BiPS

How do various symmetries show up in Bipolar space???

Bipolar harmonic coefficients catches the symmetries
Of the underlying correlation function!

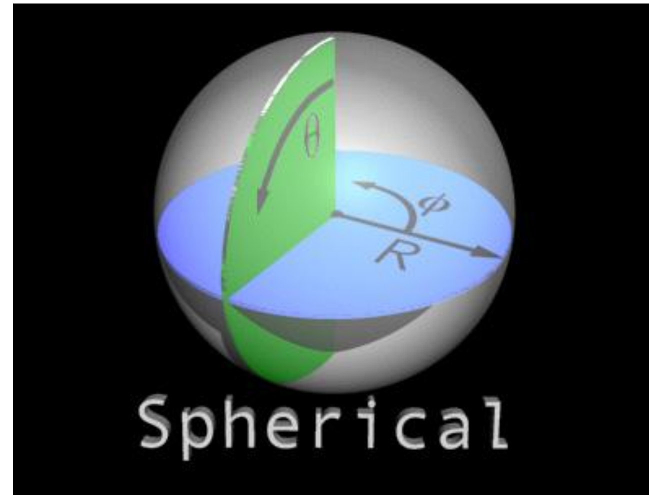
Set of all possible Bipolar coefficients indexed by l1,l2,L,M



The various symmetries…

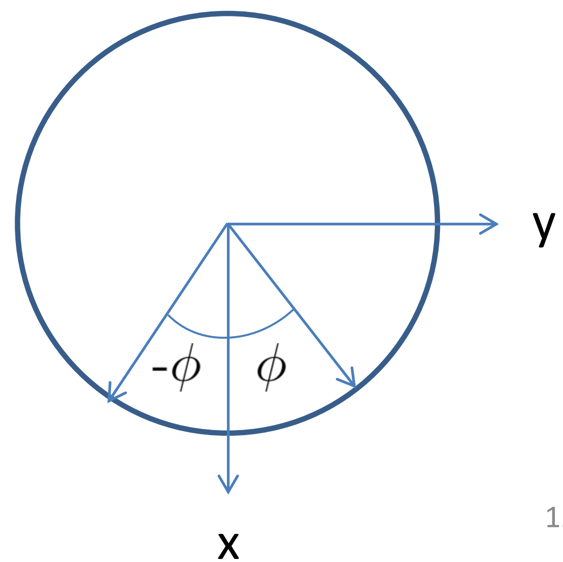
$$C(\hat{n}_1, \hat{n}_2) \text{ where } \hat{n} \to (\theta, \phi)$$

Correlation function is invariant under:



- complex conjugation(Reality)
- symmetric under exchange of n1 and n2.
- point reflection $\theta \to \pi - \theta$ and $\phi \to \pi + \phi$
- Mirror reflection about x-y plane $\theta \to \pi - \theta$ and $\phi \to \phi$
- $\theta \to \theta, \phi \to -\phi$
- $\theta \to -\theta$ or $\phi \to \phi + \pi$
- $\theta \to -\theta$ and $\phi \to -\phi$

change of argument signs



Symmetries of correlation function & Bipolar coefficients

- **Correlation Function is Real** $C(\hat{n_1}, \hat{n_2}) = C^*(\hat{n_1}, \hat{n_2})$

Unlike spherical harmonic coefficients Recall $a_{lm} = (-1)^m a^*_{l,-m}$

$$A^{JM}_{l_1 l_2} = (-1)^{l_1+l_2+J+M} A^{*J-M}_{l_1 l_2}$$

$l_1 + l_2 + J = \text{even}$	$l_1 + l_2 + J = \text{odd}$
Even parity bipolar coefficients $A^{(+)JM}_{l_1 l_2} = (-1)^M A^{*J-M}_{l_1 l_2}$	**Odd parity bipolar coefficients** $A^{(-)JM}_{l_1 l_2} = (-1)^{M+1} A^{*J-M}_{l_1 l_2}$

In rotational or azimuthal symmetries $M = 0$, non-vanishing bipolar coefficients have $l_1 + l_2 + L$=even

- **Correlation function is Symmetric**

$$C(\hat{n_1}, \hat{n_2}) = C(\hat{n_2}, \hat{n_1})$$

Even parity BipoSH are symmetric. Odd parity ones are anti-symmetric

$$A^{JM}_{l_1 l_2} = (-1)^{l_1+l_2+J} A^{JM}_{l_2 l_1}$$

$$A^{JM}_{ll} = A^{JM}_{ll}\,\delta_{J,2k} \qquad k = 0, 1, 2, 3\ldots$$

$$A^{JM}_{l0} = A^{JM}_{0l}\,\delta_{J,l}$$

- **Point Reflection** Recall $Y_{lm}(\hat{n}) = (-1)^l Y_{lm}(\hat{n})$

$$C(\theta_1, \phi_1, \theta_2, \phi_2) = C(\pi - \theta_1, \phi_1 + \pi, \pi - \theta_2, \phi_2 + \pi)$$

$$l_1 + l_2 = \text{even}$$

- **Mirror Reflection** Recall $Y_{lm}(\hat{n}) = (-1)^{l+m} Y_{lm}(\hat{n})$

$$C(\pi - \theta_1, \phi_1, \pi - \theta_2, \phi_2) = C(\theta_1, \phi_1, \theta_2, \phi_2)$$

$$l_1 + l_2 + M = \text{even}$$

$$C(\ldots.) = -C(\ldots.)$$

Anti-Symmetry leads to an odd value of above combinations.

$$C(\theta_1,\phi_1,\theta_2,\phi_2) = C(\theta_1,-\phi_1,\theta_2,-\phi_2)$$

Real Bipolar coefficients

Anti-symmetry gives purely imaginary Bipolar coefficients

$$C(\theta_1,\phi_1,\theta_2,\phi_2) = C(\theta_1,\phi_1+\pi,\theta_2,\phi_2+\pi)$$

OR

$$C(\theta_1,\phi_1,\theta_2,\phi_2) = C(-\theta_1,\phi_1,-\theta_2,\phi_2)$$

$M = \text{even}$

Anti-Symmetry restricts M to be **odd**.

Symmetry $\hat{n} \rightarrow \hat{-n}$

$$C(\theta_1,\phi_1,\theta_2,\phi_2) = C(-\theta_1,-\phi_1,-\theta_2,-\phi_2)$$

Real Bipolar coefficients & $M = \text{even}$

Bipolar coefficients & symmetries- SI and non-SI models…

Statistically Isotropic CMB

Correlation function is invariant under the rotations

$$C(\hat{n}_1, \hat{n}_2) = C(\hat{n}_1 . \hat{n}_2) = C(\Theta) = \frac{1}{4\pi} \sum_{l=2}^{\infty} (2l+1) C_l P_l(\cos\Theta)$$

$$\cos\Theta = \cos\theta_1 \cos\theta_2 + \sin\theta_1 \sin\theta_2 \cos(\phi_1 - \phi_2)$$

Correlation function doesn't depend on the two direction, only on the angle between them

- Real coefficients
- l1+l2=even
- Rotational symmetry gives M=0 hence l1+l2+J=even.
- J=0
- Symmetric coefficients.

RECALL

$$\phi \rightarrow -\phi$$

$$\theta \rightarrow \pi - \theta \text{ and } \phi \rightarrow \pi + \phi$$

$$A^{JM}_{l_1 l_2} = (-1)^{l_1 + l_2 - J} A^{JM}_{l_2 l_1}$$

Bipolar coefficients

$$A^{JM}_{l_1 l_2} = (-1)^{l_1} C_{l_1} \sqrt{2l_1 + 1} \delta_{l_1 l_2} \delta_{J0} \delta_{M0}$$

Bipolar power spectrum

$$\kappa_J = \kappa_0 \delta_{J0}$$

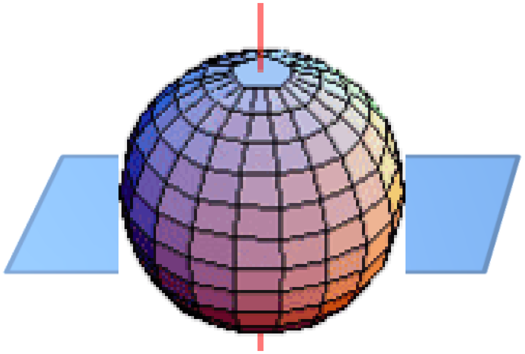
A.Hajian and T. Souradeep, ApJ **597** L5 (2003)

Cylindrical Symmetry:

Rotational symmetry about z-axis

$$C^{(A)}(\theta_1, \phi_1, \theta_2, \phi_2) = C^{(A)}(\theta_1, \phi_1 + \Delta\phi, \theta_2, \phi_2 + \underset{\hat{z}}{\Delta}\phi)$$

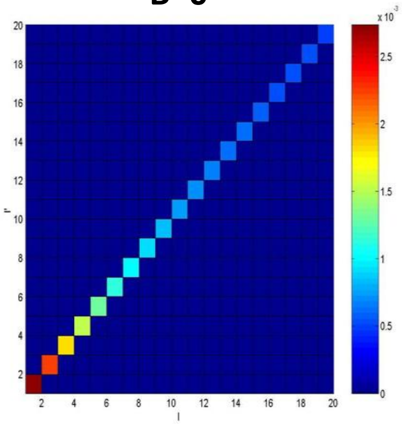
- Has a continuous ϕ symmetry
- M=0
- Hence l1+l2+J=even
- Real coefficients
- symmetric bipolar coefficients under exchange of l1 and l2.



If correlation function further has reflection symmetry(about x-y plane) then, l1+l2=even, which will imply J=even.

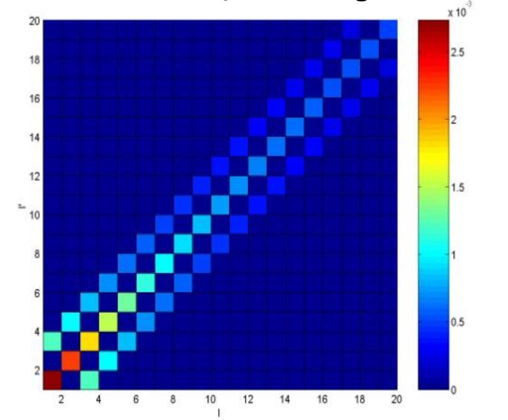
Primordial homogenous magnetic field

B=0



- Homogenous magnetic fields break Statistical Isotropy by inducing Preferred direction.
- Correlation function depends not only on the angular separation b/w two points but also on their orientation w.r.t magnetic field.

Cross Correlation b/w off diagonal terms



Magnetic field induces off-diagonal correlations **R. Durrer (1998)**

$$\langle a_{l_1m_1}a^*_{l_2m_2}\rangle = \delta_{m_1m_2}\delta_{l_1l_2}C_{l_1} + \delta_{m_1m_2}(\delta_{l_1+1,l_2-1} + \delta_{l_1-1,l_2+1})D_{l_1}$$

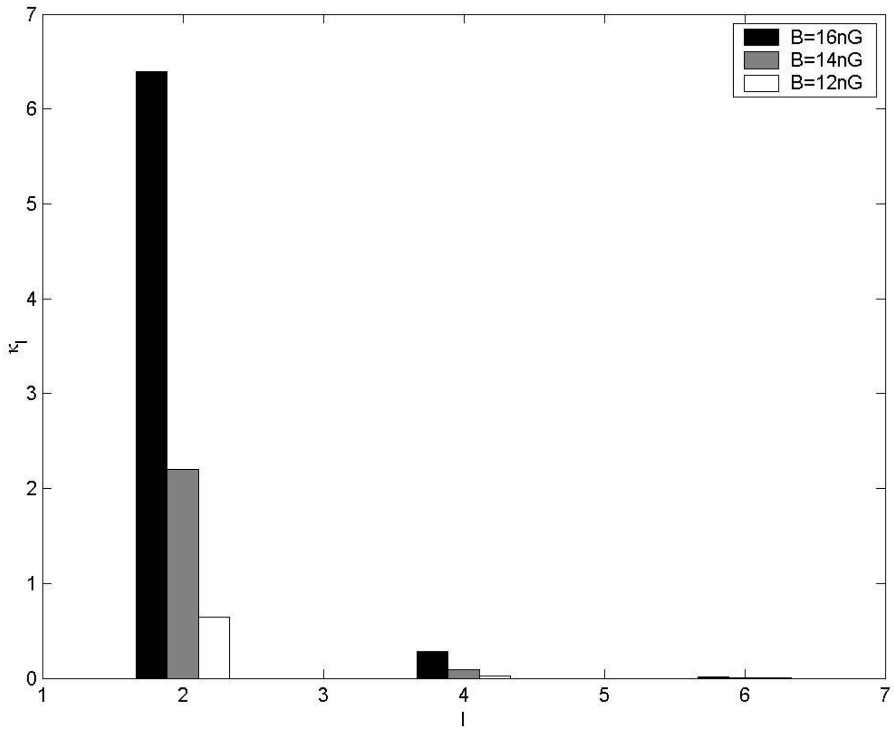
$$D_l(m) = \langle a^*_{l-1m}a_{l+1m}\rangle \equiv \langle a^*_{l+1m}a_{l-1m}\rangle$$

power spectrum of off-diagonal elements of covariance matrix.

- Azimuthally symmetric, M=0
- l1+l2+J=even
- Further l1+l2=even
- Hence in this case, J=even

$$A^{JM}_{l_1l_2} = (-1)^{l_1}(2l_1+1)^{1/2}C_{l_1}\delta_{l_1l_2}\delta_{J0}\delta_{M0} + D_{l_1}\delta_{l_1,l2\pm2}\delta_{M0}\delta_{J,2k}\sum_m(-1)^m C^{J0}_{l_1ml_2-m}$$

BiPS of Primordial Magnetic Fields



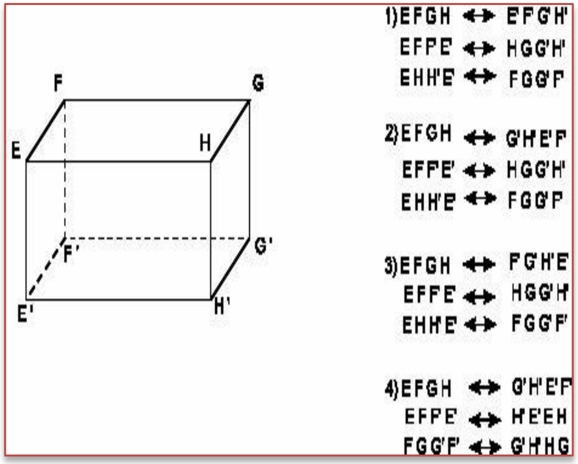
Amir Hajian, PhD thesis

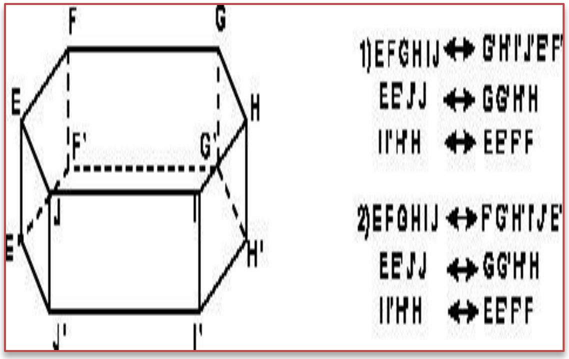
2n-fold discrete Cylindrical symmetry

- Compact universe with flat universal cover exhibits 2*n-fold* rotational symmetry about an axis.

- There are six possible compact models of the universe with flat UC by identifying opposite sides of fundamental polyhedra(parallelopiped).

1)	Identify opposite faces by translations.
2)	Opposite faces, one pair being rotated by angle π
3)	Opposite faces, one pair being rotated by $\frac{\pi}{2}$
4)	Opposite faces, all three pairs being rotated by π
5)	Opposite faces, top face rotated by an angle $\frac{2\pi}{3}$ with respect to bottom face.
6)	Opposite faces, the top face being rotated by an angle $\frac{\pi}{3}$ with respect to bottom face.





n-fold rotational symmetry

Correlation function having *n-fold(discrete)* rotational symmetry about z-axis:

$$C^{(A)}(\theta_1,\phi_1,\theta_2,\phi_2) = C^{(A)}(\theta_1,\phi_1+\frac{2\pi}{n},\theta_2,\phi_2+\frac{2\pi}{n})$$

Even-fold cases will always have a symmetry

$$C(\theta_1,\phi_1,\theta_2,\phi_2) = C(\theta_1,\phi_1+\pi,\theta_2,\phi_2+\pi)$$

Which implies , M=even

- All possible Euclidean models of compact universe exhibit reflection symmetry about x-y plane:

$$C(\theta_1,\phi_1,\theta_2,\phi_2) = C(\pi-\theta_1,\phi_1,\pi-\theta_2,\phi_2)$$

l1+l2=even

Form of correlation function under even-fold symmetry.

$$C^{(A)}(\theta_1,\phi_1,\theta_2,\phi_2) = \sum_{m_1,m_2} f_{m_1,m_2}(\theta_1,\theta_2)\delta_{m_1+m_2,nk}\cos(m_1\phi_1+m_2\phi_2)$$

- Real coefficients

Temperature Modulation

$$\Delta T(\hat{n}) = \Delta T^i(\hat{n})[1 + f(\hat{n})]$$

Statistical isotropic temperature fluctuation

Modulation function

$$C(\hat{n_1}, \hat{n_2}) = C(\Theta)[1 + f(\hat{n_1}) + f(\hat{n_2})]$$

$$f(\hat{n}) = \sum_{lm} f_{lm} Y_{lm}(\hat{n})$$

$$C(\hat{n_1}, \hat{n_2}) = \sum_{l_1 l_2 L M} A^{JM}_{l_1 l_2} \sqrt{\frac{(2l_1+1)(2l_2+1)}{4\pi(2J+1)}} C^{J0}_{l_1 0 l_2 0} \{Y_{l_1}(\hat{n}_1) \otimes Y_{l_2}(\hat{n}_2)\}_{JM}$$

$C^{J0}_{l_1 0 l_2 0}$ is non-vanishing for $l_1 + l_2 + L =$even

- Symmetry of the correlation function is guiding to have a restricted bipolar basis, having l1+l2+J=even.
- M=even
- **L=l, so the modulation in temperature space directly shows up in Bipolar space**

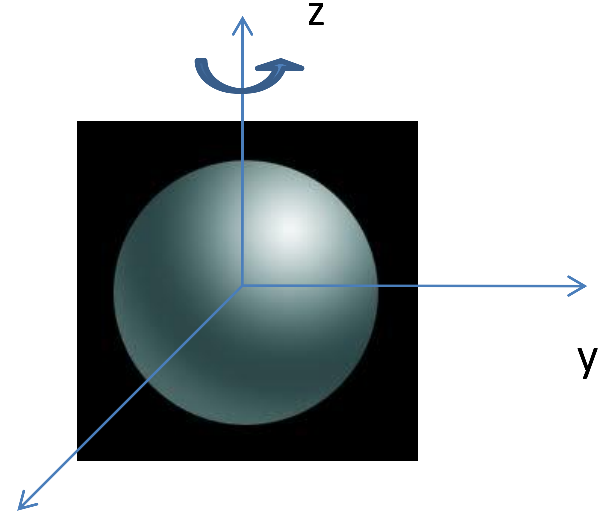
Toy Models- SI violation

❑ *Correlation function- one preferred axis*

- To break the rotational invariance of correlation function, introduce a preferred direction(with reflection symmetry).

$$C(\hat{n}_1, \hat{n}_2) = 1 - \epsilon(\hat{q}_1.\hat{z} - \hat{q}_2.\hat{z})^2 \qquad \epsilon < \frac{1}{4}$$

- **Given the correlation function, what all Bipolar coefficients one can expect???? Check for symmetries…..**



- Real coefficients
- l1+l2=even
- Azimuthal symmetry guarantees M=0, hence l1+l2+J=even.
- J=even, vanishes for J odd.
- Symmetric coefficients.

Non-vanishing independent BipoSH are $A^{00}_{00}, A^{20}_{20}, A^{00}_{11}, A^{20}_{11}$

$$\kappa_0 = \frac{16\pi^2}{27}[27 - 36\epsilon + 16\epsilon^2]$$

$$\kappa_2 = \frac{16}{15}(\frac{8\pi\epsilon}{3})^2$$

use real space representation of kappa and get the same results!!!(Amir Hajian PhD thesis)

❑ *Correlation function, having preferred direction(z-axis) as:*

$$C(\hat{n}_1, \hat{n}_2) = (1 + \epsilon \hat{n_1}.\hat{z})(1 + \epsilon \hat{n_2}.\hat{z})$$

NO reflection symmetry

- Real coefficients
- Azimuthal symmetry guarantees M=0, hence l1+l2+L=even.
- Symmetric coefficients.

Non-vanishing independent BipoSH are $A^{00}_{00}, A^{10}_{10}, A^{00}_{11}, A^{20}_{11}$

$$\kappa_0 = \frac{16\pi^2}{27}[27 + \epsilon^4] \qquad \kappa_1 = \frac{32\pi^2\epsilon^2}{3}$$

$$\kappa_2 = \frac{32\pi^2\epsilon^4}{27}$$

❑ *Correlation function with cross terms: Two preferred axes, x and y-axes*

$$C(\hat{n}_1, \hat{n}_2) = [(\hat{n_1} - \hat{n_2}).\hat{x}][(\hat{n_1} - \hat{n_2}).\hat{y}]$$

- Imaginary coefficients
- l1+l2=even
- l1+l2+M=even
- M=even
- Symmetric coefficients.

Non-vanishing independent BipoSH are A^{20}_{22}, A^{22}_{11}

$$\kappa_0 = 0$$

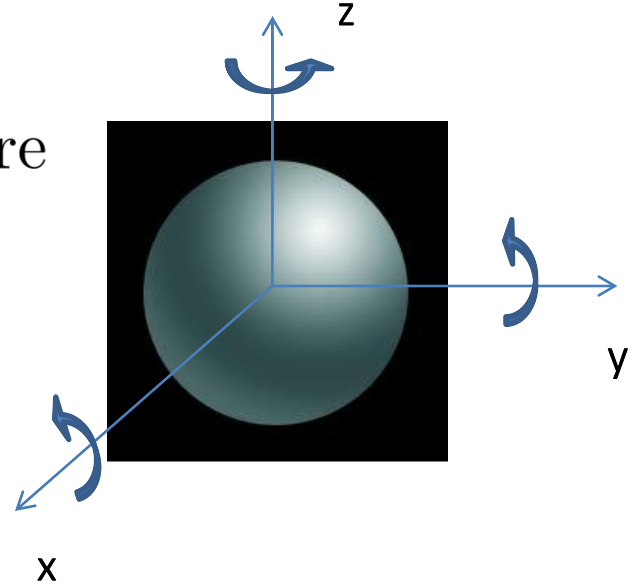
$$\kappa_2 = \frac{256\pi^2}{45}$$

Correlation function, having three preferred axis

$$C(\hat{n}_1, \hat{n}_2) = 1 - (\epsilon_1[(\hat{n_1} - \hat{n_2}).\hat{x}]^2 + \epsilon_2[(\hat{n_1} - \hat{n_2}).\hat{y}]^2 + \epsilon_3[(\hat{n_1} - \hat{n_2}).\hat{z}]^2)$$

- Real coefficients
- l1+l2+M=even
- M=even

Non-vanishing independent BipoSH are
$A^{00}_{00}, A^{20}_{20}, A^{22}_{20}, A^{22}_{11}, A^{00}_{11}, A^{22}_{11}$



$$\kappa_0 = 16\pi^2 + \frac{256\pi^2}{27}\sum_{\epsilon^2} + \frac{512\pi^2}{27}\Pi_\epsilon - \frac{64\pi^2}{3}\sum_{\epsilon}$$

$$\kappa_2 = \frac{64\pi^2}{135}(\sum_{\epsilon^2} - \Pi_\epsilon)$$

if $\epsilon_1 = \epsilon_2 = \epsilon_3$, then κ_2=0

Conclusion & Discussion

- We conclude that symmetries of the correlation function are structurally imprinted in Bipolar space.

- We studied various models of SI violation leave distinct imprints on bipolar space.

- For homogenous magnetic field, BipoSH's are restricted to even L and M=0.

- **This formalism can be used as a powerful tool to distinguish various types of SI breakdown.**


Aditya
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Tarun
Gaurav
shantanu
Sanil
Nidhi
Sucheta
Thank you