Multifield Dynamics in D-brane Inflation

Liam McAllister

Cornell

Based on:

N. Agarwal, R. Bean, L.M., G. Xu, to appear.

D. Baumann, A. Dymarsky, S. Kachru, I. Klebanov, L.M., 1001.5028

Primordial Features and Non-Gaussianities

HRI, December 18, 2010

Summary

- Inflation is sensitive to Planck-scale physics, and string theory equips us to compute Planck-suppressed contributions to the inflaton action.
- The resulting models generically involve a number of light fields in a complicated potential.
- Key question: what is the characteristic dynamics of N fields in a ‘random’ potential?
- Detailed study of a string theory toy model with N=6 reveals interesting phenomena that are plausibly, and testably, general.

Plan

Background and motivation

- Inflation and Planck-scale physics
- Moduli and inflation in string theory
- Inflation in a random potential

Lamppost: D-brane inflation

- Setup
- Results: scaling behavior

Open problems

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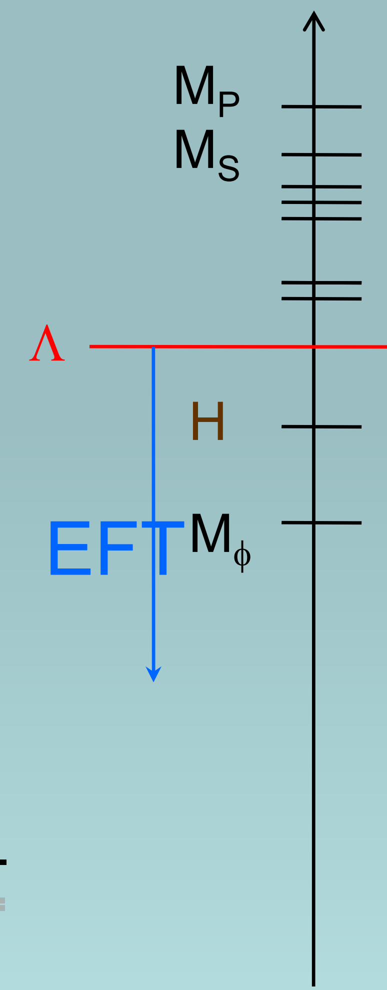
Motivation

- Inflation provides a beautiful causal mechanism to generate the observed CMB anisotropies and distribution of large-scale structure.
- Inflation is sensitive to Planck-scale physics: Planck-suppressed operators generically make critical contributions to the dynamics.
- This provides a remarkable opportunity to probe aspects of the ultraviolet completion of gravity through observation.
- To make meaningful use of this connection, we should:
  - Compute these Planck-suppressed contributions to the inflaton action in string theory
  - Search for characteristic properties: “what kind of inflation is natural in string theory?” (contrast “can I realize inflation of type X in string theory”).

Inflation in effective field theory

If we begin with a UV-complete theory, we derive an effective description by integrating out the massive fields ($M>\Lambda$).

Otherwise, we parameterize our ignorance of the UV theory by writing most general EFT consistent with (postulated) symmetries.



Result: non-renormalizable interactions among light fields induced by integrating out heavy fields, e.g.

$$V = V_0 + \mu^3\phi + m^2\phi^2 + \lambda\phi^4 + \sum_{i=1}^{\infty} c_i \frac{\phi^{\Delta_i}}{\Lambda^{\Delta_i - 4}}$$

Planck-sensitivity of inflation

$$S = \int d^4x \sqrt{-g}\left[\frac{1}{2}R + \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\,\partial_\nu\phi - V_0(\phi)\right]$$

$$\Delta V \equiv \mathcal{O}_\Delta = V_0\left(\frac{\phi}{\Lambda}\right)^{\Delta-4}$$

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$$\Delta = 6:\ \delta\eta \sim \left(\frac{M_p}{\Lambda}\right)^2 \gtrsim 1$$

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$$\Delta = 6 : \; \delta\eta \sim \left(\frac{M_p}{\Lambda} \right)^2 \gtrsim 1$$

For small inflaton excursions, $\Delta\phi \lesssim M_{pl}$, one must control corrections $\mathcal{O}_\Delta$ with $\Delta \lesssim 6$.

For large inflaton excursions, $\Delta\phi \gg M_{pl}$, one must control an infinite series of corrections, with arbitrarily large Δ.

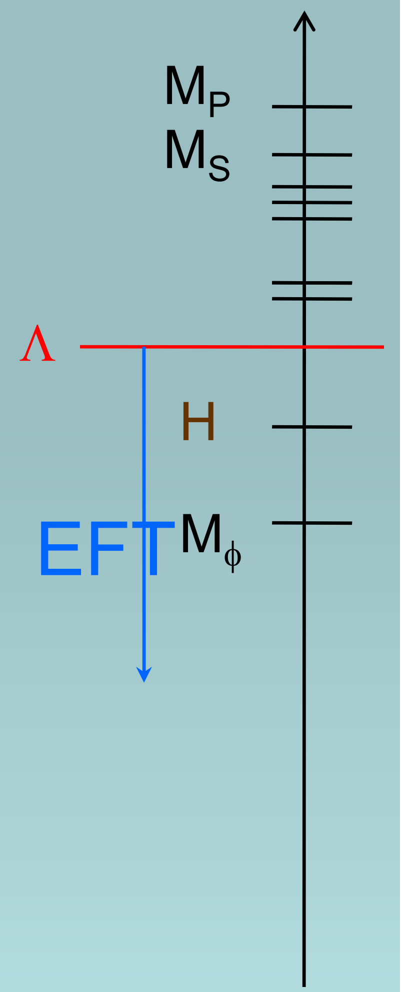
Inflation in effective field theory

Often in particle physics we are insensitive to effects arising at a sufficiently high cutoff Λ.

But for inflation, even $\Lambda = M_P$ is not high enough: even Planck-mass d.o.f. can substantially correct the inflaton action.

Moreover, we know that some new d.o.f. must appear at or below the Planck scale.

So, we should carefully examine Planck-suppressed contributions to the inflaton action in a theory of quantum gravity.



Options for dealing with the sensitivity to Planck-scale physics

I. Invoke a symmetry strong enough to forbid all such contributions.
   - i.e., forbid the inflaton from coupling to massive d.o.f.

Freese, Frieman, Olinto 1990
Arkani-Hamed, Cheng, Creminelli, Randall 2003
Kallosh, Hsu, Prokushkin 2004
Dimopoulos, Kachru, McGreevy, Wacker 2005
Conlon & Quevedo 2005
L.M., Silverstein, Westphal 2008
Flauger, L.M., Pajer, Westphal, Xu 2008

II. Enumerate all relevant contributions and determine whether fine-tuned inflation can occur.
   - i.e., arrange for cancellations. (this talk)

Baumann, Dymarsky, Klebanov, L.M., 2007
Baumann, Dymarsky, Kachru, Klebanov, L.M., 2008, 2009, 2010
Agarwal, Bean, L.M., Xu, to appear.

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In general, the contributions of interest arise from integrating out massive fields.

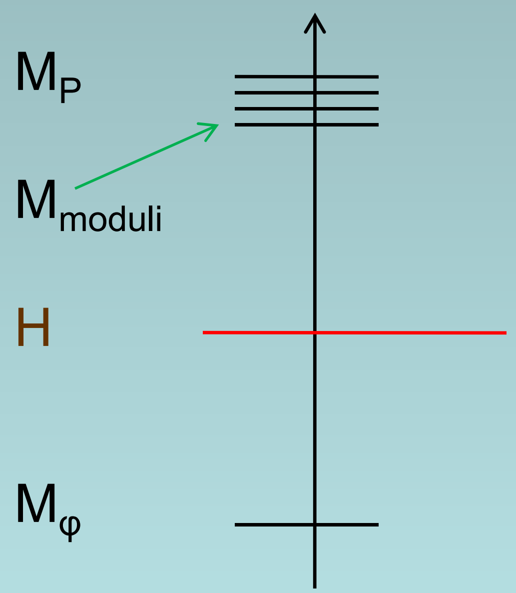
In string theory, the contributing massive fields include stabilized moduli.

So we should consider the spectrum and couplings of moduli in string theory.

Moduli and inflation in string theory

- String compactifications typically include many moduli. If massless, these are very problematic.
- Significant progress in the past decade: in flux compactifications, most moduli obtain masses.

want: light inflaton, Planck-mass moduli



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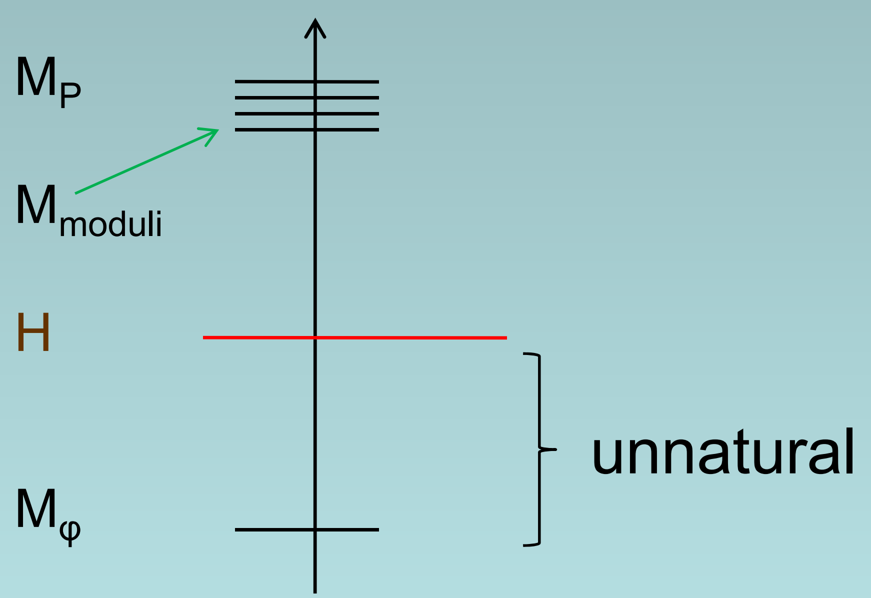

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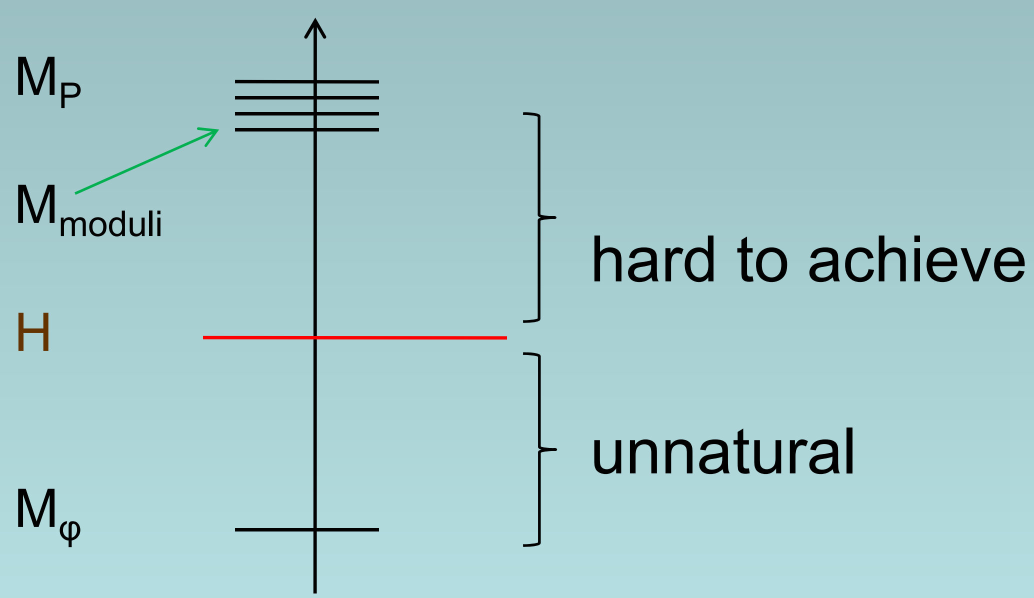
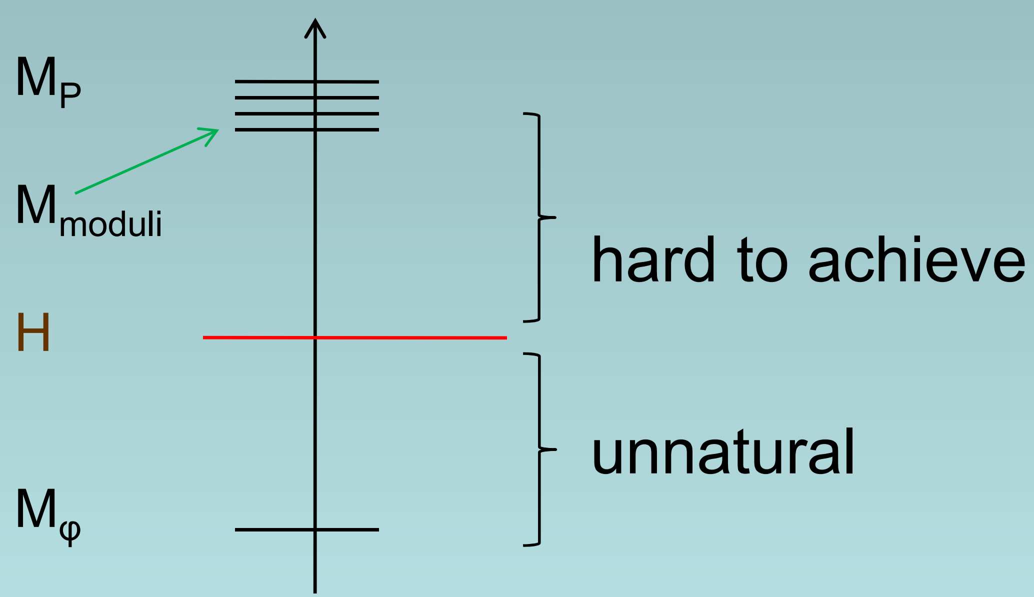


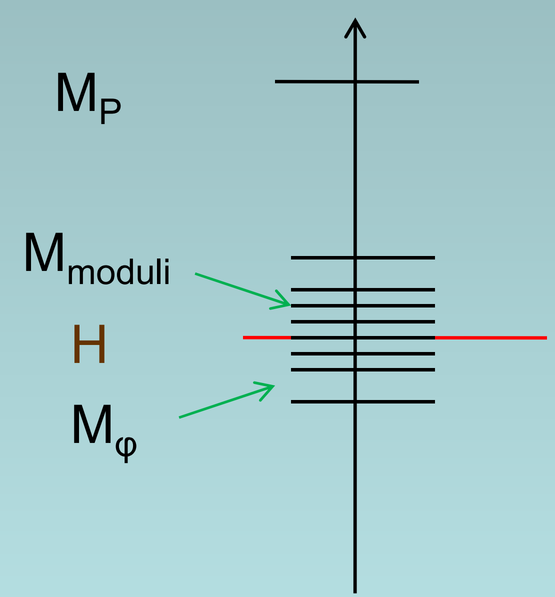
Moduli and inflation in string theory

- String compactifications typically include many moduli. If massless, these are very problematic.
- Significant progress in the past decade: in flux compactifications, most moduli obtain masses.
- But, these masses are finite!

want: light inflaton, Planck-mass moduli



get: most masses near H



Summary so far

- String theory strongly motivates scenarios involving many light fields whose potentials are controlled by Planck-suppressed contributions.
- Key question: when inflation arises in this context, what are its characteristic properties?

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- Key question: when inflation arises in this context, what are its characteristic properties?

e.g.,

- what is the probability of N_e e-folds?
- are the inflating trajectories smooth or bent?
- do features often arise in the last 60 e-folds?
- how are the CMB observables correlated with N_e?

And, how do the above characteristics depend on the number of light fields?

Simplification at large N?

- For N=1, many thousands of papers, but no simple answers: dynamics depends on the details of the potential.
- Simplifications at large N are commonplace in field theory and string theory (e.g., ‘t Hooft limit).
- Can we hope for something similar here? Might the characteristic properties depend only weakly on the details of the potential when N is large?

Random matrix theory

- Suppose we construct an ensemble of NxN matrices by drawing the entries of each matrix from a given distribution Ω .
- For large N, we can predict the spectrum of eigenvalues to excellent accuracy, and the result is independent of Ω.

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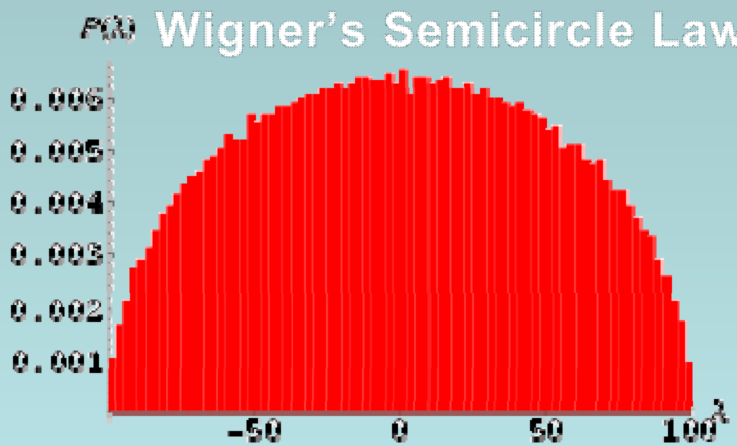
e.g., suppose **A** is an NxN matrix whose entries A_{ij} are drawn from a Gaussian distribution with mean zero and variance σ^2/N.

Let **M** = **A** + **A**^{T} .
Then the spectrum of M is

$$\rho(\lambda) = \frac{1}{2\pi\sigma^2}\sqrt{4\sigma^2 - \lambda^2}$$



Robustness of RMT

- the distribution Ω need not be Gaussian
- the mean of Ω need not be small
- the entries can have some correlations

General result:

If M is a matrix whose constituent entries are drawn from *any* distribution with appropriately bounded moments, then **in the large N limit** the spectrum of M approaches that of a matrix whose constituent entries have a Gaussian distribution with mean zero.

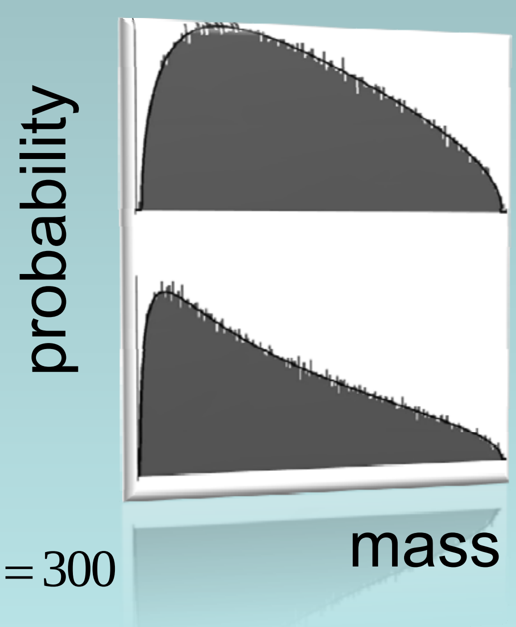
Z.D.Bai

This sort of universality is a strong motivation for studying the dynamics at large N.

Large N simplifications for inflation

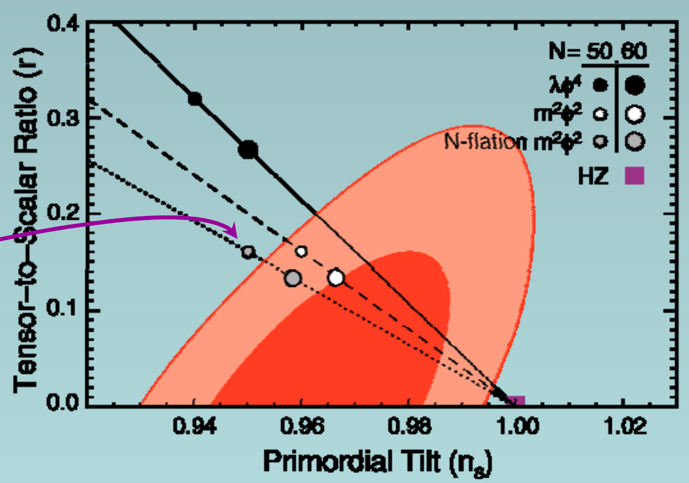
In N-flation, one has hundreds of axions whose collective excitation drives inflation.
Each individual axion mass depends on many details of the stabilized compactification, but the spectrum of masses is simple!

Marčenko-Pastur Law



$N = 300$

Easther & L.M., 2005



Komatsu et al., 2010

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A simple N-field model

Arguably simplest model: Gaussian random landscape.

$$V(\phi_1, \ldots \phi_N) = \sum_{J=1}^{\infty} c^{(J)}_{i_1 \cdots i_J} \phi^{i_1} \cdots \phi^{i_J} \Lambda^{4-J}$$

- Have used weak-coupling (integer) dimensions
- Wilson coefficients $c^{(J)}_{i_1 \cdots i_J}$ can be drawn from some distribution, e.g. a Gaussian.
- The special case J=2 was covered by RMT.

How to be general?

- At strong coupling, operator dimensions can change substantially. Should we restrict to integers? Does it matter?
- Also, we do not have a solid prior for the distribution – does it matter whether it is Gaussian?
- Are all possible terms included with equal weight?

Our approach:

- Work in a string theory construction where we can explicitly compute the form of the potential.
- 'Experimentally' check for dependence on the distribution.
- Although N will not be very large, we will find some universal behavior.

Strategy summarized

String theory motivates considering many light fields governed by a complicated potential induced by Planck-suppressed couplings.

We will study a specific example of this sort and search for simplifications when the number of fields, N, is large.

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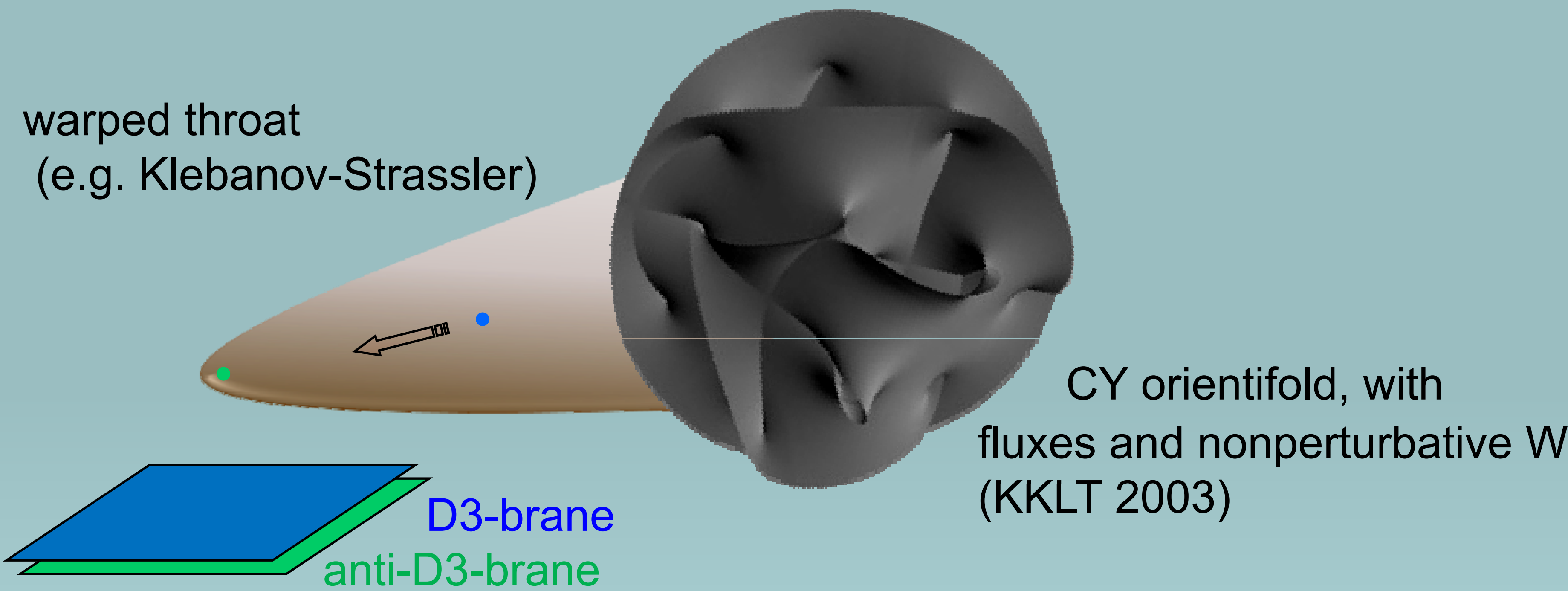
Warped D-brane inflation



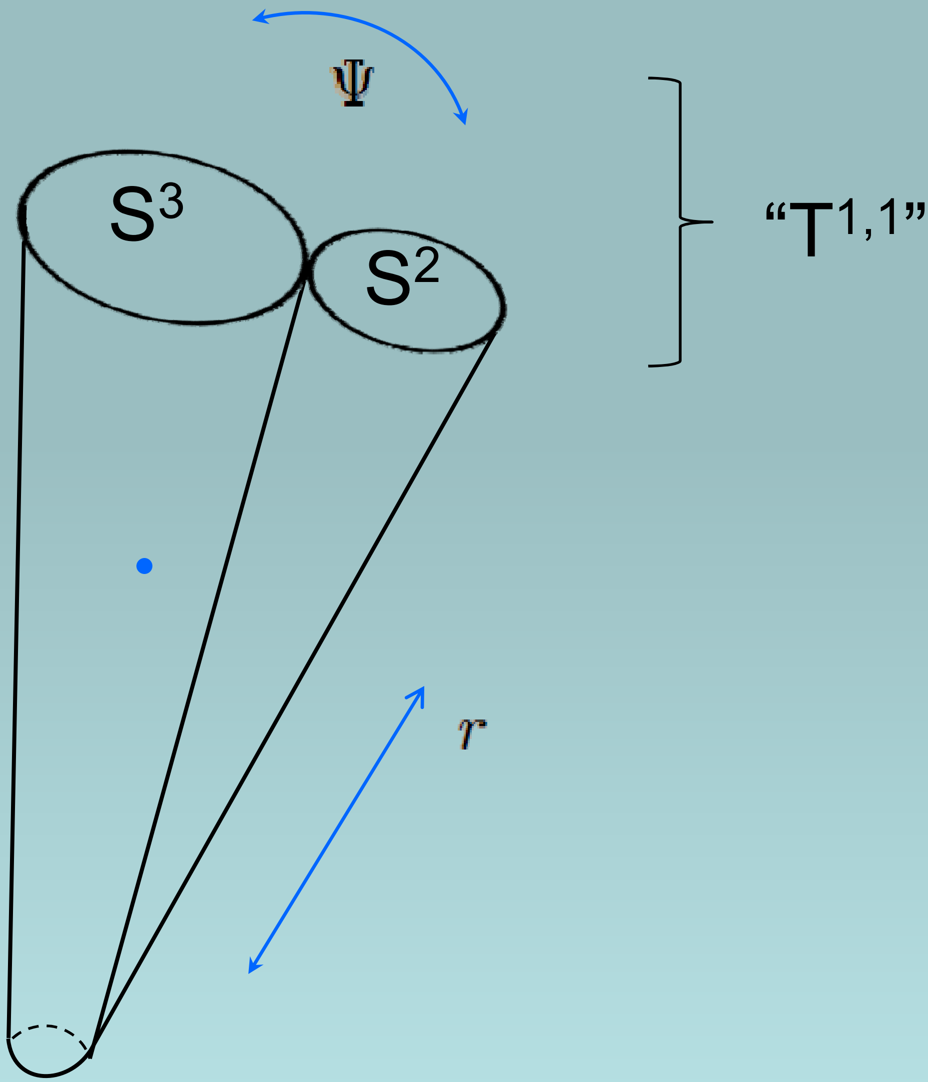
Dvali&Tye 1998

Dvali,Shafi,Solganik 2001

Burgess,Majumdar,Nolte,Quevedo,Rajesh,Zhang 2001

Kachru, Kallosh, Linde, Maldacena, L.M., Trivedi, 2003

The inflaton field space



$$ds^2 = e^{2A_{(0)}(r)}\eta_{\mu\nu}dx^\mu dx^\nu + e^{-2A_{(0)}(r)}(dr^2 + r^2 d\Omega^2_{T^{1,1}})$$

Computing the inflaton potential

- Diverse contributions: Coulomb interaction, curvature couplings, couplings to the moduli in the Kähler potential and in the superpotential.

- Tool: AdS/CFT allows us to write an arbitrary contribution to the inflaton Lagrangian, encompassing all the above effects, in terms of a supergravity solution.

- So we found the most general supergravity solution for this system and read off the structure of the potential.

Baumann, Dymarsky, Klebanov, L.M., 2007
Baumann, Dymarsky, Kachru, Klebanov, L.M., 2008, 2009, 2010

Table 7: *Matching between supergravity G_- flux modes and CFT operators.*

δ	j_1	j_2	R	Operator		Multiplet	Type	Flux Series
$\frac{5}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	-1	$[S^1]_{\theta^2}$	$[\mathrm{Tr}(AB)]_{\theta^2}$	V.I	chiral	I
3	0	0	2	$[\Phi^0_+]_{\mathrm{b}}$	$[\mathrm{Tr}(W^2_{(1)} + W^2_{(2)})]_{\mathrm{b}}$	V.IV	chiral	III
$\frac{7}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$[T^1_\alpha]_\theta$	$[\mathrm{Tr}(W_\alpha(AB))]_\theta$	G.I	chiral	II
4	0	0	0	$[\Phi^0_-]_{\theta^2}$	$[\mathrm{Tr}(W^2_{(1)} - W^2_{(2)})]_{\theta^2}$	V.III	chiral	$\star$
4	0	1	0	$[_aL^{2,0}_\alpha]_\theta$	$[\mathrm{Tr}(W_\alpha J_a)]_\theta$	G.I+G.III	semi-long	II
4	1	0	0	$[_bL^{2,0}_\alpha]_\theta$	$[\mathrm{Tr}(W_\alpha J_b)]_\theta$	G.I+G.III	semi-long	II
4	1	1	0	$[S^2]_{\theta^2}$	$[\mathrm{Tr}(AB)^2]_{\theta^2}$	V.I	chiral	I
$\sqrt{28}-1$	1	1	-2	–	$[\mathrm{Tr}(f)]_{\theta^2}$	V.I	long	I
$\frac{9}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	3	$[\Phi^1_+]_{\mathrm{b}}$	$[\mathrm{Tr}(W^2_{(1)} + W^2_{(2)})(AB)]_{\mathrm{b}}$	V.IV	chiral	III
$\frac{9}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1	$[\bar{\Phi}^1_+]_{\mathrm{b}}$	$[\mathrm{Tr}(W^2_{(1)} + W^2_{(2)})(\overline{AB})]_{\mathrm{b}}$	V.IV	–	III
$\frac{9}{2}$	$\frac{1}{2}$	$\frac{3}{2}$	-1	$[_aJ^1]_{\theta^2}$	$[\mathrm{Tr}(J_a(AB))]_{\theta^2}$	V.I	semi-long	I
$\frac{9}{2}$	$\frac{3}{2}$	$\frac{1}{2}$	-1	$[_bJ^1]_{\theta^2}$	$[\mathrm{Tr}(J_b(AB))]_{\theta^2}$	V.I	semi-long	I
5	1	1	2	$[T^2_\alpha]_\theta$	$[\mathrm{Tr}(W_\alpha(AB)^2)]_\theta$	G.I	chiral	II
5	0	1	2	$[_aI^0]_{\mathrm{b}}$	$[\mathrm{Tr}((W^2_{(1)} + W^2_{(2)})J_a)]_{\mathrm{b}}$	V.IV	semi-long	III
5	1	0	2	$[_bI^0]_{\mathrm{b}}$	$[\mathrm{Tr}((W^2_{(1)} + W^2_{(2)})J_b)]_{\mathrm{b}}$	V.IV	semi-long	III
$\sqrt{28}$	1	1	0	–	$[\mathrm{Tr}(W_\alpha f)]_\theta$	G.I+G.III	long	II
$\sqrt{40}-1$	0	2	-2	–	$[\mathrm{Tr}(f_a)]_{\theta^2}$	V.I	long	I
$\sqrt{40}-1$	2	0	-2	–	$[\mathrm{Tr}(f_b)]_{\theta^2}$			

cf. Ceresole, Dall'Agata, D'Auria 1999

Spectrum of the D3-brane potential

$$V = \sum_i c_i \phi^{\Delta_i} h_i(\Psi)$$

$$\Delta_{\mathcal{H}} = \frac{3}{2}\,,\ 2\,,3\,,\ \ldots$$

$$\Delta_{\Lambda} = 1\,,\ 2\,,\ \frac{5}{2}\,,\ \sqrt{28}-\frac{5}{2}$$

$$\Delta_{\mathcal{R}} = 2_s\,,\ 3\,,\ \frac{7}{2}\,,\ 4\,,\ \cdots$$

$$V(\phi) = V_0 + b_1\,j_1(\Psi)\,\phi^1 + a_{3/2}\,h_{3/2}(\Psi)\,\phi^{3/2} + \Big(c_2 + a_2\,h_2(\Psi) + b_2\,j_2(\Psi)\Big)\,\phi^2 + b_{5/2}\,j_{5/2}(\Psi)\,\phi^{5/2} + b_{2.79}\,j_{2.79}(\Psi)\,\phi^{2.79} + \ldots$$

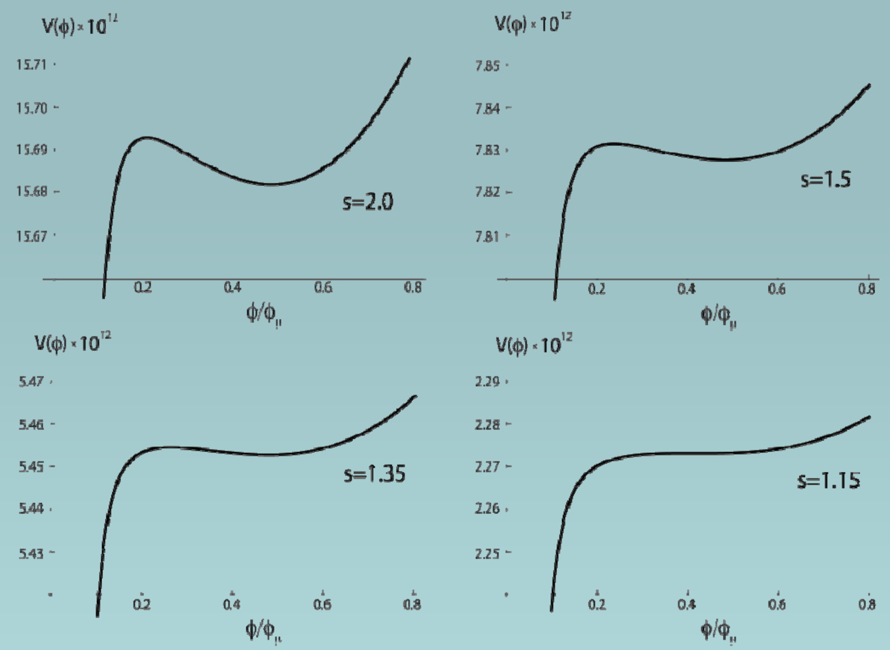
We will keep the 724 terms with $\Delta<4$.

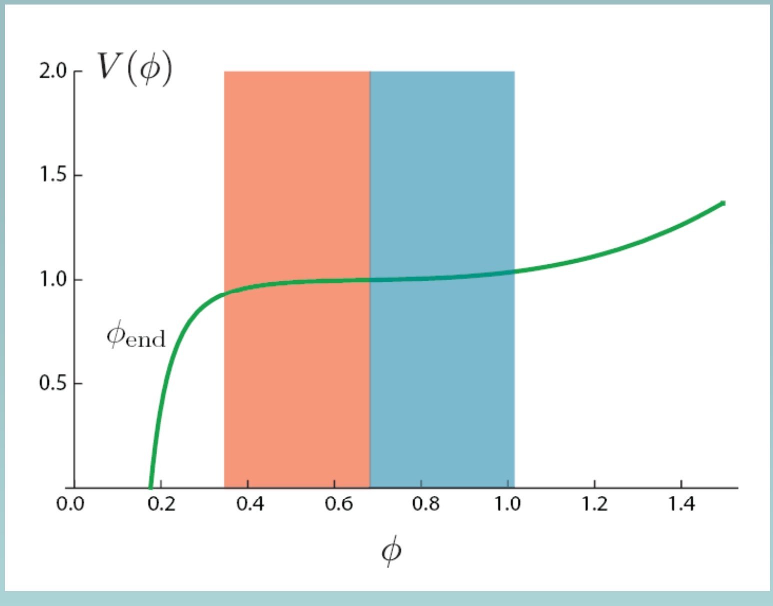
Baumann, Dymarsky, Kachru, Klebanov, L.M., 1001.5028

Single-field phenomenology:

Inflection point inflation

$$V(r) = c_1 r^1 + c_{3/2} r^{3/2} + c_2 r^2 + \ldots$$





Baumann, Dymarsky, Klebanov, L.M., 2007
Panda, Sami, Tsujikawa, 2007
Ali, Chingangbam, Panda, Sami, 2008
Ali, Deshamukhya, Panda, Sami, 2010

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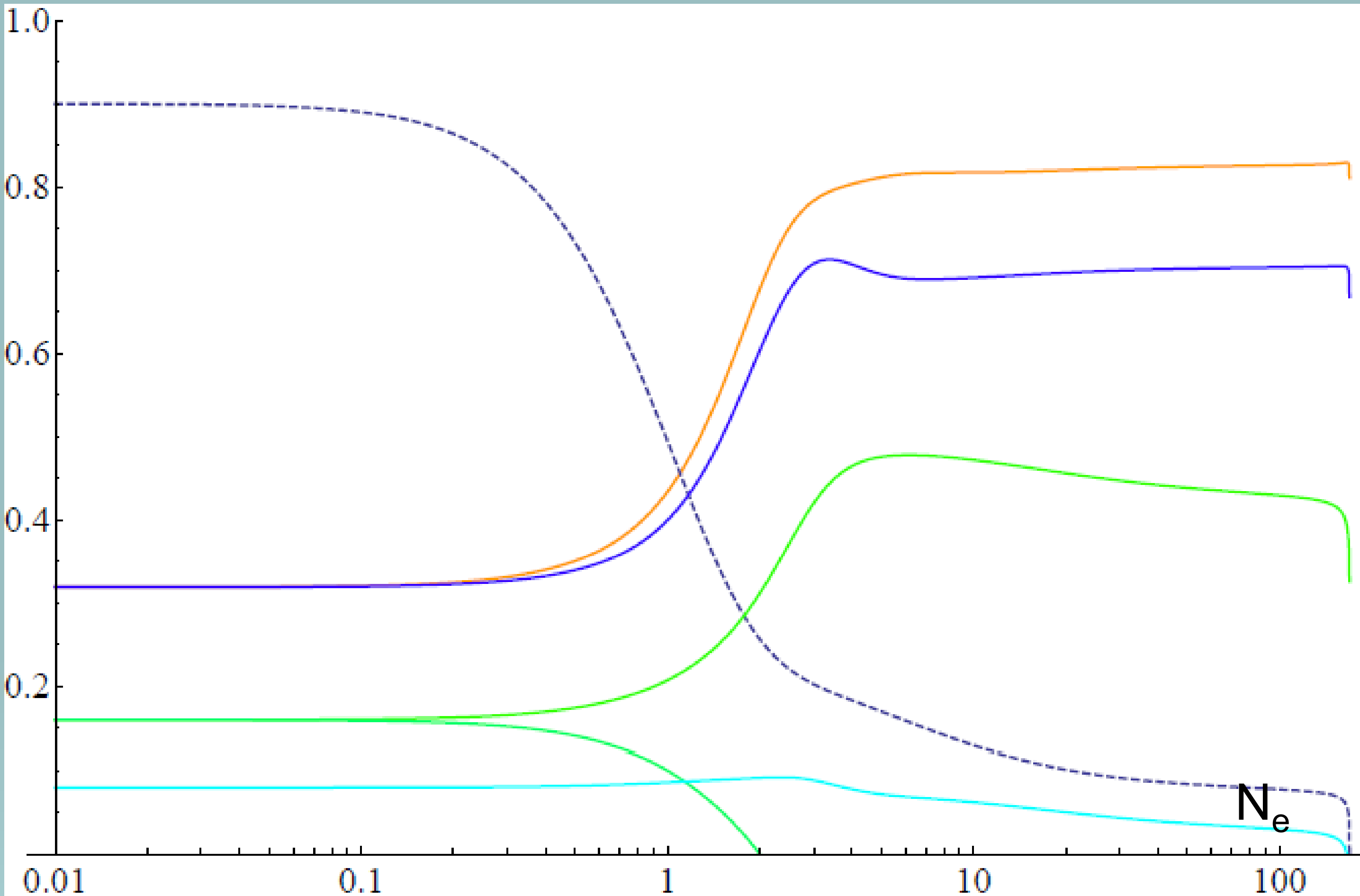
$$V = \sum_i c_i \phi^{\Delta_i} h_i(\Psi)$$

Method:

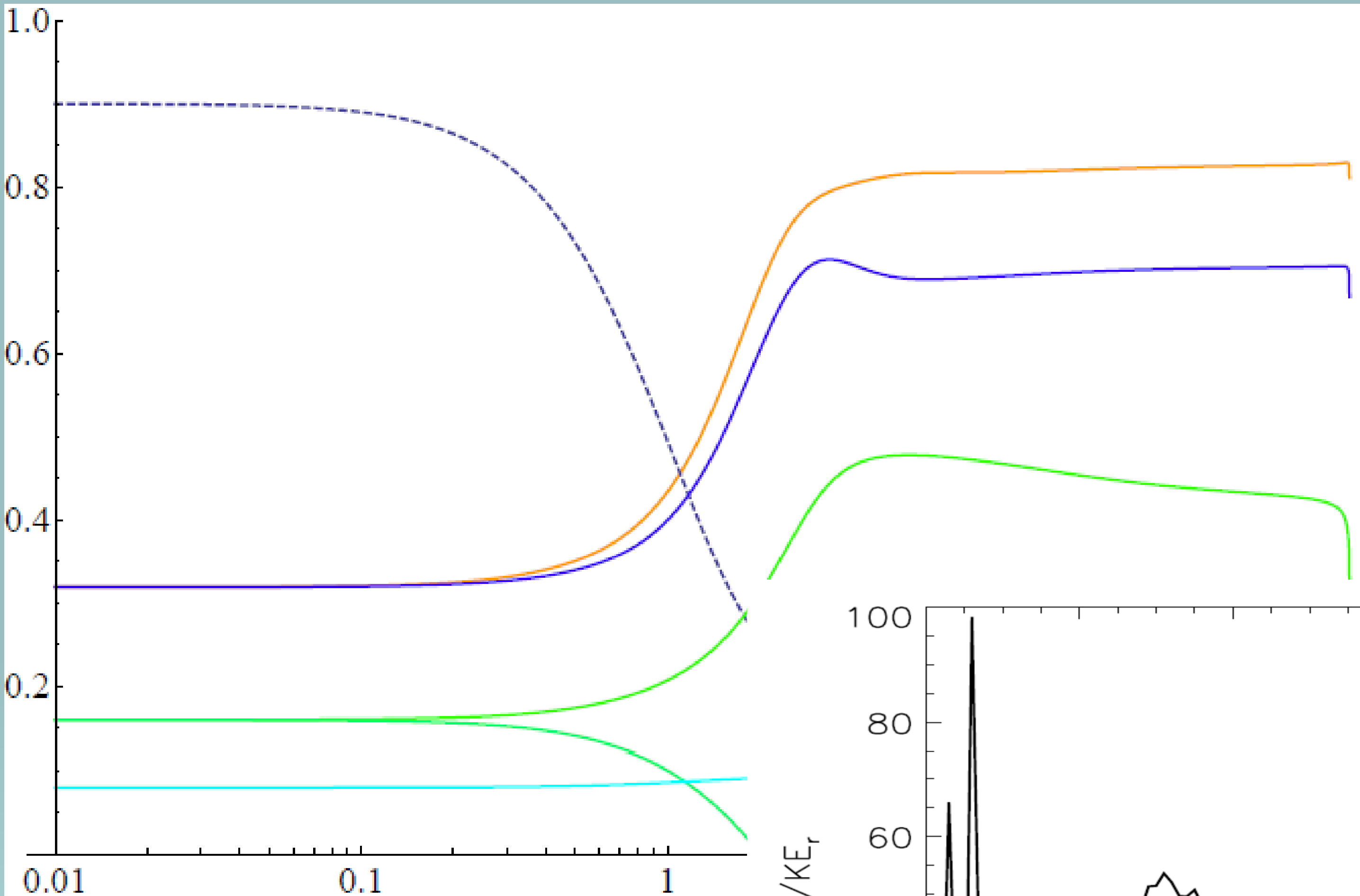
We draw the coefficients c_i from a Gaussian distribution, generating an ensemble of potentials. (We will later check that the choice of distribution is not significant.)

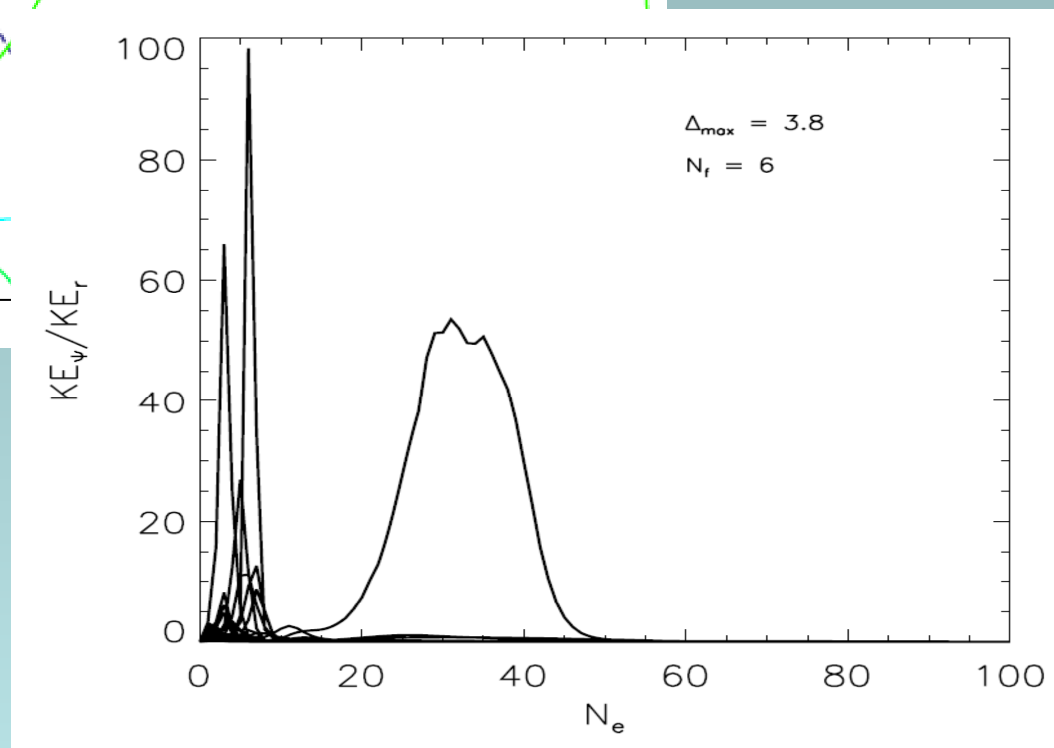
We draw a potential from the ensemble, choose a random initial condition (with appropriately bounded kinetic energy), and find the full six-field evolution of the homogeneous background.

We stare at the results and try to identify “robust observables”, i.e. quantities that depend very weakly on the details of the distribution, but may depend on N.



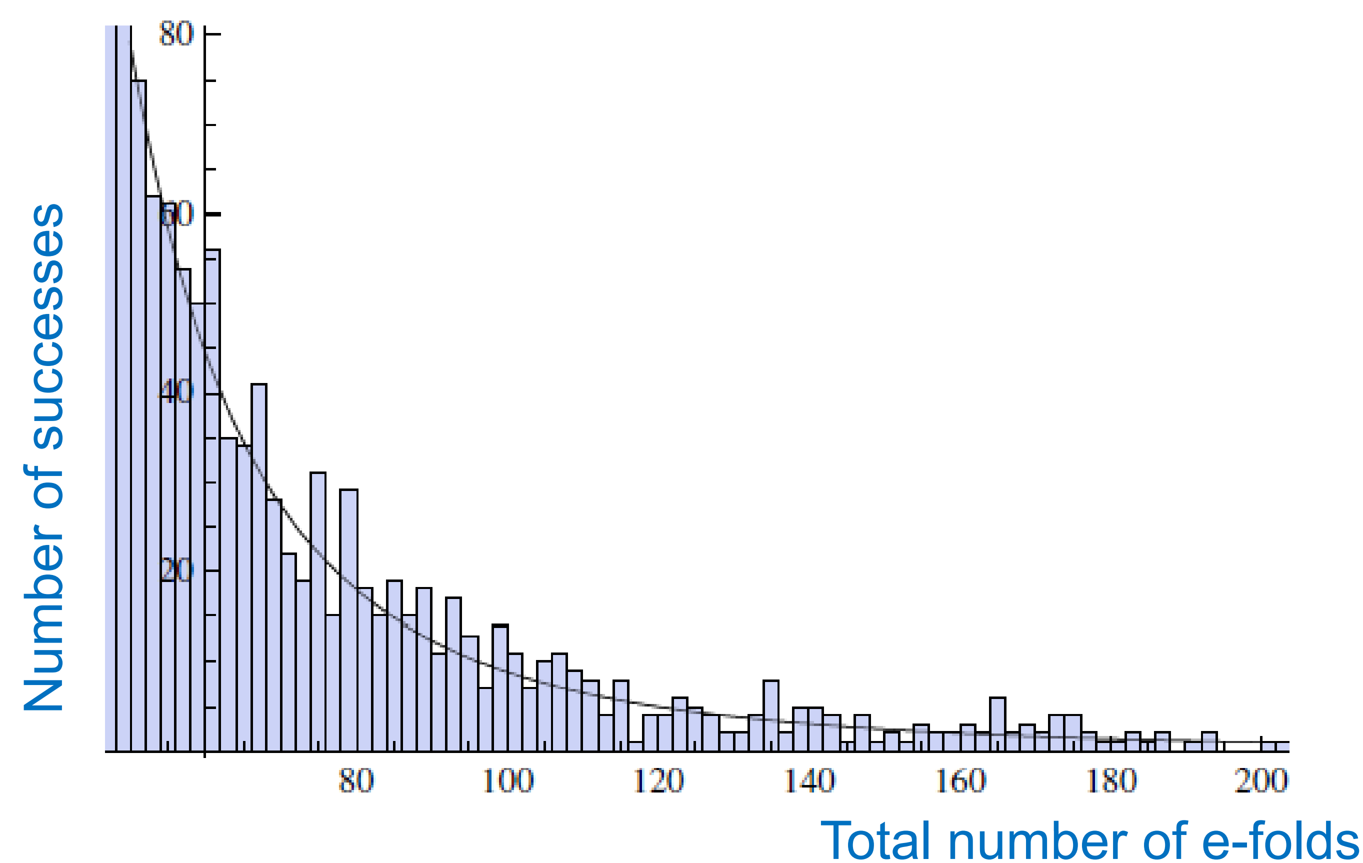
A typical successful trajectory. The angles evolve initially and then settle into an angular minimum.



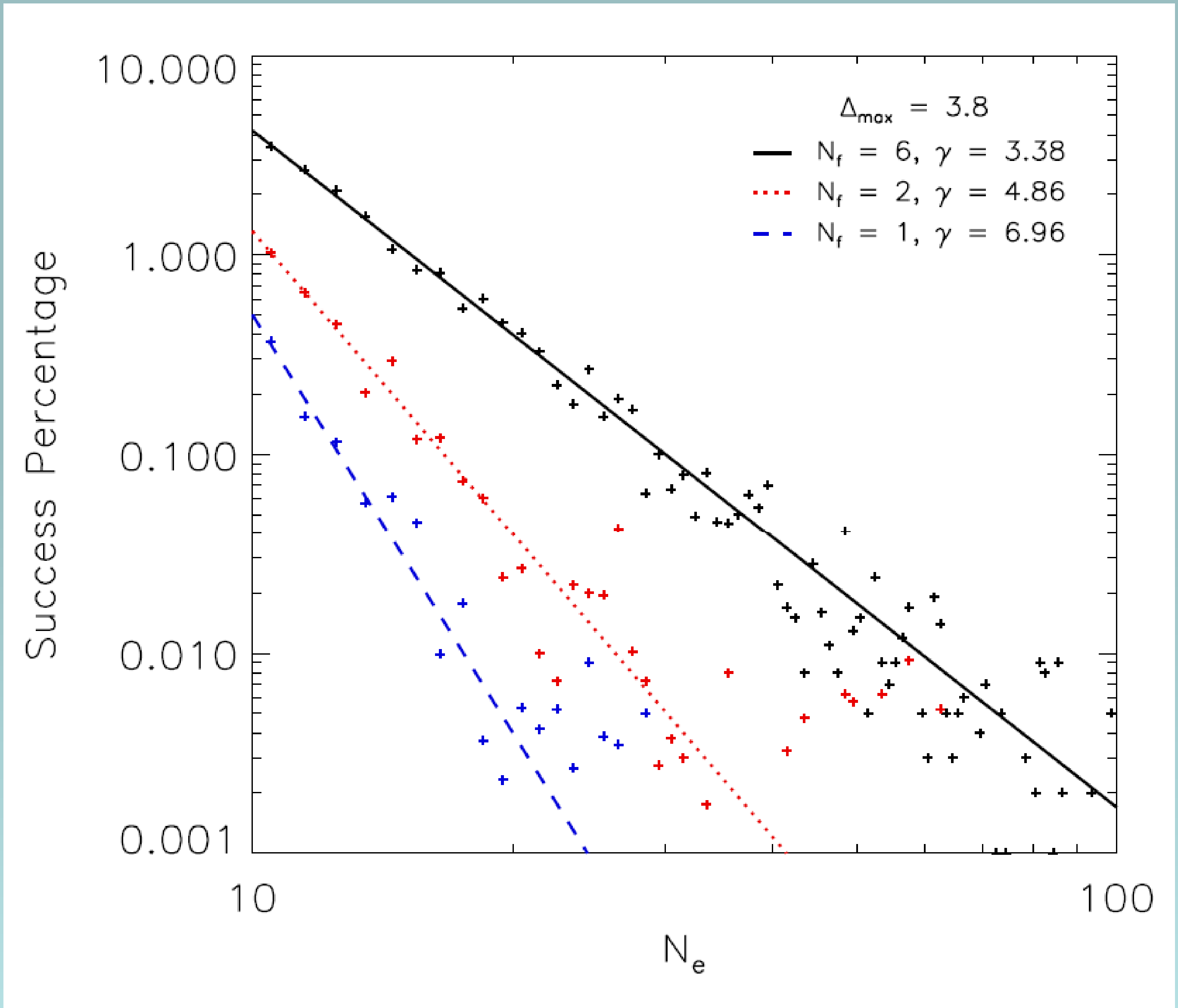


100 cases with N>60

Success rate vs. number of e-folds



Success rate is a power law



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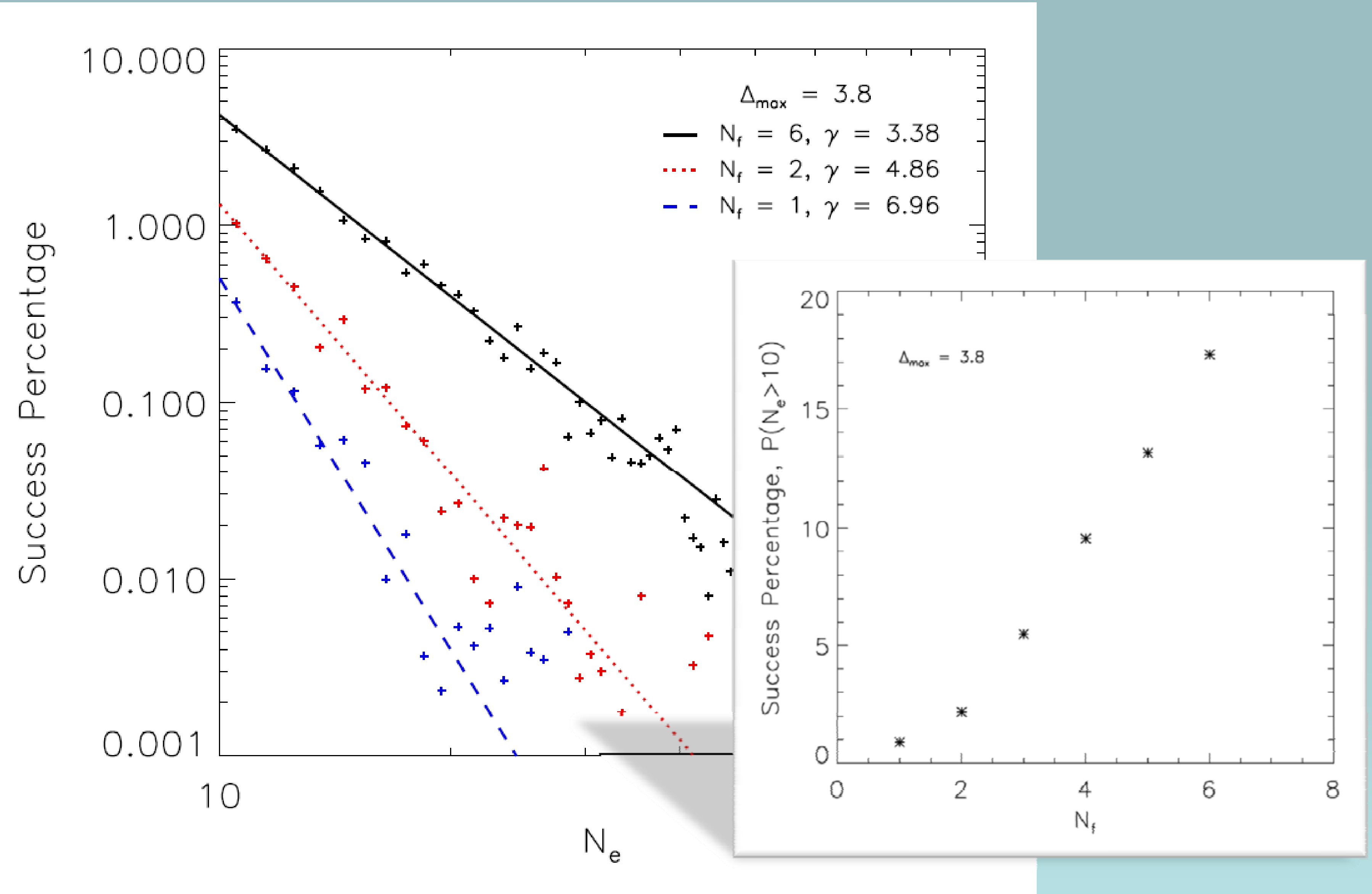
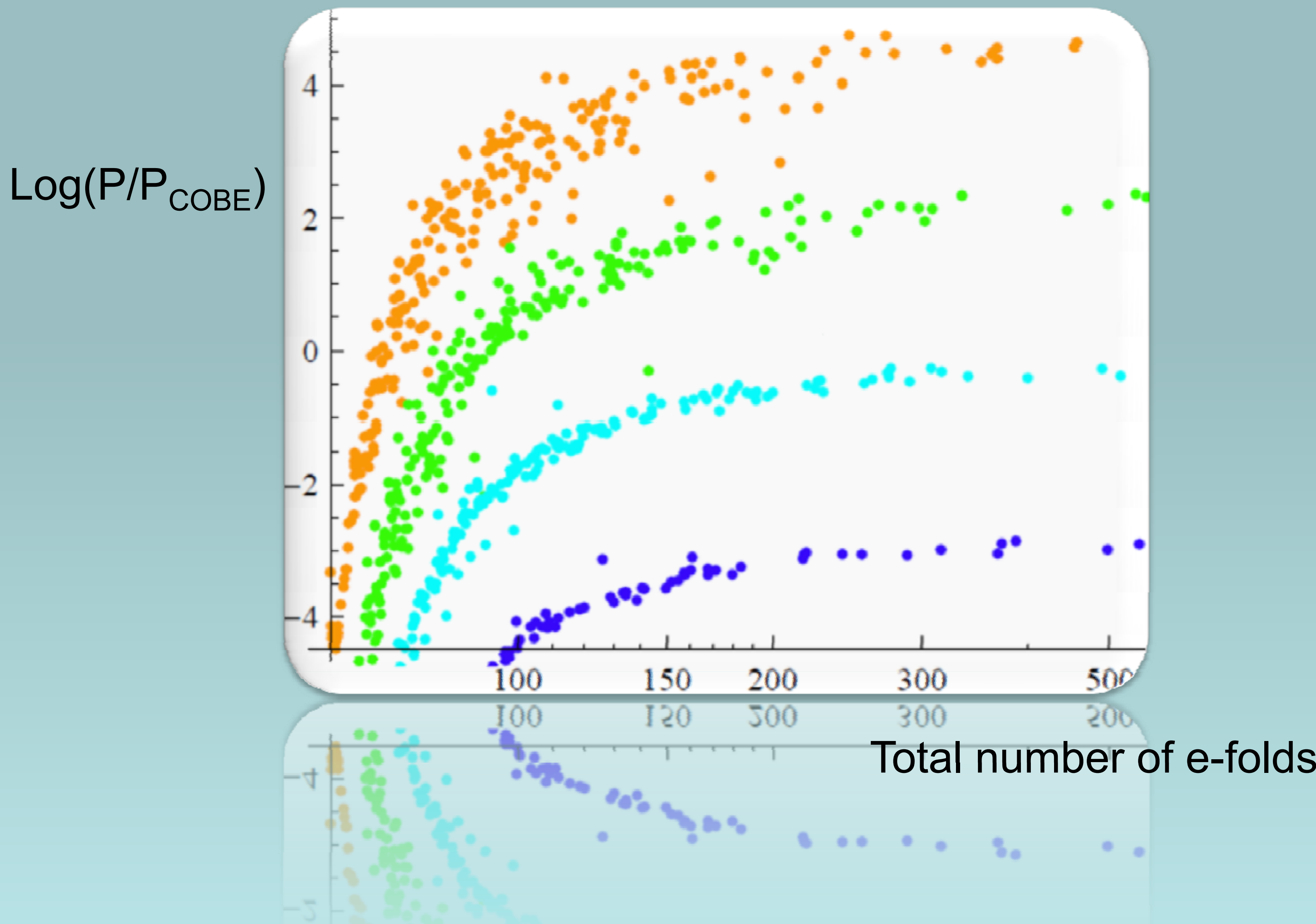
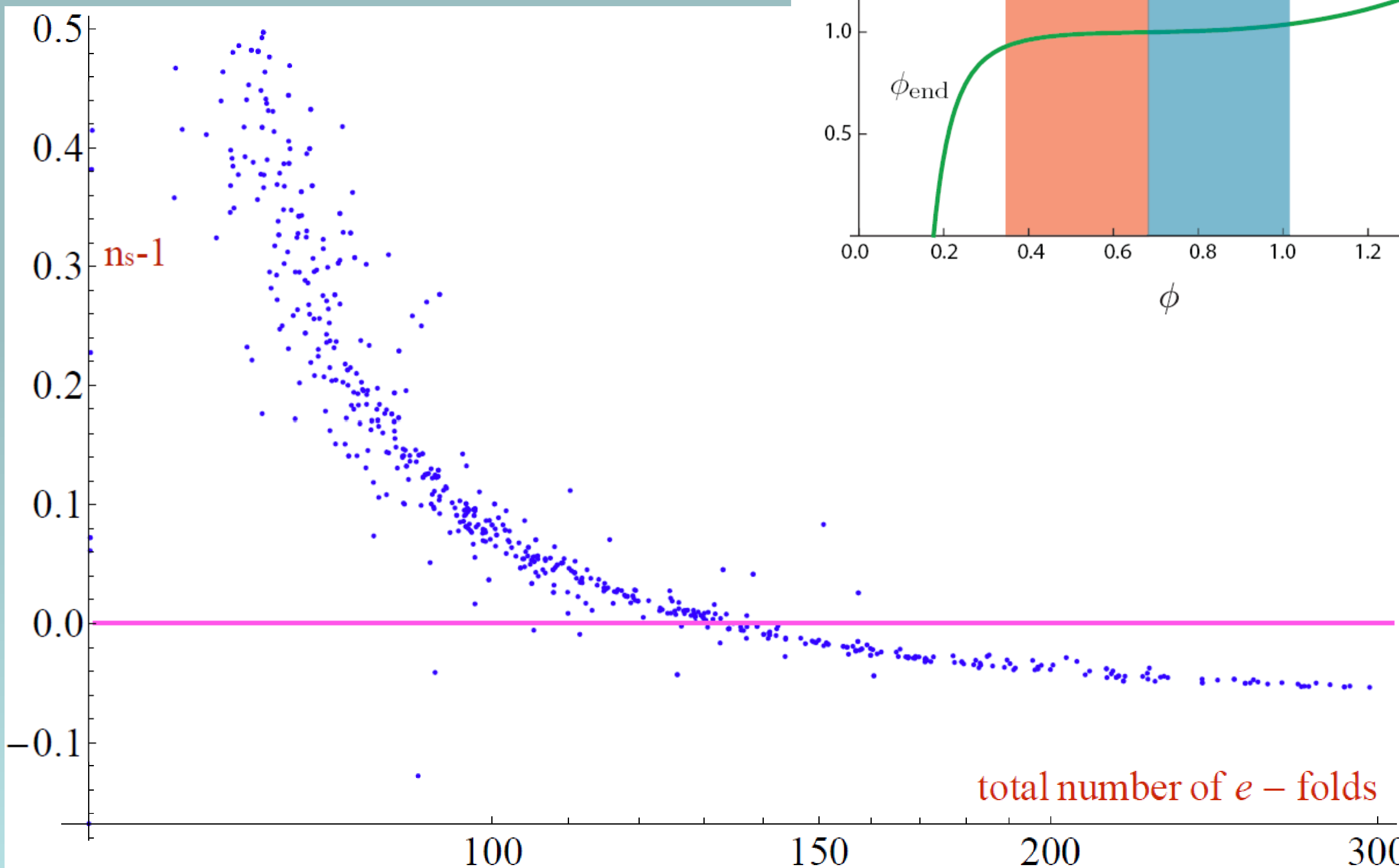


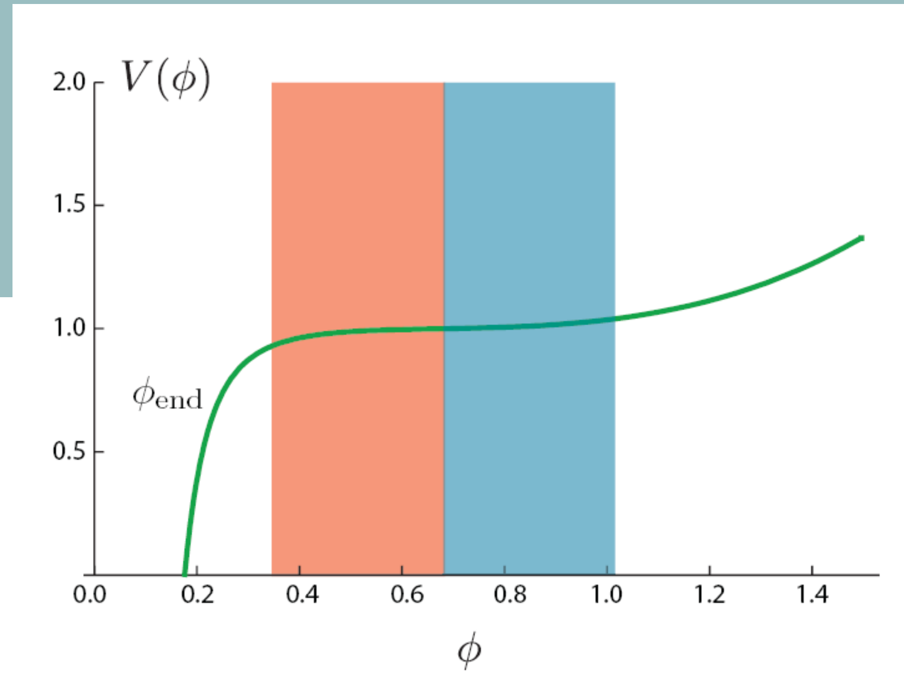
Normalization, with caveats



The tilt



0.5
0.4
0.3
0.2
0.1
0.0
−0.1
ns-1
100
150
200
300
total number of e − folds



$V(\phi)$
2.0
1.5
1.0
0.5
$\phi_{\rm end}$
0.0
0.2
0.4
0.6
0.8
1.0
1.2
1.4
ϕ

Open problems/work in progress

- Perturbations.
- Counting the likelihood of features.
- Can DBI inflation arise by chance?
- (In)significance of the truncation to $\Delta<4$
- Extension to more general, higher-dimensional systems.

Conclusions (1)

- String theory strongly motivates considering inflation models with many light fields whose potential is controlled by Planck-suppressed contributions.
- We studied a particular case with N=6 fields (D3-brane inflation), where the structure of the inflaton potential could be computed explicitly.
- Drawing the Wilson coefficients from various distributions, we constructed ensembles of potentials.
- We then studied the typical behavior in these ensembles.
- We find scaling behavior: the probability of N_e e-folds of inflation is a power law, N_e^{-3}

Conclusions (2)

- The probability of inflation increases as a power law in the number of fields.
- It would be interesting to extend our results to more general settings.
- We did not yet study the perturbations, but there is plausibly a rich story involving bending trajectories, transients, and ultimately primordial features and non-Gaussianities.