CMB Bispectrum from Magnetic Fields - Passive Mode

(Non-Gaussianity from Primordial Cosmic Magnetic Fields)

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Overview

- Found a $10^6\times$ stronger CMB bispectrum sourced by cosmic primordial magnetic fields (than earlier calculated)
- Order-of-magnitude better limits on pimordial $\vec{B}$ from CMB non-Guassianity
- The magnetic f_{NL} is appreciable & interesting, given current constraints

Why is **CMB Non-Gaussianity** from Cosmic **Magnetic** Fields Important?

NG from Inflationary models:

Small fluctuations in the field (linear order)

$\downarrow$

Gaussian statistics for Fluctuation

$\downarrow$

Gaussian statistics for CMB Temperature Anisotropy at lowest order

Primordial CMB non-Gaussianity **only** from **higher order** effects

NG from Magnetic Fields:

Magnetic energy densities & stresses inherently quadratic in $\vec{B}$ field: $\rho_B, \Pi_B \propto |\vec{B}|^2$

$\downarrow$

Even for a purely Gaussian $\vec{B}$ field, magnetic stresses non-Gaussian

$\downarrow$

Non-Gaussianity in $\vec{B}$ field induced CMB anisotropy

Magnetic CMB non-Gaussianity from $\vec{B}$ **even** at **lowest order**

Primordial Cosmic Magnetic Field - Motivation

- Magnetic fields are ubiquitous out to large scales > 10 kpc

  μGauss $\vec{B}$ observed in galaxies: both coherent & stochastic

  $\vec{B}$ growth via either dynamo amplification or flux freezing
  $\longrightarrow$ a seed $\vec{B}$ field is required

  These seed fields may be of primordial origin

- Evidence for equally strong $\vec{B}$ in high redshift ($z \sim 2$) galaxies

  [Bernet et al. 08, Kronberg et al. 08]

  Enough time for dynamo to act?

- Recent FERMI/*LAT* observations of γ-ray halos around AGN

  Detection of intergalactic $\vec{B} \approx 10^{-15} G$ [Ando & Kusenko 10]

  Lower limit: $\vec{B} \geq 10^{-16}$ G on intergalactic $\vec{B}$ [Neronov & Vovk, *Science* 10]

No compelling mechanism yet for origin of strong primordial $\vec{B}$ fields

[e.g. Martin & Yokoyama 08]

Properties of Assumed Cosmic Magnetic Field

Homogeneous cosmic $\vec{B}$ fields: v strict limits from CMB quadrupole, anisotropic homogenous model.

- Magnetic Field: Stochastic. Statistically homogeneous and isotropic.
- Assumed to be a Gaussian Random Field. Statistical properties of $\vec{B}$ specified completely by 2-point correlation function.
- Magnetic field $\longrightarrow$ velocity field
  On scales $> L_G$ (galactic scales): velocities small enough that the magnetic fields do not change.

$$\vec{B}(\vec{x}, t) = \frac{\vec{b}_0(\vec{x})}{a^2(t)}$$

Statistical & Spectra of the Magnetic Field

Field: Non helical, Gaussian and spectrum specified by
$\langle b_i(\vec{k}) b_j^*(\vec{q}) \rangle = (2\pi)^3 \delta(\vec{k} - \vec{q}) P_{ij}(\vec{k}) M(k)$
$\rightarrow$ Completely determined by $M(k)$

$P_{ij}(\vec{k}) = (\delta_{ij} - k_i k_j / k^2)$ is the projection operator that ensures $\vec{\nabla} \cdot \vec{b}_0 = 0$

$\langle \vec{b}_0 \cdot \vec{b}_0 \rangle = 2 \int \frac{dk}{k} \Delta_b^2(k)$ with $\Delta_b^2 = k^3 M(k) / 2\pi^2$

Form of $M(k)$**:**
$M(k) = Ak^n$ with a cutoff at
Alfven wave damping scale

Fixing A: In terms of variance, B_0,
of Magnetic Field at $k_G = 1 h\text{Mpc}^{-1}$

$$\Rightarrow \Delta_b^2(k) = \frac{B_0^2}{2} (n+3) \left(\frac{k}{k_g} \right)^{n+3}$$

Magnetic CMB Bispectrum - Energy Density (Compensated Mode)

[T. R. Seshadri & K. Subramanian, PRL 09]

- First calculation of magnetic CMB bispectrum
- $\frac{\Delta T}{T}$ sourced by magnetic energy density

  $\vec{\Omega}_B(\vec{k}) = \frac{1}{(2\pi)^3} \int d^3q \;\; b_i(\vec{k} - \vec{q}) b_i^*(\vec{q})/(8\pi\rho_0)$

  $\frac{\Delta T(\hat{n})}{T} \sim 0.03 \; \Omega_B(\vec{x}_0 - \hat{n}D^*)$

- Reduced bispectrum $l_1(l_1 + 1)l_3(l_3 + 1)b_{l_1 l_2 l_3} \sim 10^{-22}$ for $B_0 \sim$ 3nG and scale-invariant magnetic field spectrum.
- This is a new type of non-Gaussianity in the CMB
- This is a new probe of primordial magnetic fields.
- $\vec{B}$ bispectrum + WMAP5 f_{NL} $\rightarrow$upper limits on $B_0 \sim$ 35nG. Expected to improve significantly when vector and tensor modes also to be included.

[also Caprini et al. 09, Cai et al 10, Brown 10]

Now Consider Scalar Anisotropic Stress from $\vec{B}$

- Magnetic stress tensor

$$T^i_j(\mathbf{x}) = \frac{1}{4\pi a^4}\left(\frac{1}{2}b_0^2(\mathbf{x})\delta^i_j - b_0^i(\mathbf{x})b_{0j}(\mathbf{x})\right)$$

- in Fourier space

$$S^i_j(\mathbf{k}) = \frac{1}{(2\pi)^3}\int b^i(\mathbf{q})b_j(\mathbf{k}-\mathbf{q})d^3\mathbf{q}$$

$$T^i_j(\mathbf{k}) = \frac{1}{4\pi a^4}\left(\frac{1}{2}S^\alpha_\alpha(\mathbf{k})\delta^i_j - S^i_j(\mathbf{k})\right).$$

- Magnetic perturbations to $T^i_j(\mathbf{k})$

$$T^i_j(\mathbf{k}) = p_\gamma\left(\Delta_B(\mathbf{k})\delta^i_j + \Pi_{B}{}^i_j(\mathbf{k})\right)$$

Scalar Anisotropic Stress $\rightarrow$ Passive Mode

- Assume $\vec{B}$ stresses small compared to total ρ, Π of photons + baryons

  linear perturbations
  scalar, vector, tensor evolve independently
  we focus on the scalar part of $\Pi_{B}{}^{i}_{j}$
  as a source of CMB non-Gaussianity

- Scalar Anisotropic perturbations $\Pi_B(\mathbf{k})$ given by projection operator

$$\Pi_B(\boldsymbol{k}) = -\frac{3}{2}\left(\hat{k}_i\hat{k}_j - \frac{1}{3}\delta_{ij}\right)\Pi_B^{ij}$$

- Neutrinos: also develop scalar anisotropic stress after decoupling
- Prior to neutrino decoupling, $\Pi_B(\mathbf{k})$ only source
- After neutrino decoupling, $\Pi_\nu(\mathbf{k})$ also contributes with equal magnitude and opposite sign: rapid compensation [Lewis 04]

Magnetic anisotropic stress $\Pi_B(\mathbf{k})$ has effect only till neutrino decoupling

Passive Mode Curvature Perturbation

- After neutrino decoupling there are two types of scalar perturbation modes
- Compensated mode (studied earlier)
- **Passive mode** [J. R. Shaw & A. Lewis, PRD 10]

$$\zeta = \zeta(\tau_B) - \frac{1}{3} R_\gamma \Pi_B \left[\ln\left(\frac{\tau_\nu}{\tau_B}\right) + \left(\frac{5}{8R_\nu} - 1\right)\right].$$

  # grown logarithmically from $\vec{B}$ generation at τ_B to ν-decoupling at τ_ν

  # adiabatic-like passive evolution after ν-decoupling

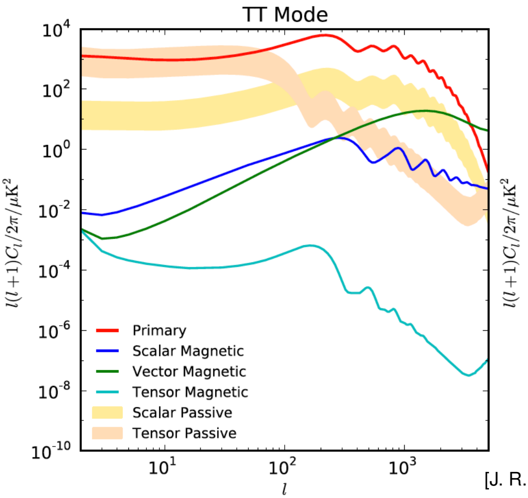
  # non-Gaussian statistics (unlike primordial adiabatic perturbations)

- For range τ_B corresponding to temperature range from $T_B \approx 10^{14}$ GeV (inflationary) to $T_B \approx 10^3$ GeV (electroweak)

$$\ln\left(\frac{\tau_\nu}{\tau_B}\right) > 10$$

$$\boxed{\zeta \simeq -\frac{1}{3} R_\gamma \Pi_B \ln\left(\frac{\tau_\nu}{\tau_B}\right)}$$

Passive Mode Power Spectra



[J. R. Shaw & A. Lewis, PRD 10]

CMB Bispectrum **Results** for Magnetic Passive Mode

- $l_1(l_1+1)l_3(l_3+1)b_{l_1 l_2 l_3} \approx 10^{-16}$ or $\times 10^6$ stronger than compensated mode

  n_B = -2.8, 3 nG field, $\tau_B \approx 10^{14}$ GeV

- Squeezed Collinear full evaluation:
  $l_1(l_1+1)l_3(l_3+1)b_{l_1 l_2 l_3} \approx -1.4 \times 10^{-16}$

  using WMAP7 $-10 < f_{NL}$ get upper limit $B_0 < 2nG$

- General configuration approximate evaluation:
  $l_1(l_1+1)l_3(l_3+1)b_{l_1 l_2 l_3} \approx 6 - 9 \times 10^{-16}$

  using WMAP7 $f_{NL} < 74$ get upper limit $B_0 < 3nG$

- Inflationary bispectrum with $f_{NL} \sim 1$ is $l_1(l_1+1)l_3(l_3+1)b_{l_1 l_2 l_3} \approx 10^{-18}$
- CAVEAT: only Sachs-Wolfe
- CAVEAT: τ_B dependence: But little change $B_0 < 2 - 4nG$

Conclusions

- Cosmological magnetic fields an interesting possibility: CMB NG unique probe
- First bispectrum calculation of $\vec{B}$ CMB magnetic anisotropy (for $B_0 \sim 3$ nG) > primordial bispectrum ($f_{NL} \sim 1$) [greater by $\times 100$]
- 10 times stronger B_0 limit of 2 nG from bispectrum
- The magnetic f_{NL}^B is at an interesting level right now $f_{NL}^B \sim 20 \left(\frac{B_{-9}}{2}\right)$

  caveat - $f_{NL}^B \propto B^6$

  but NG effects calculated are getting stronger even as B_0 upper limit falling