Multiple quantum collapse of the inflaton field and its implications on the birth of cosmic structure

Gabriel León

Institute for Nuclear Sciences, UNAM, Mexico

PFNG-2010

joint work with D. Sudarsky

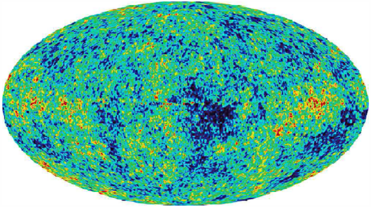
Outline

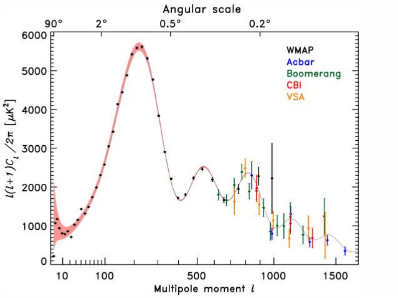
Successes of the Inflationary Paradigm

- Solves the problems of the Hot Big Bang model (Horizon, Flatness, etc.) [Guth, 1981]
- The main success...

Successes of the Inflationary Paradigm

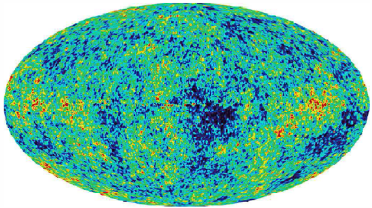
- Solves the problems of the Hot Big Bang model (Horizon, Flatness, etc.) [Guth, 1981]
- The main success...
  Quantum Fluctuations of the inflaton results in theoretical predictions of the spectrum of primordial *inhomogeneities*. [Mukhanov, 1992]
- Observational Data in agreement with such predictions!

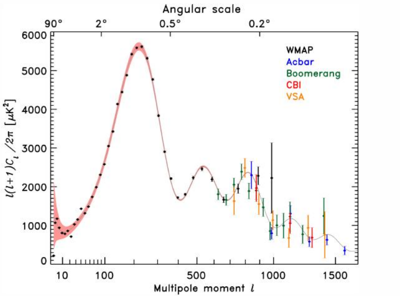




Successes of the Inflationary Paradigm

- Solves the problems of the Hot Big Bang model (Horizon, Flatness, etc.) [Guth, 1981]
- The main success...
  Quantum Fluctuations of the inflaton results in theoretical predictions of the spectrum of primordial *inhomogeneities*. [Mukhanov, 1992]
- Observational Data in agreement with such predictions!





- Note however, The oscillations have no relation with inflation but with plasma physics (inflation alone would lead to a flat line!)

Inflaton

- Action (Quantum Field Theory + General Relativity):

$$S = \int d^4x \quad \sqrt{-g}\left[\frac{1}{16\pi G}R[g] - \frac{1}{2}g^{ab}\nabla_a\phi\nabla_b\phi - V(\phi)\right] \tag{1}$$

- Separation (homogeneous-isotropic background + small fluctuations):

$$\phi(\mathbf{x},\eta) = \phi_0(\eta) + \delta\phi(\mathbf{x},\eta) \tag{2}$$

- Field perturbations induce metric perturbations:

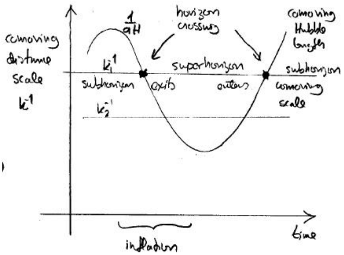
$$ds^2 = a(\eta)^2[-(1+2\Psi)d\eta^2 + (1-2\Psi)\delta_{ij}dx^i dx^j] \tag{3}$$

- Introduce a new field variable $v = a\delta\phi + \Psi\phi_0' a'$
- After choosing a vacuum state, one proceeds to quantize the theory

$$\hat{v}(\eta,x) = \frac{1}{\sqrt{2}}\int \frac{d^3k}{(2\pi)^{3/2}}[v_k^\star(\eta)e^{i\mathbf{k}\cdot\mathbf{x}}\hat{a}_k^- + v_k(\eta)e^{-i\mathbf{k}\cdot\mathbf{x}}\hat{a}_k^+] \tag{4}$$

A brief description of the origin of cosmic structure

- During the very early times of inflation, the scales associated with quantum fluctuations are inside the horizon, i.e., $a\lambda < H^{-1}$. Those quantum fluctuations are the primordial seeds of large-scale structure.
- In the inflationary stage, metric perturbations with scales $a\lambda > H^{-1}$ are "frozen", that is, its amplitud remains constant. One then is invited to treat the perturbations in a classical way.
- At the end of inflation (and after a *reheating* period), the universe follows the standard evolution as given by the Big-Bang model. The previous frozen perturbations re-enter the horizon and continue their evolution into the structure we observe today.



Theoretical Predictions

- Express v in terms of Ψ and calculate the 2-point correlation function in the vacuum state $|0\rangle$:

$$\langle 0|\hat{\Psi}(\eta,x)\hat{\Psi}(\eta,y)|0\rangle = \int \frac{dk}{k}\frac{1}{2\pi^2}|\Psi_k(\eta)|^2 k^3 \frac{\sin kr}{kr} \tag{5}$$

where $r \equiv |x-y|$.

- The predicted power spectrum of the metric perturbations is:

$$\mathcal{P}_\Psi(k,\eta) = \frac{1}{2\pi^2}|\Psi_k(\eta)|^2 k^3 \tag{6}$$

- Considering $\Psi_k(\eta) \approx \frac{2}{3}\left(H\frac{\delta\phi}{\phi_0'}\right)_{k\sim aH}$ for the scales which re-enter the horizon during the radiation dominated epoch, one obtains that the predicted power spectrum is thus:

$$\mathcal{P}_\Psi(k,\eta_r) \approx \frac{V}{\epsilon M_{pl}^4} \tag{7}$$

Something is not clear...

- It is not clear:

$$\langle 0|\hat{\Psi}(\eta,x)\hat{\Psi}(\eta,y)|0\rangle = \overline{\Psi(\eta,x)\Psi(\eta,y)}$$

> *"These are quantum averages, not averages over an ensemble of classical field configurations...some sort of decoherence must set in...It is not apparent just how this happen..."*

[S. Weinberg, *Cosmology*; 2008]

- **Remarkable**: The universe was originally in a homogeneous-isotropic state and there is a scalar field (the inflaton) which is in a vacuum state also homogeneous and isotropic (there are some irrelevant deviations of this left from an imperfect inflation at the order of e^{-80}), but the universe ended up with inhomogeneities that fit the experimental data.
- **However** The standard quantum mechanical time evolution does not break this symmetry (the symmetry being the homogeneity and isotropy)

Something is not clear...

- It is not clear:

$$\langle 0|\hat{\Psi}(\eta,x)\hat{\Psi}(\eta,y)|0\rangle = \overline{\Psi(\eta,x)\Psi(\eta,y)}$$

> *"These are quantum averages, not averages over an ensemble of classical field configurations...some sort of decoherence must set in...It is not apparent just how this happen..."*

  [S. Weinberg, *Cosmology*; 2008]

- **Remarkable**: The universe was originally in a homogeneous-isotropic state and there is a scalar field (the inflaton) which is in a vacuum state also homogeneous and isotropic (there are some irrelevant deviations of this left from an imperfect inflation at the order of e^{-80}), but the universe ended up with inhomogeneities that fit the experimental data.
- **However** The standard quantum mechanical time evolution does not break this symmetry (the symmetry being the homogeneity and isotropy)

The fundamental question

- **How did it happen?**

$$|\text{homogeneous}\rangle \quad \rightarrow \quad |\text{inhomogeneous}\rangle$$

- This question is often presented in the literature as: “How do quantum fluctuations become classical?”
- The most popular answer: *Decoherence*

The fundamental question

- **How did it happen?**

$$|\text{homogeneous}\rangle \quad \rightarrow \quad |\text{inhomogeneous}\rangle$$

- This question is often presented in the literature as: "How do quantum fluctuations become classical?"
- The most popular answer: *Decoherence*

The fundamental question

- **How did it happen?**

$$|\text{homogeneous}\rangle \quad \rightarrow \quad |\text{inhomogeneous}\rangle$$

- This question is often presented in the literature as: "How do quantum fluctuations become classical?"
- The most popular answer: *Decoherence*

Is it “just philosophy?”

V. Mukhanov in his book *Physical Foundations of Cosmology* (2005), on the issue of how do the inhomogeneities arise:

> *“Quantum mechanical unitary evolution does not destroy translational invariance... However decoherence is not sufficient to explain the breaking of translational invariance. ... we have to appeal to either to Bohr’s reduction postulate or to Everett’s many-worlds interpretation of quantum mechanics. The first possibility does not look convincing in the cosmological context.”*

- There are of course other postures, as well. However, often these issues are resolved as “just philosophy”, with no impact whatsoever on the predictions of the primordial spectrum and related issues.
- We will see that such preconception is mistaken

Our Approach

We have considered in detail one approach (inspired by Penrose's ideas): The standard paradigm + "self-induced collapse" hypothesis.

- The metric description of gravity is just an effective one. The fundamental theory describes gravity in terms of some more fundamental degrees of freedom. Quantum Mechanics is incomplete.
- A new ingredient brought into physics by quantum-gravity has an effective description as a self-induced collapse (which does not rely on an external agent to induce it).

The transition and the collapse proposal

Our approach on attempting to answer the question: "How does it happen?
$|\text{homogeneous}\rangle \rightarrow |\text{in-homogeneous}\rangle$ is described as:

- Invoking the collapse of the wave function, one could break the symmetry of the original state.

$$|\text{homogeneous}\rangle \quad \rightarrow \textbf{Collapse} \rightarrow \quad |\text{inhomogeneous}\rangle$$

- According to Quantum Mechanics the collapse of the wave function (and therefore the onset of the asymmetry) occurs only as a result of a measurement. But, how do we apply this postulate in the cosmological setting?.
- Phenomenological Model: *Something* (possibly Quantum Gravity) intrinsic to the system occurred triggering the collapse of its wave function.
- Once one hypothesizes that there is a new kind of physical process which affects the system under investigation, it seems logical to consider the possibility that it occurs more than once, and in circumstances different from those for which it was first proposed. One can parameterize the collapses.

[Sudarsky, 2006]

The transition and the collapse proposal

Our approach on attempting to answer the question: "How does it happen? $|\text{homogeneous}\rangle \rightarrow |\text{in-homogeneous}\rangle$ is described as:

- Invoking the collapse of the wave function, one could break the symmetry of the original state.

$$|\text{homogeneous}\rangle \qquad \rightarrow \textbf{Collapse} \rightarrow \qquad |\text{inhomogeneous}\rangle$$

- According to Quantum Mechanics the collapse of the wave function (and therefore the onset of the asymmetry) occurs only as a result of a measurement. But, how do we apply this postulate in the cosmological setting?.
- Phenomenological Model: *Something* (possibly Quantum Gravity) intrinsic to the system occurred triggering the collapse of its wave function.
- Once one hypothesizes that there is a new kind of physical process which affects the system under investigation, it seems logical to consider the possibility that it occurs more than once, and in circumstances different from those for which it was first proposed. One can parameterize the collapses.

[Sudarsky, 2006]

Semiclassical Gravity

- Gravity and Matter are treated differently and are coupled to the inflaton according to:

$$G_{ab} = 8\pi G \langle \hat{T}_{ab} \rangle$$

- This is supposed to hold at all times except when a quantum gravity induced collapse of the wave function occurs,
- At this *time of collapse* the excitation of the fundamental quantum-gravitational degrees of freedom must be taken into account and there is a breakdown in the semi-classical approximation.
- This possible breakdown is represented by the presence of a term Q_{ab} in the semi-classical Einstein's equation which is supposed to become non-zero **only** during the collapse of the quantum mechanical wave function.

$$G_{ab} + Q_{ab} = 8\pi G \langle \hat{T}_{ab} \rangle$$

Detailed analysis of the collapse proposal

- Quantum field: $\hat{y} = a\hat{\delta\phi}$;
- Canonical conjugate momentum $\hat{\pi}_y = a\hat{\delta\phi}' = \hat{y}' - \hat{y}a'/a$
- Einstein semiclassical equations:

$$G_{ab} = 8\pi G\langle \hat{T}_{ab}\rangle \qquad \Rightarrow \qquad \Psi_k(\eta) = \frac{-4\pi G\phi_0'}{ak^2}\langle\hat{\pi}_k(\eta)\rangle_{c_n}$$

- **Before** the collapse $|0\rangle$:

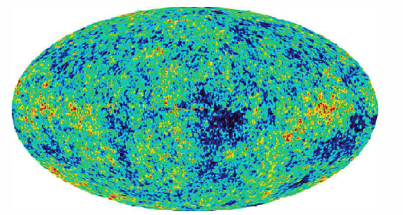
$$\langle \hat{y}_k^{R,I}\rangle_0 = 0; \qquad \langle \hat{\pi}_k^{R,I}\rangle_0 = 0 \qquad \Rightarrow \qquad \Psi_k(\eta) = 0$$

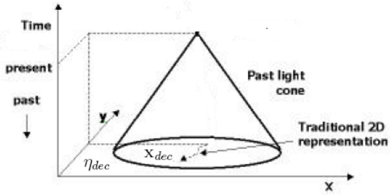
- Assume that at time $\eta_k^{c_n}$ (during the inflationary regime) the mode k has collapsed n times according to the scheme:

$$\langle \hat{y}_k^{(R,I)}(\eta_k^{c_n})\rangle_{c_n} = x_{k,I}^{(n)(R,I)}\sqrt{(\Delta\hat{y}_k^{(R,I)}(k\eta_k^{c_n}))^2_{c_{n-1}}} + \langle \hat{y}_k^{(R,I)}(\eta_k^{c_n})\rangle_{c_{n-1}}, \qquad (8a)$$

$$\langle \hat{\pi}_k^{(R,I)}(\eta_k^{c_n})\rangle_{c_n} = x_{k,II}^{(n)(R,I)}\sqrt{(\Delta\hat{\pi}_k^{(R,I)}(k\eta_k^{c_n}))^2_{c_{n-1}}} + \langle \hat{\pi}_k^{(R,I)}(\eta_k^{c_n})\rangle_{c_{n-1}}. \qquad (8b)$$

- The measured quantity is $\frac{\delta T}{T_0}(\theta, \varphi)$ which is related with Ψ





- The quantity of interest is:

$$a_{lm} = \int d\Omega \Psi(\eta_D, \mathbf{x}_D) Y^{\star}_{lm}(\theta, \varphi) \quad (9)$$

- One identifies the theoretical prediction of $|a_{lm}|^2$ with the observed value of $|a^{obs}_{lm}|^2$. Ignoring the physics of the plasma, the observations are recovered if the theoretical prediction for $|a_{lm}|^2$ is:

$$|\alpha_{lm}|^2 = \frac{V}{\epsilon M^4_{pl}} \int \frac{dk}{k^2} |j_l(kR_D)|^2 C \quad (10)$$

Theoretical predictions under the multiple collapse scheme

- The multiple collapse scheme predicts $|a_{lm}|^2$ to be:

$$|\alpha_{lm}|^2_{M.L.} = \frac{V}{\epsilon M_{pl}^4} \int \frac{dk}{k^2} |j_l(kR_D)|^2 \sum_{n=1}^{N} C_l^{(n)}(k) \tag{11}$$

where

$$\begin{aligned} C_l^{(n)}(k) &= \left(\sin(k\eta_k^{c_n})\right)^2 \left(Y_{\mathbf{k}}^{+} + (-1)^l Y_{\mathbf{k}}^{-}\right) \\ &+ \left(\cos(k\eta_k^{c_n}) - \frac{\sin(k\eta_k^{c_n})}{k\eta_k^{c_n}}\right)^2 \left(\Pi_{\mathbf{k}}^{+} + (-1)^l \Pi_{\mathbf{k}}^{-}\right) \end{aligned}$$

and

$$Y_{\mathbf{k}}^{\pm} \equiv (\Delta\hat{y}_k^R(k\eta_k^{c_n}))^2_{c_{n-1}} \pm (\Delta\hat{y}_k^I(k\eta_k^{c_n}))^2_{c_{n-1}}, \tag{12a}$$

$$\Pi_{\mathbf{k}}^{\pm} \equiv (\Delta\hat{\pi}_k^R(k\eta_k^{c_n}))^2_{c_{n-1}} \pm (\Delta\hat{\pi}_k^I(k\eta_k^{c_n}))^2_{c_{n-1}}, \tag{12b}$$

Gravitational Waves from inflation

An important observation follows directly from the point of view adopted to relate the metric effective description of gravity with the quantum aspect of the matter field, i.e., $G_{ab} = 8\pi G\langle\hat{T}_{ab}\rangle$:

- The source of the fluctuations that lead to anisotropies and inhomogeneities lies in the quantum uncertainties for the scalar field, which collapses, due to some unknown quantum gravitational effect. Once collapsed, these density inhomogeneities and anisotropies feed into the gravitational degrees of freedom leading to nontrivial perturbations in the metric functions, in particular, the newtonian potential. However, the metric itself is not a source of the quantum gravitational induced collapse.
- Therefore, as the scalar field does not act as a source for the gravitational tensor modes -at least not at the lowest order considered here-, the tensor modes can not be excited. Thus, the scheme naturally leads to the prediction of a zero -or at least a strongly suppressed- amplitude of gravitational waves to the CMB.

Conclusions

- The standard inflationary approach does not provides a satisfactory explanation on how does the transition from a symmetrical state, corresponding to the primordial universe, to a non-symmetrical one.
- The collapse hypothesis was originally proposed to deal with the fundamental problem of the standard approach. This work, shows that, even though in principle we do not know precisely what is the nature of the physics behind what we call the collapse, we can, in fact, obtain some insights on the 'rules' that govern it, i.e., those determining the nature of post-collapse states, the time of collapse and the number of collapses of each mode, by comparing the observations with our theoretical predictions.
- This work shows that the ideas tied to the collapse proposal are not mere philosophical in nature, but are susceptible to standard theoretical analysis.

References

A. Perez, H. Sahlmann, and D.Sudarsky, *Classical and Quantum Gravity* **23** 2317, (2006) [arXiv: gr-qc/0508100].

A. De Unanue and D. Sudarsky, *Phys. Rev. D* **78** 043510, (2008) [arXiv: 0801.4702]

G. León and D. Sudarsky, *Classical and Quantum Gravity* **27** 225017, (2010) [arXiv: 1003.5950]