Cosmological singularities in string theory

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[arXiv: 1012.0113, 0909.4731 [hep-th], KN; arXiv:0904.4532, Kallingalthodi Madhu, KN;
arXiv:0807.1517, Adel Awad, Sumit Das, Suresh Nampuri, KN, Sandip Trivedi;
arXiv:0711.2994, Awad, Das, KN, Trivedi;
hep-th/0602107, 0610053, Das, Jeremy Michelson, KN, Trivedi.]

- AdS/CFT with cosmological singularities: gauge theories with time-dep couplings and spacelike singularities
- worldsheet: null singularities and free strings

Big-Bang singularities, strings . . .

- Big Bang/Crunch singularities, time, in string theory (toy) models?
  Understand spacelike, null singularities — events in time.

General Relativity breaks down at singularities:
curvatures, tidal forces divergent, notions of spacetime break down.
Want "stringy" description, eventually towards smooth quantum (stringy) completion of classical spacetime geometry.

Previous examples: "stringy phases" in $e.g.$ 2-dim worldsheet (linear sigma model) descriptions (including time-dep versions, e.g. tachyon dynamics in (meta/)unstable vacua), dual gauge/Matrix theories, . . .

In what follows, we'll use (i) the AdS/CFT framework,
(ii) worldsheet string spectrum analysis near null singularities.

AdS/CFT and deformations

Bulk string theory on $AdS_5 \times S^5$ with dilaton (scalar) $\Phi = const$, 5-form field strength, and metric (Poincare coords)

$$ds^2 = \frac{1}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu + dz^2) + ds^2_{S^5} \ ,$$

dual to boundary $d = 4$ $\mathcal{N}{=}4$ (large N) $SU(N)$ Superconformal Yang-Mills theory, coupling $g^2_{YM} = e^\Phi$.

Assume AdS/CFT: study *time-dependent* deformations of AdS/CFT.

Bulk: time-dependent sources, time evolution (thro Einstein eqns) eventually gives cosmological singularity. Breaks down.

Boundary: Gauge theory dual is a sensible Hamiltonian quantum system in principle, subject to time-dependent sources. Response?

Deform metric, dilaton (non-normalizable time-dep defmns):

$$ds^2 = \frac{1}{z^2}(\tilde{g}_{\mu\nu}dx^\mu dx^\nu + dz^2) + ds^2_{S^5} \ , \qquad \Phi = \Phi(t) \text{ or } \Phi(x^+).$$

Solution if: $\tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi \ , \ \ \frac{1}{\sqrt{-\tilde{g}}}\,\partial_\mu(\sqrt{-\tilde{g}}\;\tilde{g}^{\mu\nu}\partial_\nu\Phi) = 0.$

AdS cosmologies cont'd

In many cases, possible to find new coordinates such that boundary metric $ds_4^2 = \lim_{z\to 0} z^2 ds_5^2$ is flat, at least as an expansion about boundary ($z = 0$): Penrose-Brown-Henneaux (PBH) transformations, subset of bulk diffeomorphisms leaving metric invariant (in Fefferman-Graham form), acting as Weyl transformation on boundary. Thus dual gauge theory lives on flat space. So sharp sub-question: Gauge theory with time-dependent coupling $g_{YM}^2 = e^{\Phi}$, subject to Hamiltonian time evolution through this external time-dependent source. Response?

Sources approaching $e^{\Phi} \to 0$ at some finite point in time, e.g. $g_{YM}^2 = e^{\Phi} \ \to \ (-t)^p \ , \ p > 0 \ \ [t < 0]$, give rise to bulk singularity

$R_{tt} = \frac{1}{2}\dot{\Phi}^2 \ \sim \frac{1}{t^2} \ .$ Curvatures, tidal forces diverge near $t = 0$.

We'd specially like to understand gauge theory response near $t = 0$.

Gauge theories, time-dep couplings

Gauge theory kinetic terms $\int e^{-\Phi} F^2$ not canonical.

As in usual perturbation theory, try absorbing coupling $g_{YM}^2 = e^{\Phi}$ into the gauge field A_μ: now g_{YM} appears only in interaction terms.

Toy scalar theory: $L[\tilde{X}] = -e^{-\Phi}\left(\frac{1}{2}(\partial\tilde{X})^2 + \tilde{X}^4\right)$.

Redefining $\tilde{X} = e^{\Phi/2}X$: $L \to -(\partial X)^2 - m^2(\Phi)X^2 - e^{\Phi}X^4$,

dropping a boundary term, and $m^2(\Phi) = \frac{1}{4}\partial_\mu\Phi\partial^\mu\Phi - \frac{1}{2}\partial_\mu\partial^\mu\Phi$.

Time-dep $\Phi = \Phi(t)$: *e.g.* $g_{YM}^2 = e^{\Phi} = (-t)^p$, $p > 0$ $[t < 0]$

gives $m^2(\Phi) = -\frac{1}{4}(\dot{\Phi})^2 + \frac{1}{2}\ddot{\Phi} = -\frac{p(p+2)}{4\ t^2}$.

Can study time-dep quantum mechanics of single momentum-k modes.

- X variables canonical: tachyonic divergent mass forces $X \sim \frac{1}{t^{p/2}}$. Extra information required as $X \to \infty$: X description not good.
- $\tilde{X}$ variables finite near $t = 0$: interaction terms $e^{-\Phi}\tilde{X}^4|_{t\sim 0}$ large.

Time-dep field theory wave-fnal

General field theory Schrodinger picture analysis possible near $t = 0$.

[Lagrangian $\int d^3x\, e^{-\Phi}(\frac{1}{2}(\partial_t\tilde{X})^2 - \frac{1}{2}(\partial_i\tilde{X})^2 - \tilde{X}^4) \equiv \int d^3x\, e^{-\Phi}(\frac{1}{2}(\partial_t\tilde{X})^2 - V[\tilde{X}])$]

$\Pi(x) \rightarrow \frac{1}{i}\frac{\delta}{\delta\tilde{X}}$; Hamiltonian: $H = e^{-\Phi}V[\tilde{X}] + e^{\Phi}\int d^3x(-\frac{1}{2}\frac{\delta^2}{\delta\tilde{X}^2})$,

Schrodinger eqn: $i\partial_t\psi[\tilde{X}(x), t] = H\psi[\tilde{X}(x), t]$.

$e^{\Phi} = (-t)^p \rightarrow 0$ as $t \rightarrow 0$, so potential term $e^{-\Phi}V$ dominates in H $\Rightarrow$ $i\partial_t\psi \sim\ e^{-\Phi(t)}V[\tilde{X}(x)]\psi$. This gives near-singularity time-dep of wave-functional (generic state) as

$$\psi[\tilde{X}(x), t] \sim\ e^{-i(\int dt\ e^{-\Phi(t)})V[\tilde{X}(x)]}\ \psi_0[\tilde{X}(x)]\ .$$

Phase $\sim\ \frac{(-t)^{1-p}}{1-p}V[\tilde{X}(x)]$. If $p > 1$, “wildly” oscillating ($t \rightarrow 0$).

Energy diverges for generic states $(\langle V\rangle \neq 0)$ [no time-dep in $\langle V\rangle$]

$$\langle H\rangle \sim e^{-\Phi}\langle V\rangle = \frac{1}{(-t)^p}\ \int D\tilde{X}\ V[\tilde{X}]\ |\psi_0[\tilde{X}(x)]|^2\ .$$

The gauge theory

Analyzing various dilaton couplings to scalars, fermions, gauge fields, dominant contributions as $e^\Phi = (-t)^p \to 0$ are gauge fields.

Gauge fields: KE terms have dilaton coupling $\int e^{-\Phi}\,\mathrm{Tr}F^2$. Determine the behaviour of the system near $t \sim 0$.

(Coulomb gauge) Residual action for two physical transverse components A^i becomes $\int e^{-\Phi}(\partial A^i)^2$, (i.e. two copies of the scalar theory earlier). Cubic/quartic interactions: no time derivatives, contribute only to potential energy $V[A^i(x)] = \frac{1}{4}\int d^3x\,\mathrm{Tr}F_{ij}^2$.

Near singularity $(t \sim 0)$ time-dep of wave functional

$$\psi[A^i(x), t] \sim e^{-i(\int dt\ e^{-\Phi})V[A^i(x)]}\,\psi_0[A^i(x)] \,.$$

"Wildly" oscillating phase ($p > 1$). Energy diverges $\langle H\rangle \sim e^{-\Phi}\langle V\rangle$.

Thus if $g_{YM}^2 = e^\Phi \to 0$ strictly, gauge theory response singular.
For cutoff e^Φ, large energy production due to time-dep source.

AdS cosmologies with spacelike singularities

Recall: $ds^2 = \frac{1}{z^2}(\tilde{g}_{\mu\nu}dx^\mu dx^\nu + dz^2) + ds^2_{S^5}\ , \ \ \Phi = \Phi(x^\mu)\ .$

Solution if: $\tilde{R}_{\mu\nu} = \frac{1}{2}\partial_\mu\Phi\partial_\nu\Phi\ , \quad \frac{1}{\sqrt{-\tilde{g}}}\,\partial_\mu(\sqrt{-\tilde{g}}\ \tilde{g}^{\mu\nu}\partial_\nu\Phi) = 0\ .$

Solutions with spacelike Big-Bang (Crunch) singularities:

- $ds^2 = \frac{1}{z^2}\left[dz^2 - dt^2 + \sum_{i=1}^{3} t^{2p_i}(dx^i)^2\right]\ ,$
  $e^\Phi = |t|^{\sqrt{2(1-\sum_i p_i^2)}}, \qquad \sum_i p_i = 1\ .$ [Kasner cosmologies]

- $ds^2 = \frac{1}{z^2}\left[dz^2 + |\sinh(2t)|(-dt^2 + \frac{dr^2}{1+r^2} + r^2(d\theta^2 + sin^2\theta d\phi^2))\right],$
  $e^\Phi = g_s\ |\tanh t|^{\sqrt{3}}\ .$ [$k = -1$ (hyperbolic) FRW boundary]

Dilaton bounded, asymptotic spacetime $AdS_5 \times S^5$ (using coord transf).

- $ds^2 = \frac{1}{z^2}\left[dz^2 - dt^2 + \eta_{ab}(t)(e^a_\alpha dx^\alpha)(e^b_\beta dx^\beta)\right]\ , \quad e^\Phi = e^{\Phi(t)}\ .$

AdS-BKL cosmologies: spatial metric is a homogenous space in Bianchi classification.

Singularities, gauge theory

Families of AdS-cosmologies with similar leading near singularity behaviour: essentially dilaton-driven, symmetric AdS-Kasner ($p_i = \frac{1}{3}$). With flat bndry metric: gauge coupling $g_{YM}^2 = e^\Phi = |t|^{\sqrt{3}}$ as $t \to 0$. $p = \sqrt{3} > 1 \Rightarrow$ wave-fnal phase "wildly" oscillating, ill-defined (from earlier). Energy divergent if $g_{YM}^2 = e^\Phi \to 0$ strictly as $t \to 0$.

In gauge theory, deform gauge coupling so that $g_{YM}^2 = e^\Phi$ small but nonzero near $t = 0$. Now finite but large phase oscillation and energy production. $\dot{\Phi} \sim \frac{\dot{g}_{YM}}{g_{YM}}$ finite so bulk also nonsingular (but stringy). Eventual gauge theory endpoint depends on details of energy production. On long timescales, expect that gauge theory thermalizes: then reasonable to imagine that late-time bulk is AdS-Schwarzschild black hole.

See also arXiv:0906.3275, Awad, Das, Ghosh, Oh, Trivedi: slowly varying dilaton cosmologies and their gauge theory duals.

Null singularities, gauge theory

$$g_{YM}^2 = e^{\Phi(x^+)}, \;\; ds^2 = \frac{1}{z^2}[e^{f(x^+)}dx_\mu dx^\mu + dz^2] + ds_{S^5}^2 \;.$$

There exist gauge theory variables where the interaction terms unimportant near $e^\Phi \to 0$. Near singularity lightcone Schrodinger wavefunctional resembles that for weakly coupled Yang-Mills theory at location in null time ($x^+ = 0$) of bulk singularity [e.g. $e^\Phi \sim g_s(-x^+)^p$]. These variables appear to be dual to stringy objects in bulk.

This suggests that while classical bulk sugra variables are bad, lightcone Hamiltonian time evolution of the gauge theory is sensible.

Renormalization effects: introduce “short-time” (momentum) cutoff. Sufficiently high frequency modes in gauge theory (relative to $\dot{\Phi}$) might give nontrivial contributions to gauge theory effective action/Hamiltonian, so previous arguments might be modified.

Null singularities and strings

Expectation: stringy effects (beyond GR) are important.
AdS string technically difficult. Possible to construct simpler toy models with no fluxes or dilaton, where the singularity is $purely$ $gravitational$ so more tractable by string worldsheet methods.

Consider $ds^2 = e^{h_0(x^+)}\left(-2dx^+dx^- + (dx^i)^2\right) + e^{h_m(x^+)}(dx^m)^2,$
with $i = 1, 2,\ m = 3, \ldots, D-2$.
Simple classes of null Kasner-like cosmological singularities at $x^+ = 0$ arise as $e^{h_I} \to (x^+)^{p_I}$.

Einstein equations $\Rightarrow$ algebraic relations between Kasner exponents.

[Integer-valued exponents exist, but restrictive: for $D = 10$ (critical superstring), $(p_0, p_1) = (0, 2), (12, -2), (12, 28), (180, -28), (180, 390), \ldots$]

Can recast these as $ds^2 = -2dy^+dx^- + (y^+)^{A_I}(dx^I)^2.$

Null Kasner singularities, plane waves

Null Kasner: $ds^2 = -2dy^+dx^- + (y^+)^{A_I}(dx^I)^2.$ [Rosen]

By coord transf $y^I = (y^+)^{A_I/2}x^I,\ y^- = x^- + (\frac{\sum_I A_I(y^I)^2}{4y^+})$,

these can be recast as anisotropic plane waves with singularities

$$ds^2 = -2dy^+dy^- - \sum_I \chi_I(y^I)^2\frac{(dy^+)^2}{(y^+)^2} + (dy^I)^2, \quad \text{[Brinkman]}$$

with $A_I = 1 \pm \sqrt{1-4\chi_I}$.

Einstein eqns: $R_{++} = \frac{1}{(y^+)^2}\sum_I \chi_I = \frac{1}{(y^+)^2}\sum_I \frac{A_I(2-A_I)}{4} = 0.$

All χ_I equal: solutions only with other matter fields.

Curvature invariants finite in these null backgrounds.

Diverging tidal forces: from deviation of null geodesic congruences.

No nonzero covariant contraction $\Rightarrow$ no α' stringy corrections.

Various previous investigations of singular plane waves.

We will mostly focus here on Rosen (null Kasner cosmology) form.

Free string worldsheet theory

Closed string action $S = -\int \frac{d\tau d\sigma}{4\pi\alpha'}\sqrt{-h}h^{ab}\,\partial_a X^\mu \partial_b X^\nu g_{\mu\nu}(X)$.

[Lightcone gauge $y^+ = \tau$. Set $h_{\tau\sigma} = 0$, with $E(\tau,\sigma) = \sqrt{-\frac{h_{\sigma\sigma}}{h_{\tau\tau}}}$:

Rosen action $S_R = -\frac{1}{4\pi\alpha'}\int d^2\sigma\,(-2Eg_{+-}\partial_\tau X^- - Eg_{II}(\partial_\tau x^I)^2 + \frac{1}{E}\,g_{II}(\partial_\sigma x^I)^2)$.

Lightcone momentum conjugate to x^- $p_- = \frac{Eg_{+-}}{2\pi\alpha' l} = -\frac{1}{2\pi\alpha' l} = const$ gives $E = -\frac{1}{g_{+-}}$.]

Lightcone gauge Hamiltonian $H = -p_+$, satisfying physical state condition $(m^2 = -2g^{+-}p_+p_- - g^{II}(p_{I0})^2)$: $[l = -2\pi p_-\alpha',\ p_- < 0]$

$$H_R = \frac{1}{4\pi\alpha'}\int_0^l d\sigma\,\Big((2\pi\alpha')^2\frac{(\Pi^I)^2}{\tau^{A_I}} + \tau^{A_I}(\partial_\sigma x^I)^2\Big). \quad \text{[Rosen]}$$

containing only physical transverse string degrees of freedom x^I.
Quadratic worldsheet theory, external time-dependent coefficients.

By comparison, Brinkman Hamiltonian is

$$H_B = \frac{1}{4\pi\alpha'}\int d\sigma\,\Big((2\pi\alpha')^2(\Pi^I_y)^2 + (\partial_\sigma y^I)^2 + \sum_I \frac{\chi_I}{\tau^2}(y^I)^2\Big)\ .$$

Lightcone string wavefunctional

Want to understand near singularity time-dependence of lightcone string Schrodinger wavefunctional.

Schrodinger eqn: $\quad i\partial_{y^+}\Psi = i\partial_\tau \Psi[x^I,\tau] = H[x^I,\tau]\Psi[x^I,\tau].$

$[H_R = \frac{1}{4\pi\alpha'}\int_0^l d\sigma\ \left((2\pi\alpha')^2 \frac{(\Pi^I)^2}{\tau^{A_I}} + \tau^{A_I}(\partial_\sigma x^I)^2\right), \qquad \Pi^I[\sigma] = -i\frac{\delta}{\delta x^I[\sigma]}.]$

$A_I > 0$: as $\tau \to 0$, kinetic terms dominate,

$$i\partial_\tau \Psi[x^I,\tau] \sim\ -\pi\alpha'\tau^{-A_I}\int d\sigma\ \frac{\delta^2}{\delta x^{I\,2}}\ \Psi[x^I,\tau]$$

giving $\quad \Psi[x^I,\tau] \sim\ e^{i\pi\alpha' \frac{\tau^{1-A_I}}{1-A_I}\int d\sigma\ \frac{\delta^2}{\delta x^{I\,2}}}\ \Psi[x^I].$

Wavefunctional has nonsingular time-dep if $A_I \le 1$.

[Alternatively, can recast this as flat space free Schrodinger eqn in terms of τ^{1-A_I}.]

$A_I < 0$: $\quad \Psi[x^I,\tau] \sim\ e^{-i\frac{\tau^{1-|A_I|}}{4\pi\alpha'(1-|A_I|)}\int d\sigma\ (\partial_\sigma x^I)^2}\ \Psi[x^I]$, well-defined if $|A_I| \le 1$.

[Brinkman] $\quad \Psi[y^I,\tau] \sim\ e^{-\frac{i}{\tau}\sum_I \chi_I (y^I)^2}\ \Psi[y^I], \quad$ ill-defined.

Singular plane waves, Rosen patches

Thus Rosen frames with $|A_I| \leq 1$ encode string wavefunctional having nonsingular time-dependence near singularity. Multiple such Rosen-Kasner exponents: does such a Rosen-Kasner frame exist with $|A_I| \leq 1$ for each x^I-dirn, consistent with eqns of motion?

Simplest case: two Kasner exponents A_1, A_2.

$$[ds^2 = -2dy^+dx^- + \tau^{A_1}(dx_2^2 + dx_3^2) + \tau^{A_2}(dx_4^2 + \ldots + dx_{D-2}^2)$$
$$2\chi_1 + (D-4)\chi_2 = 2A_1(2-A_1) + (D-4)A_2(2-A_2) = 0.]$$

$A_I = 1 \pm \sqrt{1-4\chi_I}$ $\Rightarrow$ four Rosen frames/patches.

Can show that $\quad 0 < \chi_1 \leq \frac{1}{4}, \quad 0 < A_1 = 1 - \sqrt{1-4\chi_1} \leq 1\,,$

$$-\frac{3}{4} \leq -\frac{1}{2(D-4)} \leq \chi_2 < 0, \quad -1 \leq A_2 = 1 - \sqrt{1-4\chi_2} < 0,$$

gives a Rosen patch (A_1^-, A_2^-) with $0 < |A_1|, |A_2| \leq 1$.

With more Kasner exponents ($i.e.$ more anisotropy), space of possibilities increases.

Detailed string quantization

Classical string modes can be exactly solved for from worldsheet EOM: Free string quantization can be carried out in great detail.

$$f^I_{Rn}(\tau) = \sqrt{n}(\tfrac{\tau}{l})^{\frac{1-A_I}{2}} (c^I_{n1} J_{\frac{1-A_I}{2}}(\tfrac{n\tau}{l}) + c^I_{n2} Y_{\frac{1-A_I}{2}}(\tfrac{n\tau}{l}))$$

$$\Big[x^I(\tau,\sigma) = x^I_0(\tau) + \sum_{n=1}^{\infty} \big(k^I_n f^I_{Rn}(\tau)(a^I_n e^{in\sigma/l} + \tilde{a}^I_n e^{-in\sigma/l}) + h.c.\big)\Big]$$

String spectrum, oscillator masses can be calculated explicitly.

Toy model: time-dep harmonic oscillator arising as 1-dim single momentum mode of string. Allows explicit calculations of wavefunctions, observables.

Observables without time derivatives can be shown to be identical between Rosen/Brinkman variables.

Observables with time derivatives are different: both diverge, but Rosen ones are milder for singularities with $|A_I| \leq 1$.

[Momentum exp. values: $\langle \frac{1}{l} \int d\sigma\, g^{II}(\Pi^I)^2 \rangle \sim \sum_n |k^I_n|^2 g_{II} |\dot{f}^I_n|^2 (\{a^I_n, a^I_{-n}\} + h.c.)$ for single excitation states. Since $\frac{(\Pi^I_x)^2}{\tau^{A_I}} = \left(\Pi^I_y - \frac{A_I}{2\tau} y^I\right)^2$, these are different.

$\langle \frac{1}{l} \int d\sigma\, \tau^{-A_I} (\Pi^I_x)^2 \rangle \sim \tau^{-A_I}$ [Rosen], $\quad \langle \frac{1}{l} \int d\sigma\, (\Pi^I_y)^2 \rangle \sim \tau^{A_I - 2}$ [Brinkman].]

Mode asymptotics, oscillator masses

$[f^I_{Rn}(\tau) = \sqrt{n}(\frac{\tau}{l})^{\frac{1-A_I}{2}}(c^I_{n1}J_{\frac{1-A_I}{2}}(\frac{n\tau}{l}) + c^I_{n2}Y_{\frac{1-A_I}{2}}(\frac{n\tau}{l}))]$

Cutoff const-(null)time surface $y^+ \equiv \tau = \tau_c = y^+_c$:

local energy density (curvature) $\sim \frac{1}{(y^+_c)^2}$.

Low-lying (small n): $f^I_{Rn} \sim \lambda^I_{2n} + \lambda^I_{1n}(\frac{\tau_c}{l})^{1-A_I}, \qquad n \ll \frac{l}{\tau_c}$.

Highly stringy (large n): $f^I_{Rn} \sim \frac{e^{-in\tau_c/l}}{(\tau_c/l)^{A_I/2}}, \quad n\tau_c \gg l.$

These ultra-high oscillation number modes exist for any infinitesimal regularization of near-singularity region.

[Masses $m^2 = \sum_n \frac{p_- l}{2\alpha'}|k^I_{Rn}|^2\tau^{A_I}\left((\{a^I_n, a^I_{-n}\} + \{\tilde{a}^I_n, \tilde{a}^I_{-n}\})\left(|\dot{f}^I_{Rn}|^2 + \frac{n^2}{l^2}|f^I_{Rn}|^2\right)\right.$

$\left. - \{a^I_n, \tilde{a}^I_n\}\left((\dot{f}^I_{Rn})^2 + \frac{n^2}{l^2}(f^I_{Rn})^2\right) + h.c.\right), \quad k^I_{Rn} = \frac{i}{n}\sqrt{\frac{\pi\alpha' l^{-A_I}}{2|c^I_{n0}|}}$.

Oscillator algebra: $[a^I_n, a^J_{-m}] = [\tilde{a}^I_n, \tilde{a}^J_{-m}] = n\delta^{IJ}\delta_{nm}$]

Single string states light near singularity: using mode asymptotics,

low-lying: $m^2 \sim \frac{l^{A_I}}{\alpha' \tau_c^{A_I}} \ll \frac{1}{\tau_c^2}$, highly stringy: $m^2 \sim \frac{n}{\alpha'} \ll \frac{1}{\tau_c^2}$.

[This requires $\frac{p_-\alpha'}{(y^+_c)} \ll n \ll \frac{\alpha'}{(y^+_c)^2}$.]

Oscillator masses

Highly stringy oscillators light if $\frac{p_- \alpha'}{(y_c^+)} \ll n \ll \frac{\alpha'}{(y_c^+)^2}$.

Implicitly implies $p_- \ll \frac{1}{y_c^+}$.

Estimate for number of such oscillator levels excited: $\frac{\alpha'}{(y_c^+)^2}(1 - p_- y_c^+)$.

On a Planck scale cutoff surface, highest oscillator level turned on is of order $n \sim (\frac{l_s}{l_p})^2 \sim \frac{1}{g_s^{2/(D-2)}}$ [using naive relation for Newton const $G_D = l_P^{D-2} = g_s^2 l_s^{D-2}$]. In free string limit $g_s \to 0$, large number $n \gg 1$ of highly stringy oscillator states.

As $y_c^+ \to 0$, all oscillators light, excited.

Large proliferation of light string states in near singularity region.

String is highly excited in the vicinity of the singularity.

Conclusions, questions

Null: Free string wavefunctional has nonsingular time-dep for certain singularities. However, proliferation of light string oscillator states near singularity. Possibly large backreaction due to highly excited strings. String interactions, 2nd quantized (string field theory) framework? Dual to renormalization effects (for corresponding AdS cosmologies)?

Spacelike: If gauge coupling $g_{YM}^2(t) \to 0$ strictly, then gauge theory response singular: energy diverges. Deform g_{YM}^2 to be small but nonzero near $t = 0$. Now finite but large phase oscillation and energy production. $\dot{\Phi} \sim \frac{\dot{g}_{YM}}{g_{YM}}$ finite now, so bulk also nonsingular. Likely string oscillators highly excited.

Continuing past singularity, eventual endpoints ?

Futuristic: hints of very early universe?

. . .