Oscillations in the Primordial Bispectrum

From an excited initial state

Daan Meerburg, University of Amsterdam

P. D. Meerburg, J. P. van der Schaar and P. S. Corasaniti, "**Signatures of Initial State Modifications on Bispectrum Statistics**," JCAP **0905**, 018 (2009) [arXiv:0901.4044 [hep-th]],
P. D. Meerburg, J. P. van der Schaar and M. G. Jackson, "**Bispectrum signatures of a modified vacuum in single field inflation with a small speed of sound**," ,JCAP **1002**, 001 (2010), arXiv:0910.4986 [hep-th].

P.D. Meerburg, "Oscillations in the Primordial Bispectrum I: Mode Expansion", Phys. Rev. D **82**, 063517 (2010) [arXiv:astro-ph/1006.2771].

P. D. Meerburg and J. P. van der Schaar, "Minimal cut-off vacuum state constraints from CMB bispectrum statistics," arXiv:1009.5660 [hep-th].

PFNG 14-12-2010

In this talk I'll consider possible deviation from a Bunch Davies Vacuum condition at the onset of inflation.

In this talk I'll consider possible deviation from a Bunch Davies Vacuum condition at the onset of inflation.
Such modifications will have consequences for the primordial in-in matrix elements such as the primordial bispectrum.

In this talk I’ll consider possible deviation from a Bunch Davies Vacuum condition at the onset of inflation.
Such modifications will have consequences for the primordial in-in matrix elements such as the primordial bispectrum.
The source of (large) non-Gaussian effects could be traced to non-zero particle density during inflation (excited states).

In this talk I'll consider possible deviation from a Bunch Davies Vacuum condition at the onset of inflation.
Such modifications will have consequences for the primordial in-in matrix elements such as the primordial bispectrum.
The source of (large) non-Gaussian effects could be traced to non-zero particle density during inflation (excited states).
Constraining the resulting bispectra will constrain deviations from BD.

In this talk I'll consider possible deviation from a Bunch Davies Vacuum condition at the onset of inflation.
Such modifications will have consequences for the primordial in-in matrix elements such as the primordial bispectrum.
The source of (large) non-Gaussian effects could be traced to non-zero particle density during inflation (excited states).
Constraining the resulting bispectra will constrain deviations from BD.
The presence of oscillations (due to a mixing of positive and negative plane wave solutions) makes it hard to constrain these bispectra. Mode expansion and extraction could offer a possibility.

Motivation:

Motivation:

1. **Theoretical.** Inflation is an effective theory in the Sitter space. Assuming BD would assume knowledge of theory at all (UV) physical scales.

Motivation:

1. **Theoretical.** Inflation is an effective theory in the Sitter space. Assuming BD would assume knowledge of theory at all (UV) physical scales.

2. **Observational.** BD mods predict consequences for the power spectrum as well (oscillations). Some studies have suggested that this might represent a better reconstruction (Shafieloo++, 2004;2007a;2007b, Kogo++2004a; 2004b, Sealfon++2005, Verde&Peiris 2008).

Motivation:

1. **Theoretical.** Inflation is an effective theory in the Sitter space. Assuming BD would assume knowledge of theory at all (UV) physical scales.

2. **Observational.** BD mods predict consequences for the power spectrum as well (oscillations). Some studies have suggested that this might represent a better reconstruction (Shafieloo++, 2004;2007a;2007b, Kogo++2004a; 2004b, Sealfon++2005, Verde&Peiris 2008).

3. **Pragmatic.** What if?

Standard way to choose initial state (during inflation) is as follows:

Standard way to choose initial state (during inflation) is as follows:

-At small enough (subhorizon) scales space is flat

Standard way to choose initial state (during inflation) is as follows:
-At small enough (subhorizon) scales space is flat

-In the limit $\eta \rightarrow -\infty$ the solution of the mode equation that approaches a positive plane wave is known as Bunch Davies (BD) vacuum in de Sitter.

Standard way to choose initial state (during inflation) is as follows:
-At small enough (subhorizon) scales space is flat

-In the limit $\eta \to -\infty$ the solution of the mode equation that approaches a positive plane wave is known as Bunch Davies (BD) vacuum in de Sitter.

Given inflation certainly is an effective description it seems problematic to consider a state of which the definition requires us to go to arbitrarily small scales/high energies.

Standard way to choose initial state (during inflation) is as follows:
-At small enough (subhorizon) scales space is flat

-In the limit $\eta \to -\infty$ the solution of the mode equation that approaches a positive plane wave is known as Bunch Davies (BD) vacuum in de Sitter.

Given inflation certainly is an effective description it seems problematic to consider a state of which the definition requires us to go to arbitrarily small scales/high energies.

A proposal is to define an initial state at the cutoff time $\eta_0(k)$, the earliest time our effective theory can be trusted.

$$v_k^* = \alpha_k u_k^*(\eta) + \beta_k u_k(\eta) \qquad \beta = 0 \leftrightarrow \text{ BD}$$

This choice has consequences for the primordial spectra.

This choice has consequences for the primordial spectra.

For the power spectrum:

This choice has consequences for the primordial spectra.

For the power spectrum:

$$|N(k)|^2 = \frac{1}{1-|b(k)|^2} \qquad b^*(k) = \beta_k/\alpha_k \qquad N^*(k) = \alpha_k$$

This choice has consequences for the primordial spectra.

For the power spectrum:

$$|N(k)|^2 = \frac{1}{1-|b(k)|^2} \qquad b^*(k) = \beta_k/\alpha_k \qquad N^*(k) = \alpha_k$$

$$P(k) \propto |v_k|^2 \qquad u_k = e^{i\delta}|u_k|$$

This choice has consequences for the primordial spectra.

For the power spectrum:

$$|N(k)|^2 = \frac{1}{1-|b(k)|^2} \qquad b^*(k) = \beta_k/\alpha_k \qquad N^*(k) = \alpha_k$$

$$P(k) \propto |v_k|^2 \qquad u_k = e^{i\delta}|u_k|$$

$$P(k) \propto \frac{1}{1-|b(k)|} \times \left((1+|b(k)|^2 + e^{2i\delta}b(k)^* + e^{-2i\delta}b(k))|u_k|^2\right).$$

This choice has consequences for the primordial spectra.

For the power spectrum:

$$P(k) \simeq P_{BD}(k)\left(1 + 2|b(k)|^2 \cos(\alpha(k) + \delta)\right).$$

This choice has consequences for the primordial spectra.

For the power spectrum:

$$P(k) \simeq P_{BD}(k)\left(1 + 2|b(k)|^2 \cos(\alpha(k) + \delta)\right).$$

For the bispectrum:

This choice has consequences for the primordial spectra.

For the power spectrum:

$$P(k) \simeq P_{BD}(k)\left(1 + 2|b(k)|^2 \cos(\alpha(k) + \delta)\right) .$$

For the bispectrum:

Measures interactions at 3 level. Contributions are coming from non-zero particle density and potentially strong(er) interactions at the early stages of inflation

This choice has consequences for the primordial spectra.

For the power spectrum:

$$P(k) \simeq P_{BD}(k)\left(1 + 2|b(k)|^2 \cos(\alpha(k) + \delta)\right).$$

For the bispectrum:

Measures interactions at 3 level. Contributions are coming from non-zero particle density and potentially strong(er) interactions at the early stages of inflation

Results depend on theory of inflation. Generally (single field):

This choice has consequences for the primordial spectra.

For the power spectrum:

$$P(k) \simeq P_{BD}(k)\left(1 + 2|b(k)|^2 \cos(\alpha(k) + \delta)\right).$$

For the bispectrum:

Measures interactions at 3 level. Contributions are coming from non-zero particle density and potentially strong(er) interactions at the early stages of inflation

Results depend on theory of inflation. Generally (single field):

$$B^{pr}(k_1, k_2, k_3) \propto |\beta|(\Lambda_c/H^*)^n F(k_1, k_2, k_3) \times L(\cos(\tilde{k}\Lambda_c/H^* + \delta))$$

This choice has consequences for the primordial spectra.

For the power spectrum:

$$P(k) \simeq P_{BD}(k)\left(1 + 2|b(k)|^2 \cos(\alpha(k) + \delta)\right).$$

For the bispectrum:

$$B^{pr}(k_1, k_2, k_3) \propto |\beta|(\Lambda_c/H^*)^n F(k_1, k_2, k_3) \times L(\cos(\tilde{k}\Lambda_c/H^* + \delta))$$

Chen et al 2007, Holman&Tolley 2008, Meerburg++ 2009a, 2009b

This choice has consequences for the primordial spectra.

For the power spectrum:

$$P(k) \simeq P_{BD}(k)\left(1 + 2|b(k)|^2 \cos(\alpha(k) + \delta)\right).$$

For the bispectrum:

$$B^{pr}(k_1, k_2, k_3) \propto |\beta|(\Lambda_c/H^*)^n F(k_1, k_2, k_3) \times L(\cos(\tilde{k}\Lambda_c/H^* + \delta))$$

Amplitude

$$k\eta_0(k) \sim \Lambda_c/H^*$$

Chen et al 2007, Holman&Tolley 2008, Meerburg++ 2009a, 2009b

This choice has consequences for the primordial spectra.

For the power spectrum:

$$P(k) \simeq P_{BD}(k)\left(1 + 2|b(k)|^2 \cos(\alpha(k) + \delta)\right).$$

For the bispectrum:

$$B^{pr}(k_1, k_2, k_3) \propto |\beta|(\Lambda_c/H^*)^n F(k_1, k_2, k_3) \times L(\cos(\tilde{k}\Lambda_c/H^* + \delta))$$

Amplitude

$$k\eta_0(k) \sim \Lambda_c/H^*$$

Scale invariant part

Chen et al 2007, Holman&Tolley 2008, Meerburg++ 2009a, 2009b

This choice has consequences for the primordial spectra.

For the power spectrum:

$$P(k) \simeq P_{BD}(k) \left(1 + 2|b(k)|^2 \cos(\alpha(k) + \delta)\right) .$$

For the bispectrum:

$$B^{pr}(k_1, k_2, k_3) \propto |\beta|(\Lambda_c/H^*)^n F(k_1, k_2, k_3) \times L(\cos(\tilde{k}\Lambda_c/H^* + \delta))$$

Amplitude

Scale invariant part

Scale dependence and running

$$k\eta_0(k) \sim \Lambda_c/H^*$$

Chen et al 2007, Holman&Tolley 2008, Meerburg++ 2009a, 2009b

Example: Non-canonical single field inflation + BD mod

Example: Non-canonical single field inflation + BD mod

$x_i = k_i/k_{max} \quad k_{max} \sim 10^{-1}\mathrm{Mpc}^{-1}$

Example: Non-canonical single field inflation + BD mod

$x_i = k_i/k_{max} \quad k_{max} \sim 10^{-1}\text{Mpc}^{-1}$

$$F \times L = \frac{\omega_v k_{max}^{-6}}{x_1 x_2 x_3} \sum_j \frac{1}{x_j^3} \left(\frac{1}{2} \frac{\cos\left(\omega_v \frac{x_{j+1}+x_{j+2}}{x_j} + \delta\right)}{\omega_v \left(\frac{x_{j+1}+x_{j+2}}{x_j} - 1\right)} - \frac{\sin \omega_v \left(\omega_v \frac{x_{j+1}+x_{j+2}}{x_j} + \delta\right)}{\omega_v^2 \left(\frac{x_{j+1}+x_{j+2}}{x_j} - 1\right)^2} \; \frac{\cos\delta - \cos\left(\omega_v \frac{x_{j+1}+x_{j+2}}{x_j} + \delta\right)}{\omega_v^3 \left(\frac{x_{j+1}+x_{j+2}}{x_j} - 1\right)^3} \right)$$

Here we defined a frequency: $\omega_v = \Lambda_c / H^*$

Example: Non-canonical single field inflation + BD mod

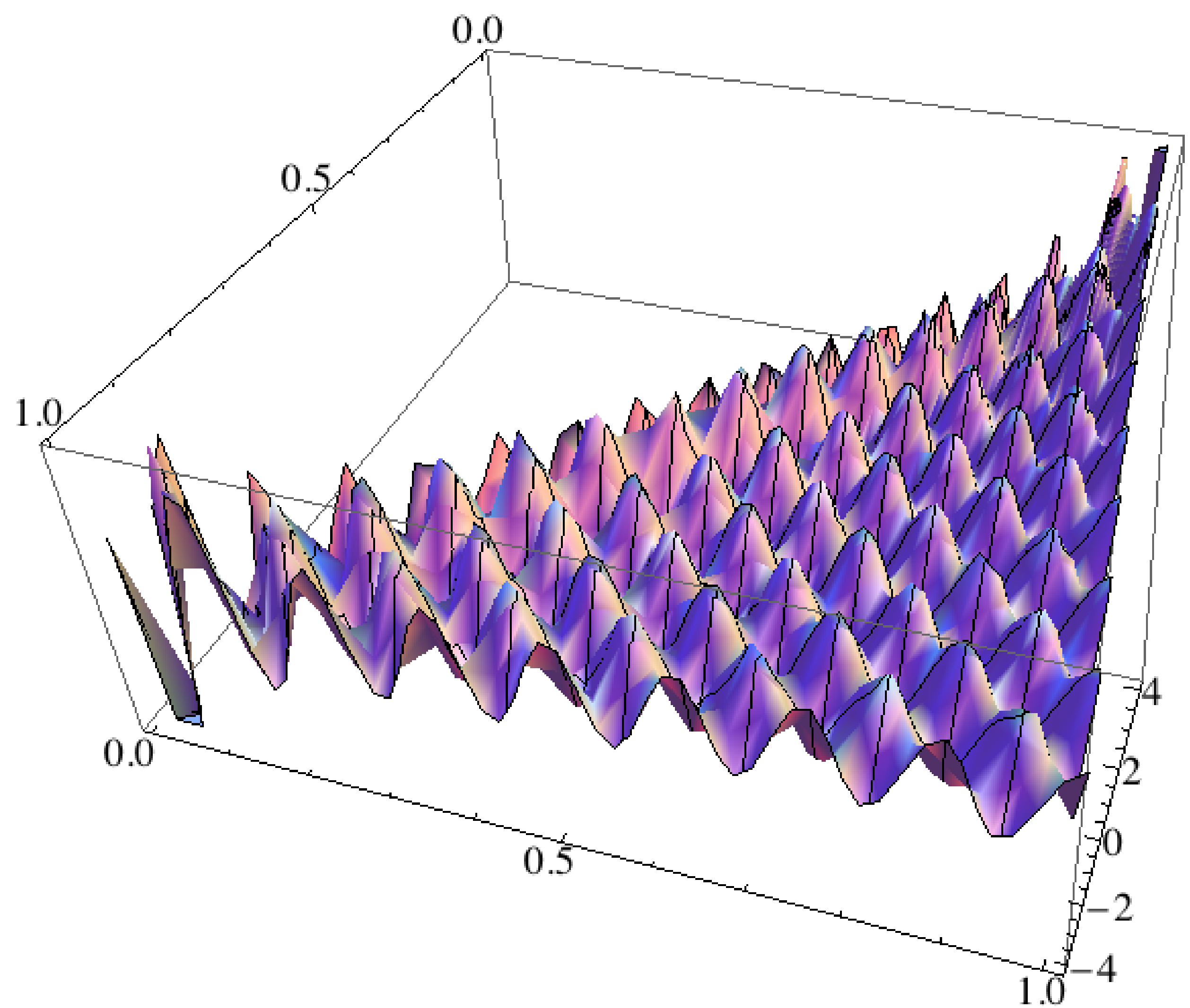
$$x_i = k_i/k_{max} \quad k_{max} \sim 10^{-1}\mathrm{Mpc}^{-1}$$

$$F \times L = \frac{\omega_v k_{max}^{-6}}{x_1 x_2 x_3} \sum_j \frac{1}{x_j^3} \left(\frac{1}{2} \frac{\cos\left(\omega_v \frac{x_{j+1}+x_{j+2}}{x_j} + \delta\right)}{\omega_v \left(\frac{x_{j+1}+x_{j+2}}{x_j} - 1\right)} - \frac{\sin \omega_v \left(\omega_v \frac{x_{j+1}+x_{j+2}}{x_j} + \delta\right)}{\omega_v^2 \left(\frac{x_{j+1}+x_{j+2}}{x_j} - 1\right)^2} \frac{\cos\delta - \cos\left(\omega_v \frac{x_{j+1}+x_{j+2}}{x_j} + \delta\right)}{\omega_v^3 \left(\frac{x_{j+1}+x_{j+2}}{x_j} - 1\right)^3} \right)$$

Here we defined a frequency: $\omega_v = \Lambda_c / H^*$

Out[400]=

Enfolded



$$f_{NL}^{nBD} \sim \frac{1}{c_s^2} \omega_v^3 |\beta|$$

$$k_1 = k_{max}$$

Meerburg++ 2009a, 2009b

Theoretical and Observational:

Theoretical and Observational:

-Transplanckian

-Backreaction

-Power Spectrum

-Bispectrum

Theoretical and Observational:

-Transplanckian

-Backreaction

-Power Spectrum

-Bispectrum

Transplanckian:

Can not excite modes beyond scale associated with Λ_c:

$$\beta_k \longrightarrow 0 \; \forall \; k > \Lambda_c a(\eta_0)$$

Theoretical and Observational:

-Transplanckian

-Backreaction

-Power Spectrum

-Bispectrum

Transplanckian:

Can not excite modes beyond scale associated with Λ_c:

$$\beta_k \longrightarrow 0 \;\forall\; k > \Lambda_c a(\eta_0)$$

Backreaction:

$\delta\rho \sim |\beta|^2 \Lambda_c^4$ **+ slow roll:** $|\beta| < \sqrt{\epsilon\eta'} H M_{pl}/\Lambda_c^2$

Observational:

Power spectrum.

$$V(\phi) = \frac{1}{2}m^2\phi^2[1+\alpha\sin\left(\frac{\phi}{\beta M}+\delta\right)]$$ Pahud, Kamionkowski & Liddle, 2008

WMAP 3: $\alpha \lesssim 3\times 10^{-5}$ **PLANCK:** $\mathcal{O}(10^{-6})$

Best constraints on the largest frequencies ($\beta = 5\times 10^{-3}$)

$$V(\phi) = \mu^3\left[\phi + bf\sin\left(\frac{\phi}{f}\right)\right] \qquad \Delta^2_{\mathcal{R}}(k) = \Delta^2_{\mathcal{R}}(k_*)\left(\frac{k}{k_*}\right)^{n_s-1}\left[1+\delta n_s\cos\left(\frac{\phi_k}{f}\right)\right]$$

WMAP 5: $f = 6.67\times 10^{-4}$ and $\delta n_s = 0.17$ Flauger et al, 2009a

$bf \lesssim 10^{-4}$

BD would then be constrained as $|\beta| < 10^{-1}$

WMAP7: See talk by Christophe Ringeval. Working on that. (several models)

Observational:

Power spectrum.

$$V(\phi) = \frac{1}{2}m^2\phi^2[1+\alpha\sin\left(\frac{\phi}{\beta M}+\delta\right)]$$ Pahud, Kamionkowski & Liddle, 2008

WMAP 3: $\alpha \lesssim 3\times 10^{-5}$ **PLANCK:** $\mathcal{O}(10^{-6})$

Best constraints on the largest frequencies $(\beta = 5\times 10^{-3})$

Recall Christophe Ringeval Talk!

$$V(\phi) = \mu^3\left[\phi + bf\sin\left(\frac{\phi}{f}\right)\right] \qquad \Delta^2_{\mathcal{R}}(k) = \Delta^2_{\mathcal{R}}(k_*)\left(\frac{k}{k_*}\right)^{n_s-1}\left[1+\delta n_s\cos\left(\frac{\phi_k}{f}\right)\right]$$

WMAP 5: $f = 6.67\times 10^{-4}$ and $\delta n_s = 0.17$ Flauger et al, 2009a

$bf \lesssim 10^{-4}$

BD would then be constrained as $|\beta| < 10^{-1}$

WMAP7: See talk by Christophe Ringeval. Working on that. (several models)

Observational:

Bispectrum.

Observational:

Bispectrum.

-Compute full bispectrum

Observational:

Bispectrum.

-Compute full bispectrum
-Build templates (e.g. local, equilateral..)

Observational:

Bispectrum.

-Compute full bispectrum
-Build templates (e.g. local, equilateral..)
-Easy: find a way to compare bispectra

Observational:

Bispectrum.

-Compute full bispectrum
-Build templates (e.g. local, equilateral..)
-Easy: find a way to compare bispectra ←

Observational:

Bispectrum.

-Compute full bispectrum
-Build templates (e.g. local, equilateral..)
-Easy: find a way to compare bispectra ←

How? Define correlation. Like off diagonal fisher matrix elements in k-space.

Observational:

Bispectrum.

-Compute full bispectrum
-Build templates (e.g. local, equilateral..)
-Easy: find a way to compare bispectra ←

How? Define correlation. Like off diagonal fisher matrix elements in k-space.

$$F_X \star F_Y \equiv \int_{\Delta_k} dk_1 dk_2 dk_3 \frac{k_1^4 k_2^4 k_3^4}{k_t} F_X F_Y$$

$$B_X \cdot B_Y = \sum_{l_1, l_2, l_3} \frac{B^X_{l_1 l_2 l_3} B^Y_{l_1 l_2 l_3}}{\Delta_{l_1 l_2 l_3} C_{l_1} C_{l_2} C_{l_3}} \equiv \mathcal{F}_{XY}$$

Observational:

Bispectrum.

-Compute full bispectrum
-Build templates (e.g. local, equilateral..)
-Easy: find a way to compare bispectra ←

How? Define correlation. Like off diagonal fisher matrix elements in k-space.

$$F_X \star F_Y \equiv \int_{\Delta_k} dk_1 dk_2 dk_3 \frac{k_1^4 k_2^4 k_3^4}{k_t} F_X F_Y$$

$$\mathrm{C}(F_X, F_Y) \equiv \frac{F_X \star F_Y}{(F_X \star F_X)^{1/2} (F_Y \star F_Y)^{1/2}}$$

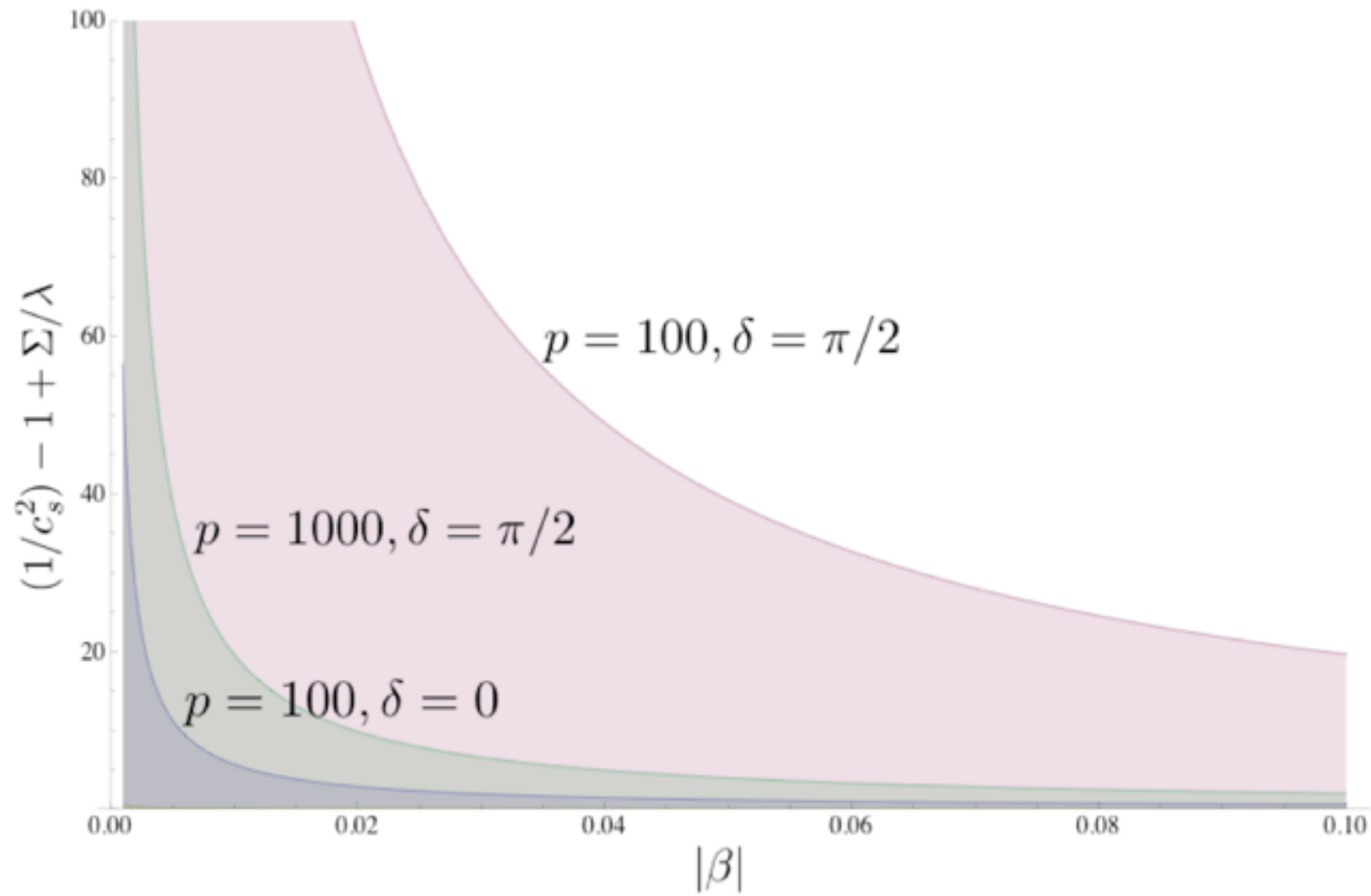
Δ_k : tetrahedral domain; domain in which comoving bispectrum 'lives': $k_a \leq k_b + k_c$ for $k_a \geq k_b, k_c$

$$k_a, k_b, k_c \leq k_{max}, \quad a, b, c = \{1, 2, 3\} \quad a \neq b \neq c$$

Bispectrum $\mathrm{C}(F_X, F_Y) \equiv \frac{F_X \star F_Y}{(F_X \star F_X)^{1/2}(F_Y \star F_Y)^{1/2}}$

For non-BD bispectrum (single field, non-canonical action)

$\omega_v = p$



Meerburg et al, 2009b

Danielsson, Meerburg&JpSchaar, 2010b

What is β? As said it is the Bogolyubov transformation.

Danielsson, Meerburg&JpSchaar, 2010b

What is β? As said it is the Bogolyubov transformation.
Can it be qualified? Yes, but the derived constraints are independent of the model that describes the transformation (Besides Hadamard and weakly scale dependent for long distance).

Danielsson, Meerburg&JpSchaar, 2010b

What is β? As said it is the Bogolyubov transformation.
Can it be qualified? Yes, but the derived constraints are independent of the model that describes the transformation (Besides Hadamard and weakly scale dependent for long distance).

We can derive an expression for β , that can be considered the ‘minimal vacuum modification’: each mode is excited separately with minimum uncertainty in field and field momentum

Danielsson, Meerburg&JpSchaar, 2010b

What is β? As said it is the Bogolyubov transformation.
Can it be qualified? Yes, but the derived constraints are independent of the model that describes the transformation (Besides Hadamard and weakly scale dependent for long distance).

We can derive an expression for β, that can be considered the 'minimal vacuum modification': each mode is excited separately with minimum uncertainty in field and field momentum

It is known as the new physics hypersurface (NPH) scenario, since it is derived from assuming a high energy cut off scale Λ_c at a time η_0

$$\beta_k = \frac{i}{2k\eta_0 + i} e^{-2ik\eta_0}$$

$$\beta_k = \frac{i}{2kc_s\eta_0 + i} e^{-2ikc_s\eta_0}$$

Danielsson, Meerburg&JpSchaar, 2010b

What is β? As said it is the Bogolyubov transformation.
Can it be qualified? Yes, but the derived constraints are independent of the model that describes the transformation (Besides Hadamard and weakly scale dependent for long distance).

We can derive an expression for β , that can be considered the 'minimal vacuum modification': each mode is excited separately with minimum uncertainty in field and field momentum
It is known as the new physics hypersurface (NPH) scenario, since it is derived from assuming a high energy cut off scale Λ_c at a time η_0

$$\beta_k \sim \frac{1}{2c_s\Lambda_c} e^{i\left(\frac{3}{2}\pi - \frac{2c_s\Lambda_c}{H}\right)} \qquad \frac{10^2}{c_s} \leq \frac{\Lambda_c}{H} \leq 8.5\times 10^4 c_s$$

Danielsson, Meerburg&JpSchaar, 2010b

Although bounds on non-Gaussianity exist, constraints are relatively weak (that we obtain by comparison) due to presence of oscillations!

Although bounds on non-Gaussianity exist, constraints are relatively weak (that we obtain by comparison) due to presence of oscillations!

Excited states are not the only scenario that lead to oscillations. We have seen today that there are several models, e.g. sharp features, axion-monodromy, multifield . . .(more to follow)

Although bounds on non-Gaussianity exist, constraints are relatively weak (that we obtain by comparison) due to presence of oscillations!

Excited states are not the only scenario that lead to oscillations. We have seen today that there are several models, e.g. sharp features, axion-monodromy, multifield . . .(more to follow)

Can we improve constraints by looking at oscillatory spectra?

Although bounds on non-Gaussianity exist, constraints are relatively weak (that we obtain by comparison) due to presence of oscillations!

Excited states are not the only scenario that lead to oscillations. We have seen today that there are several models, e.g. sharp features, axion-monodromy, multifield . . .(more to follow)

Can we improve constraints by looking at oscillatory spectra?

1) It is helpful if the primordial spectra is factorizable (reduces the number of computations), i.e.

Although bounds on non-Gaussianity exist, constraints are relatively weak (that we obtain by comparison) due to presence of oscillations!

Excited states are not the only scenario that lead to oscillations. We have seen today that there are several models, e.g. sharp features, axion-monodromy, multifield . . .(more to follow)

Can we improve constraints by looking at oscillatory spectra?

1) It is helpful if the primordial spectra is factorizable (reduces the number of computations), i.e.

$$F(k_1, k_2, k_3) = \sum f_1(k_i) f_2(k_{i+1}) f_3(k_{i+2})$$

Although bounds on non-Gaussianity exist, constraints are relatively weak (that we obtain by comparison) due to presence of oscillations!

Excited states are not the only scenario that lead to oscillations. We have seen today that there are several models, e.g. sharp features, axion-monodromy, multifield . . .(more to follow)

Can we improve constraints by looking at oscillatory spectra?

1) It is helpful if the primordial spectra is factorizable (reduces the number of computations), i.e.

$$F(k_1, k_2, k_3) = \sum f_1(k_i) f_2(k_{i+1}) f_3(k_{i+2})$$

2) One could try to build a template, but there is another problem arising. Both the frequency and the phase can be considered free parameters of the theory.

Factorization.

General question: Can we factorize?

Not trivial; for example, equilateral shape

$$F \propto \frac{1}{k_1 k_2 k_3 k_t^3}$$

has been factorized with the equilateral template, but 'shear luck'. Creminelli

Alternative is (so-called) mode expansion:

$$F x_1^2 x_2^2 x_3^2 \simeq \sum_{n=0}^{N} \alpha_n R_n(x_1, x_2, x_2)$$

Fergusson et al 2009-2010

Rewrite original spectrum as a sum of functions that are factorized from scratch and are orthonormal on tetrahedral

INTERMEZZO

Building the orthonormal basis.

In k space define:

$$\mathcal{T}[f] = \int_{\Delta_k} f(k_1, k_2, k_3) w(k_1, k_2, k_3) d\Delta_k$$

INTERMEZZO

Building the orthonormal basis.

In k space define:

$$\mathcal{T}[f] = \int_{\Delta_k} f(k_1, k_2, k_3) w(k_1, k_2, k_3) d\Delta_k$$

$$\tilde{w} = \frac{1}{2} x(4 - 3x) \quad \mathcal{T}[f] = \int_0^1 f(x) \tilde{w}(x)$$

INTERMEZZO

Building the orthonormal basis.

In k space define:

$$\mathcal{T}[f] = \int_{\Delta_k} f(k_1, k_2, k_3) w(k_1, k_2, k_3) d\Delta_k$$

$$w_n \equiv \mathcal{T}[x^n] = \frac{n+6}{2(n+3)(n+2)}$$

$$q_n(x) = \frac{1}{\mathcal{N}} \begin{vmatrix} 1/2 & 7/24 & 1/5 & \cdots & w_n \\ 7/24 & 1/5 & 3/20 & \cdots & w_{n+1} \\ \vdots & \vdots & \vdots & & \vdots \\ w_{n-1} & w_n & w_{n+1} & \cdots & w_{2n-1} \\ 1 & x & x^2 & \cdots & x^n \end{vmatrix} \qquad \mathcal{T}[q_n q_m] = \delta_{mn}$$

INTERMEZZO

Building the orthonormal basis.

Build 3 dimensional basis:

$$\mathcal{Q}_n(x, y, z) = \frac{1}{6\mathcal{N}} q_{\{p} q_r q_{s\}} \quad \text{6 terms}$$

$n = 0 \rightarrow 000$	$n = 4 \rightarrow 111$	$n = 8 \rightarrow 022$	$n = 12 \rightarrow 113$	$n = 16 \rightarrow 222$	$n = 20 \rightarrow 024$	$n = 24 \rightarrow 133$
$n = 1 \rightarrow 001$	$n = 5 \rightarrow 012$	$n = 9 \rightarrow 013$	$n = 13 \rightarrow 023$	$n = 17 \rightarrow 123$	$n = 21 \rightarrow 015$	$n = 25 \rightarrow 124$
$n = 2 \rightarrow 011$	$n = 6 \rightarrow 003$	$n = 10 \rightarrow 004$	$n = 14 \rightarrow 014$	$n = 18 \rightarrow 033$	$n = 22 \rightarrow 006$	$n = 26 \rightarrow 034$
$n = 3 \rightarrow 002$	$n = 7 \rightarrow 112$	$n = 11 \rightarrow 112$	$n = 15 \rightarrow 005$	$n = 19 \rightarrow 114$	$n = 23 \rightarrow 223$	$n = 27 \rightarrow 115$

Orthonormalize these by Gramm-Schmidt to end up with a set that obeys:

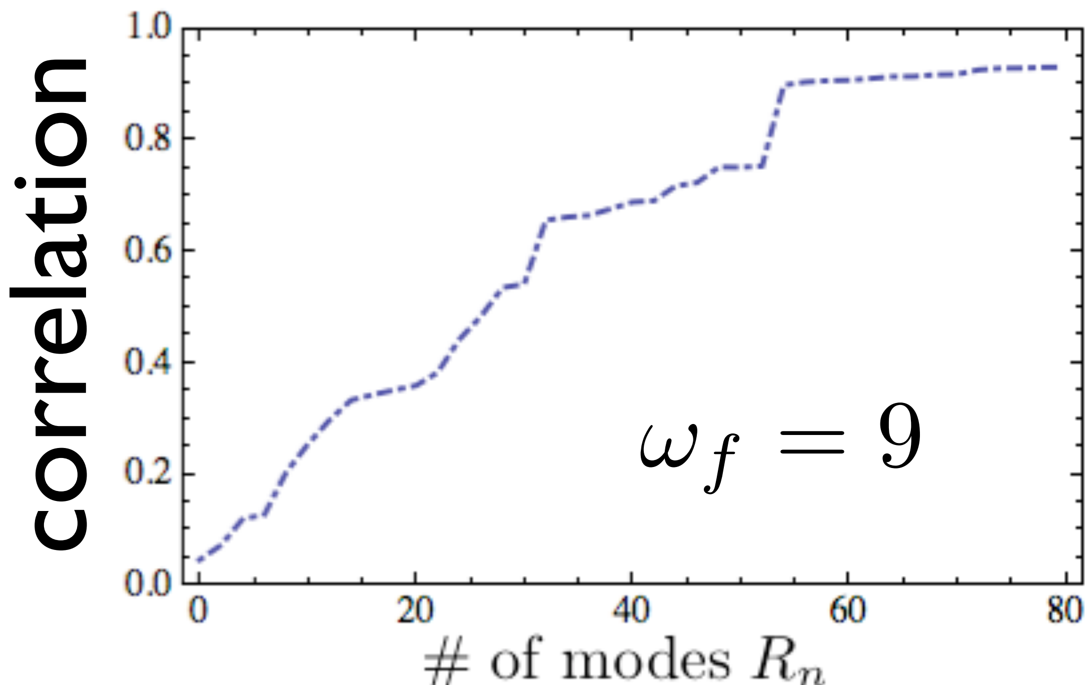
$$R_n \star R_m = \delta_{mn}$$

Using polynomial modes orthogonalized on the tetrahedral domain:

$$F x_1^2 x_2^2 x_3^2 \simeq \sum_{n=0}^{N} \alpha_n R_n(x_1, x_2, x_2)$$

Using polynomial modes orthogonalized on the tetrahedral domain:

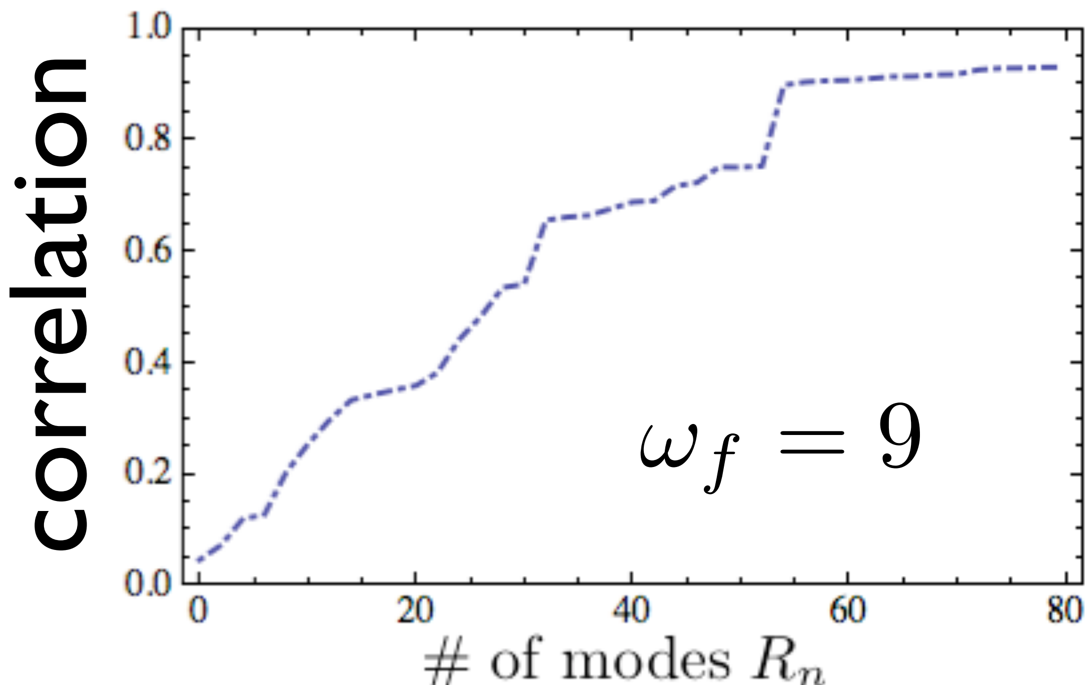
$$F x_1^2 x_2^2 x_3^2 \simeq \sum_{n=0}^{N} \alpha_n R_n(x_1, x_2, x_2)$$



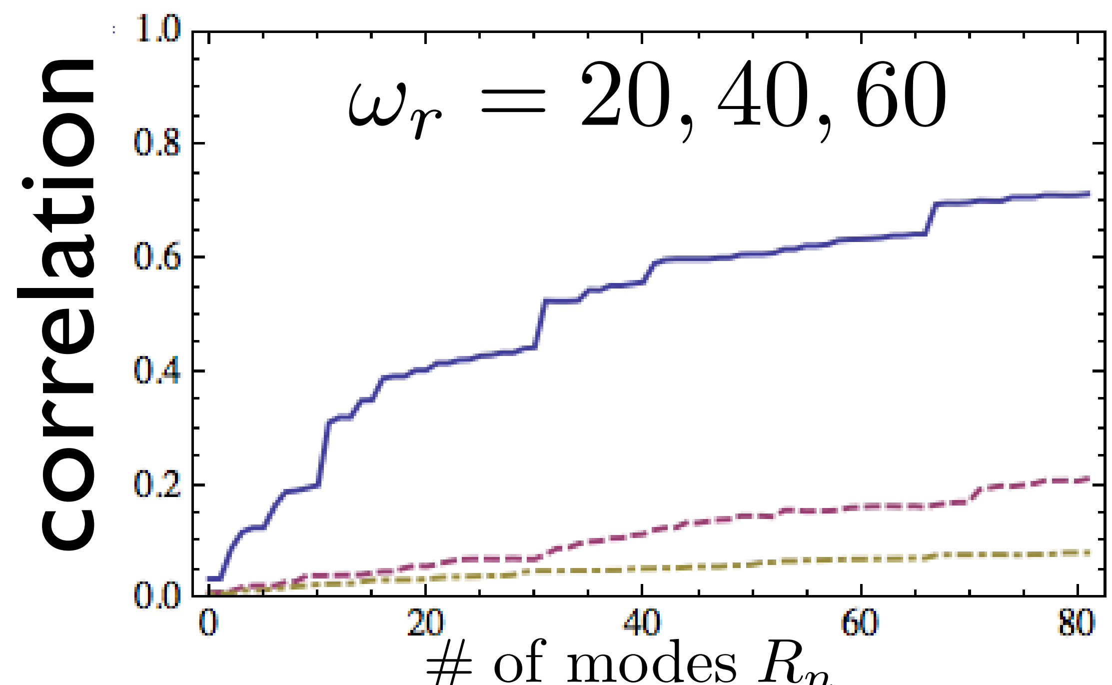
Feature bispectrum

Using polynomial modes orthogonalized on the tetrahedral domain:

$$F x_1^2 x_2^2 x_3^2 \simeq \sum_{n=0}^{N} \alpha_n R_n(x_1, x_2, x_2)$$



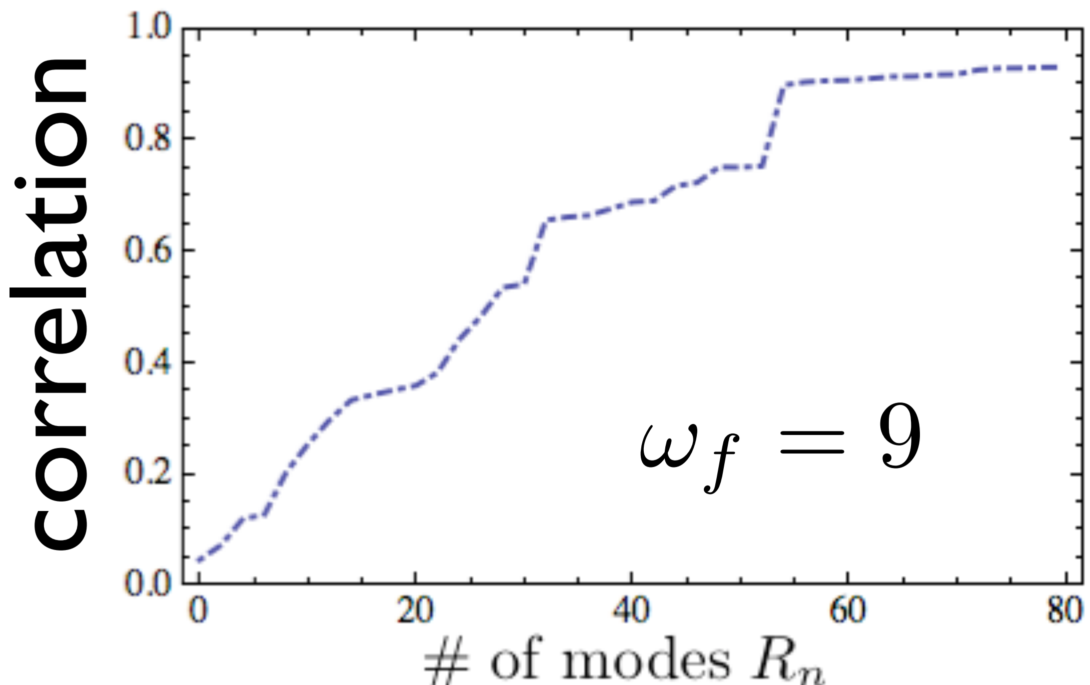
Feature bispectrum



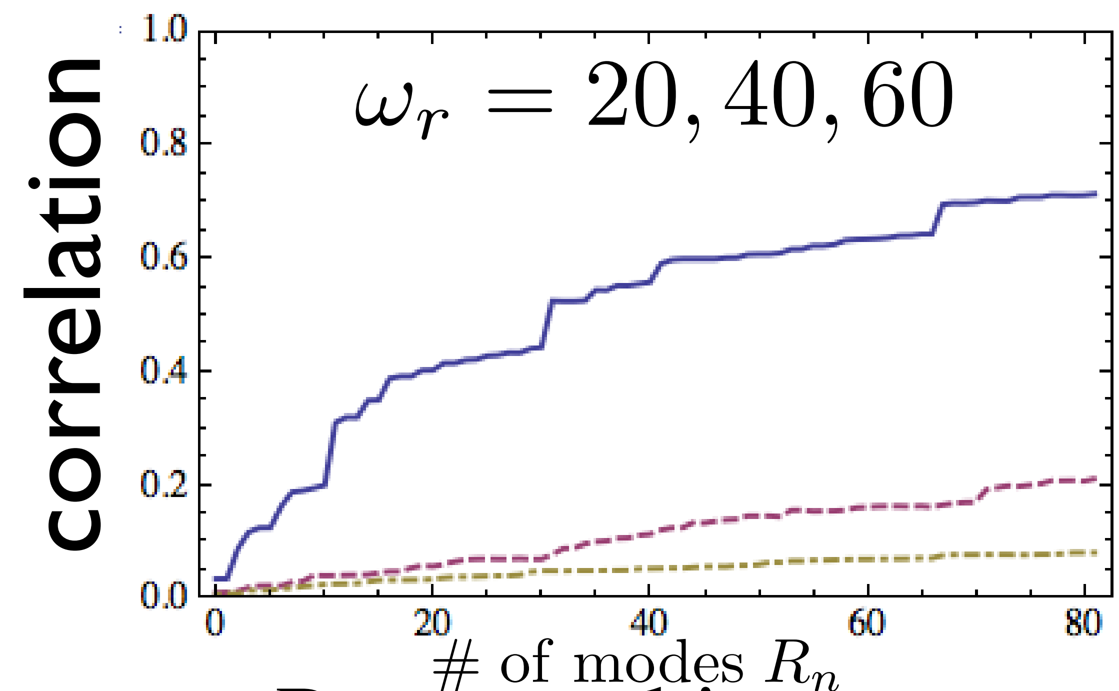
Resonant bispectrum

Using polynomial modes orthogonalized on the tetrahedral domain:

$$F x_1^2 x_2^2 x_3^2 \simeq \sum_{n=0}^{N} \alpha_n R_n(x_1, x_2, x_2)$$



Feature bispectrum

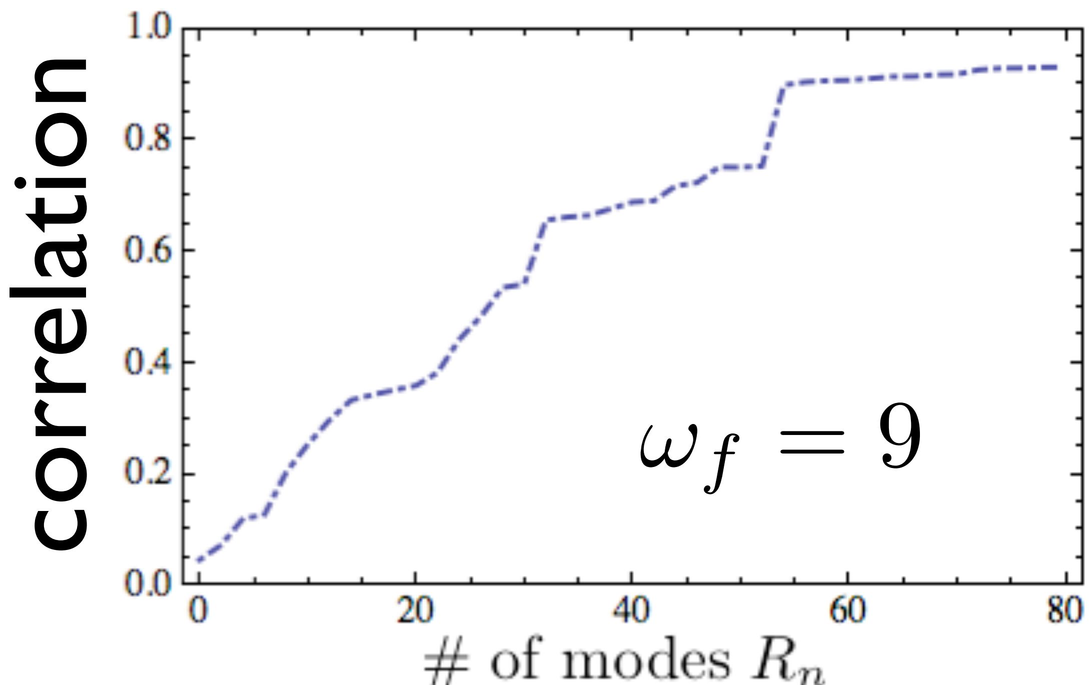


Resonant bispectrum

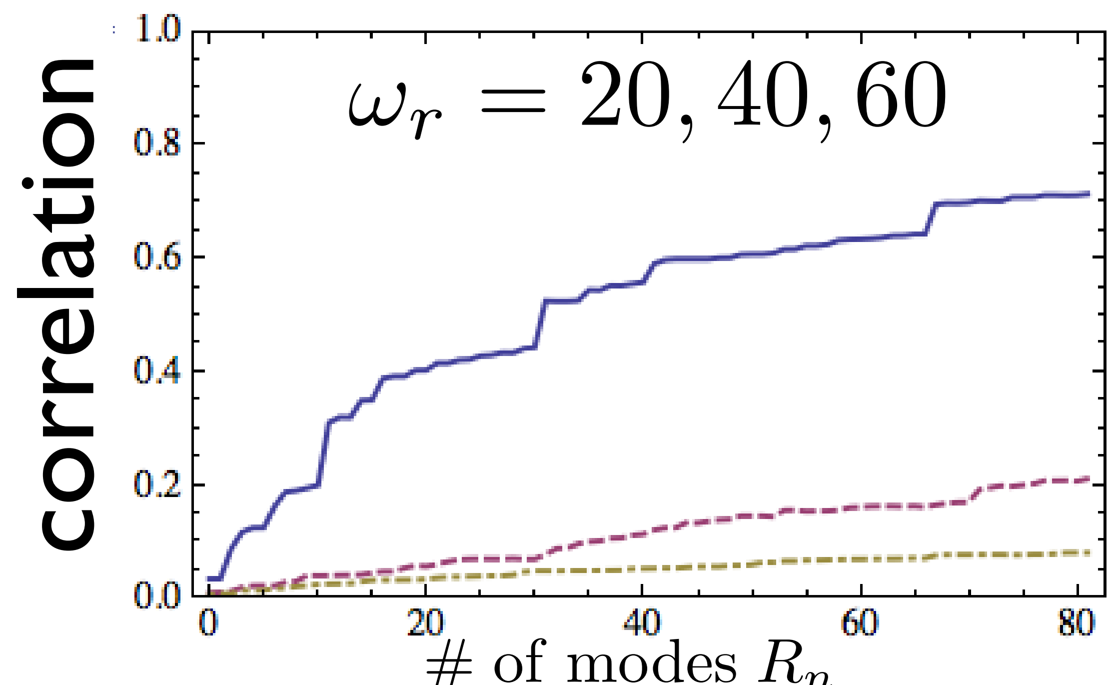
Need many modes to achieve `good' correlation with original spectrum

Using polynomial modes orthogonalized on the tetrahedral domain:

$$F x_1^2 x_2^2 x_3^2 \simeq \sum_{n=0}^{N} \alpha_n R_n(x_1, x_2, x_2)$$



Feature bispectrum



Resonant bispectrum

Need many modes to achieve `good' correlation with original spectrum

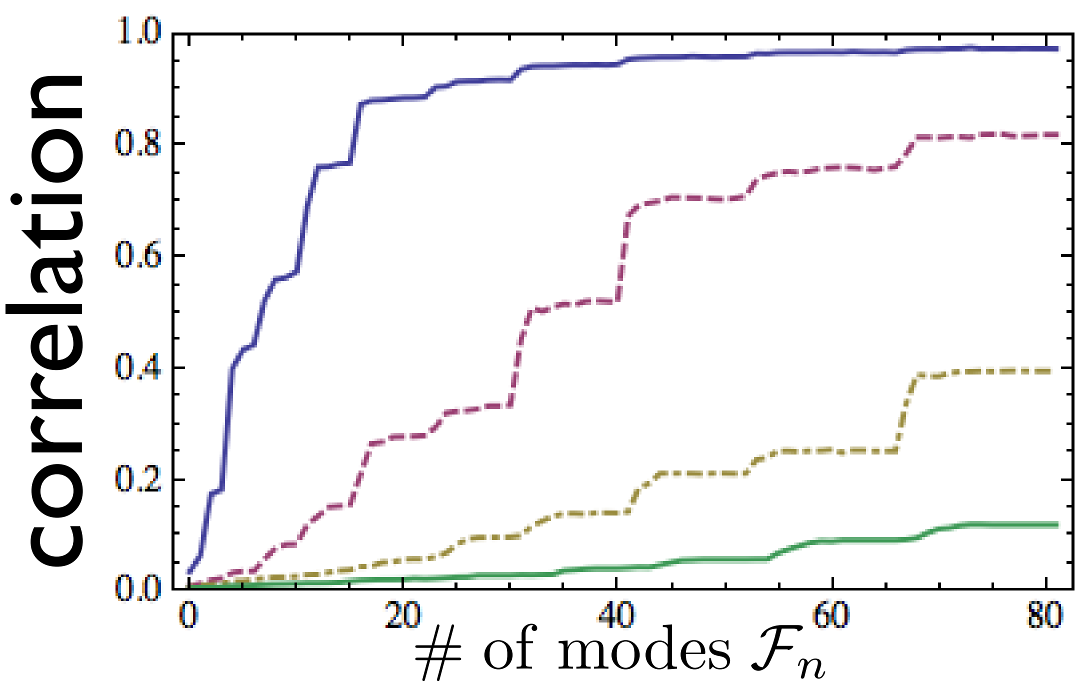
Recall however that the correlation used to be of order 1percent!

Maybe if we use another basis? Fourier instead of polynomials

Meerburg 2010a

$$x^n \rightarrow e^{i2\pi x}$$
$$R_n \rightarrow \mathcal{F}_n$$

$$\omega_r = 20, 40, 60, 80$$



Decrease number of modes necessary by a factor ~5 for resonant bispectrum and feature bispectrum.

What about other oscillatory bispectra?

For non-BD bispectra it only works for a canonical single field. For non-canonical models correlation increases faster than polynomial but stops growing altogether after a while. Probably because of many small features near edge of tetrahedral domain, even at small frequency.

For non-BD bispectra it only works for a canonical single field. For non-canonical models correlation increases faster than polynomial but stops growing altogether after a while. Probably because of many small features near edge of tetrahedral domain, even at small frequency.

However, also considered toy-spectra:

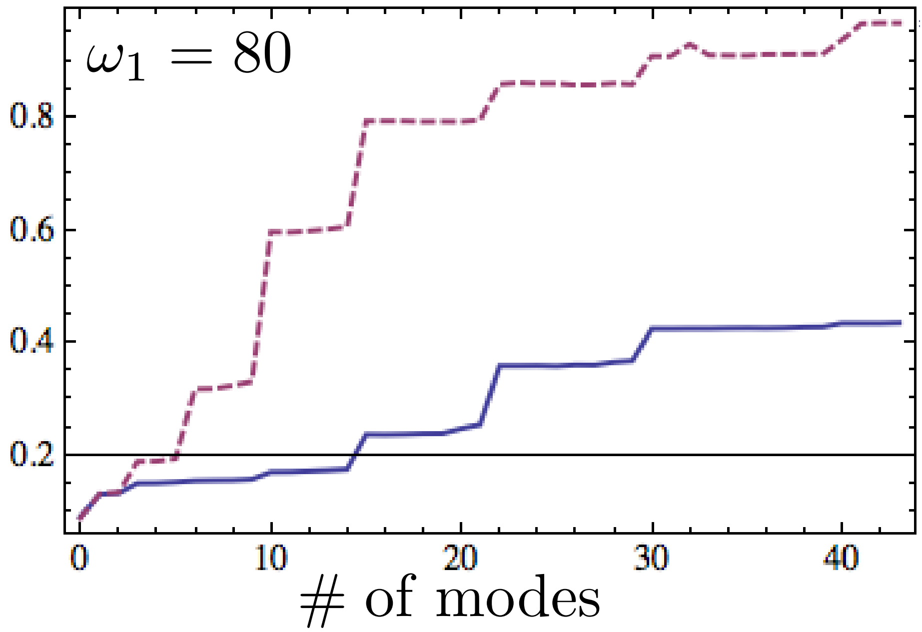
$$\begin{aligned} F_1 &= \frac{1}{k_1^2 k_2^2 k_3^2}\left(\sin\frac{\omega_1}{k_1+1} + \sin\frac{\omega_1}{k_2+1} + \sin\frac{\omega_1}{k_3+1}\right) \\ F_2 &= \frac{1}{k_1^2 k_2^2 k_3^2}\sin\omega_2 k_1 k_2 k_3 \end{aligned}$$

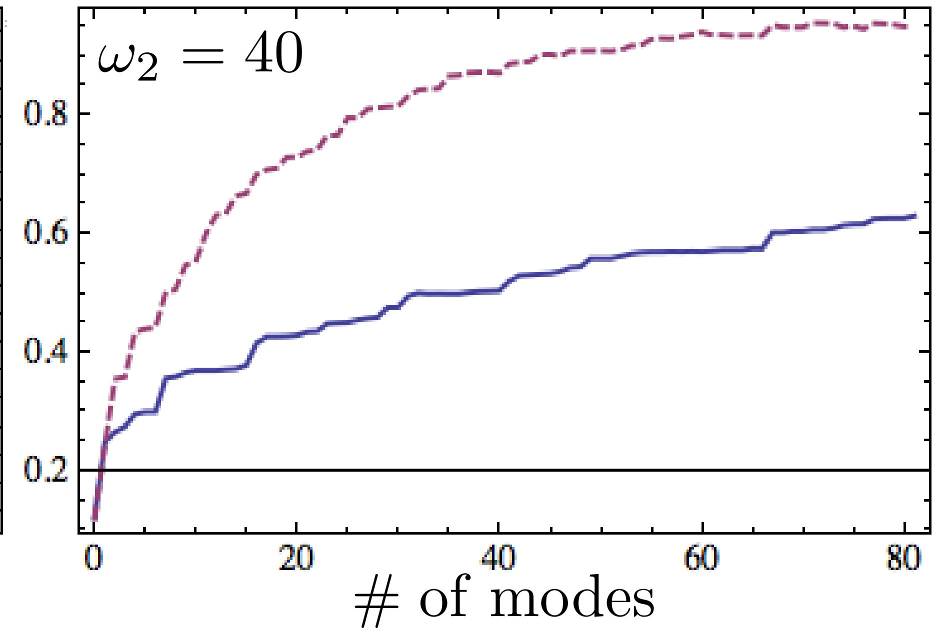
For non-BD bispectra it only works for a canonical single field. For non-canonical models correlation increases faster than polynomial but stops growing altogether after a while. Probably because of many small features near edge of tetrahedral domain, even at small frequency.

However, also considered toy-spectra:

$$F_1 = \frac{1}{k_1^2 k_2^2 k_3^2}\left(\sin\frac{\omega_1}{k_1+1} + \sin\frac{\omega_1}{k_2+1} + \sin\frac{\omega_1}{k_3+1}\right)$$

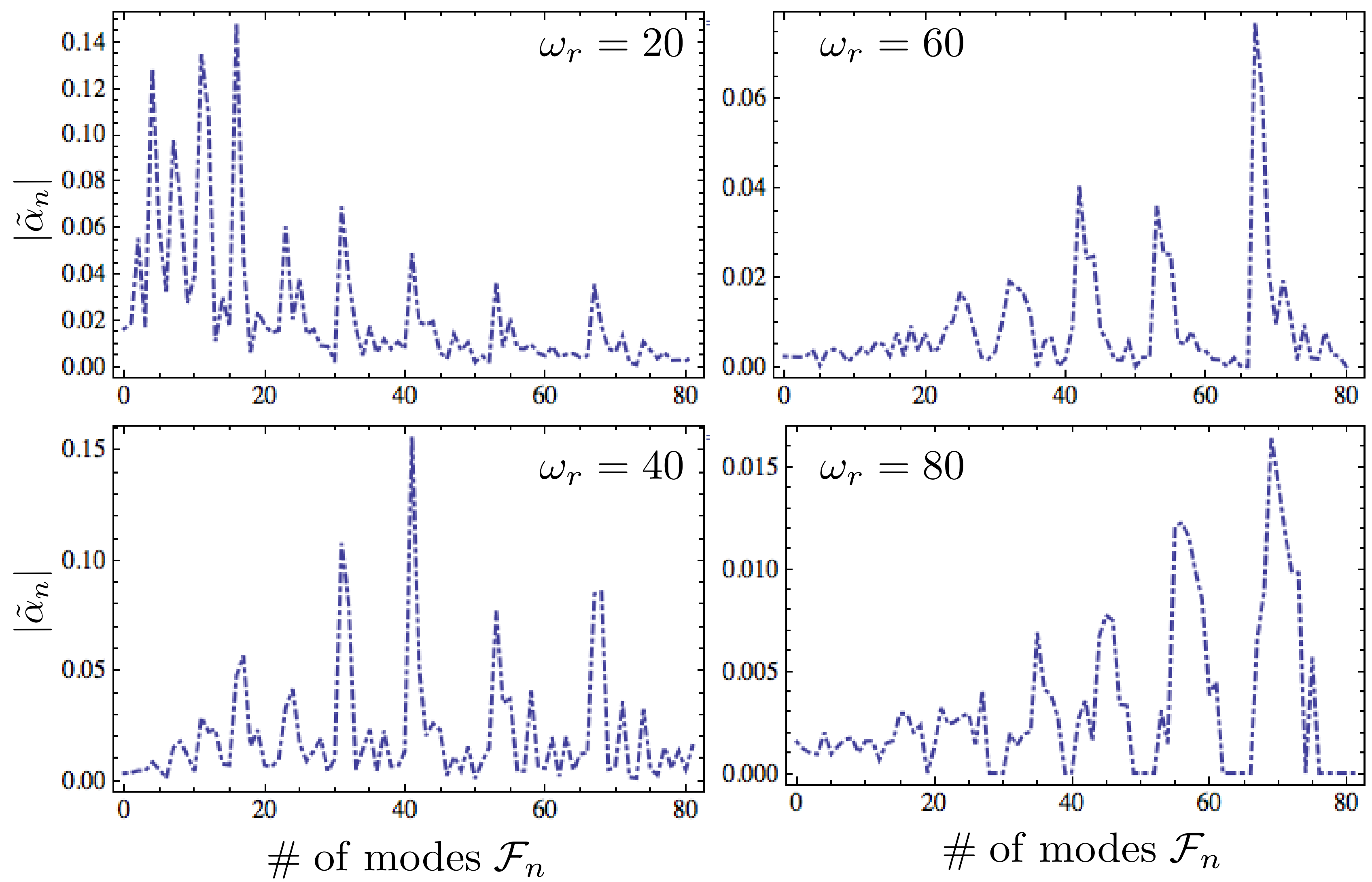
$$F_2 = \frac{1}{k_1^2 k_2^2 k_3^2}\sin\omega_2 k_1 k_2 k_3$$





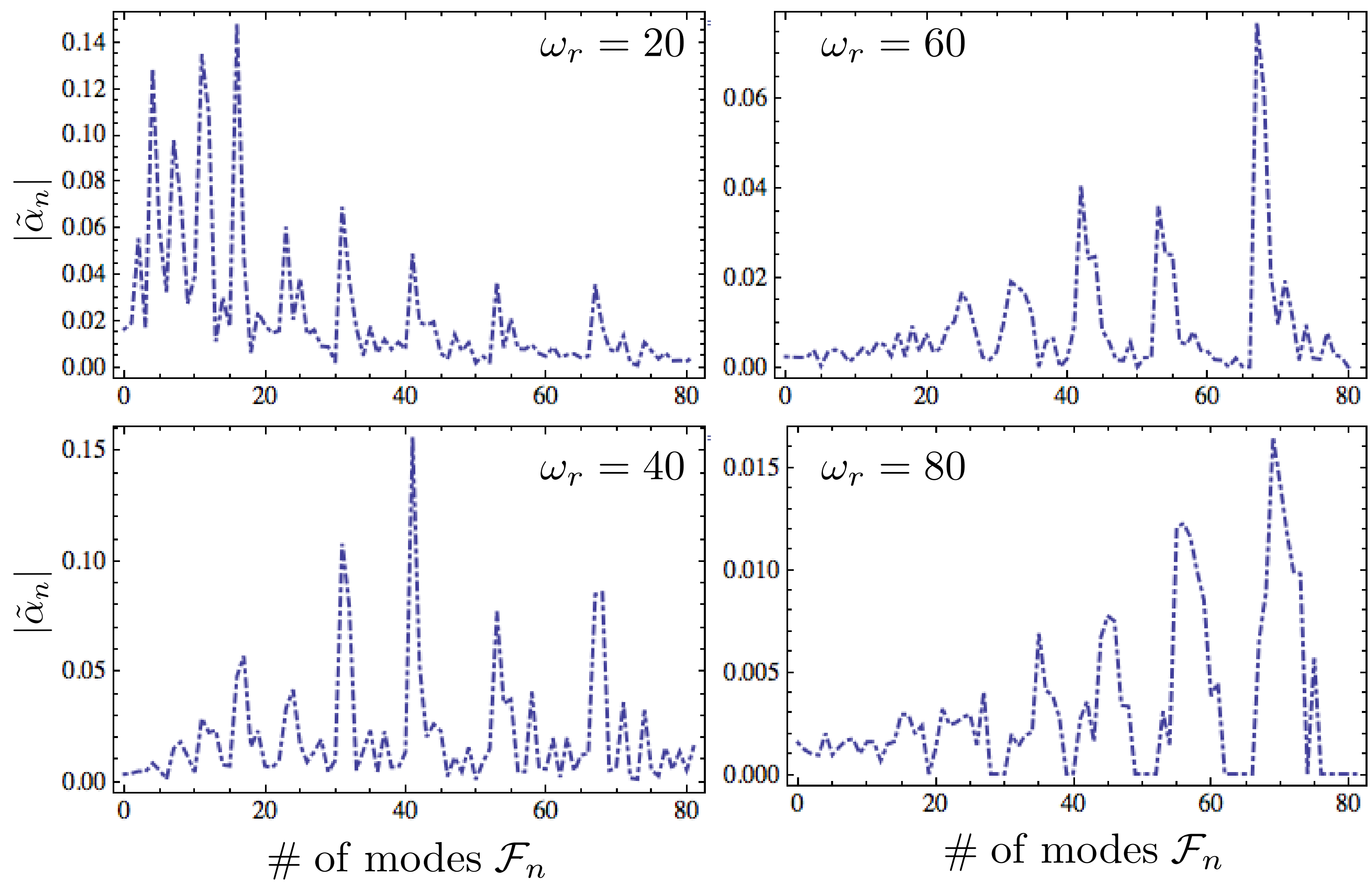
Not only is Fourier expansion more efficient, there is **another advantage.**

Consider the modes (alpha) for different bispectra



Not only is Fourier expansion more efficient, there is **another advantage.**

Consider the modes (alpha) for different bispectra

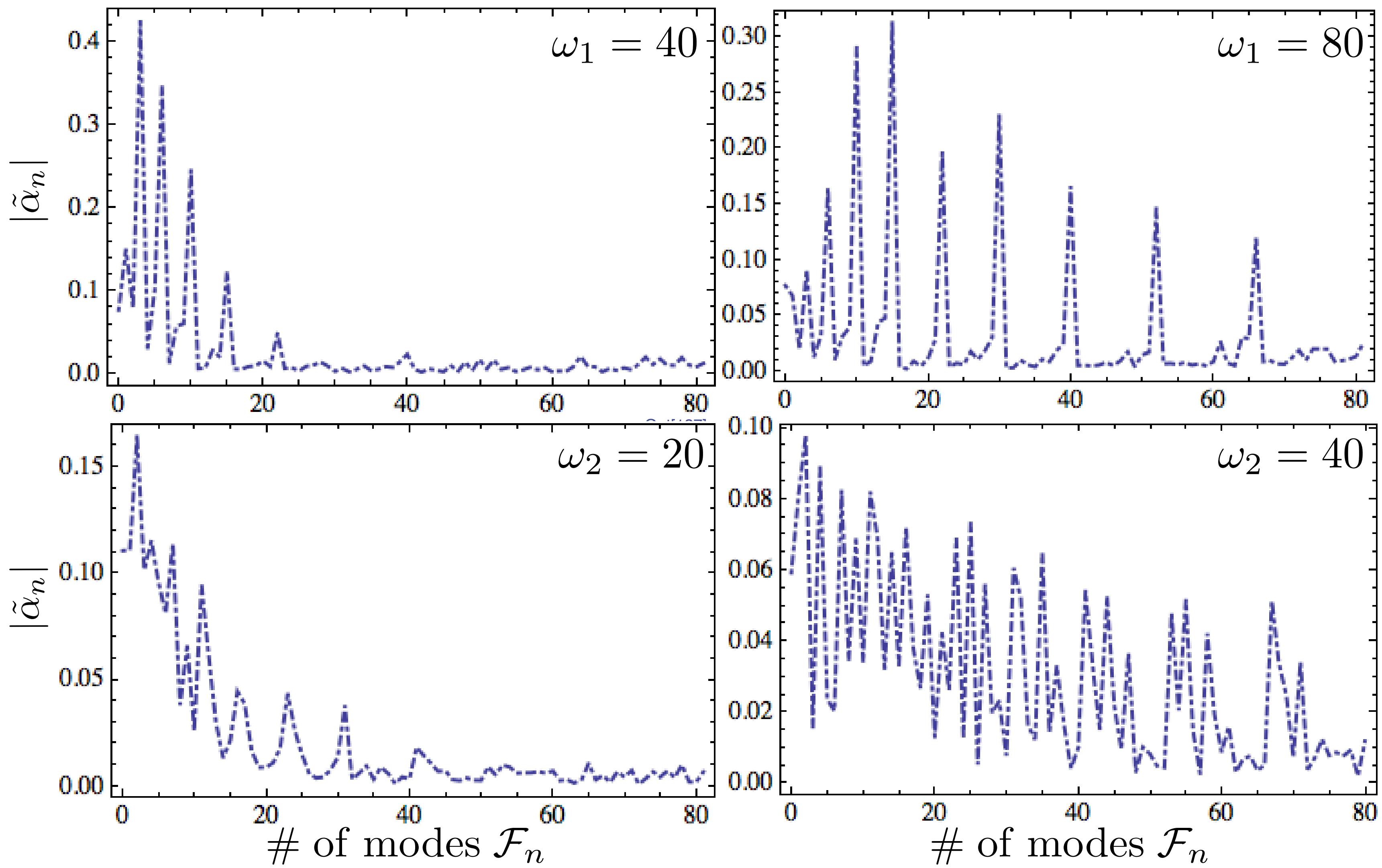


They all peak at similar mode numbers, for same shape bispectra.

And, these appear already at low mode number for high frequencies

Not only is Fourier expansion more efficient, there is **another advantage.**

Consider the modes (alpha) for different bispectra

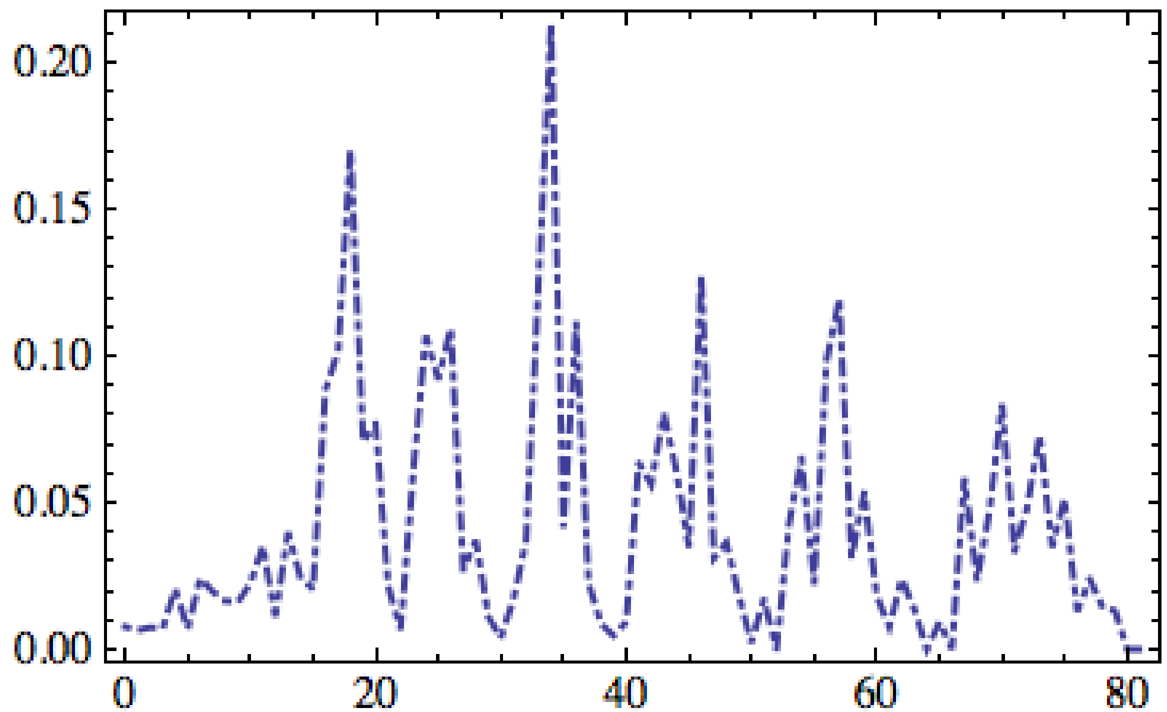


They all peak at similar mode numbers, for same shape bispectra.

And, these appear already at low mode number for high frequencies

Not only is Fourier expansion more efficient, there is **another advantage.**

Consider the modes (alpha) for different bispectra



$$F_3 = \frac{1}{k_1^2 k_2^2 k_3^2}\left(\sin\frac{\omega_3 k_t}{k_1+1} + \sin\frac{\omega_3 k_t}{k_2+1} + \sin\frac{\omega_3 k_t}{k_3+1}\right)$$

They all peak at similar mode numbers, for same shape bispectra.
And, these appear already at low mode number for high frequencies

What does this mean?

What does this mean?

For some bispectra it means you would need even less modes to achieve significant correlation with the original spectrum

What does this mean?

For some bispectra it means you would need even less modes to achieve significant correlation with the original spectrum

Observationally it is even more helpful: measuring a few modes could give an indication of:

What does this mean?

For some bispectra it means you would need even less modes to achieve significant correlation with the original spectrum

Observationally it is even more helpful: measuring a few modes could give an indication of:

-shape of primordial bispectrum

What does this mean?

For some bispectra it means you would need even less modes to achieve significant correlation with the original spectrum

Observationally it is even more helpful: measuring a few modes could give an indication of:

-shape of primordial bispectrum

-frequency of the oscillations

What does this mean?

For some bispectra it means you would need even less modes to achieve significant correlation with the original spectrum

Observationally it is even more helpful: measuring a few modes could give an indication of:

-shape of primordial bispectrum

-frequency of the oscillations

Even frequencies larger than discussed could be indicated as they would already have some modes appear at low mode number.

Conclusions

Conclusions

-I have discussed the appearance of **oscillations in the primordial power spectrum and the bispectrum due to excited initial states**

Conclusions

-I have discussed the appearance of **oscillations in the primordial power spectrum and the bispectrum due to excited initial states**

-The amplitude of the bispectrum could be **observably large**

Conclusions

-I have discussed the appearance of **oscillations in the primordial power spectrum and the bispectrum due to excited initial states**

-The amplitude of the bispectrum could be **observably large**

-The observed power spectrum and bispectrum can put **constraints** on these models

Conclusions

-I have discussed the appearance of **oscillations in the primordial power spectrum and the bispectrum due to excited initial states**

-The amplitude of the bispectrum could be **observably large**

-The observed power spectrum and bispectrum can put **constraints** on these models

-The constraints are relatively weak because oscillatory spectra are **hard to measure** and for bispectra they have not been measured at all!.

Conclusions

-I have discussed the appearance of **oscillations in the primordial power spectrum and the bispectrum due to excited initial states**

-The amplitude of the bispectrum could be **observably large**

-The observed power spectrum and bispectrum can put **constraints** on these models

-The constraints are relatively weak because oscillatory spectra are **hard to measure** and for bispectra they have not been measured at all!.

-Using leakage factors you can derive constraints. I have suggested mode expansion as a way to **improve constraints**.

Suggestions

Suggestions

-The method of mode expansion should be further investigated.
In particular it should be applied to multipole spectra. In progress

Suggestions

-The method of mode expansion should be further investigated. In particular it should be applied to multipole spectra. In progress

-One could try and see if it possible to develop a consistency mechanism between different spectra

Suggestions

-The method of mode expansion should be further investigated. In particular it should be applied to multipole spectra. In progress

-One could try and see if it possible to develop a consistency mechanism between different spectra

-One could consider the real space correlation function.

Suggestions

-The method of mode expansion should be further investigated. In particular it should be applied to multipole spectra. In progress

-One could try and see if it possible to develop a consistency mechanism between different spectra

-One could consider the real space correlation function.

-It would be preferable to develop a simple test that could yield insight into whether there are oscillations/features in the first place. In progress

Suggestions

-The method of mode expansion should be further investigated. In particular it should be applied to multipole spectra. In progress

-One could try and see if it possible to develop a consistency mechanism between different spectra

-One could consider the real space correlation function.

-It would be preferable to develop a simple test that could yield insight into whether there are oscillations/features in the first place. In progress

-Parallel develop methods of detection for LSS

Suggestions

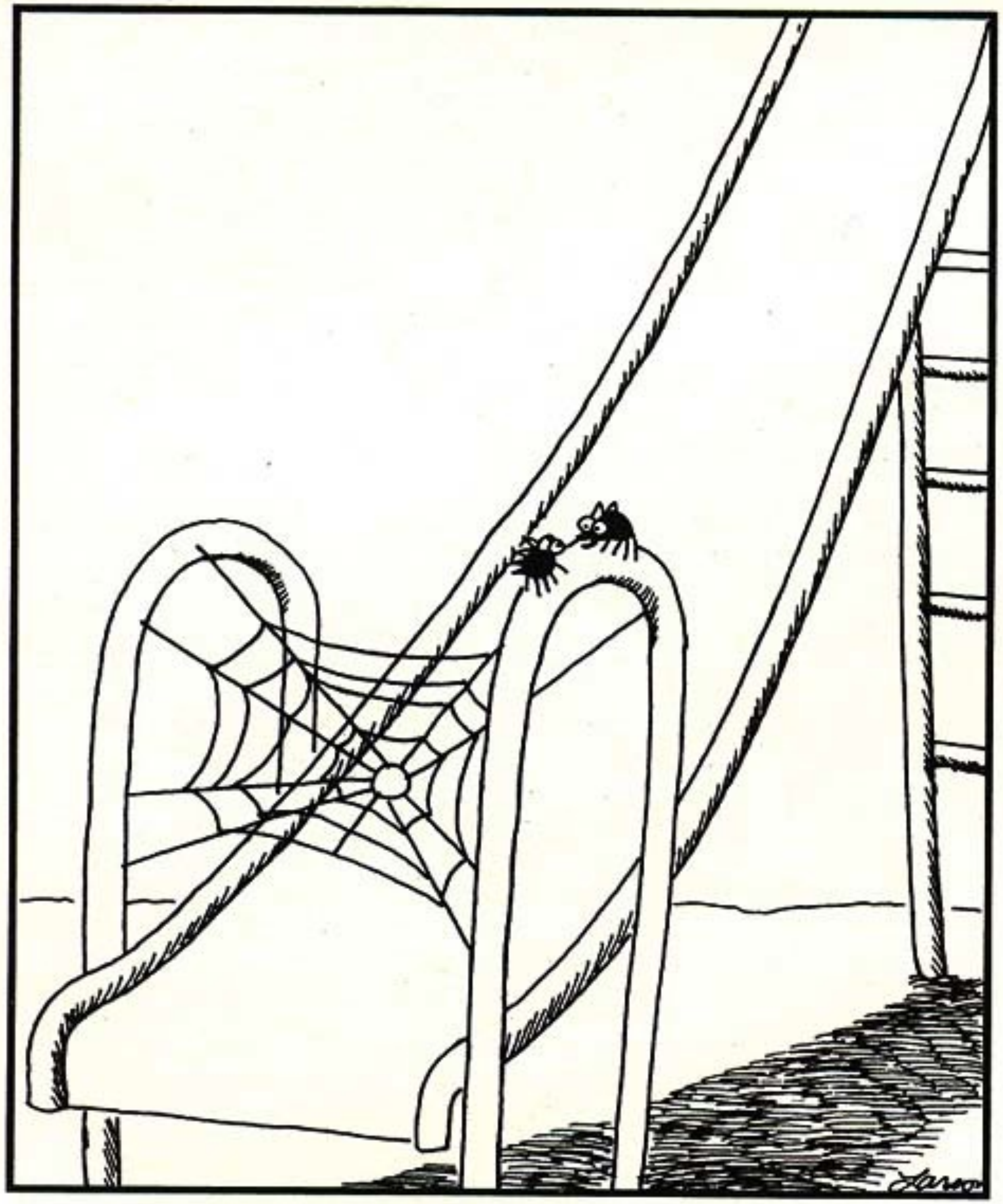
-The method … er investigated. In particular … ectra. In progress

-One could t… consistency mechanism …

-One could … unction.

-It would be … that could yield insight into … es in the first place. In pro…

-Parallel dev…



"If we pull this off, we'll eat like kings."