Non-gaussianity from Inflationary model with a Step in the Spectral index

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1. Introduction

CMB Observations
Strong constraints on inflationary models

More detailed properties of inflationary scenario
Cosmological parameters

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Cosmological parameters

Standard slow-roll inflation
Scale invariant spectrum, $\mathcal{P} \propto k^{(n-1)}$ with $n \simeq 1$
Gaussian

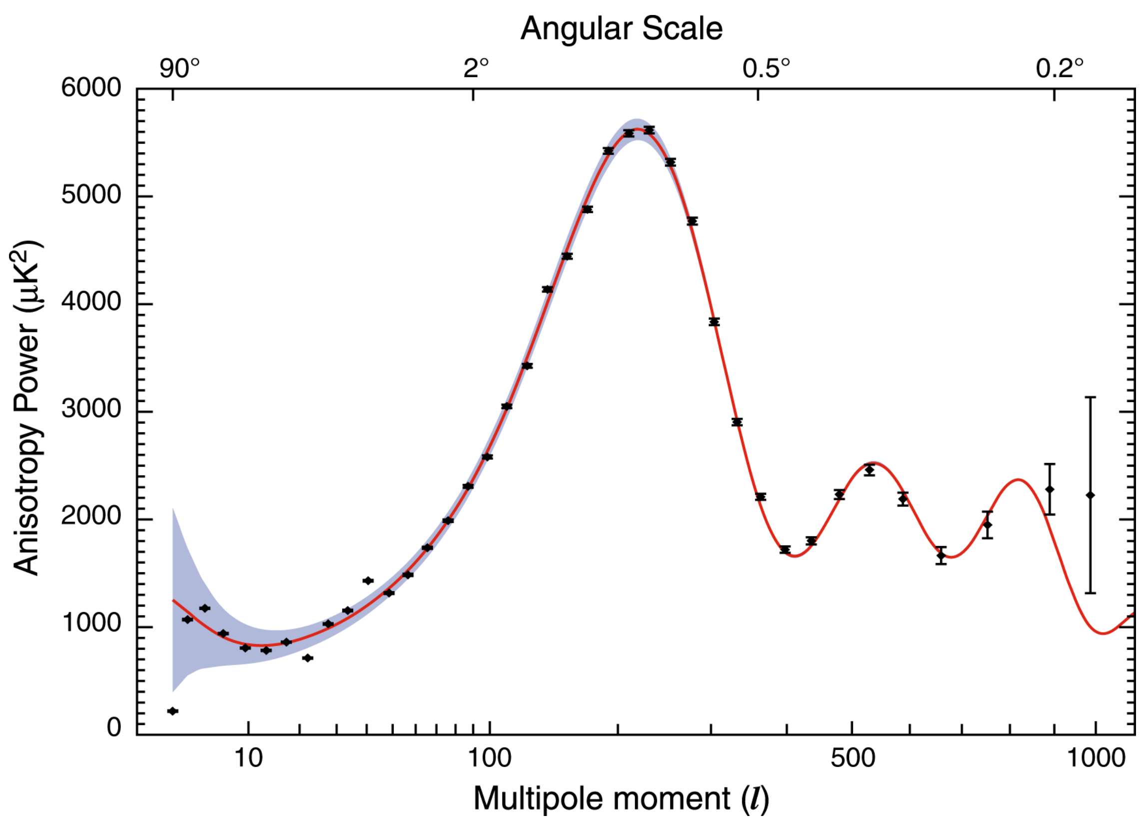
"What kind of perturbation spectra can be produced by inflation?"

Spectral Index, $n = 1 + \frac{d \ln P(k)}{d \ln k}$

Relative distribution of power on various scales

Running of the Spectral Index $\frac{dn}{d \ln k}$

Deviation of the primordial power specturm from power law



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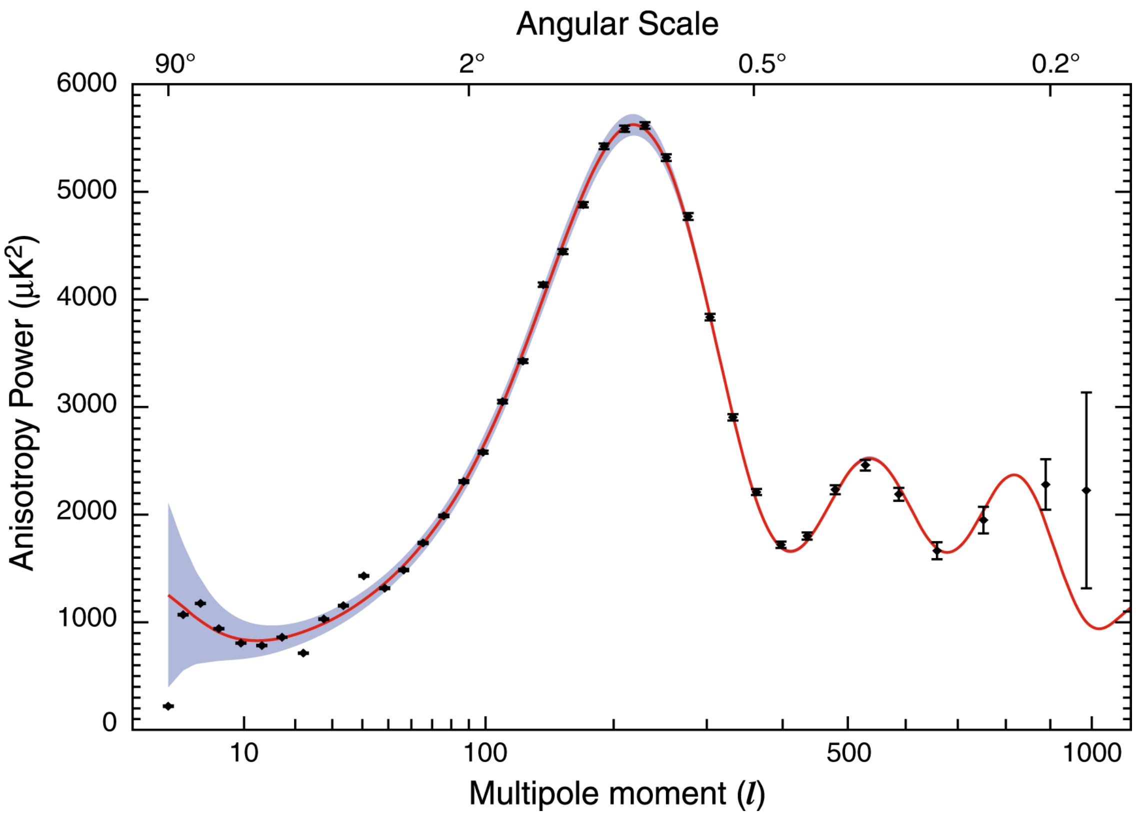

$$\frac{dn_s}{d \ln k} = -0.048 \pm 0.029$$

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probe of non-linear physics in the very early universe

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all the odd corr. functions vanish, even corr. functions in terms of the 2-point functions

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Non-linearity parameter f_{NL}

$$\Phi = \Phi_L + f_{NL} * (\Phi_L)^2$$

2. Primordial Non-Gaussianity

$P(\varphi)$ of quantum fluctuations φ in the ground state of Bunch-Davies vacuum - Gaussian
$\Rightarrow$ $P(\mathcal{R})$ of curvatute perturbations $\mathcal{R}$ - Gaussian $\mathcal{R} = -[\frac{H(\phi)}{\dot{\phi}_0}]\varphi$

ϕ_0 is the mean field $\phi = \phi_0 + \varphi$

Non-Gaussianity can be generated when

- (i) scalar fields are not free, but have some interactions,
- (ii) there are non-linear corrections to the relation between the primordial curvature perturbation $\mathcal{R}$ and the quantum fluctuations φ
- (iii) the initial state is not in the Bunch-Davies vacuum

(i)Non-linear coupling - compute action to cubic order in perturbation

$$V(\phi) = V_0 + V'\varphi + \frac{1}{2}V''\varphi^2 + \frac{1}{6}V'''\varphi^3 + \cdots$$

Cubic or higher order interaction terms yield non-Gaussianity in φ (Falk *et al* 1993)

(ii) the leading term of Taylor expansion of non-linear relation between $\mathcal{R}$ and φ
Salopek & Bond 1990 Even if φ is Gaussian $\mathcal{R}$ can be non-Gaussian due to non-linear terms in Taylor series expansion of the relation

More observationally relevant quantity, the curvature perturbation during the matter era Φ

At the linear order $\Phi = (3/5)\mathcal{R}_L$. The actual relation is more complicated at the non-linear order.

$$\Phi = \Phi_L + f_{NL}\Phi_L^2$$

$\Phi \sim 10^{-5}$, the secod term is smaller than he first by $10^{-5} f_{NL} \Rightarrow$ second term is only 0.1$\%$ of the first term for $f_{NL} \sim 10^2$

tiny deviation from Gaussian fluctuations

($\Delta T/T$) $\leftrightarrow \Phi$ as $\Delta T/T = -\Phi/3$ at the linear order on very large angular scales
Non-linear corrections add terms of order unity to f_{NL} by the time we observe it in CMB

(iii) presence of particles at the beginning of inflation $\Rightarrow$ departure of the initial state of quantum fluctuations from the Bunch Davies vacuum $\rightarrow$ enhanced non-Gaussinaity

Standard single field inflation - scalar field is slowly rolling down the potential
Hubble parameter H, Inflaton Potential V and Field - changing slowly

f_{NL} small $\mathcal{O}(\epsilon, \eta) \longrightarrow 10^{-2}$ or smaller
(Salopek $\&$ Bond 1990, Falk *et al*. 1993, Gangui *et al*. 1994, Maldacena 2003, Seery $\&$ Lidsey 2005)

How can a large f_{NL} be generated?

break either the slow-roll or single field concept!!

More general models – multiple scalar fields, features in inflaton potential, non-adiabatic fluctuations, non-canonical kinetic terms, deviations from Bunch-Davies vacuum
$\longrightarrow$ substantially higher amounts of non-Gaussianity
(Bartolo *et al*. 2004, for a review and references therein)

3. Inflaton Potential with a localised feature

break the slow-roll temporarily

$\longrightarrow$ large non-Gaussianity at certain limited scale at which the feature exists

- local deviations from the approximately flat spectrum
- violation of standard slow-roll conditions
- additional scalar fields
- corrections in the inflaton effective potential

Local non-analytic features of $V(\phi)$

A. A. Starobinsky, astro-ph/9808152

$[\,]$ a jump in the relevant quantity, $[A] \equiv A(\varphi_0 + 0) - A(\varphi_0 - 0)$

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- $[V] = [V'] = 0, [V''] \neq 0$
  A sudden change in the slope of the potential leads to step in its second derivative $V''(\varphi)$
  -leads to a step in the primordial spectral index n_s, accompanied by small oscillations with decreasing amplitude.

Inflationary model with a step-like discontinuity in the evolution of the effective mass

M. Joy, V. Sahni and A. A.Starobinsky, PRD 77, 023514 (2008)

Hybrid inflationary scenario

$$V(\psi,\phi) = \frac{1}{4\lambda}\left(M^2 - \lambda\psi^2\right)^2 + \frac{1}{2}m^2\phi^2 + \frac{g^2}{2}\phi^2\psi^2$$

Effective mass of the field ψ

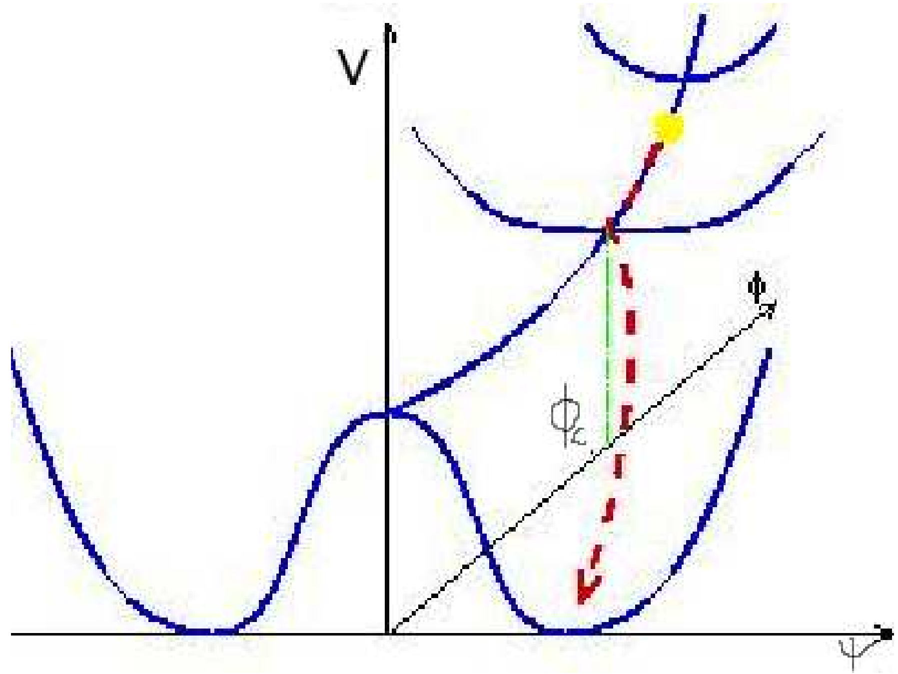
$$m_\psi^2 \equiv \frac{d^2V}{d\psi^2}\Big|_{\psi=0} = g^2\phi^2 - M^2$$

critical value $\phi_c = M/g$

$m_\psi^2 > 0$ if $\phi > \phi_c$ and $m_\psi^2 < 0$ if $\phi < \phi_c$

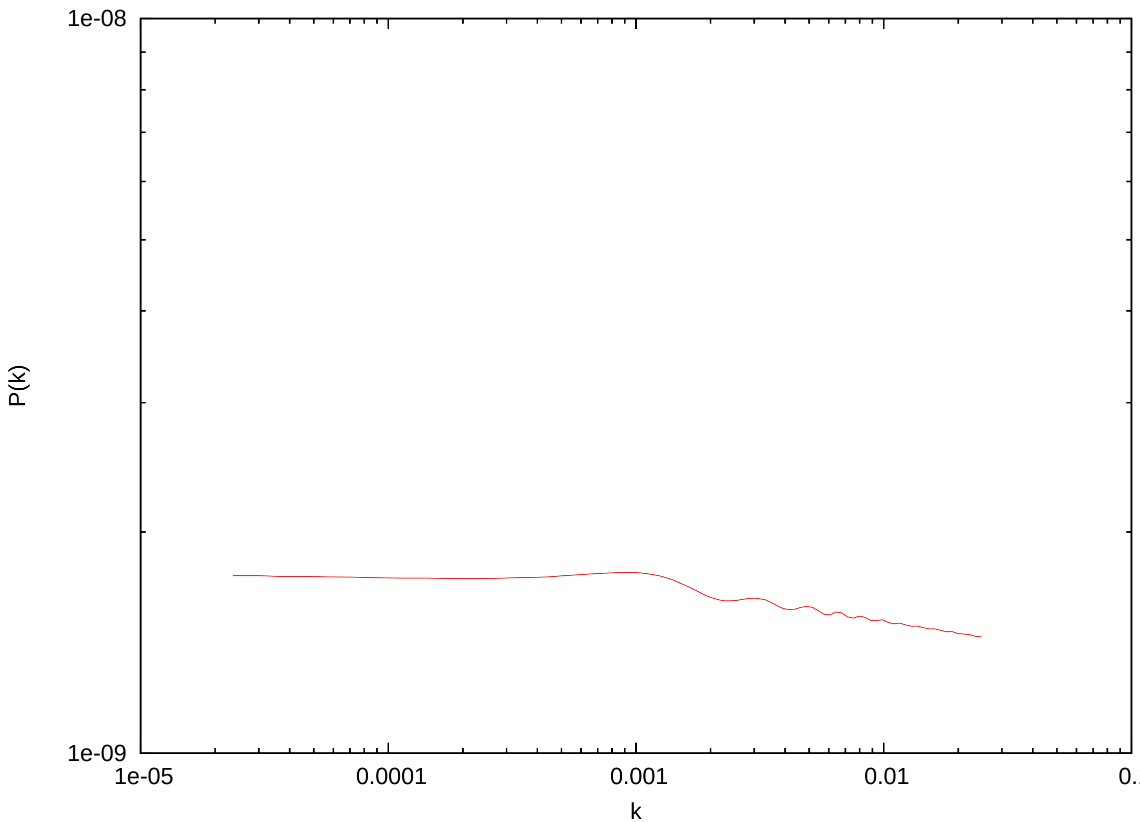
The inflaton potential experiences a sudden small change in the second derivative, V'' (the effective mass of the inflaton).

Inflationary model with a step-like discontinuity in the evolution of the effective mass



✓ $\phi > \phi_c$ the only minimum of the effective potential is at $\psi = 0$
✓ $\phi = \phi_c$ waterfall - rapid cascade of ψ towards the minimum of its potential
✓ $\phi < \phi_c$ phase transitions with symmetry breaking

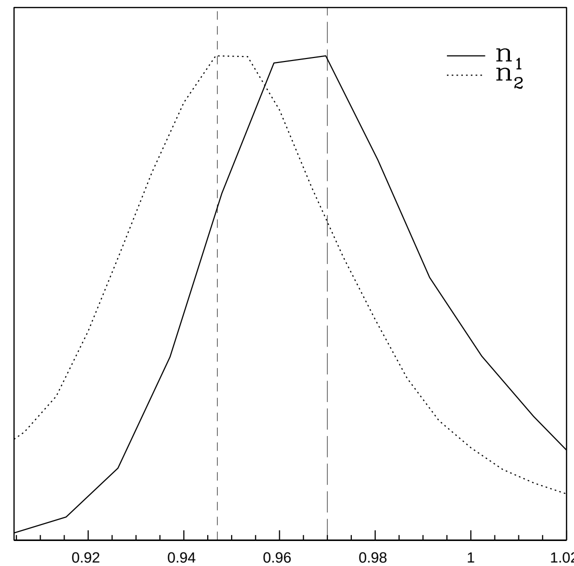
Resulting power spectrum has certain small oscillations superimposed on the almost flat spectrum and the spectral index has a localised step, followed by damped oscillations.



Power Spectrum

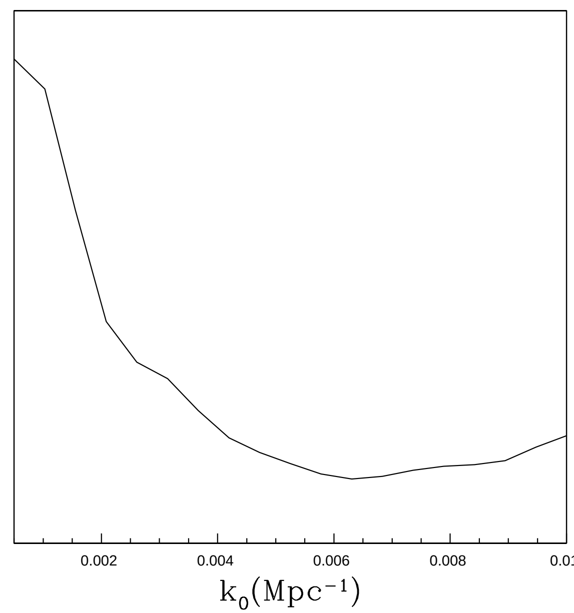
5. The model with a feature is tested against CMB data

CosmoMC to confront our model with the observational data from WMAP
Better fit with $\Delta\chi^2_{eff} = -3.052$ compared to the best PL model with a constant n_s



Marginalized posterior distributions for the spectral indices n_1 and n_2
n_1 slightly larger than that without a step is favoured by the data
The marginalised probability for n_2 peaks at 0.947, while the marginalized probability for n_1 peaks at 0.97.

The probability distribution for k_0 (location of the feature)



The feature, should preferentially lie on large scales: the marginalized upper limit for k_0 is 0.00355 Mpc^{-1} at 95% CL.

Main evidence for large running in the WMAP dataset comes from sufficiently low multi-poles with $l \lesssim 40$.

Non-Gaussianity for the model with the feature

Chen et al. (2006) - the non-gaussianity generated for a different feature in the potential

The impact on the 3-point function is localised around the wavenumbers that are most affected by the feature

$$S = \int dx^4 \sqrt{g} \left[\frac{M_p^2}{2} R + \frac{1}{2} (\partial\phi)^2 - V(\phi) \right] \tag{-2}$$

The slow roll parameters

$$\epsilon = \frac{\dot{H}}{H^2} \,, \quad \eta = \frac{\dot{\epsilon}}{H\epsilon}$$

The power spectrum - 2-point correlation function of the curvature perturbation

$$\begin{aligned} \langle \zeta(x) \zeta(x) \rangle &= \int \frac{d^3 k_1}{(2\pi)^3} \frac{d^3 k_2}{(2\pi)^3} \langle \zeta(k_1) \zeta(k_2) \rangle e^{i(k_1 + k_2)\cdot x} \\ &= \int \frac{dk}{k} P_\zeta \end{aligned} \tag{-3}$$

Non-Gaussianities $\leftarrow$ arise from departures from nonlinear couplings - the action to cubic order in the perturbation

Expand the full action order by order in perturbation variable ζ, the interaction Hamiltonian that results from the third order action

$$\begin{aligned} H_{int}(\tau) &= -\int d^3x \Big\{ a\epsilon^2\zeta\zeta'^2 + a\epsilon^2\zeta(\partial\zeta)^2 - 2\epsilon\zeta'(\partial\zeta)(\partial\chi) \\ &\quad + \frac{a}{2}\epsilon\eta'\zeta^2\zeta' + \frac{\epsilon}{2a}(\partial\zeta)(\partial\chi)(\partial^2\chi) + \frac{\epsilon}{4a}(\partial^2\zeta)(\partial\chi)^2 \Big\} \end{aligned}$$

The three-point function will be non-zero due to this interaction term. Transforming to Fourier space

$$\langle\zeta(\mathbf{x})\zeta(\mathbf{x})\zeta(\mathbf{x})\rangle = \int \frac{d^3k_1}{(2\pi)^3}\frac{d^3k_2}{(2\pi)^3}\frac{d^3k_3}{(2\pi)^3}\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle e^{i(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\cdot\mathbf{x}}$$

The 3-point correlation function at some time τ after horizon exit - the vacuum expectation value of the three point function in the interaction vacuum

$$\langle\zeta(\tau,\mathbf{k}_1)\zeta(\tau,\mathbf{k}_2)\zeta(\tau,\mathbf{k}_3)\rangle = -i\int_{\tau_0}^{\tau} d\tau'\ a\ \langle[\zeta(\tau,\mathbf{k}_1)\zeta(\tau,\mathbf{k}_2)\zeta(\tau,\mathbf{k}_3), H_{int}(\tau')]\rangle$$

Quantising ζ

$$\zeta(\tau,x) = \int \frac{d^3p}{(2\pi)^3}\zeta(\tau,k)e^{ip\cdot x}\ ,$$

with associated operators and mode functions, $\zeta(\tau,\mathbf{k}) = u(\tau,\mathbf{k})a(\mathbf{k}) + u^*(\tau,-\mathbf{k})a^\dagger(-\mathbf{k})$
3-point correlation function as a sum of integrals of the form

$$I_{\epsilon^2} \propto \Re\left[\prod_i u_i(\tau_{end})\int_{\tau_0}^{\tau_{end}} d\tau\epsilon^2 a^2\xi_1(\tau)\xi_2(\tau)\xi_3(\tau) + \mathcal{O}(\epsilon^3)\right]$$

$$I_{\epsilon\eta'} \propto \Re\left[\prod_i u_i(\tau_{end})\int_{\tau_0}^{\tau_{end}} d\tau\epsilon\eta' a^2\xi_1(\tau)\xi_2(\tau)\xi_3(\tau)\right]$$

ξ_n is either $u^*_{k_n}$ or $du^*_{k_n}/d\tau$

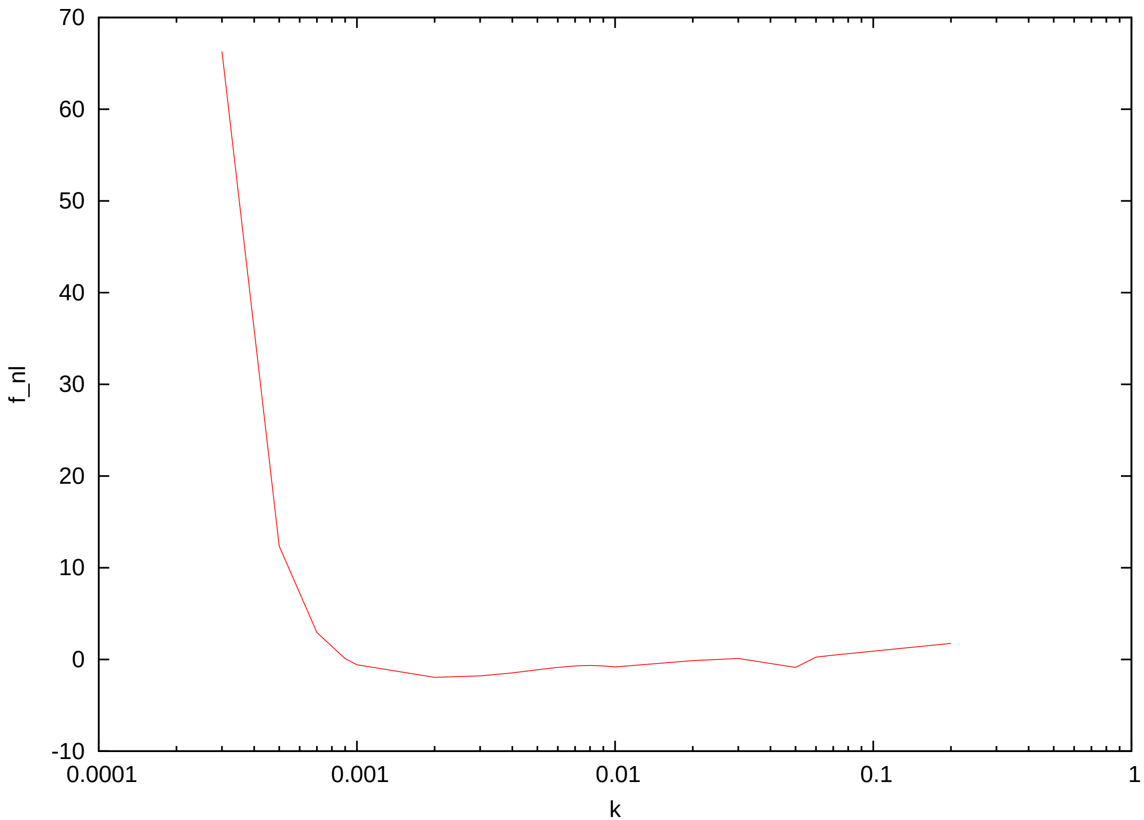
The integral is dominated by the range of τ during which the modes leave the horizon.

$$\epsilon\eta' = 6aH\left(2\epsilon^2 - \frac{\epsilon\eta}{2} - \frac{5}{6}\epsilon^2\eta + 2\epsilon^3 - \frac{\epsilon\eta^2}{12} - \epsilon\frac{V_{\phi\phi}}{3H^2}\right) \qquad \text{(-10)}$$

We compute these extra terms and study the shape and scale dependence of the 3-point function.

$$\langle\zeta(\mathbf{k}_1)\zeta(\mathbf{k}_2)\zeta(\mathbf{k}_3)\rangle = (2\pi)^7\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)\left(-\frac{3}{10}f_{NL}(P_k^\zeta)^2\right)\frac{\Sigma_i k_i^3}{\Pi_i k_i^3} \qquad \text{(-10)}$$

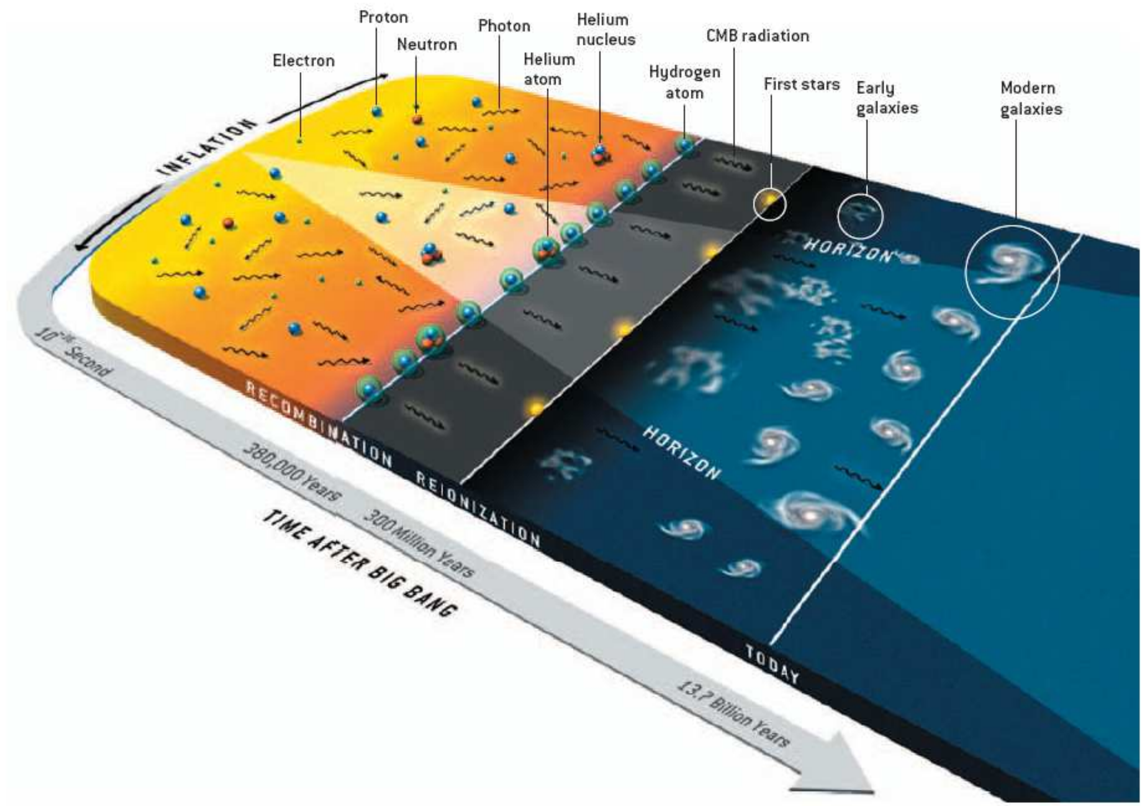
For the best fit parameter values



Summary

- The best χ^2_{eff} for this featured model shows an improvement by 3.052 over the best fit obtained assuming a power law for the primordial spectrum.
- Such a feature in the primordial spectrum, if exists at all, should lie on large scales $k_0 \lesssim 0.003$ Mpc^{-1}.
- Three-point correation function for the 'mini-waterfall' hybrid inflationary model
- For the best fit potential parameter values, the non-Gaussianity associated with the featured model is larger than those in standard slow-roll infaltion and may even be within the range of next generation CMB experiment such as Planck.

Thank You !



Proton
Photon
Helium
nucleus
Neutron
Electron
Helium
atom
CMB radiation
Hydrogen
atom
First stars
Early
galaxies
Modern
galaxies
INFLATION
HORIZON
HORIZON
RECOMBINATION
REIONIZATION
TODAY
10^{-36} Second
380,000 Years
300 Million Years
13.7 Billion Years
TIME AFTER BIG BANG

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Thank You !

2. Background cosmology near a feature in the potential

a step-like discontinuity in V'' when $\varphi(t_0) = \varphi_0$ at t_0

smoothed in a small neighborhood of φ_0 denote by $\tilde{\epsilon}$

$$V(\varphi) = V(\varphi_0) + t\,\dot{\varphi}_0\,V'(\varphi_0) + \frac{t^2}{2!}\left[\ddot{\varphi}_0\,V'(\varphi_0) + \dot{\varphi}_0{}^2 V''_{\pm}\right] + \cdots$$

$$H(t) = H_0 + t\,\dot{H}_0 + \frac{t^2}{2!}\,\ddot{H}_0 + \frac{t^3}{3!}\,\dddot{H}_{\pm} + \cdots$$

$$\varphi(t) = \varphi_0 + t\,\dot{\varphi}_0 + \frac{t^2}{2!}\,\ddot{\varphi}_0 + \frac{t^3}{3!}\,\dddot{\varphi}_{\pm} + \cdots$$

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$$\begin{aligned}
\ddot{\varphi} &+ 3H\dot{\varphi} + V'(\varphi) = 0\ , \\
H^2 &= \frac{8\pi G}{3}\left(\frac{\dot{\varphi}^2}{2} + V(\varphi)\right) \\
\dddot{\varphi}_{\pm} &= -3H_0\,\ddot{\varphi}_0 - 3\dot{H}_0\,\dot{\varphi}_0 - V''_{\pm}\,\dot{\varphi}_0
\end{aligned}$$

Slow roll parameters

$$\epsilon = \frac{8\pi G}{2}\left(\frac{\dot\varphi}{H}\right)^2 \ , \ \ \eta = \frac{\ddot\varphi}{\dot\varphi\, H} \ , \ \ \zeta^2 = \frac{1}{H^2}\left[\frac{\dddot\varphi}{\dot\varphi} - \frac{\ddot\varphi^2}{\dot\varphi^2}\right]$$

Corrections to the slow-roll parameters

$$\begin{aligned}\epsilon(t) \quad &= \quad \epsilon_0 + tH_0\ \left[2\epsilon_0^2 + 2\epsilon_0\eta_0\right] \\ &\quad + t^2 H_0^2\, \epsilon_0 \left[3\epsilon_0 - 3\eta_0 - 3\epsilon_0^2 + 6\epsilon_0\eta_0 + \eta_0^2 - \frac{V''_\pm}{H_0^2}\right]\end{aligned}$$

$$\begin{aligned}\eta(t) \quad &= \quad \eta_0 + tH_0 \left[3\epsilon_0 - 3\eta_0 + \epsilon_0\,\eta_0 - \eta_0^2 - \frac{V''_\pm}{H_0^2}\right] \\ &\quad + \frac{t^2 H_0^2}{2}\ \left[9\epsilon_0 + 9\eta_0 + 6\epsilon_0^2 + 3\epsilon_0\,\eta_0 + 9\eta_0^2 + 2\epsilon_0^2\,\eta_0 + 2\eta_0^3\right. \\ &\quad \left. + (3 - 2\epsilon_0 + 2\eta_0)\,\frac{V''_\pm}{H_0^2} - \sqrt{\frac{2\epsilon_0}{8\pi G}}\,\frac{V'''_\pm}{H_0^2}\right]\end{aligned}$$

$$
\begin{aligned}
\zeta^2(t) \quad = \quad & 3\epsilon_0 - 3\eta_0 - \eta_0^2 - \frac{V''_\pm}{H_0^2} \\
+ \quad & tH_0 \left[-9\epsilon_0 + 9\eta_0 + 6\epsilon_0^2 + 9\eta_0^2 - 3\epsilon_0\,\eta_0 - 2\epsilon_0\,\eta_0^2 + 2\eta_0^3 \right. \\
+ \quad & \qquad \left. (3 - 2\epsilon_0 + 2\eta_0)\frac{V''_\pm}{H_0^2} - \sqrt{\frac{2\epsilon_0}{8\pi G}}\,\frac{V'''_\pm}{H_0^2} \right] \\
+ \quad & \frac{t^2 H_0^2}{2} \left[27\epsilon_0 - 27\eta_0 - 18\epsilon_0^2 - 63\eta^2 + 45\epsilon_0\,\eta_0 \right. \\
+ \quad & \qquad 18\epsilon_0^3 + 6\epsilon_0^2\,\eta_0 + 36\epsilon_0\,\eta_0^2 - 6\epsilon_0^2\,\eta_0^2 - 36\eta_0^3 \\
+ \quad & \qquad 4\epsilon_0\,\eta_0^3 - 6\eta_0^4 - \left(9 - 12\epsilon_0 + 24\eta_0 + 6\epsilon_0^2 + 8\eta_0^2 - 4\epsilon_0\,\eta_0^2 + \frac{V'''_\pm}{H_0^2} \right) \frac{V'''_\pm}{H_0^2} \\
+ \quad & \qquad \left. (3 - 4\epsilon_0 + \eta_0)\sqrt{\frac{2\epsilon_0}{8\pi G}}\,\frac{V'''_\pm}{H_0^2} - \frac{2\epsilon_0}{8\pi G}\,\frac{V''''_\pm}{H_0^2} \right]
\end{aligned}
$$

$$\ddot{\xi}_k + 3H\,\dot{\xi}_k + \left(\frac{k^2}{a^2} + m^2_{eff}\right)\xi_k = 0$$

the effective mass m^2_{eff}

$$m^2_{eff} = \frac{d^2V}{d\varphi^2} + 8\,\pi\,G\,\frac{\dot{\varphi}}{H}\,\frac{dV}{d\varphi} + H\,\frac{d}{dt}\left(\frac{\dot{H}}{H^2}\right)$$

$$\begin{aligned}\frac{m^2_{eff}(t)}{H_0^2} \quad = \quad & \frac{V''_{\pm}}{H_0^2} + tH_0\sqrt{\frac{2\epsilon_0}{8\pi G}}\,\frac{V'''_{\pm}}{H_0^2} \\ & - 2\epsilon_0\,(3 + \epsilon_0 + 2\,\eta_0) \\ & - 4\,\epsilon_0\,tH_0\left[\left(3\,\epsilon_0 + \epsilon_0^2 + 3\,\epsilon_0\,\eta_0 + \eta_0^2\right) + \frac{V''_{\pm}}{H_0^2}\right]\end{aligned}$$

$m^2_{-} = V''_{-}$ when $t_0 < 0$ and $m^2_{+} = V''_{+}$ when $t_0 > 0$

3. Perturbation spectrum and spectral index

$\xi_k = \chi_k / a$

$$\chi'' + \left(k^2 + m_{\rm eff}^2 a^2 - \frac{a''}{a} \right) \chi = 0 \; ,$$

solution at $\eta \ll \eta_0$

$$\chi_{\rm in}(\eta) = \frac{\sqrt{\pi\eta}}{2} H_{\mu_1}^{(2)}(k\eta)$$

${\mu_1}^2 = \frac{9}{4} - \frac{V_-''}{H^2}$

after the feature is crossed $\eta \gg \eta_0$

$$\chi_{\rm out}(\eta) = \frac{\sqrt{\pi\eta}}{2} \left(\alpha H_{\mu_2}^{(2)}(k\eta) + \beta H_{\mu_2}^{(1)}(k\eta) \right) \; ,$$

${\mu_2}^2 = \frac{9}{4} - \frac{V_+''}{H^2}$

$$\mu_{1,2} = 2 - \frac{n_{1,2}}{2}$$

$n_1 (n_2)$ the spectral index in the ‘in’ (‘out’) region.

Power Spectrum

$$P(k) \propto P_0(k) \times |\alpha - \beta|^2$$

$P_0(k)$, the power specrum of the background model

$$\begin{aligned}
\frac{4}{\pi^2}\,|\alpha - \beta|^2 &= \Delta^2 J_{\mu_2}^2 \left(Y_{\mu_1}^2 + J_{\mu_1}^2\right) \\
&+ (k\eta_0)^2 \left\{J_{\mu_2}^2 \left(Y_{\mu_1+1}^2 + J_{\mu_1+1}^2\right) + J_{\mu_2+1}^2 \left(Y_{\mu_1}^2 + J_{\mu_1}^2\right)\right. \\
&- \left. 2J_{\mu_2}J_{\mu_2+1}\left(Y_{\mu_1}Y_{\mu_1+1} + J_{\mu_1}J_{\mu_1+1}\right)\right\} \\
&+ 2\Delta\,(k\eta_0)J_{\mu_2}\left\{J_{\mu_2}\left(Y_{\mu_1}Y_{\mu_1+1} + J_{\mu_1}J_{\mu_1+1}\right) - J_{\mu_2+1}\left(Y_{\mu_1}^2 + J_{\mu_1}^2\right)\right\}
\end{aligned}$$

$\Delta = \mu_2 - \mu_1$

the Bessel functions are evaluated at $x = k/k_0$

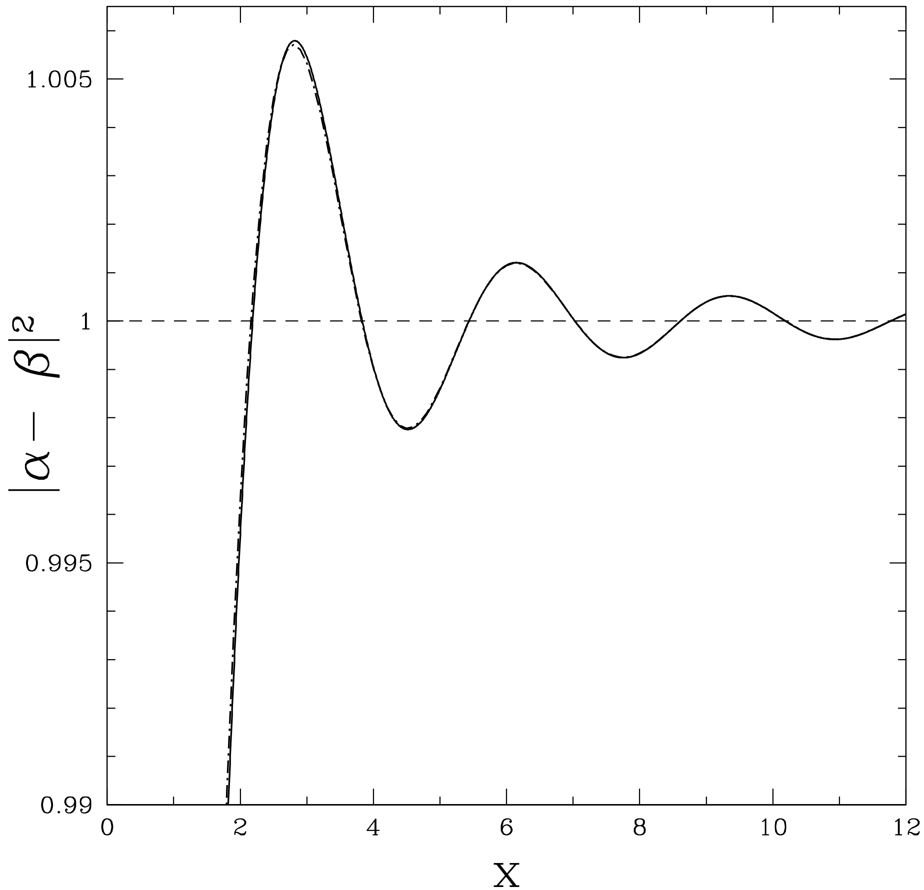
Asymptotic forms

1. For $x = k/k_0 \gg 1$

$$
\begin{aligned}
|\alpha - \beta|^2 &\simeq 1 - \frac{\tilde{\Delta}}{2x^2} \sin(\pi\mu_2 - 2x) + \frac{\tilde{\Delta}(4\mu_2^2 - 5)}{8x^3} \cos(\pi\mu_2 - 2x) \, , \\
\tilde{\Delta} &= \mu_2^2 - \mu_1^2 \, .
\end{aligned}
$$

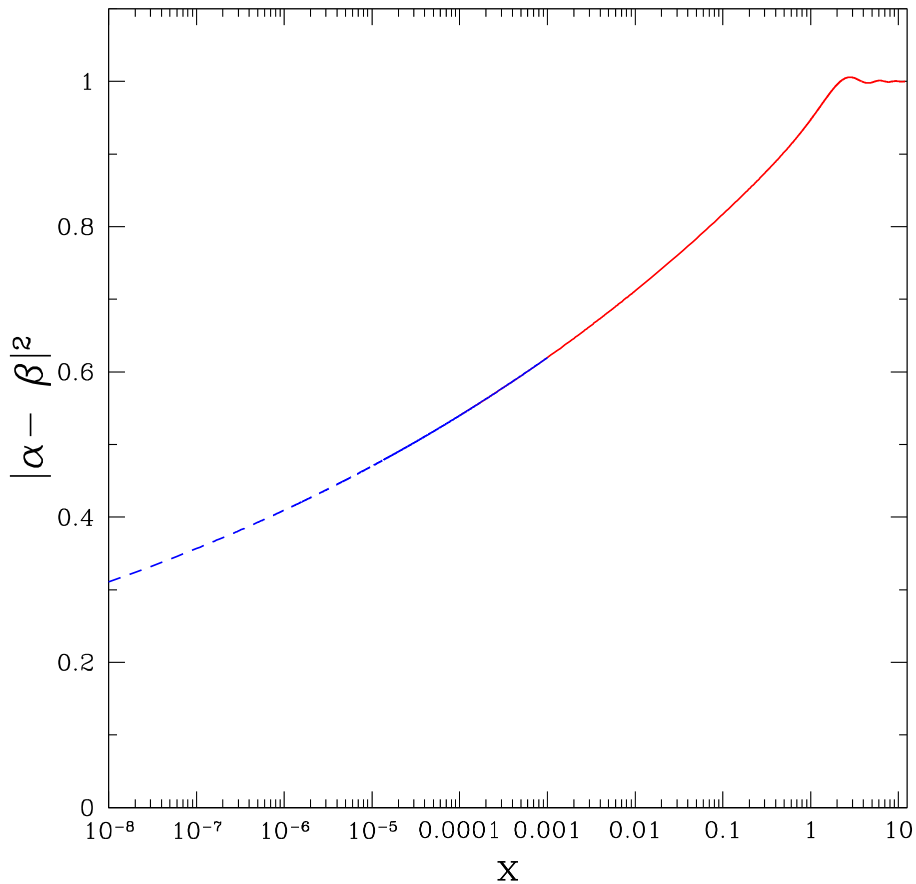
2. For $x \ll 1$

$$
|\alpha - \beta|^2 \simeq \frac{\Gamma^2(\mu_1)}{\Gamma^2(1+\mu_2)} \left(\frac{\mu_1 + \mu_2}{2} \right)^2 \left(\frac{x}{2} \right)^{2(\mu_2 - \mu_1)}
$$



$|\alpha - \beta|^2$ is shown as a function of $x = k/k_0$

The relevant values of the parameters are $\mu_1 = 1.49, \mu_2 = 1.52$.



1
0.8
0.6
0.4
0.2
0
|α − β|²
10⁻⁸
10⁻⁷
10⁻⁶
10⁻⁵
0.0001
0.001
0.01
0.1
1
10
x

The effective spectral index $n_s(k)$

$$n_s(k) - 1 = n_2 - 1 + \frac{d \log \left(|\alpha - \beta|^2 \right)}{d \log k}$$

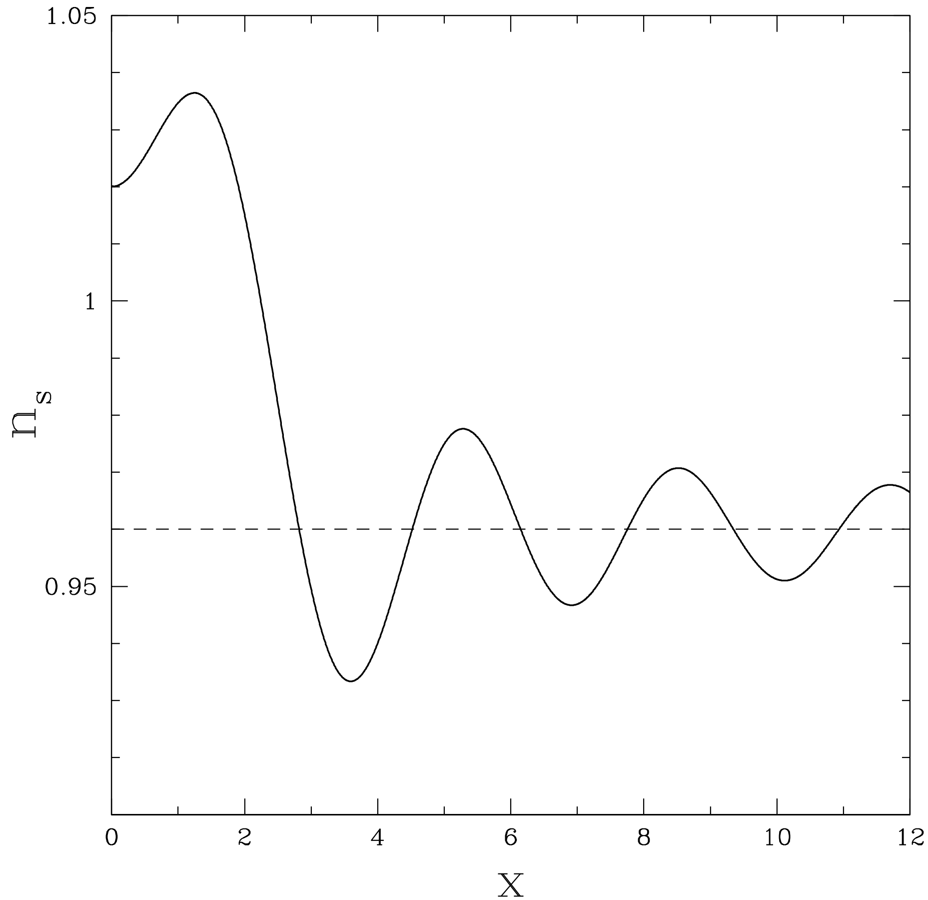
the spectral index - asymptotic forms

1. For $x = k/k_0 \gg 1$

$$n_s \simeq n_2 + \tilde{\Delta} \, \frac{\cos\left(\pi\mu_2 - 2x\right)}{x} + \tilde{\Delta} \left(\mu_2^2 - \frac{1}{4} \right) \frac{\sin\left(\pi\mu_2 - 2x\right)}{x^2}$$

2. For $x \ll 1$

$$n_s \simeq n_2 + 2\left(\mu_2 - \mu_1\right)$$



The primordial spectral index n_s is shown as a function of $x = k/k_0$ for an inflationary model in which the potential has a sudden change in its second derivative. Such a discontinuity in V'' leads to step in n_s at $x \sim 1$ which is followed by oscillations with decreasing amplitude. $\mu_1 = 1.49$, $\mu_2 = 1.52$ which correspond to $n_1 = 1.02$, $n_2 = 0.96$

Discontinuity in the second derivative of the inflaton potential (a step) leads to a step in the spectral index

The present numerical model
- drops from $n_s = 1.02$ at $k/k_0 \ll 1$ to $n_s = 0.96$ at $k/k_0 \gg 1$. The step in n_s is accompanied by 'ringing' – slowly decreasing oscillations in n_s about the mean (asymptotic) value of $n_s = 0.96$

Running of the spectral index

Inflationary model
???

4. Inflationary model with a step-like discontinuity in the evolution of the effective mass

Hybrid inflationary scenario

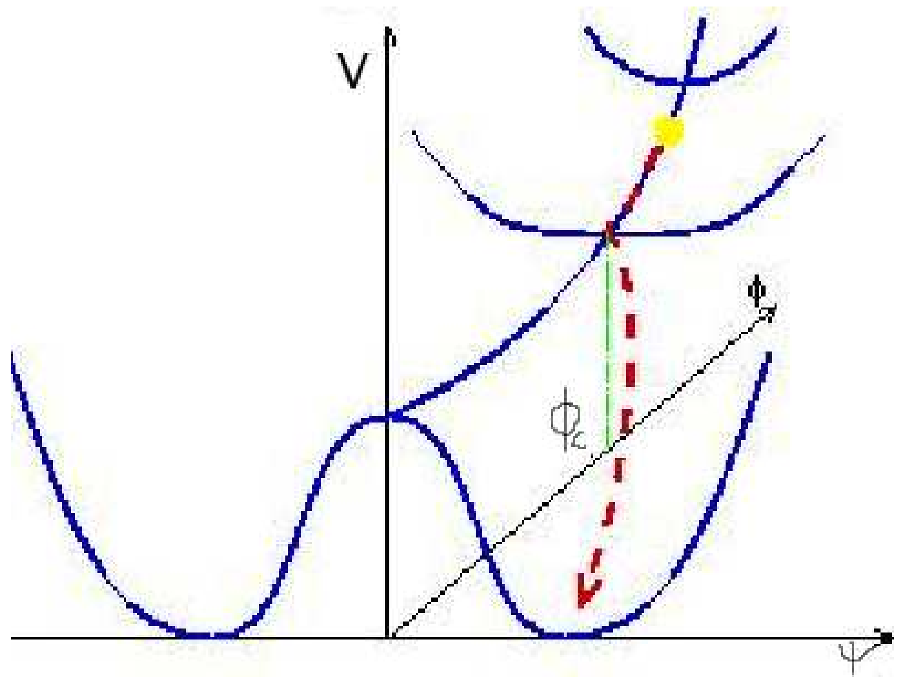
$$V(\psi,\phi) = \frac{1}{4\lambda}\left(M^2 - \lambda\psi^2\right)^2 + \frac{1}{2}m^2\phi^2 + \frac{g^2}{2}\phi^2\psi^2 \ .$$

Effective mass of the field ψ

$$m_\psi^2 \equiv \frac{d^2V}{d\psi^2}|_{\psi=0} = g^2\phi^2 - M^2$$

critical value $\phi_c = M/g$

$m_\psi^2 > 0$ if $\phi > \phi_c$ and $m_\psi^2 < 0$ if $\phi < \phi_c$



✓ $\phi > \phi_c$ the only minimum of the effective potential is at $\psi = 0$
✓ $\phi = 0$ waterfall - rapid cascade of ψ towards the minimum of its potential

✓ $\phi < \phi_c$ phase transitions with symmetry breaking

before the phase transition $\phi > M/g$, $\psi = 0$

$$V(\phi) = \frac{M^4}{4\lambda} + \frac{m^2\phi^2}{2} \ ,$$

and $\partial^2 V/\partial\phi^2 = m^2$

$$\alpha = 2\lambda m^2/g^2 M^2$$

at the instant of transition

$$V(\phi_c) = \frac{M^4}{4\lambda}(1+\alpha)$$

Condition for slow-roll inflation - prior to the transition

$$M^2 \gg \sqrt{\frac{3\lambda}{2\pi}}\,\frac{m\,m_p}{(1+\alpha)^{1/2}}\ .$$

Soon after the transition
$\phi < M/g, \quad \psi^2 = (M^2 - g^2\phi^2)/\lambda$

$$V(\phi) = \frac{1}{2}(m^2 + \frac{g^2M^2}{\lambda})\phi^2 - \frac{g^4\phi^4}{4\lambda}$$

slow-roll remain valid immediately after the transition

$$\frac{M}{g m_P}\,(1+\alpha)^{\frac{1}{2}} \gg 1$$

$\frac{|\partial^2 V/\partial\psi^2|}{H^2} \gg 1$ applied imediately after the transition

$$M^3 \ll \frac{\lambda m\, m_P^2}{(1+\alpha)^2}$$

$g^2 \ll \lambda$

Spectral Index

$$n-1=-\frac{3}{8\pi G}\left(\frac{V'}{V}\right)^2+\frac{1}{4\pi G}\left(\frac{V''}{V}\right) ,$$

before the transition

$$n_1-1=\frac{1}{2\pi}\left(\frac{gm_P}{M}\right)^2\frac{\alpha\,(1-2\alpha)}{(1+\alpha)^2}$$

after the transition

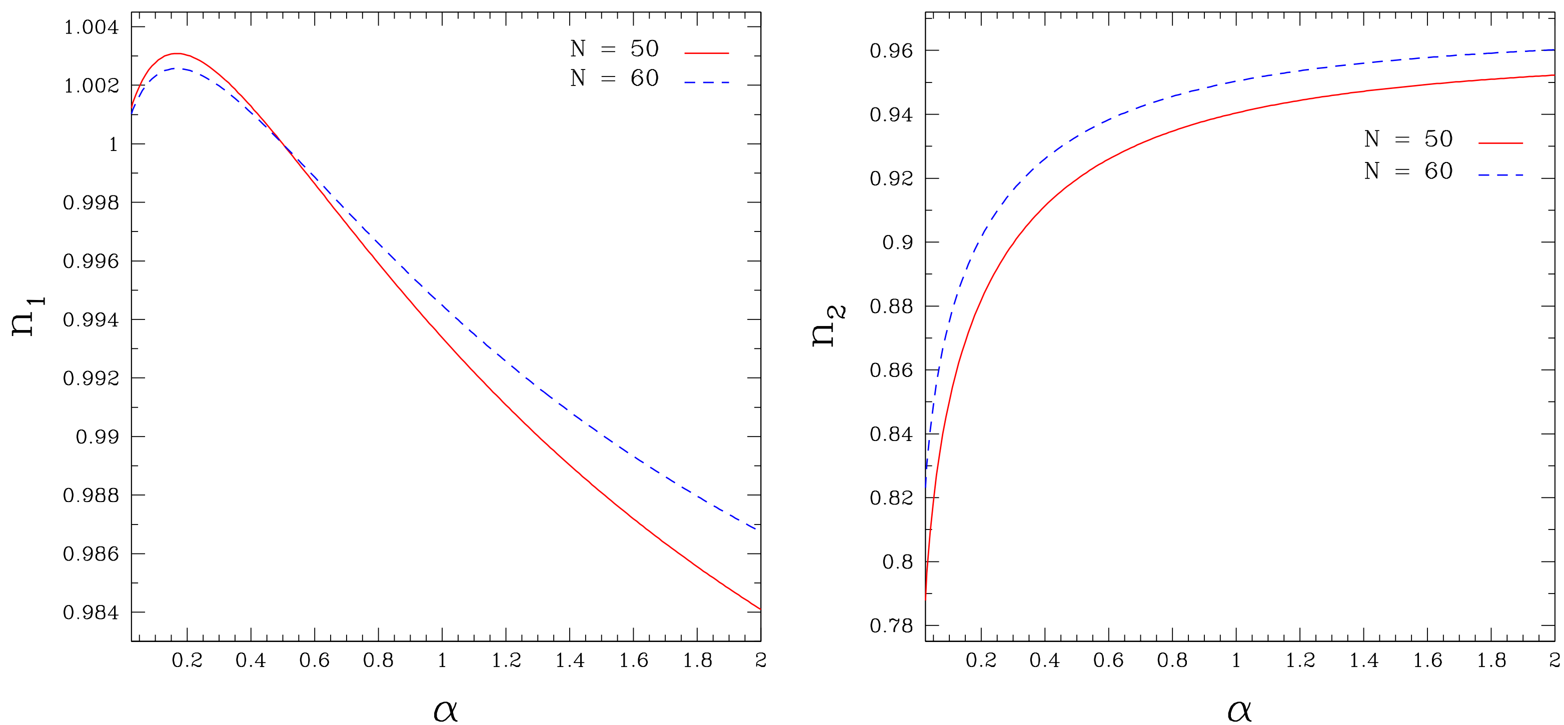
$$n_2-1=\frac{1}{2\pi}\left(\frac{gm_P}{M}\right)^2\left\{\frac{\alpha-4}{1+\alpha}-3\left(\frac{\alpha}{1+\alpha}\right)^2\right\}$$

the inflationary spectrum on large scales has a red (blue) tilt if $\alpha > 1/2$ ($\alpha < 1/2$)
$\alpha = 1/2$ results in precise scale-invariance for the initial spectrum, $n_1 = 1$

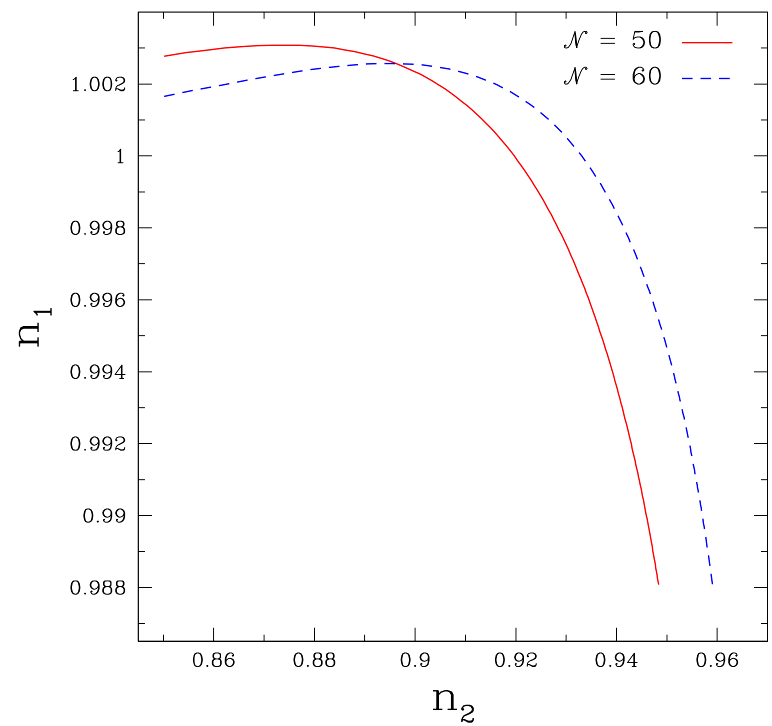
Total change in the spectral index during the course of the transition

$$\Delta n \equiv n_1 - n_2 = \frac{2}{\pi(1+\alpha)}\left(\frac{gm_P}{M}\right)^2 .$$

α must not be too large since otherwise $n_1 \simeq n_2$



The spectral index just before (n_1) the phase transition in hybrid inflation and immediately after it (n_2), is shown as a function of $\alpha = 2\lambda m^2/g^2 M^2$



Spectral indices for perturbations generated just before (n_1) and immediately after (n_2) the phase transition in hybrid inflation

α and gm_P/M are also related to the number of inflationary e-folds which take place after the phase transition has occurred

$$\begin{aligned}\mathcal{N} &= \frac{8\pi}{m_p^2}\int_{\varphi_{end}}^{\varphi_c}\frac{V}{V'}\,d\varphi \\ &= \pi\left(\frac{M}{gm_P}\right)^2\left[1-\left(1+\frac{\alpha}{2}\right)\log\frac{\alpha}{2+\alpha}\right] \ .\end{aligned}$$

Values of parameters (M, m, g) ?

comparing the inflationary curvature fluctuation on large scales with the observed CMB fluctuation

4 year COBE data implies, for an LCDM Universe

Bunn & White (1996)

$$\delta_H = 1.91\times10^{-5}\ \frac{\exp\left[1.01(1-n)\right]}{\sqrt{1+f(\Omega_m)r}}\,\Omega_m^{-0.8-0.05\log\Omega_m}\times\{1-0.18(1-n)\Omega_\Lambda-0.03r\Omega_\Lambda\}\ ,$$

where $f(\Omega_m) = 0.75 - 0.13\Omega_\Lambda^2$

setting $\mathcal{N} = 60$ and $\lambda = 0.1$, for a spatially flat LCDM universe with $\Omega_m = 0.22$ and $\Omega_\Lambda = 0.78$,

α	g	M/m_p	m/m_p
1	3.09018 $\times10^{-4}$	8.29911 $\times10^{-4}$	5.73456 $\times10^{-7}$
0.5	2.97899 $\times10^{-4}$	7.50167 $\times10^{-4}$	3.53344 10^{-7}
0.25	2.68416 $\times10^{-4}$	6.29546 $\times10^{-4}$	1.88926 $\times10^{-7}$

- A model in which inflaton potential experiences a sudden change in its second derivative
- Background cosmology near the feature in the potential is studied. The small corections in the effective mass of the inflaton filed is obtained.
- Exact solution for the density perturbation spectrum is obtained and it is found that the solution has a quasi-flat power spectrum with a change of slope (step in n_s) which occurs at some scale and is modulated by characteristic oscillations.
- A field theoretic model which can give rise the rapid change in V'' is discussed.