Primordial features induced by a non-singular cosmology

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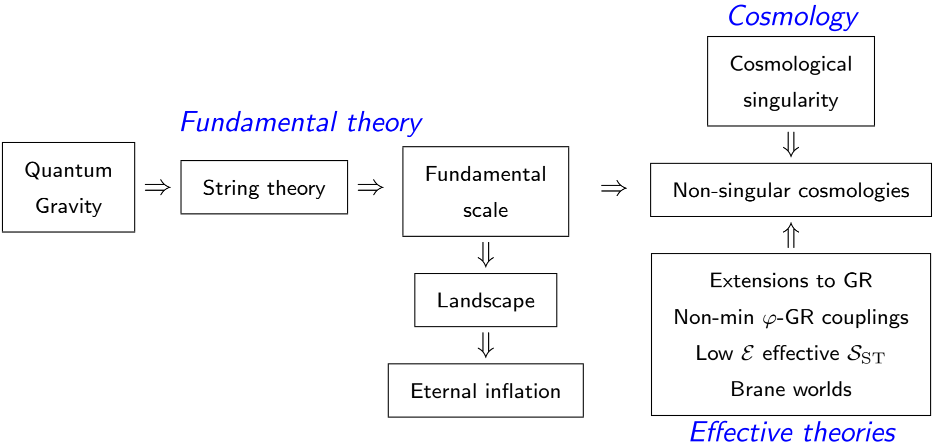
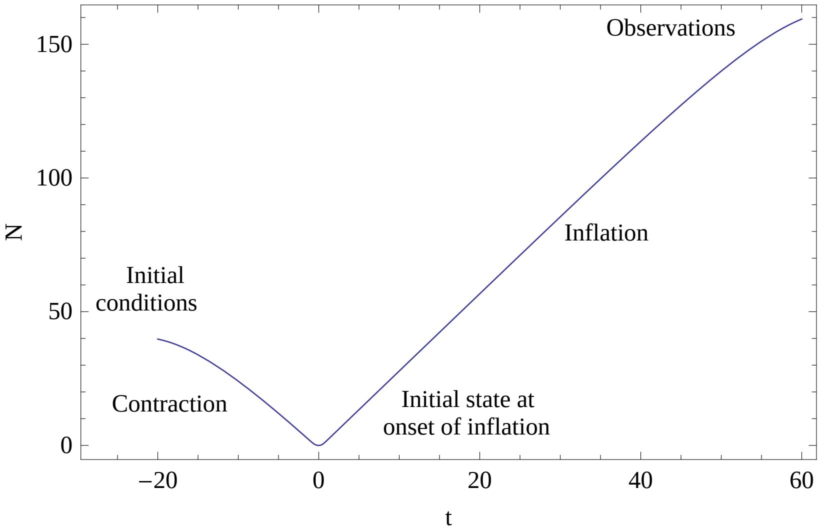
Outline

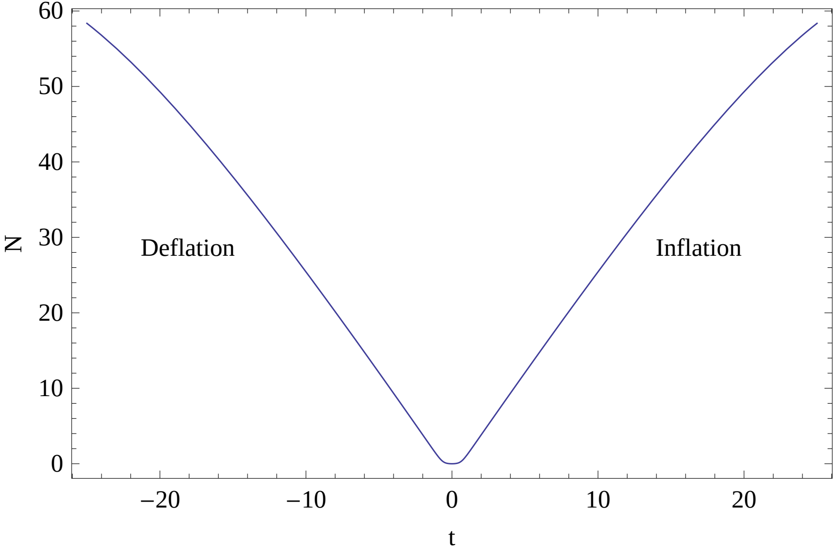
Motivation


Cosmology
Cosmological singularity
Fundamental theory
Quantum Gravity
String theory
Fundamental scale
Non-singular cosmologies
Landscape
Eternal inflation
Extensions to GR
Non-min φ-GR couplings
Low $\mathcal{E}$ effective $\mathcal{S}_{\mathrm{ST}}$
Brane worlds
Effective theories

Asymmetric cosmology



Symmetric cosmology (the subject of this talk)



Energy conditions and spatial curvature

- At the bounce :

$$a > 0 \quad \text{and} \quad \mathcal{H} = 0 \quad \text{and} \quad a'' > 0$$

- Energy conditions at the bounce :

$$\frac{\kappa}{2} a^2 (\rho + P) = \mathcal{K} - \frac{a''}{a} \qquad \text{(NEC)}$$

$$\frac{\kappa}{6} a^2 (\rho + 3P) = -\frac{a''}{a} \qquad \text{(SEC)}$$

$\Rightarrow$ If $\mathcal{K} > 0$, only the SEC is violated

- Additional tools : φ, standard kinetic term, and $V(\varphi)$

Potential for φ

- dS limit, Λ and $\mathcal{K} > 0$:

$$a(t) = a_\mathrm{b} \cosh\left(\frac{t}{a_\mathrm{b}}\right)$$

- Departure from dS :

$$a(t) = a_\mathrm{b} \cosh(\omega t) \qquad \omega \neq a_\mathrm{b}^{-1}$$

- Time expansion of a, φ and $V(\varphi)$ for $\eta \sim 0$ and solve FE's and KGE :

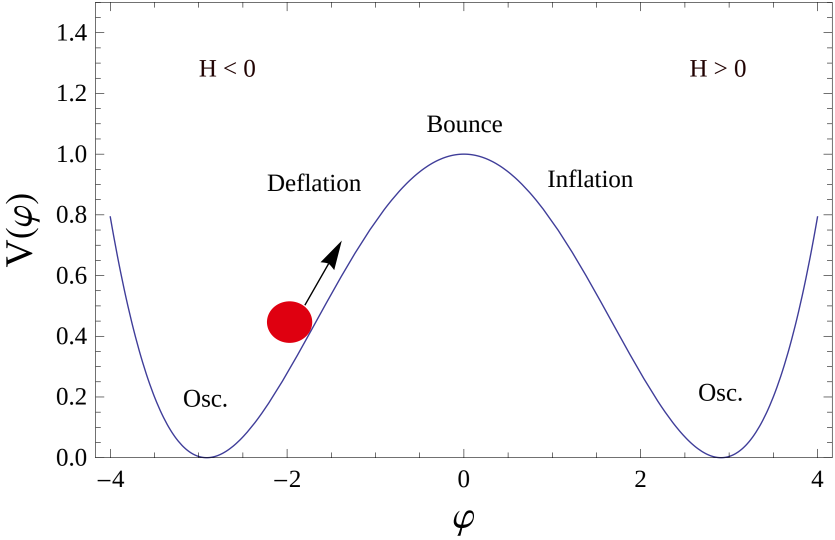
$$\varphi_\mathrm{b} = 0, \quad \varphi_\mathrm{b}'' = 0, \quad V(\varphi_\mathrm{b}) = V_0, \quad \left.\frac{\mathrm{d}V}{\mathrm{d}\varphi}\right|_\mathrm{b} = 0, \quad \left.\frac{\mathrm{d}^2 V}{\mathrm{d}\varphi^2}\right|_\mathrm{b} \leq 0$$

- Form of $V(\varphi)$:

$$V(\varphi) = V_0 - \frac{\mu^2}{2}\varphi^2 + \frac{\lambda}{24}\varphi^4$$

Martin & Peter (2003); Falciano, M.L. & Peter (2008)

Classical φ field evolution



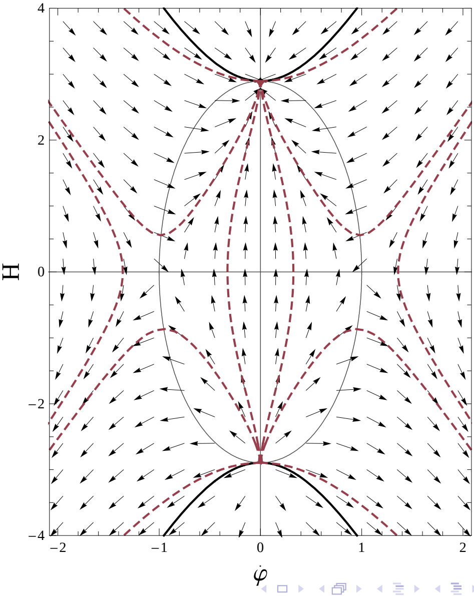
Phase space

Inflation $+\ \mathcal{K} > 0$

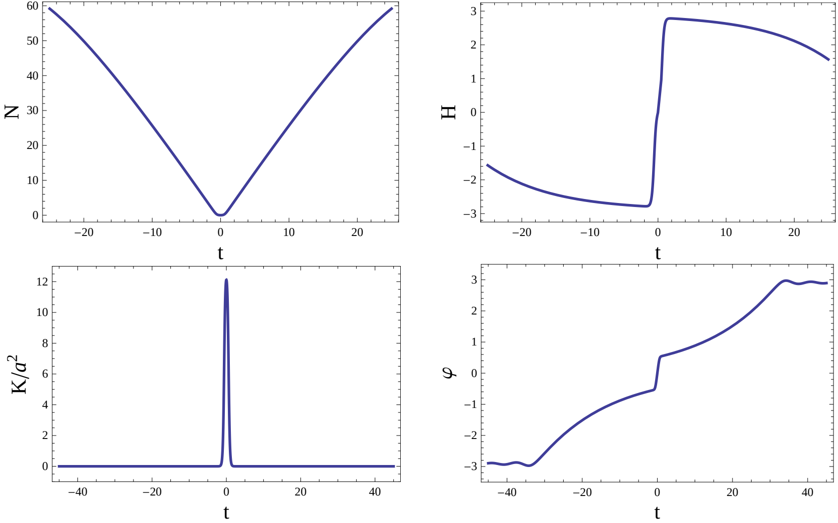
$\Downarrow$

Trajectories cross

$H = 0$ plane

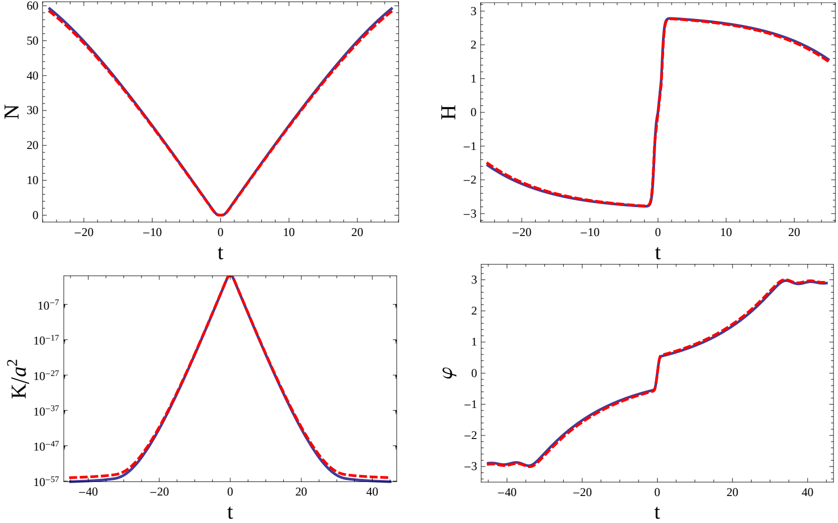


Cosmological evolution $\Omega_{\mathcal{K}} = -0.002$



$N_{\rm inf} = 65$	$a_0 = 70h^{-1}\rm Gpc$	$\epsilon_1 = 2\times 10^{-3}$	$n_{\rm s} = 0.953$	$\varsigma \ll 1$
$N_{\rm tot} = 130$	$\Omega_{\mathcal{K}} = -0.002$	$\epsilon_2 = 4\times 10^{-2}$	$a_{\rm b} = 3\times 10^5 \ell_{\rm Pl}$	$\Delta\eta = 6.04$

Cosmological evolution $\Omega_{\mathcal{K}} = -0.02$



$N_{\rm inf} = 64$ $\quad a_0 = 18 h^{-1}\mathrm{Gpc}$ $\quad \epsilon_1 = 4 \times 10^{-2}$ $\quad n_{\rm s} = 0.953$ $\quad \varsigma \ll 1$

$N_{\rm tot} = 129$ $\quad \Omega_{\mathcal{K}} = -0.02$ $\quad \epsilon_2 = 4 \times 10^{-2}$ $\quad a_{\rm b} = 2.8 \times 10^5 \ell_{\rm Pl}$ $\quad \Delta\eta = 6.285$

Bardeen potential vs Mukhanov-Sasaki variable

- Bardeen potential : $\Phi \to u$

$$u_k'' + \left[k^2 - V_u(\eta)\right] u_k = 0$$

$$V_u(\eta) = \mathcal{H}^2 + 2\left(\frac{\varphi''}{\varphi'}\right)^2 - \frac{\varphi'''}{\varphi'} - \mathcal{H}' + 4\mathcal{K}$$

- Mukhanov-Sasaki variable v :

$$v_k = -\frac{a}{\chi_k}\left(\delta\varphi_k^{(\mathrm{gi})} + \frac{\varphi'}{\mathcal{H}}\Phi_k - \frac{2\mathcal{K}}{\kappa\mathcal{H}\varphi'}\Phi_k\right)$$

$$V_v(\eta) = \frac{z_k''}{z_k} + 3\mathcal{K}(1 - c_\mathrm{s}^2)$$

$$z_k = a\frac{\varphi'}{\mathcal{H}\chi_k}$$

$$\chi_k = \begin{cases} 1 & \text{if} \quad \mathcal{K}/a^2 \ll 1 \\ f(k^2, c_\mathrm{s}^2) & \text{otherwise} \end{cases}$$

Garriga & Mukhanov (1999); Hwang & Noh (2002); Martin & Peter (2003)

Far from the bounce $v \to v^{\mathrm{flat}}$

Strategy

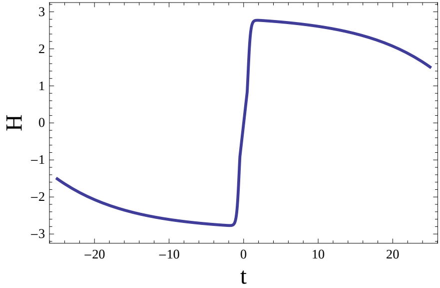
Deflation $\mathcal{K}/a^2 \ll 1$

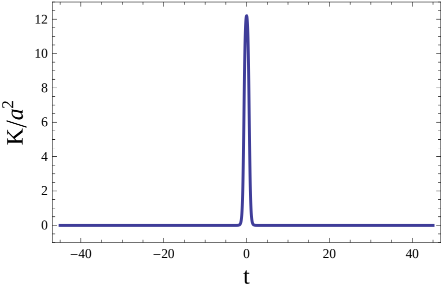
Initial conditions for v $\Rightarrow$ Relate v to u

$\Rightarrow$ Bounce: Evolve u $\Rightarrow$

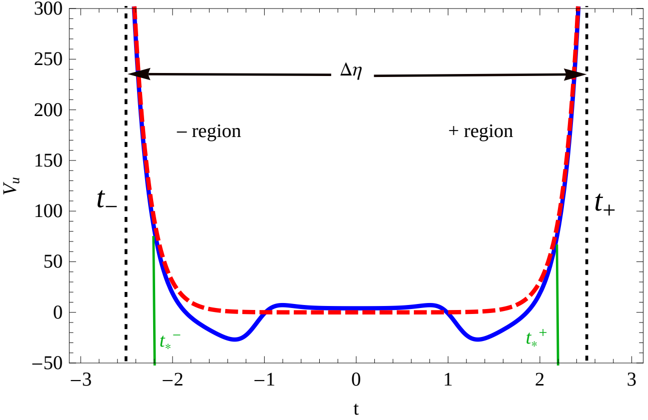
Inflation $\mathcal{K}/a^2 \ll 1$

Relate u to v $\Rightarrow$ $\mathcal{P}_\zeta(k)$, $\mathcal{C}_\ell$'s, $P_\delta(k)$





Potential for $u_k(\eta)$



$$\mathcal{H}_\pm(\eta) = -\frac{1+\epsilon_1}{\eta-\eta_\pm} = \pm\frac{1+\epsilon_1}{x_\pm} \quad \text{and} \quad V_u(\eta) = \frac{1}{x_\pm^2}\left(\epsilon_1 + \frac{\epsilon_2}{2}\right)$$

Solving for $u_k(\eta)$ through the bouncing phase

- EOM's for $u_k(\eta)$:

$$u_k'' + \left[k^2 - \frac{1}{x_\pm^2}\left(\epsilon_1 + \frac{\epsilon_2}{2}\right)\right] u_k = 0$$

- Solutions for $u_k(\eta)$:

$$u^-(\eta) = \sqrt{kx_-}\left[U_1^- H_\nu^{(1)}(kx_-) + U_2^- H_\nu^{(2)}(kx_-)\right]$$

and

$$u^+(\eta) = \sqrt{kx_+}\left[U_1^+ H_\nu^{(1)}(kx_+) + U_2^+ H_\nu^{(2)}(kx_+)\right]$$

- Matching at $\eta = 0$:

$$\begin{aligned} U_1^+ &= U_2^- \left(\sigma_k + i\right) e^{-i(k\Delta\eta - \pi\nu)} \\ U_2^+ &= U_1^- \left(\sigma_k - i\right) e^{i(k\Delta\eta - \pi\nu)} \end{aligned} \quad \text{with} \left\{ \begin{array}{l} \sigma_k = \dfrac{2\epsilon_1 + \epsilon_2}{k\Delta\eta} \\ \Delta\eta = \eta_+ - \eta_- \end{array} \right.$$

$v_k(\eta)$ far from the bounce

- Solutions for v_k $(\mathcal{K}/a^2 \sim 0)$:

$$\begin{aligned} v^-(\eta) &= \sqrt{kx_-}\left[V_1^- H_\varrho^{(1)}(kx_-) + V_2^- H_\varrho^{(2)}(kx_-)\right] \\ v^+(\eta) &= \sqrt{kx_+}\left[V_1^+ H_\varrho^{(1)}(kx_+) + V_2^+ H_\varrho^{(2)}(kx_+)\right] \end{aligned}$$

- Relate u and v :

$$v = \left(\frac{3}{\kappa}\right)^{1/2} \theta \left(\frac{u}{\theta}\right)' \quad \text{and} \quad k^2 u = -z\left(\frac{v}{z}\right)' \quad \text{for} \quad \mathcal{K} = 0$$

- Yields for v_k^- and v_k^+ :

$$\begin{aligned} V_1^+ &= V_2^- \left(\sigma_k + i\right) e^{-i(k\Delta\eta - \pi\varrho)} \\ V_2^+ &= V_1^- \left(\sigma_k - i\right) e^{i(k\Delta\eta - \pi\varrho)} \end{aligned}$$

Initial conditions for $v_k(\eta)$: a worked example

- Ansatz for V_1^- and V_2^- :

$$V_1^- = \frac{\sqrt{\pi}}{2}\varsigma_1 k^{-\alpha/2} e^{i\theta_1} \qquad \text{and} \qquad V_2^- = \frac{\sqrt{\pi}}{2}\varsigma_2\, k^{-\beta/2} e^{i\theta_2}$$

- Wronskian condition :

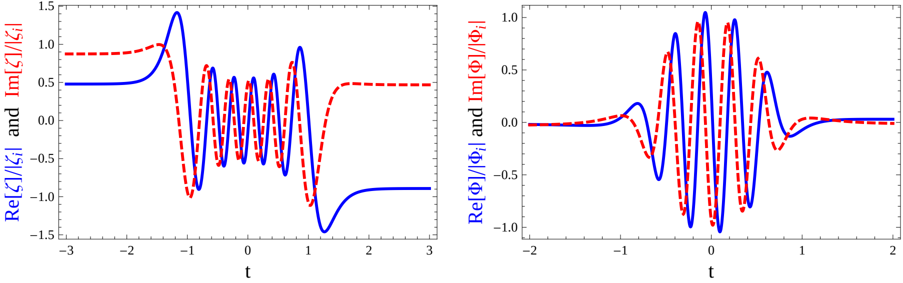
$$V_1^- = \frac{\sqrt{\pi}}{2}|\varsigma| k^{-1/2} e^{i\theta_1} \qquad \text{and} \qquad V_2^- = \frac{\sqrt{\pi}}{2}\left|1-\varsigma^2\right|^{1/2} k^{-1/2} e^{i\theta_2}$$

- Bunch-Davies limit :

$$\begin{aligned} \varsigma &= 0 \\ \theta_2 &= \frac{\pi}{2}\varrho + \frac{\pi}{4} \end{aligned}$$

Curvature perturbation and Bardeen potential

$$\zeta = \Phi - \frac{2}{\kappa}\frac{\mathcal{H}}{\varphi'^2}\left\{\Phi' + \left[1 - \frac{\mathcal{K}}{\mathcal{H}^2} + \frac{1}{3}\left(\frac{k}{\mathcal{H}}\right)^2\right]\mathcal{H}\Phi\right\}$$



Primordial power spectrum (1/2)

- Modification to $\mathcal{P}_\zeta^{\rm std}$:

$$\mathcal{P}_\zeta^{\rm nstd} = \varsigma^2 + |1-\varsigma^2| - 2\varsigma|1-\varsigma^2|^{1/2}\left[\cos(2k\Delta\eta+\theta_1-\theta_2) - \pi(2\epsilon_1+\epsilon_2)\sin(2k\Delta\eta+\theta_1-\theta_2)\right]$$

- Oscillatory frequency :

$$k_{\rm c}\Delta\eta = k_{\rm phys}\left\{\frac{4\,(1+z_{\rm b})}{\omega}\arctan\left[\tanh\frac{\omega(t_*^+-t_{\rm b})}{2}\right]+2\,(1+\epsilon_1)\,\frac{(1+z_*)}{H_*}\right\}$$

- Bounce time interval :

$$t_*^+ - t_{\rm b} = \frac{1}{\omega}\cosh^{-1}\left[\frac{1+z_{\rm b}}{1+z_{\rm end}}e^{-N_{\rm inf}}\right]$$

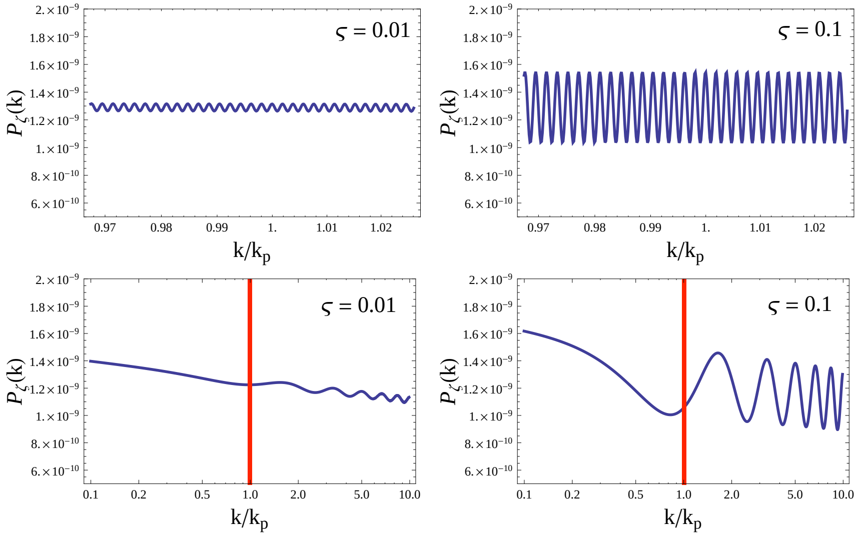
- Parameter ω

$$\omega = \sqrt{\kappa V_0 - \frac{2}{3}|1-\Omega|\,(1+z_{\rm b})\,H_0^2}$$

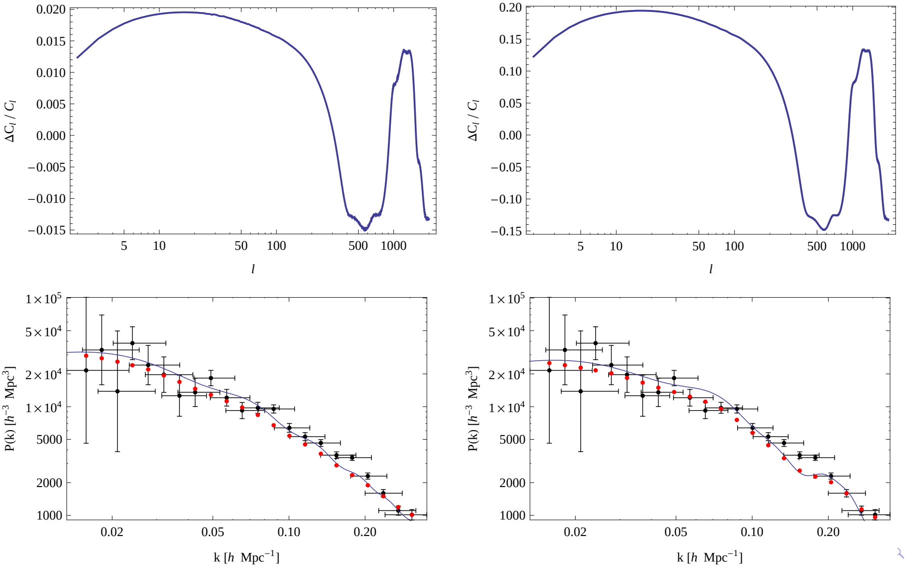
- Low frequency oscillation :

$$\mathcal{K}>0\,\Bigg|\;\begin{array}{c}\Rightarrow k\simeq n+1-1/2n\\ \text{for }(n\gg 1)\end{array}\;\left\{\begin{array}{l}\Delta\eta = m\pi + m\delta\eta\\ |\Omega_{\mathcal{K}}|\to\text{large}\end{array}\right.$$

Primordial power spectrum (2/2)



$\mathcal{C}_\ell$'s and $P_\delta(k)$ for $\Omega_\mathcal{K} = -0.02$



Conclusions and outlook

- Non-singular cosmologies are well-motivated by fundamental theory (quantum gravity), effective theories, and are desired for cosmological reasons.
- If we believe in inflation and allow $\mathcal{K} > 1$, bouncing cosmologies are a distinct possibility.
- This explicit example demonstrates that, in principle, features reflecting the bouncing timescale can be present in data.
- There exists a direct connection between the redshift of the bounce and the frequency of oscillations.
- Oscillations are linear in k ($\neq$ transplanckian).
- The amplitude of oscillations is almost decoupled from their frequency.
- The amplitude of oscillations is proportional to the deviation of the initial state from BDV.