Persistent Betti Topology

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PFNG 2010
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LSS

CMB

Rien van de Weygaert (KAI)
Herbert Edelsbrunner (IST/Duke)
Changbom Park (KIAS)

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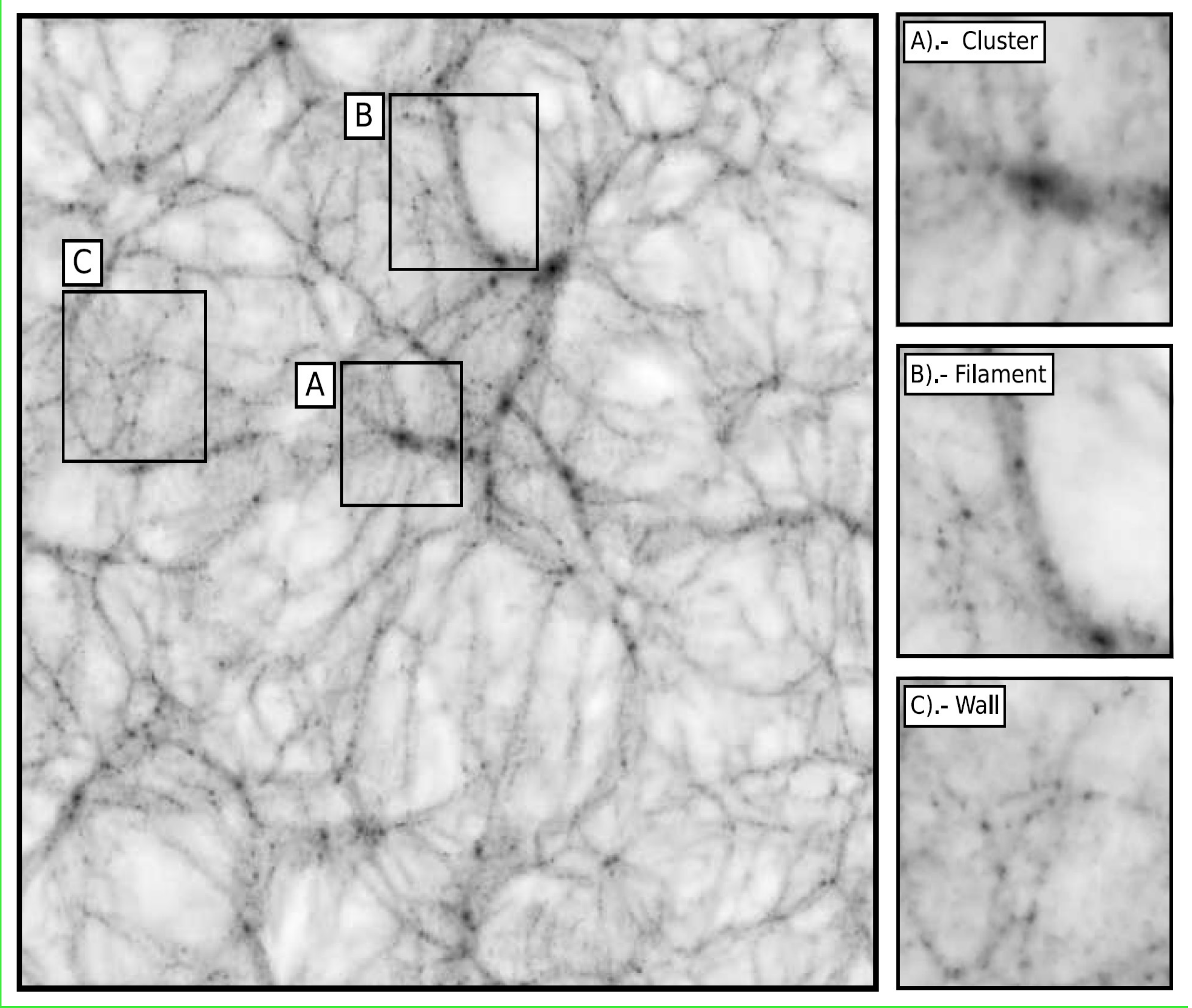
The Cosmic Web

Hierarchical Structure formation

Anisotropic collapse of matter



The Cosmic Web

Web Discretely Sampled:

By far, most information

on the Cosmic Web concerns

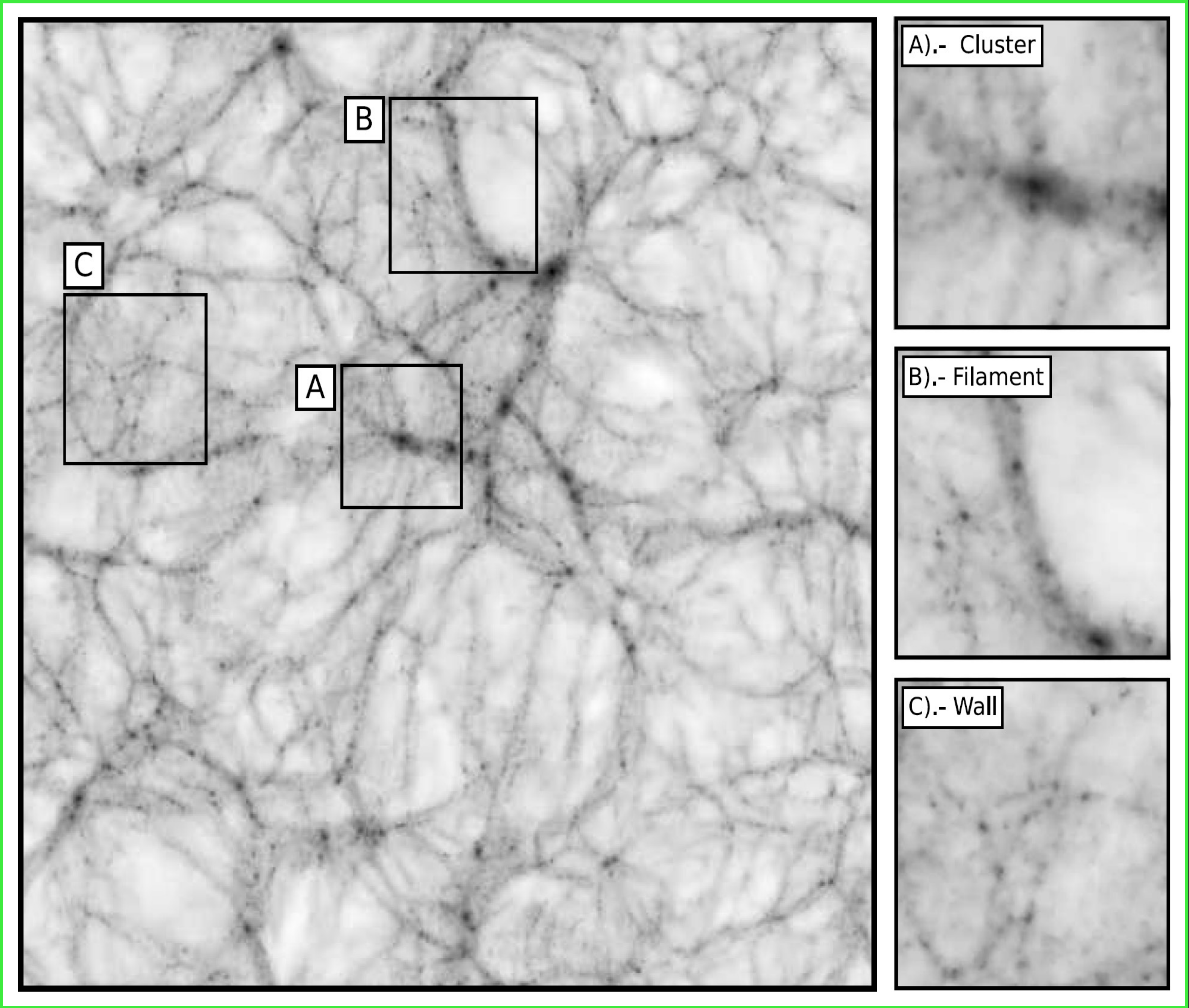
discrete samples:

observational:

Galaxy Distribution

theoretical:

N-body simulation particles



Delaunay Triangulation

Reconstruction of shapes represented by a discrete point sample

Unique for a non-degenerate point sample (Empty circumcircle)

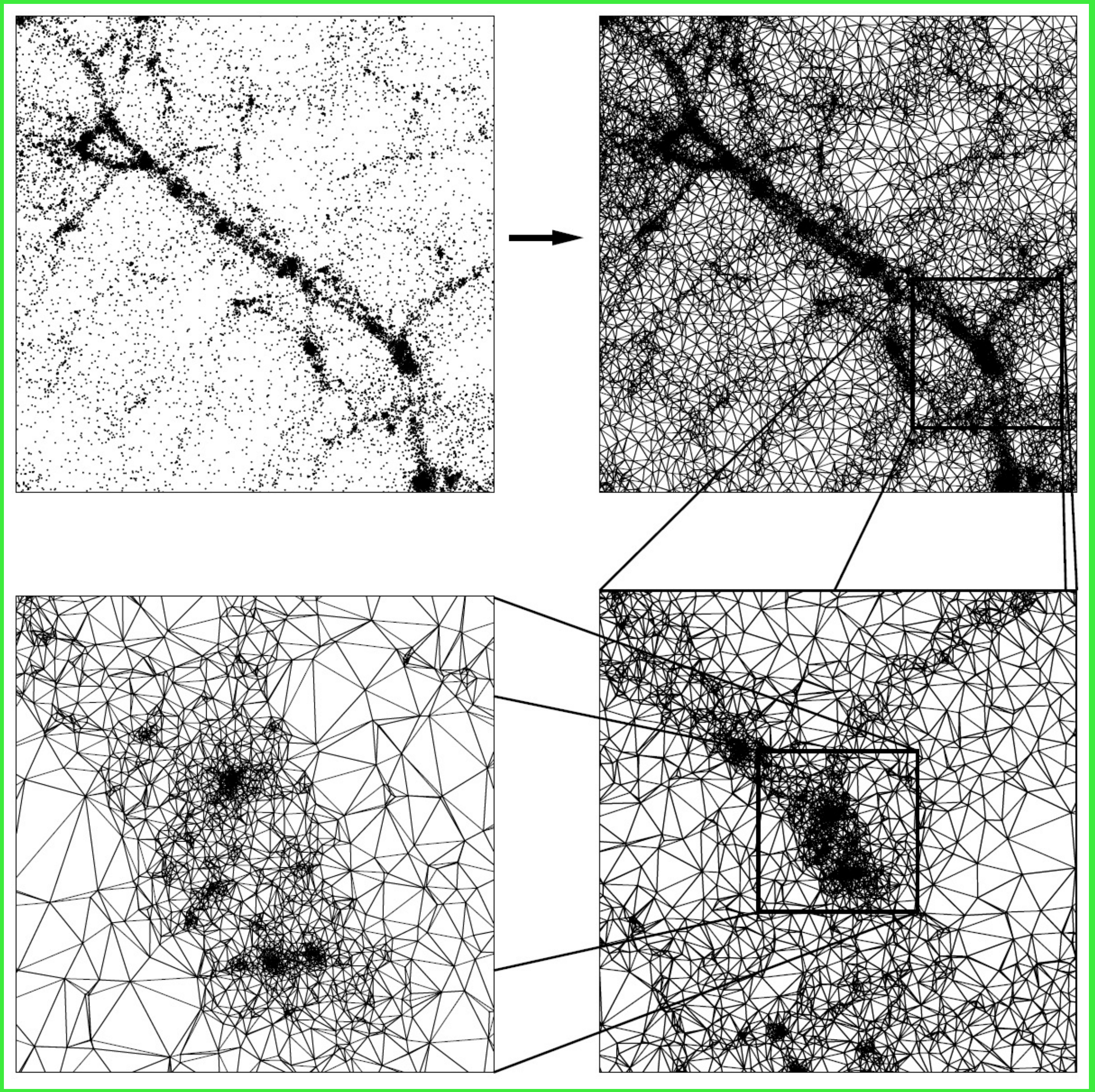
Handles multi-scale distribution naturally

↓

suggestion for exploiting this to explore the topology of the cosmic mass Distribution

↓

Alphashapes



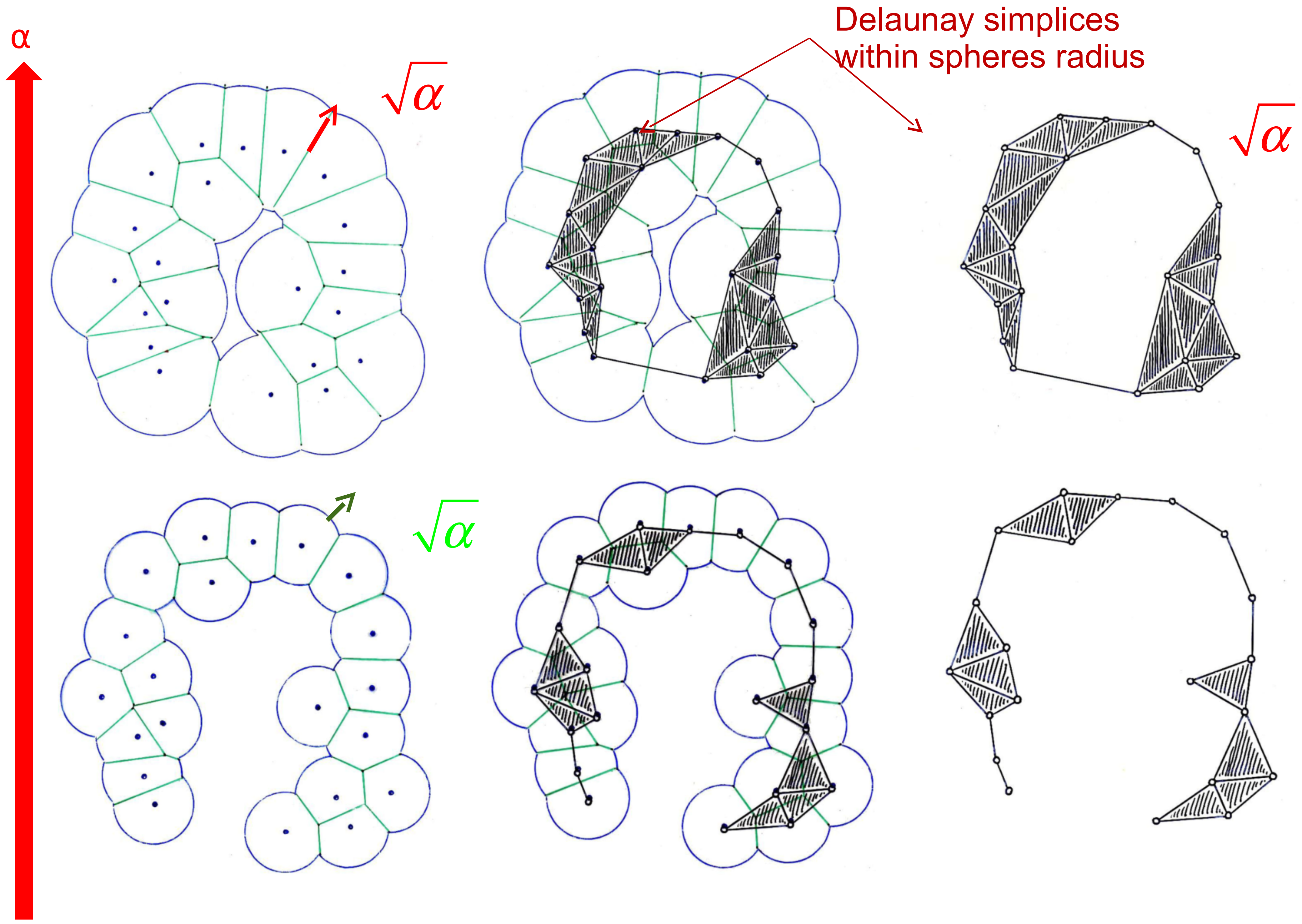
Cosmic Structure Topology

A new approach:
Exploit the topological information contained in the Delaunay Tessellations of the galaxy/halo/density distribution

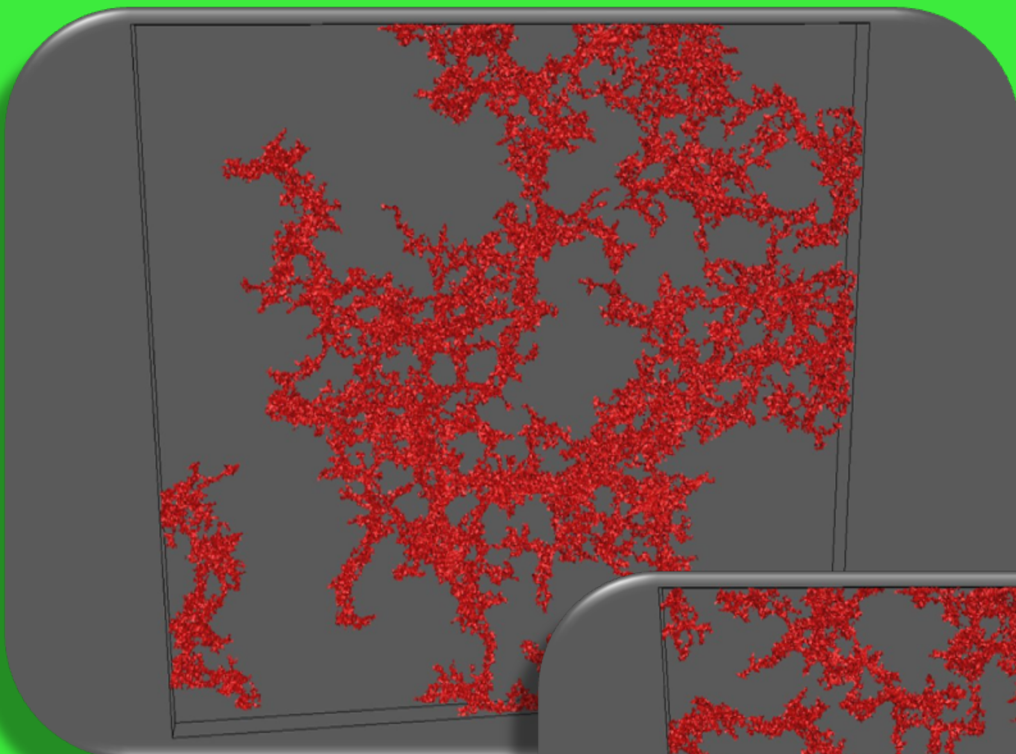
Alpha (α) Shapes

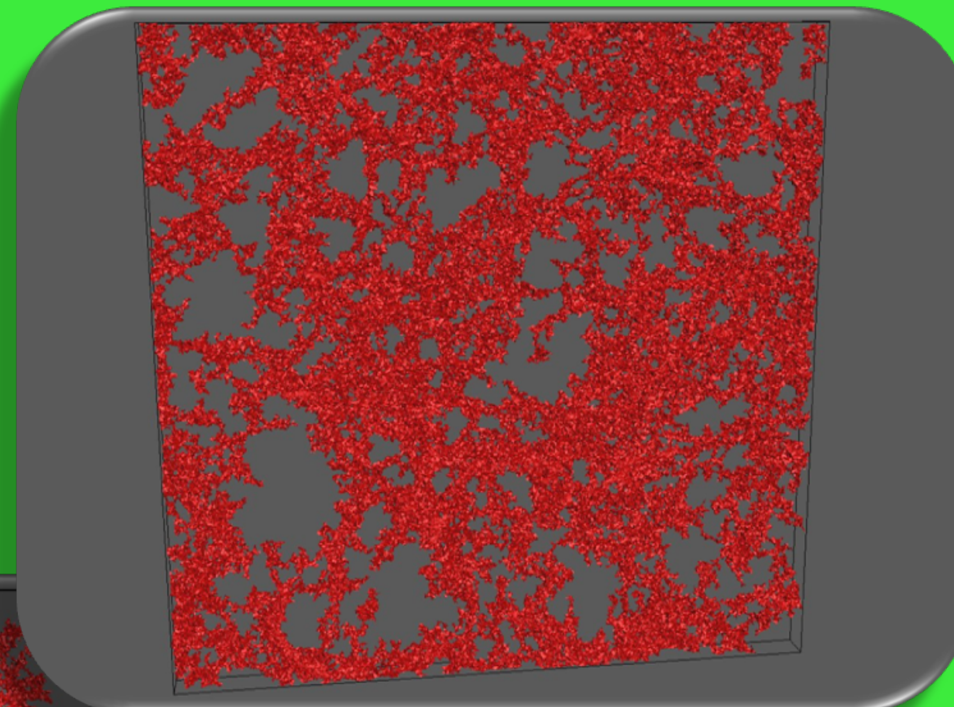
Introduced by H. Edelsbrunner & collab. (1983,1994)

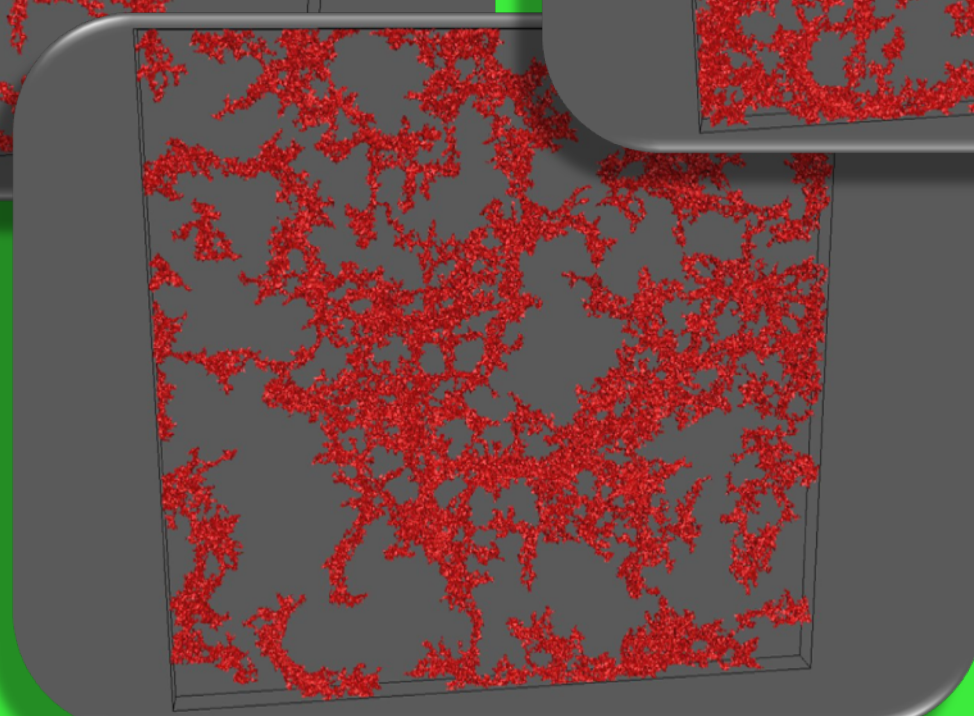
Description of intuitive notion of the shape of a discrete point set


α
Delaunay simplices
within spheres radius
$\sqrt{\alpha}$
$\sqrt{\alpha}$
$\sqrt{\alpha}$

Cosmic Structure Topology







LSS Topology

Alpha Shape of cosmological simulation

left to right: alpha value increases.

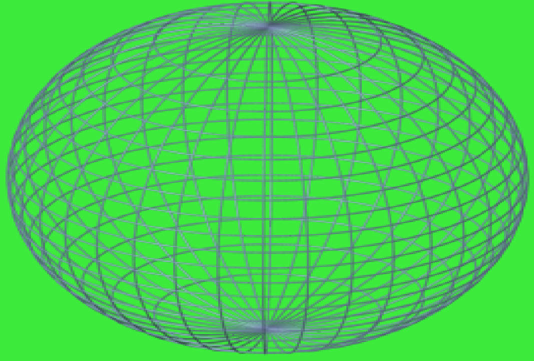
Betti Numbers

- **Provide complete quantitative characterization of the topology**
- **Can be inferred from the set of alphashapes for varying alpha**

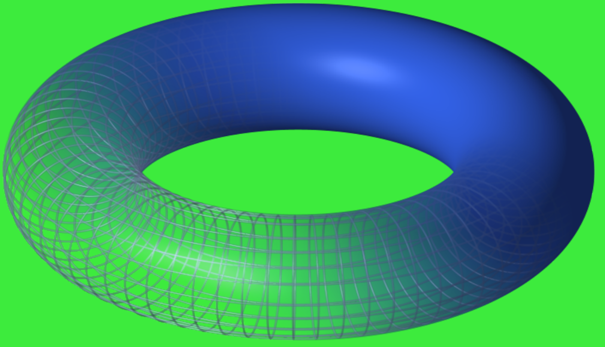
Definition

β_k – a number of k-dimensional holes of an object or shape.
Parameter k can take values from 0 < k < N, for a n-dimensional euclidean space. For N=3 we have

β_0 - the number of independent components
β_1 - the number of tunnels/loops
β_2 - the number of enclosed voids



$\beta_0 = 1, \beta_1 = 0, \beta_2 = 1$



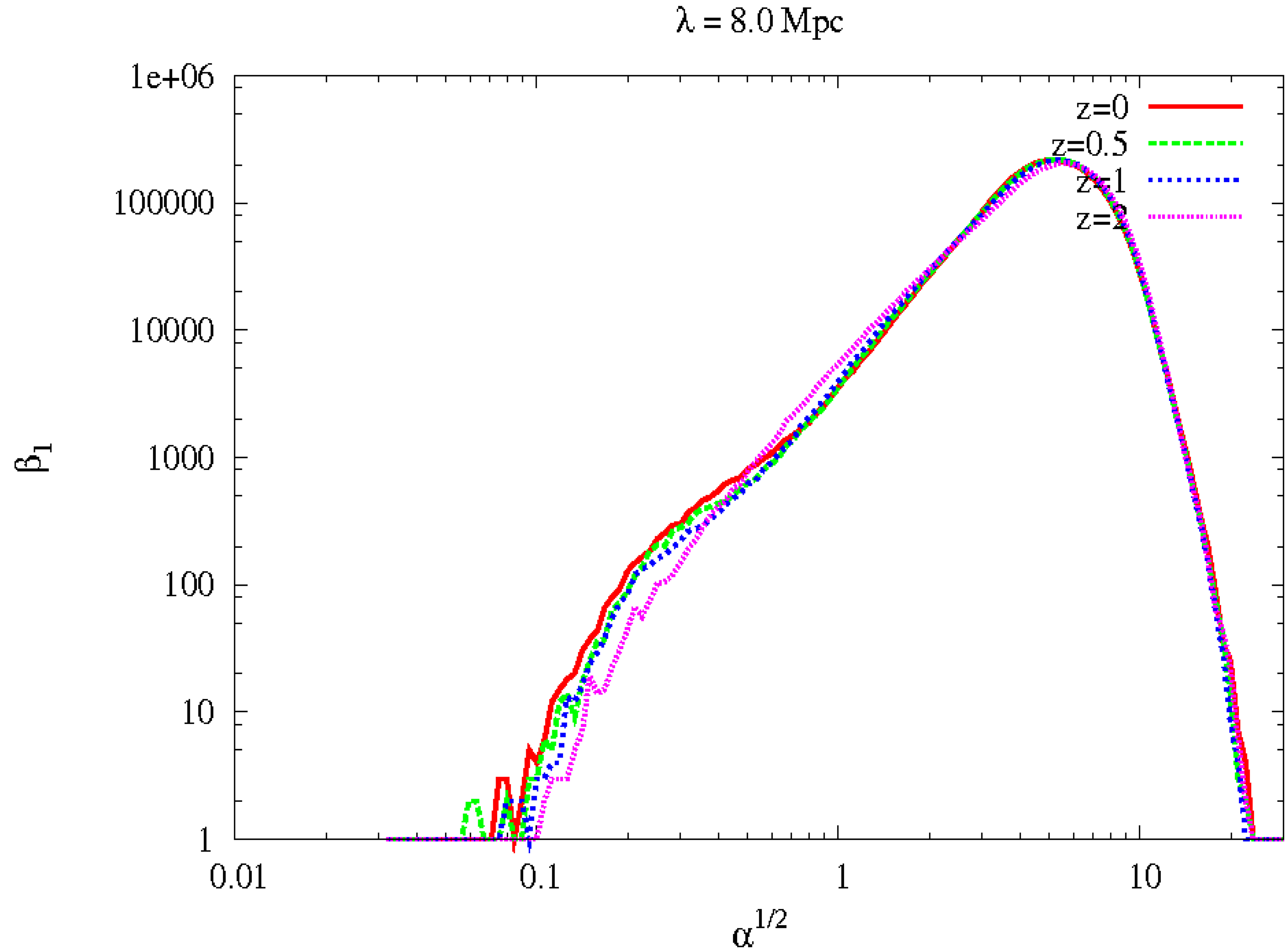
$\beta_0 = 1, \beta_1 = 2, \beta_2 = 1$

Betti Numbers and Genus

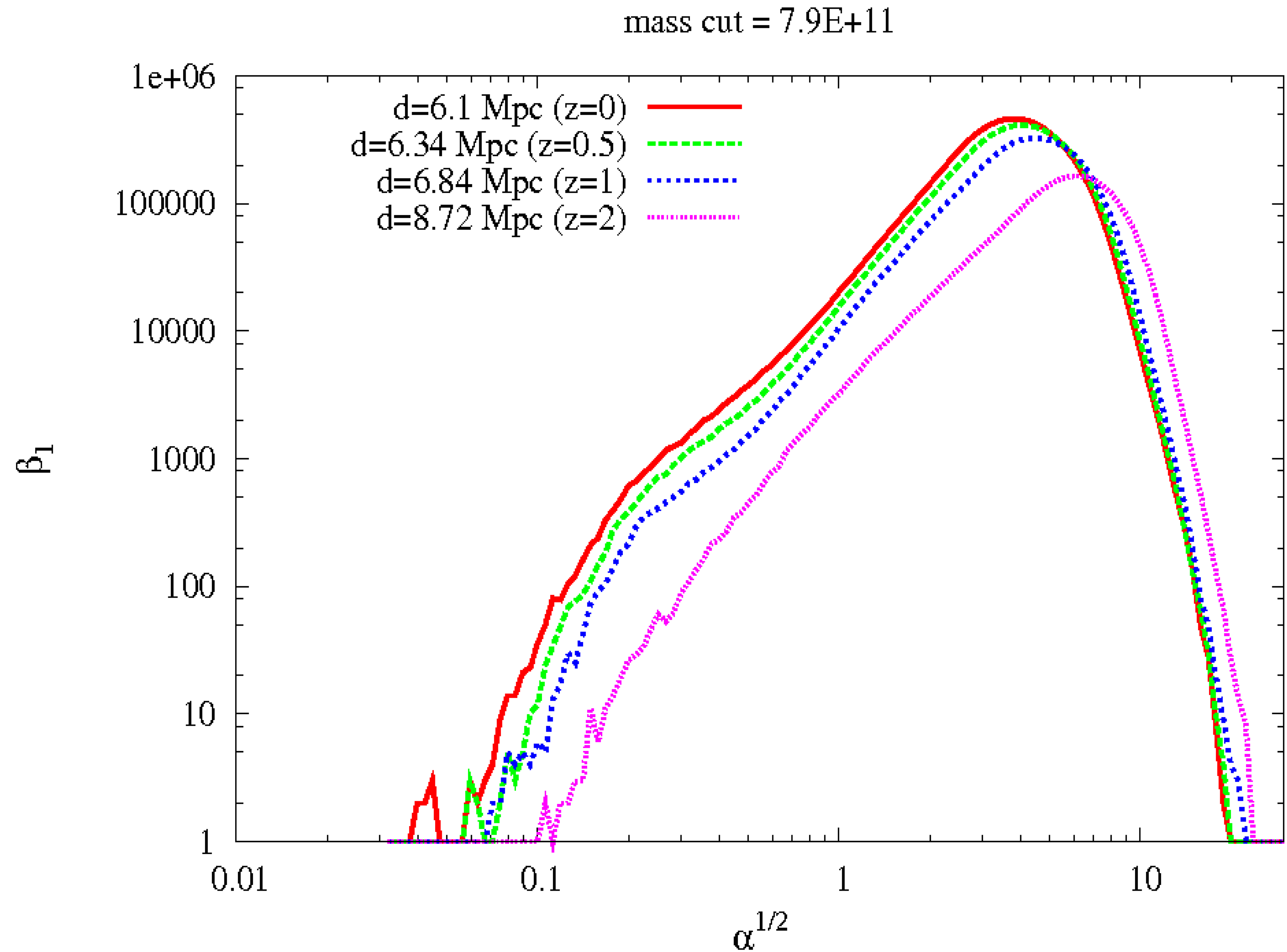
For a body with c components, the genus g specifies the number of handles on surface, and is related to the Euler characteristic via:

$$g = c - \frac{1}{2}\chi$$

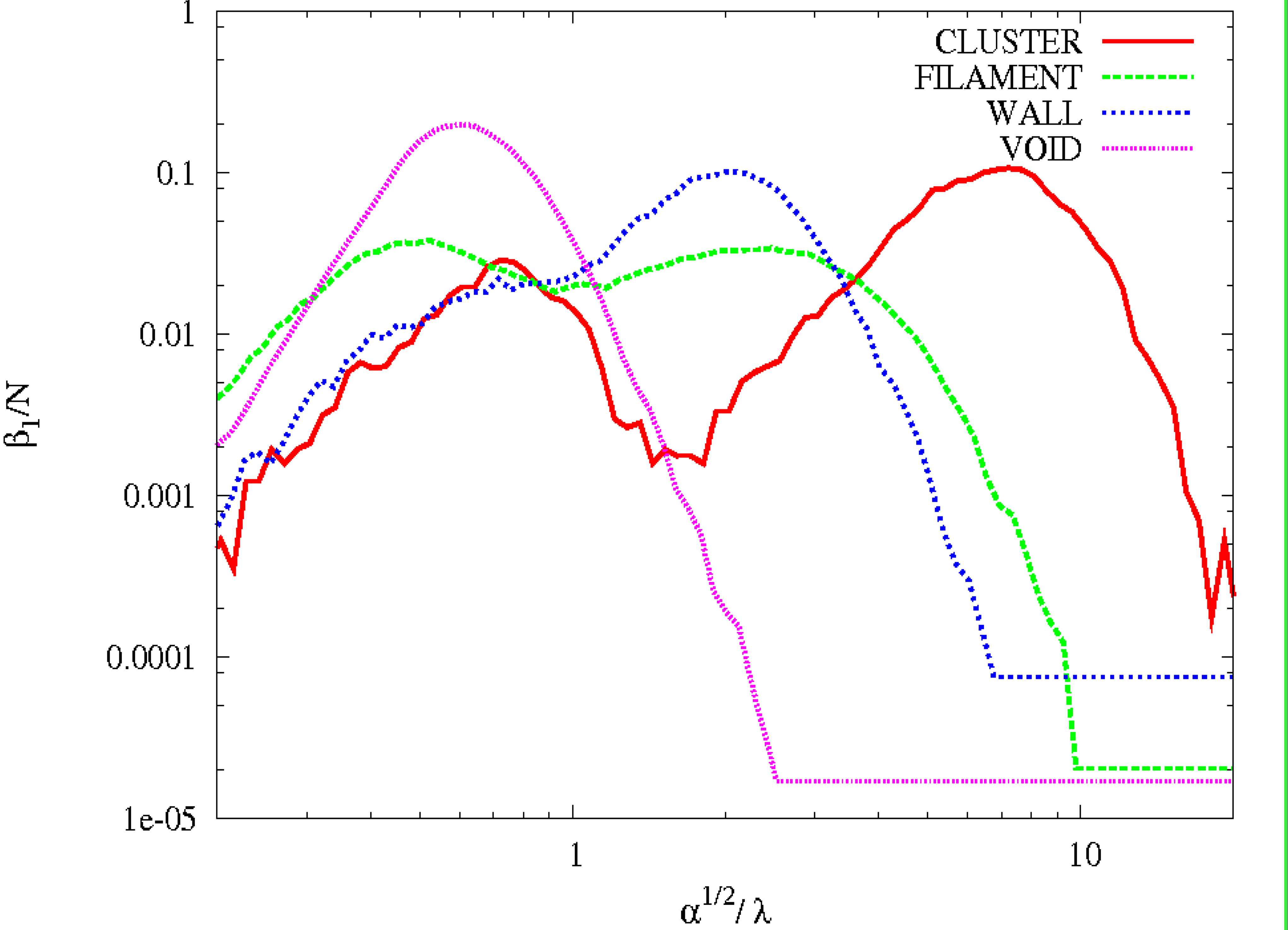
$$\chi = 2\left(\beta_0 - \beta_1 + \beta_2\right)$$



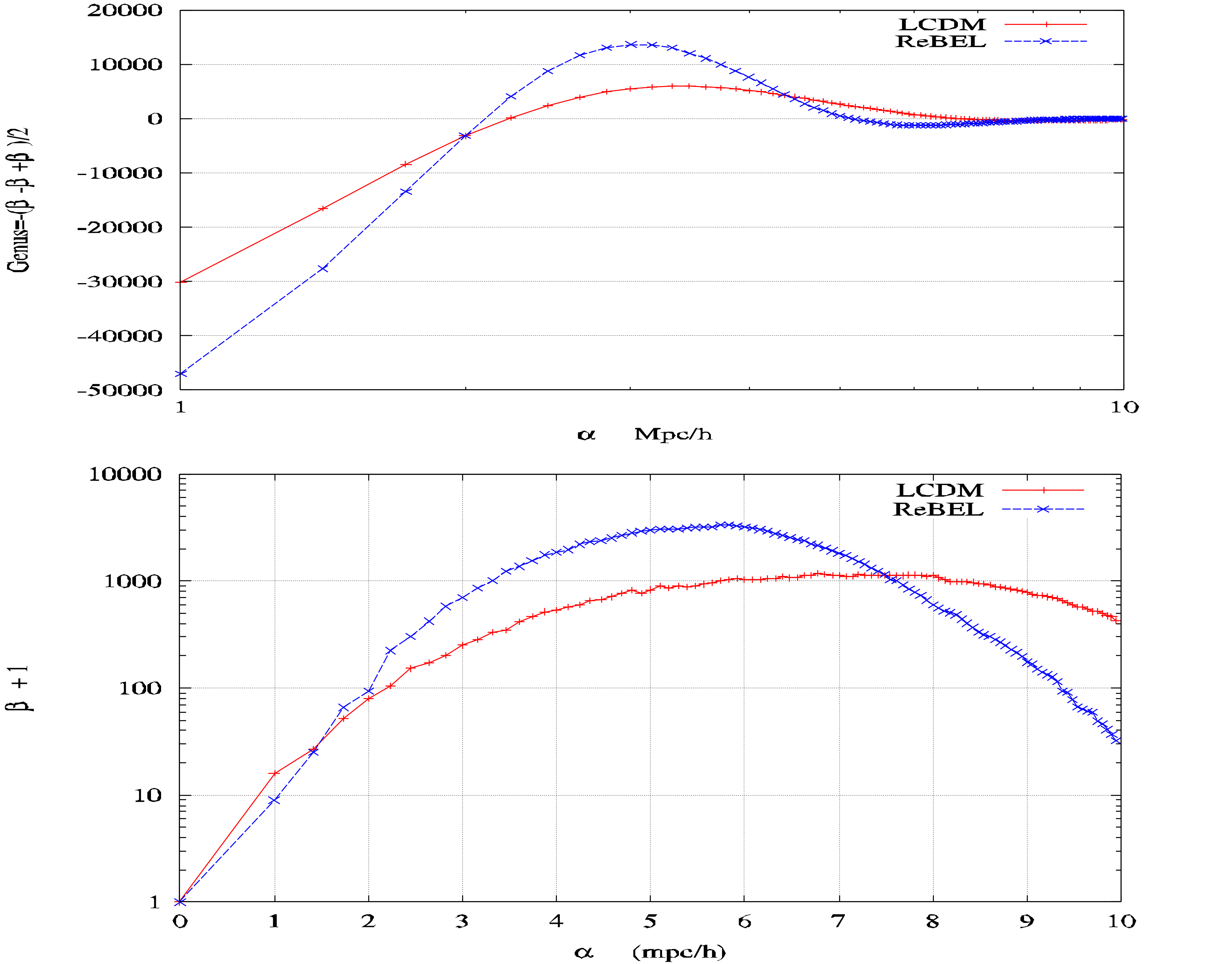
λ = 8.0 Mpc
z=0
z=0.5
z=1
z=2
1e+06
100000
10000
1000
100
10
1
0.01
0.1
1
10
β1
α1/2



mass cut = 7.9E+11
d=6.1 Mpc (z=0)
d=6.34 Mpc (z=0.5)
d=6.84 Mpc (z=1)
d=8.72 Mpc (z=2)
1e+06
100000
10000
1000
100
10
1
0.01
0.1
1
10
β_1
$\alpha^{1/2}$



CLUSTER
FILAMENT
WALL
VOID
1
0.1
0.01
0.001
0.0001
1e-05
β1/N
1
10
α1/2/ λ



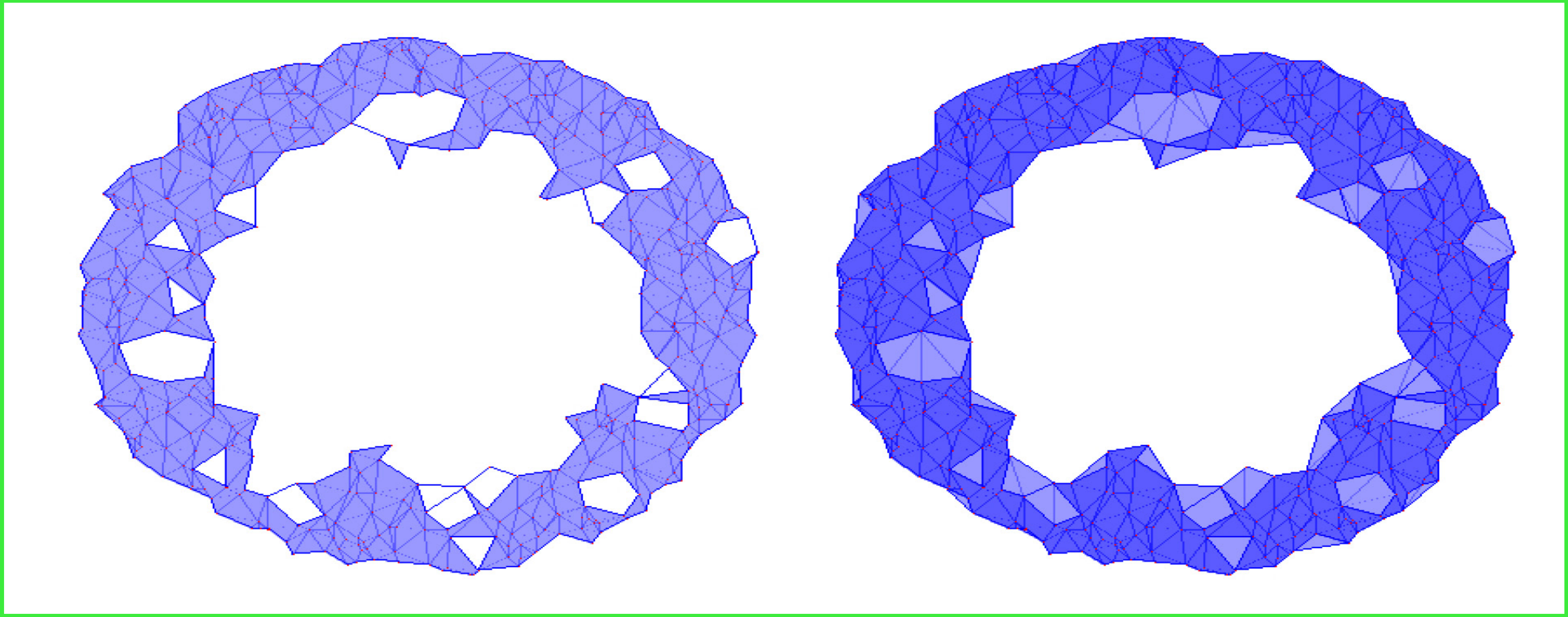
20000
10000
0
-10000
-20000
-30000
-40000
-50000
1
10
LCDM
ReBEL
Genus=-(β -β +β)/2
α Mpc/h
10000
1000
100
10
1
0
1
2
3
4
5
6
7
8
9
10
LCDM
ReBEL
β +1
α (mpc/h)

Persistence: Search for topological reality

Formalism to quantize the “life-span” of structures

Segregate real structure from noise for a single-scale distribution

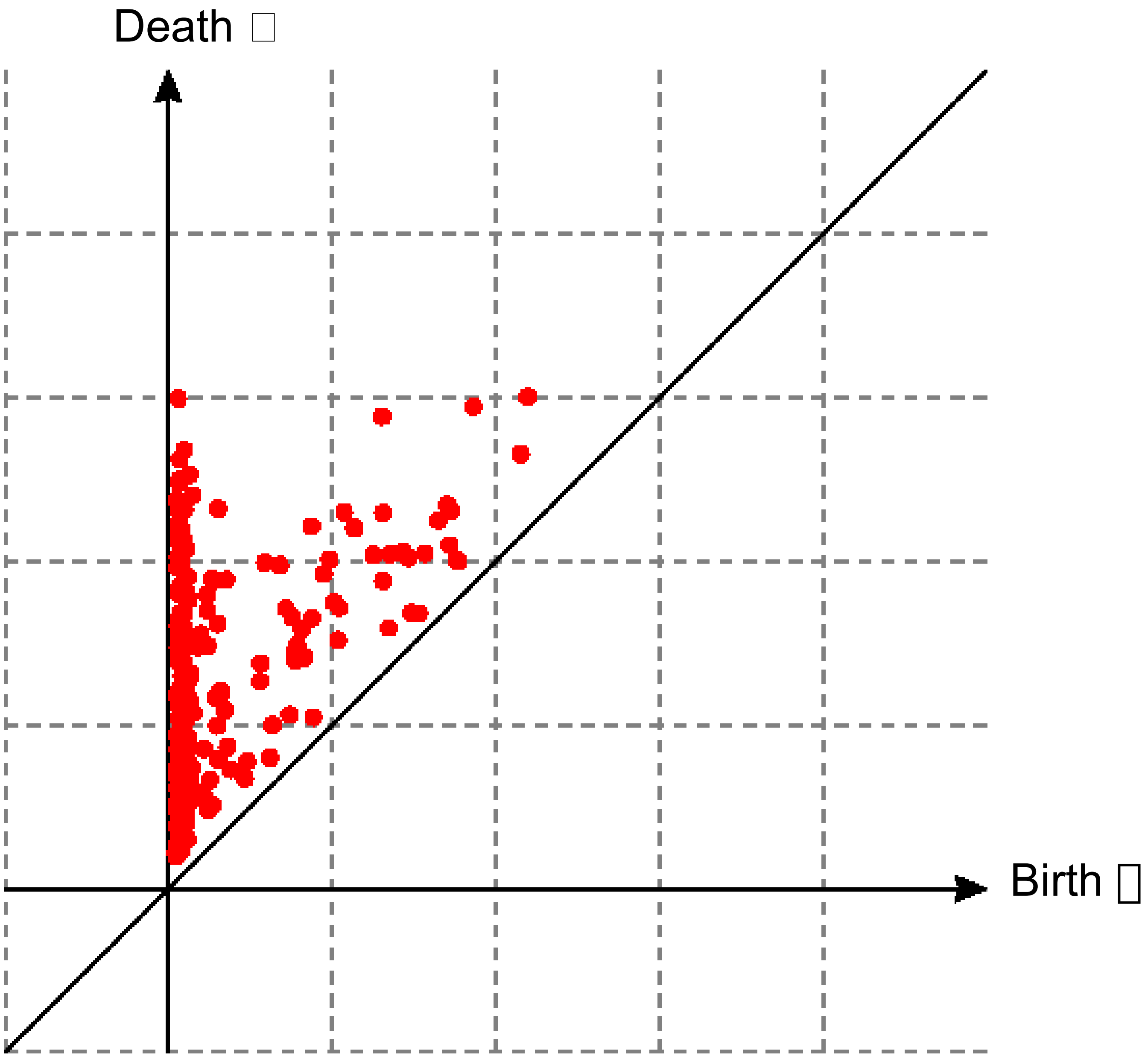
Investigate structures at a particular scale for multi-scale distributions

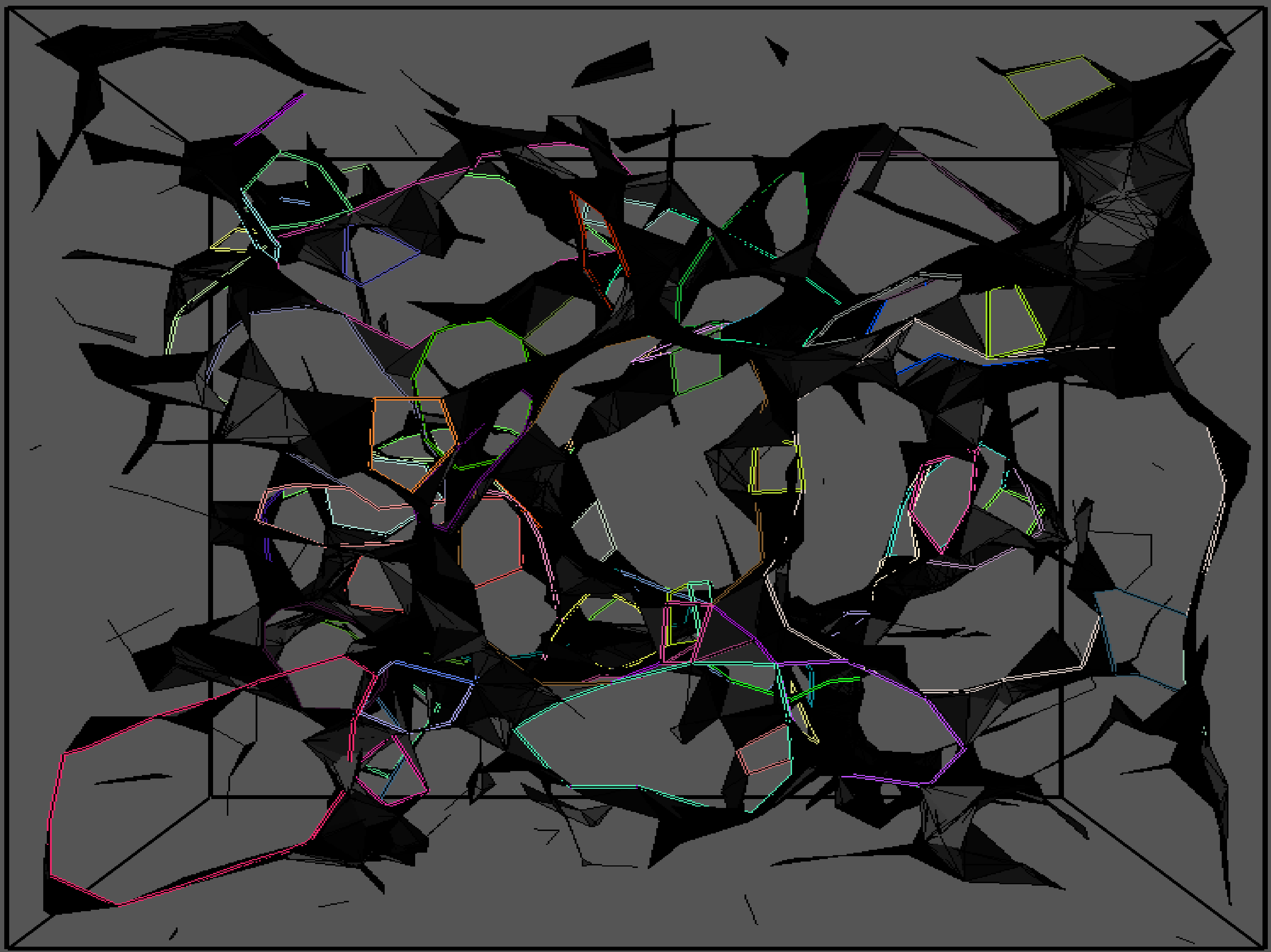


Concept introduced by Edelsbrunner:

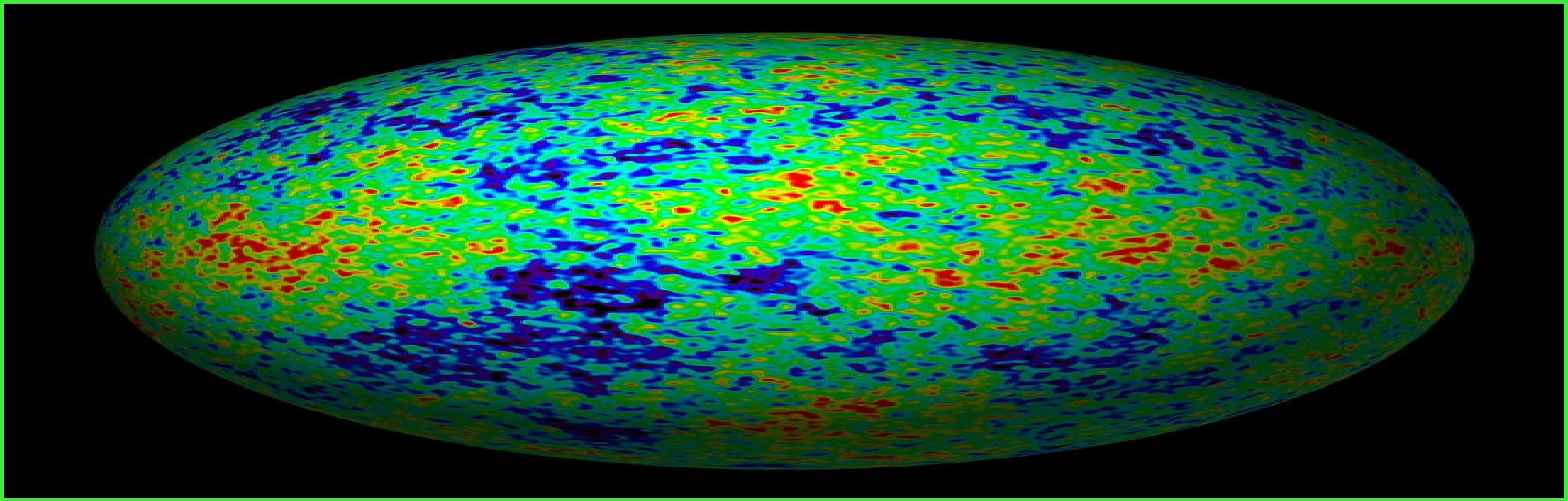
Reality of features (eg. voids) determined on the basis of interval between “birth” and “death” of features

Persistence Diagram





CMB Topology



Concepts for analysis similar as described above

Points replaced by *Pixels*

Topology as a function of level-sets