Optimal Binning of the Primordial Power Spectrum

Paniez Paykari

CEA - Saclay

Paykari & Jaffe - APJ 2009

PFNG - India

Primordial Power Spectrum

- Primordial PS → physics of the structure formation in the early Universe
- Our path to its measurements is through the different surveys such as CMB and LSS surveys
- Surveys outcome → a PS that is a **convolution** of the primordial PS and a transfer function (which holds the cosmological parameters) → induces degeneracy between them
- We want to quantify this degeneracy

CMB & Matter Power Spectra

Primordial PS

$$\Delta^2_\zeta(k) \simeq A k^{n_s - 1}$$

Matter PS

$$P_\delta(k) = T^2(k)\Delta^2_\zeta(k)$$

Measurable quantity → galaxy PS $P_g(k) = b^2(k) P_\delta(k)$

CMB PS

Radiation TF

$$C_\ell = 2\pi \int_0^\infty d\ln k\, \Delta^2_\zeta(k) \Delta^2_\ell(k)$$

Combine all Available Information

- Above PS cannot provide enough constraints on parameters on their own - inherent degeneracies
- Use them all → they depend on parameters in different ways → can constrain parameters in different directions
- One interesting quantity to measure is the primordial PS - measured in bins

Data Analysis

- Bayesian Statistics; takes into account prior knowledge

Posterior distribution

Likelihood function

$$P[H|DI] = \frac{P[D|HI]P[H|I]}{P[D|I]}$$

Prior probability

Uniform → assume constant

Evidence

Ignored

(independent of parameters)

D=data
H=Hypothesis
I=Background

- A proportionality relation

$$P[H|DI] \propto P[D|HI] = \mathcal{L}$$

Optimal Quadratic Estimator

- Likelihood function is a
  - Multivariate Gaussian (around the peak)
  - Maximum in the parameter space where the parameters have the best estimated values
  - $\ln \mathcal{L}$ is maximised at the same place in the parameter space as $\mathcal{L}$

$$\frac{\partial \ln \mathcal{L}}{\partial \lambda} \Big|_{\lambda=\bar{\lambda}} = 0$$

- Taylor Expansion around the peak

$$F = \langle \mathcal{F} \rangle = \left\langle -\frac{\partial^2 \ln \mathcal{L}}{\partial \lambda^2} \right\rangle$$

Fisher Matrix
Covariance Matrix

- Cramer-Rao equality → smallest error for a parameter is
  - Marginalised $\sqrt{(F^{-1})_{\alpha\alpha}}$
  - Non-marginalised $1/\sqrt{F_{\alpha\alpha}}$
- Fisher matrix
  - → generally used to determine the sensitivity of a particular survey to a set of parameters
  - → suitable for forecasting and optimizing future experiments
- Obtain a Fisher matrix for CMB and galaxy surveys → add them to take both into account

CMB Fisher Matrix

Fisher matrix for

$$F_{\ell\ell'} = f_{sky}\frac{2\ell+1}{2}\delta_{\ell\ell'}[C_\ell + w^{-1}e^{\ell^2\sigma^2}]^{-2}$$

Window Fn

Weight function

$$w = (\sigma^2_{pix}\Omega_{pix})^{-1}$$

Fisher matrix for other parameters

$$F_{\alpha\beta} = \sum_\ell \frac{1}{(\delta C_\ell)^2}\frac{\partial C_\ell}{\partial\lambda_\alpha}\frac{\partial C_\ell}{\partial\lambda_\beta}$$

For primordial power spectrum we have

$$\frac{\partial \mathcal{C}_\ell}{\partial \Delta^2_\zeta(k)} = 2\ell(\ell+1)\int_{k^B_{min}}^{k^B_{max}} dk\,|\Delta_\ell(k)|^2$$

Galaxy Surveys Fisher Matrix

Fisher Matrix for galaxy PS

$$F_{\alpha\beta} = \delta_{\alpha\beta} \frac{k_\alpha^2 \Delta k V}{(2\pi)^2 (P_\alpha + \frac{1}{\bar{n}})^2}$$

Volume of the survey

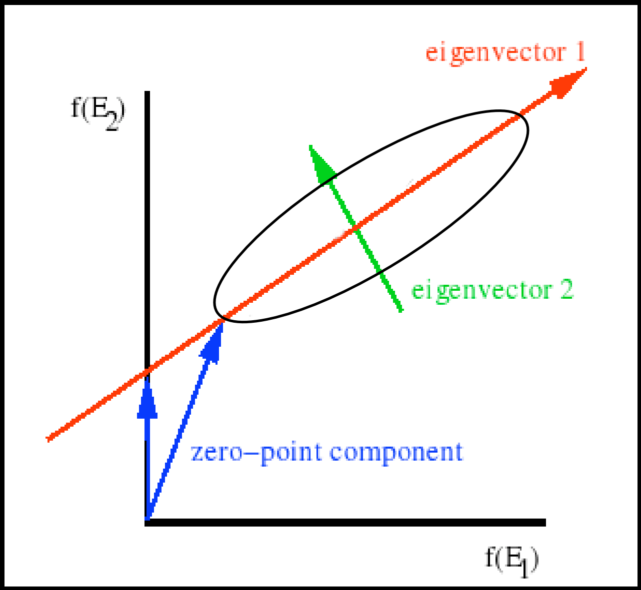
Selection fn of the survey

For primordial PS

$$\frac{\partial P_g(k)}{\partial \Delta_\zeta^2(k')} = 4 \times 2\pi^2 k T^2(k) \delta_{kk'}$$

Principal Component Analysis

- Inverse of FM → Covariance matrix
  - → hard to interpret errors
  - → construct a new set of **uncorrelated** parameters which are orthogonal linear combination of original parameters



- A new set of variables X_i → principal component of the experiment

$$C = E^T \Lambda E \qquad X = EO$$

Hermitian Square Root

- Another approach to remove correlation between parameters → Hermitian sqrt of the FM → linear transformation on the parameter space
- Does not give orthogonal basis

  → think of it as a window function

$$\underline{F}^{1/2} = \underline{E}^T \underline{\Lambda}^{1/2} \underline{E}$$

$$(\underline{F}^{-1/2})^T \underline{F} (\underline{F}^{-1/2}) = diag$$

$$H_{nm} = \frac{(F^{1/2})_{nm}}{\sum_n (F^{1/2})_{nm}}$$

$$\sum_n H_{nm} = 1$$

Define a **windowed PS**

$$\tilde{P}_m = \sum_n H_{nm} P(k_n)$$

Model of the Universe
(Parameter Space)

- A geometrically flat, adiabatic, LCDM - no dark energy
- parameter space
  - Primordial PS bins
  - Cosmological parameters

$$\Delta^2_\zeta(k) = \sum_B w_B(k) Q_B, \quad w_B = \begin{cases} 1 & k \in B \\ 0 & k \notin B \end{cases}$$

- **SDSS(BRG)**

$$V = (1\mathrm{h}^{-1}\mathrm{Gpc})^3 \;\&\; \overline{n} = \frac{10^5}{V}$$

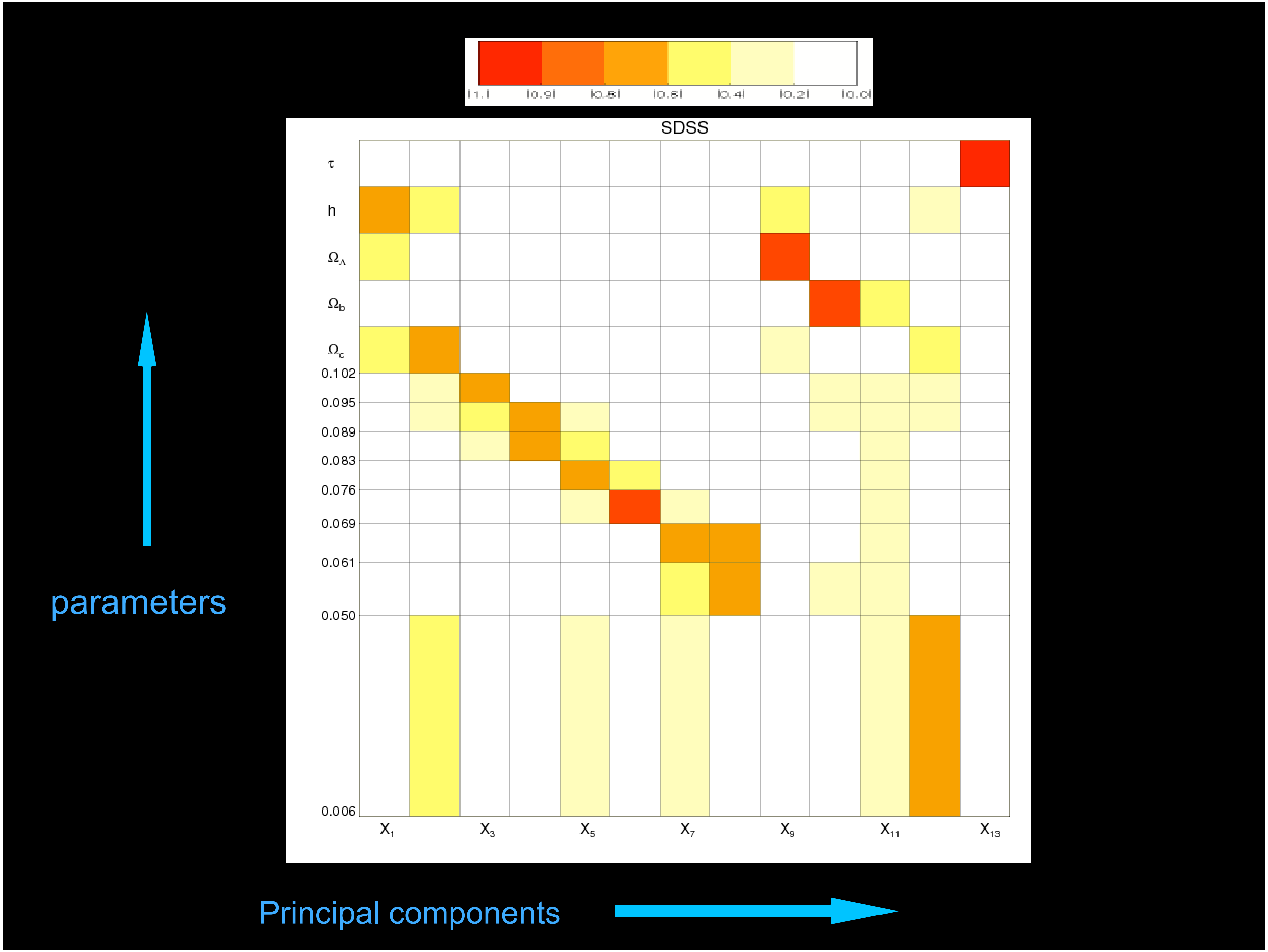
- **Planck(HFI)**

$$\nu = 100\mathrm{GHz},\ \theta_{fwhm} = 10.7',\ \sigma_{pix} = 1.7\times10^{-15},$$
$$w^{-1} = 0.028\times10^{-15},\ f_{sky} = 0.5$$

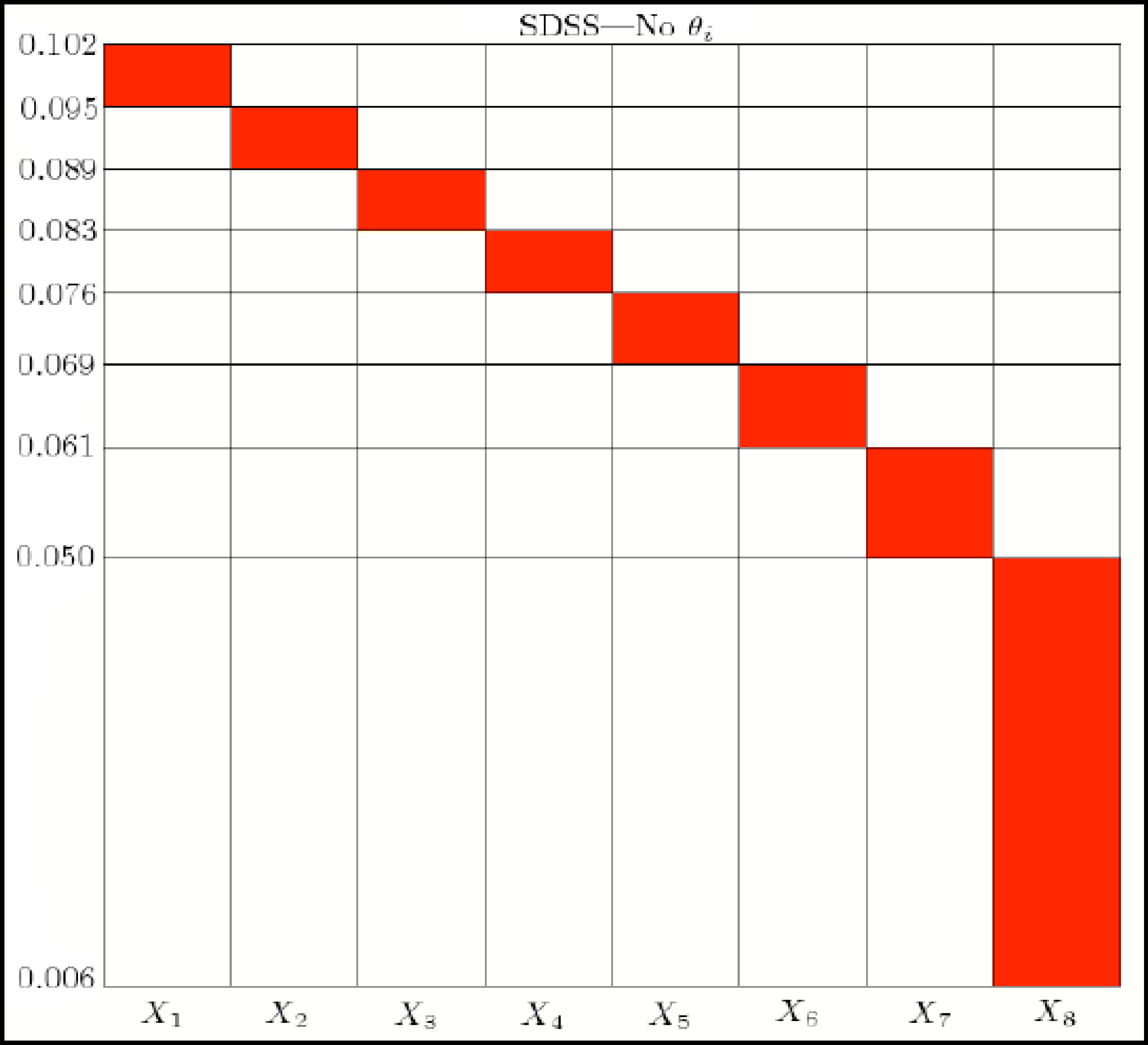
Optimal Binning

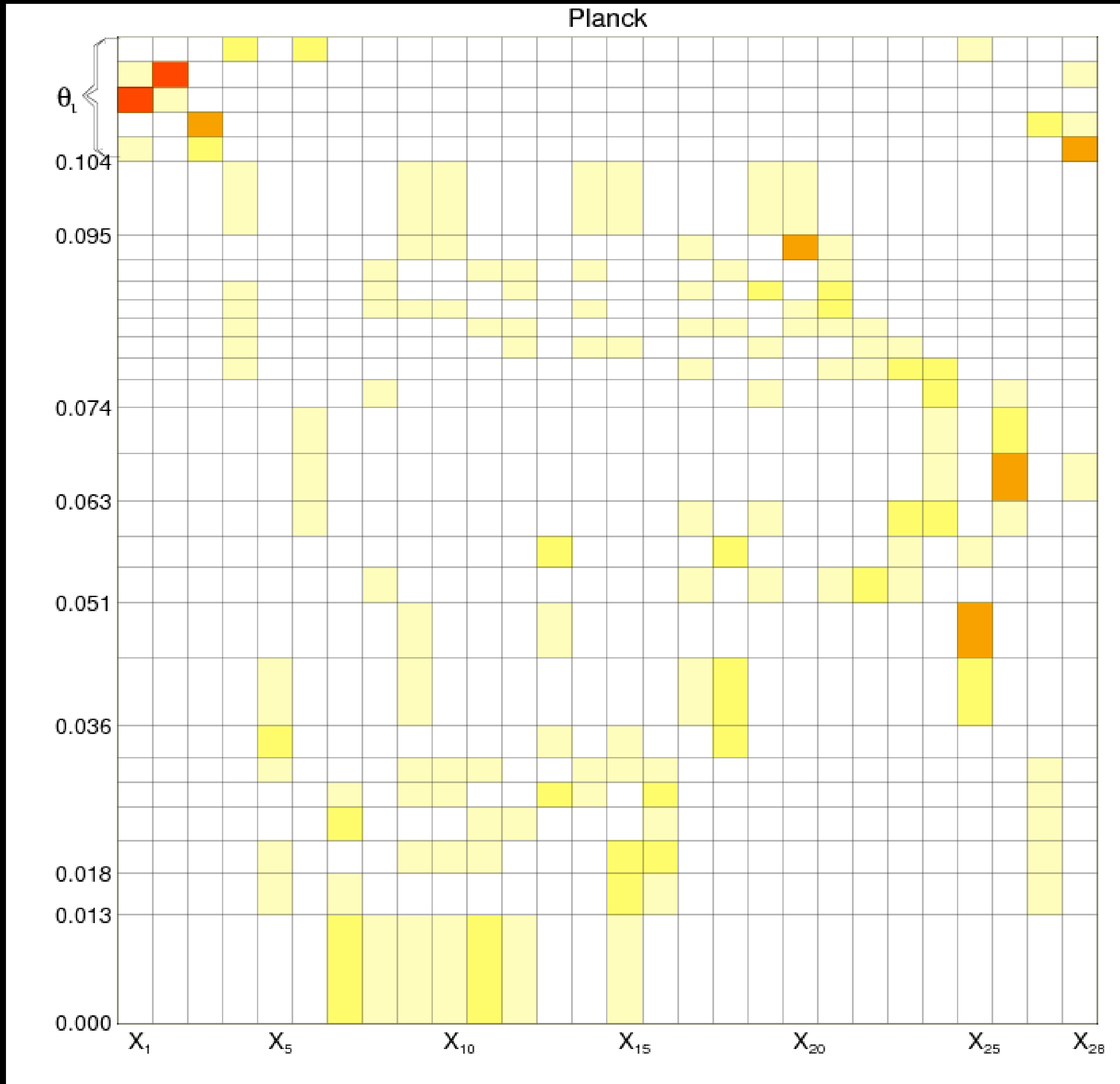
- Bins with same contribution to the Fisher matrix → same S/N
  - Signal → amplitude of the primordial PS in the bin
  - Noise → sqrt of diagonal elements of Fisher matrix for that bin

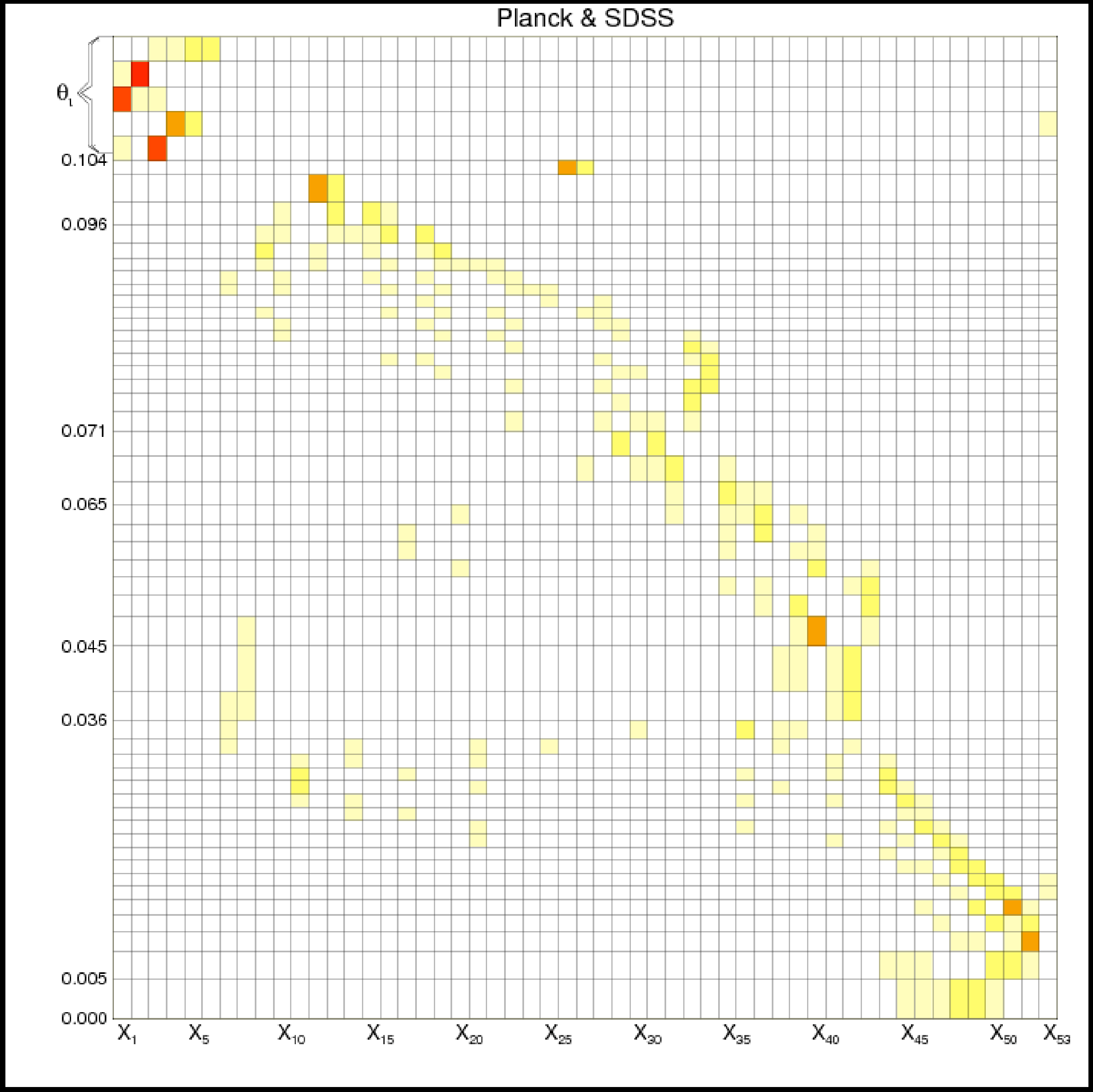
- SDSS → 8 bins
- Planck → 23 bins



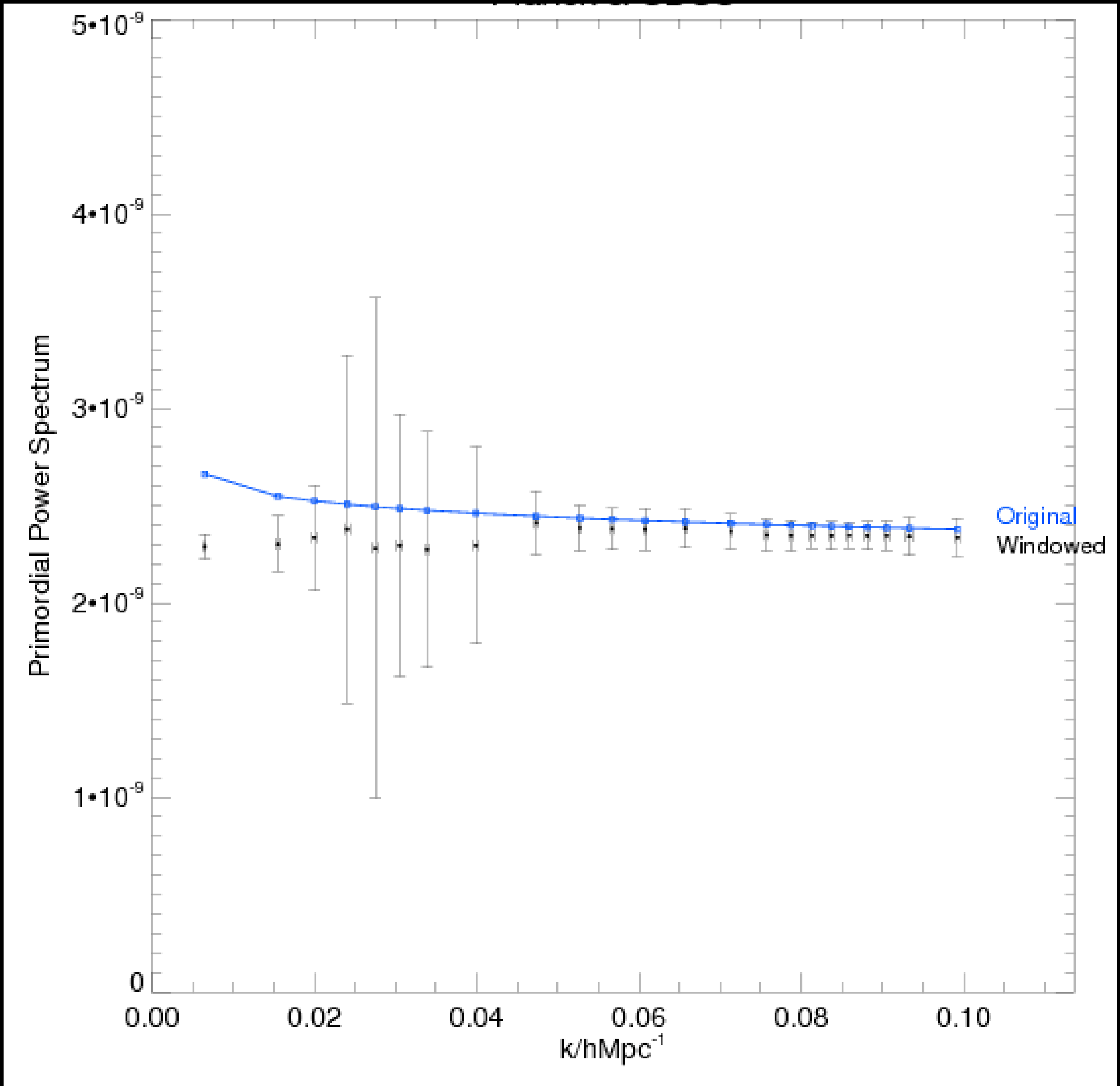
|1.1 |0.9| |0.8| |0.6| |0.4| |0.2| |0.0|
SDSS
τ
h
Ω_Λ
Ω_b
Ω_c
0.102
0.095
0.089
0.083
0.076
0.069
0.061
0.050
0.006
X_1
X_3
X_5
X_7
X_9
X_{11}
X_{13}
parameters
Principal components


SDSS—No θ_i
0.102
0.095
0.089
0.083
0.076
0.069
0.061
0.050
0.006
X_1
X_2
X_3
X_4
X_5
X_6
X_7
X_8

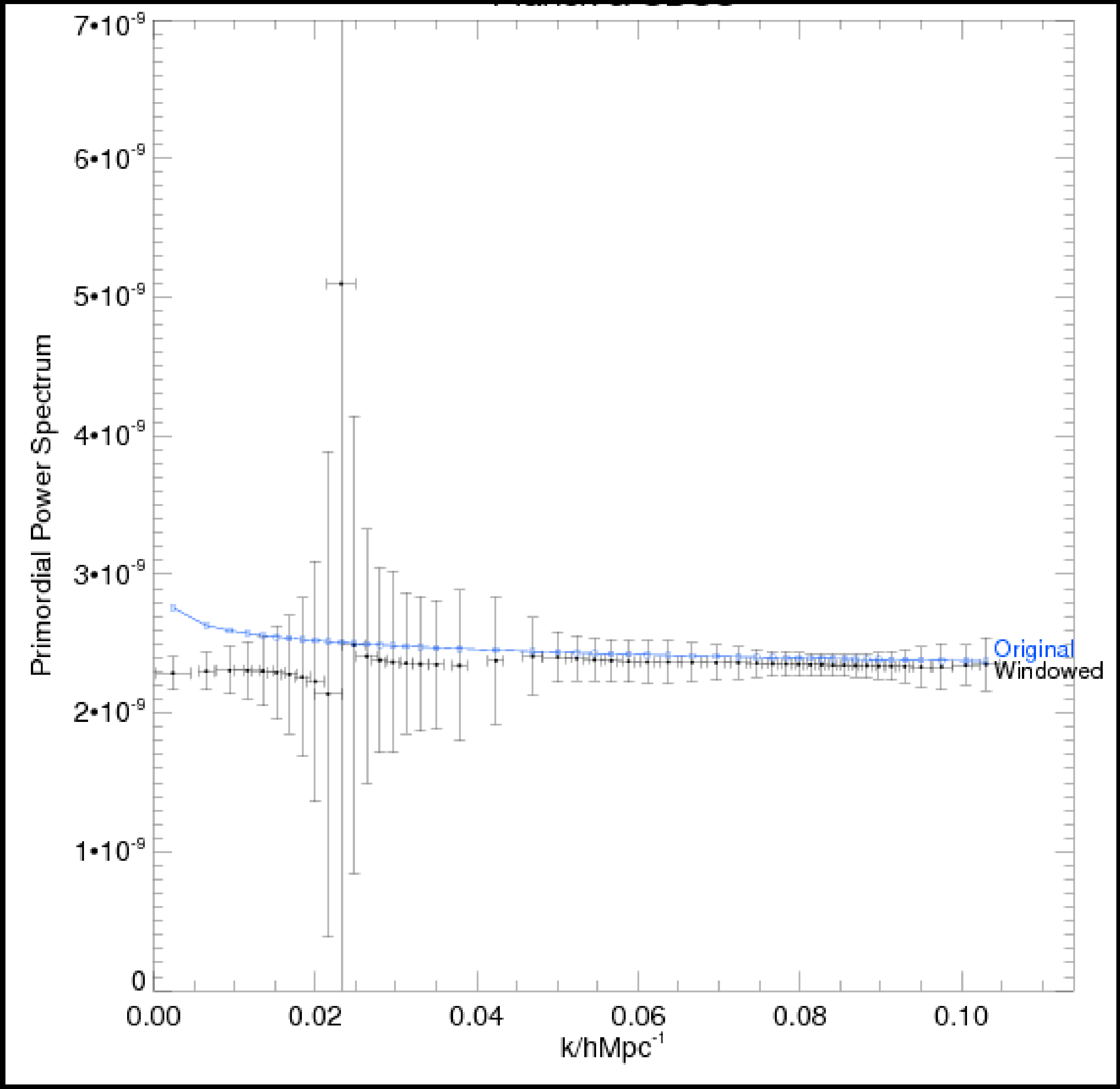

Planck
θ
0.104
0.095
0.074
0.063
0.051
0.036
0.018
0.013
0.000
X1
X5
X10
X15
X20
X25
X28


Planck & SDSS
θ_i
0.104
0.096
0.071
0.065
0.045
0.036
0.005
0.000
X1
X5
X10
X15
X20
X25
X30
X35
X40
X45
X50
X53

Windowed Power Spectrum
Planck



Windowed Power Spectrum
Planck & SDSS



Conclusions

- Induced degeneracy between cosmological parameters and PPS limit our ability to recover this PS completely
  - In case of Planck, even a perfect survey could recover the primordial PS completely, due to the projection effects.
- Quantify the degeneracy → parameter space with ***carefully chosen*** bins of the PPS and a set of cosmological parameters
- PCA:
  - measure cosmological parameters better
  - constrain small scales better than large scales
  - SDSS & Planck
    - break degeneracy between them
    - increase the resolution of the PPS significantly
- Hsqrt:
  - divert correlation between errors of the bins to correlation between bins themselves
  - quantify the difference between the actual and measured PPS

Thank you