NON-GAUSSIAN FLUCTUATIONS OF THE INFLATON AND CONSTANCY OF CORRELATIONS OF ζ OUTSIDE THE HORIZON

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NON-GAUSSIANITIES IN SINGLE FIELD INFLATION

- NON-GAUSSIANITIES IN THE CURVATURE PERTURBATION ζ CAN ARISE FROM

  1) SELF INTERACTIONS OF THE INFLATON, AND

  2) NON-LINEARITIES IN COSMOLOGICAL PERTURBATION THEORY.

- USUALLY THE SELF INTERACTION CONTRIBUTION IS IGNORED.

SELF INTERACTION CONTRIBUTION

FOR EXAMPLE, CONSIDER CUBIC SELF INTERACTIONS,

$\mu\, \phi^3$.

- THE SELF INTERACTION CONTRIBUTION TO $< \zeta^3 >$ $\sim < (\delta\phi)^3 >$ WHICH IS PROPORTIONAL TO μ, AND THUS TO THE SLOW ROLL PARAMETER $\xi \sim V'''$

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- IS IGNORED COMPARED TO TERMS PROPORTIONAL TO SLOW ROLL PARAMETERS $\varepsilon \sim V'^2$ AND $\eta \sim V''$.

[MALDACENA 2003]

BUT .....

NON-GAUSSIANITIES IN SINGLE FIELD INFLATION

1. FOR MANY MODELS (NEW INFLATION, SMALL FIELD NATURAL INFLATION AND RUNNING MASS INFLATION), $\xi >> \varepsilon$.

2. THE SELF INTERACTION CONTRIBUTION IS ACTUALLY ~ ξN_e . THIS IS COMPARABLE TO η.

   (N_e (t) is the number of e-foldings since horizon exit, and increases from 0 to 60 by the end of inflation for our current horizon scale).

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THEREFORE SELF INTERACTIONS SHOULD NOT BE IGNORED OUTRIGHT

[MALDACENA 2003 - CHAOTIC]

DO SELF INTERACTIONS IMPLY GROWTH OUTSIDE THE HORIZON ?

- BUT A CONTRIBUTION TO $< \zeta(k)^3 > \sim N_e(t)$ IMPLIES THAT THE 3-POINT FUNCTION IS GROWING AFTER HORIZON EXIT ! THIS IS CONTRARY TO ONE'S EXPECTATIONS.

IS THE CALCULATION OF THE INFLATON SELF INTERACTION CONTRIBUTION TO $< \zeta^3 >$ INCORRECT?

- IT HAS BEEN DONE INDEPENDENTLY BY FALK, RR, SREDNICKI (1993), ZALDARRIAGA (2004), BERNARDEAU, BRUNIER, UZAN (2004) AND SEERY, MALIK, LYTH(2008) IN DIFFERENT CONTEXTS.

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- IT HAS BEEN DONE INDEPENDENTLY BY FALK ET AL (1993), ZALDARRIAGA (2004), BERNARDEAU ET AL (2004) AND SEERY ET AL (2008) IN DIFFERENT CONTEXTS.

- LET US RE-EXAMINE THE ARGUMENT THAT CORRELATIONS OF THE CURVATURE PERTURBATION $\zeta(k)$ ARE CONSTANT OUTSIDE THE HORIZON.

- IN THE LITERATURE, FUNCTION $\zeta(k)$ IS SHOWN TO BE CONSTANT OUTSIDE THE HORIZON. (CLASSICAL)
- WHAT DOES THIS IMPLY FOR QUANTUM n-POINT FUNCTIONS?

- IN THE LITERATURE, FUNCTION $\zeta(k)$ IS SHOWN TO BE CONSTANT OUTSIDE THE HORIZON. (CLASSICAL)
- WHAT DOES THIS IMPLY FOR QUANTUM n-POINT FUNCTIONS?
- THE 2-POINT FUNCTION

$$< \zeta(k_1)\, \zeta(k_2) > = (2\pi)^3\, |\zeta(k_1)|^2\, \delta^3(\mathbf{k_1} - \mathbf{k_2})$$

IS CONSTANT AFTER HORIZON EXIT.

- **WHAT ABOUT HIGHER POINT FUNCTIONS ?**

CONSTANCY CONDITION FOR ζ(k) 3-POINT FUNCTION

$$\langle \zeta(t)^3 \rangle = i \int_{t_0}^{t} dt' \Big\langle \Big[H_I(t'), \zeta_I(t)^3 \Big] \Big\rangle$$

$I =$ INTERACTION PICTURE

- FOR < ζ(t)^3 > TO BE CONSTANT AFTER HORIZON EXIT THE CONTRIBUTION TO THE INTEGRAL FOR t AFTER HORIZON EXIT SHOULD BE SUPPRESSED.

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CONSTANCY CONDITION FOR ζ(k) 3-POINT FUNCTION

- ζ IS CLASSICAL AND CONSTANCY OUTSIDE THE HORIZON IMPLIES CORRELATIONS ALSO CONSTANT OUTSIDE THE HORIZON

- WEINBERG (2006) ARGUES THAT PERTURBATIONS OUTSIDE HORIZON CLASSICAL IN THE SENSE THAT

$$[\zeta, \zeta], \quad [\zeta, \dot{\zeta}] \to 0 \qquad \text{for large t}$$

n-POINT FUNCTIONS OF CAN GROW AS (ln a) OR POWERS OF (ln a), BUT NOT POWERS OF a.

$(\ln a) = N_e$

CONSTANCY CONDITION FOR ζ(k) 3-POINT FUNCTION

- WE INVESTIGATE WHETHER THE 3-POINT FUNCTION IS CONSTANT OUTSIDE THE HORIZON, IN LIGHT OF THE TIME DEPENDENT CONTRIBUTION PROPORTIONAL TO N_e FROM INFLATON SELF INTERACTIONS

CALCULATION OF THE 3-POINT FUNCTION OF $\zeta(k)$ AND f_{NL}

- WE CALCULATE $< (\delta\phi)^3 >$ IN THE $\delta\phi \neq 0$ GAUGE, USING THE CANONICAL FORMALISM FOR CUBIC NEW INFLATION $V(\phi) = V_0 - \mu\, \phi^3$.

- WE THEN RELATE $\zeta(k,t)$ TO $\delta\phi(k,t)$. AND CALCULATE $< \zeta^3(t) >$, AND THE NON-GAUSSIANITY PARAMETER f_{NL}.

- t IS ARBITRARY, UNLIKE IN THE δN FORMALISM WHERE t ~ TIME OF HORIZON EXIT SO AS TO STUDY POSSIBLE GROWTH AFTER HORIZON EXIT

CALCULATION OF THE 3-POINT FUNCTION OF $\zeta(k)$ AND f_{NL}

$< (\delta\phi)^3 >$ IN THE $\delta\phi \neq 0$ GAUGE, FOR CUBIC NEW INFLATION $V(\phi) = V_0 - \mu\,\phi^3$.

$$\langle \varphi(t)^3 \rangle = i \int_{t_0}^{t} dt' \left\langle \left[H_I(t'), \varphi_I(t)^3 \right] \right\rangle \qquad \varphi \equiv \delta\phi$$

$$S_3 = \int dt\, d^3x\, a^3 \left[-\frac{\dot{\varphi}}{4H} \varphi \dot{\varphi}^2 - \frac{1}{a^2} \frac{\dot{\phi}}{4H} \varphi (\partial_i \varphi)^2 \right. \\ \left. + \frac{\dot{\phi}}{2H} \partial_i \varphi (\partial_i^{-1} \dot{\varphi}) \dot{\varphi} - \frac{1}{6} V'''(\phi) \varphi^3 \right]$$

CALCULATION OF THE 3-POINT FUNCTION OF ζ(k) AND f_{NL}

$$\langle\varphi(\vec{k}_1,t)\hat{\varphi}(\vec{k}_2,t)\hat{\varphi}(\vec{k}_3,t)\rangle = (2\pi)^3\delta(\vec{k}_1+\vec{k}_2+\vec{k}_3)$$

$$\times\left[\frac{H^2V'''}{4\prod_i k_i^3}\left(-\frac{4}{9}k_t^3+k_t\sum_{i<j}k_ik_j+\frac{1}{3}\left\{\frac{1}{3}+\gamma+\ln|k_t\tau|\right\}\sum_i k_i^3\right)\right.$$

$$\left.+\frac{H^4}{8\prod_i k_i^3}\frac{\dot{\phi}}{H}\left(\frac{1}{2}\sum_i k_i^3-\frac{4}{k_t}\sum_{i<j}k_i^2k_j^2-\frac{1}{2}\sum_{i\neq j}k_ik_j^2\right)\right]$$

$$i,j=1,2,3,\ k_i=|\mathbf{k}_i|$$

$k_t = \Sigma\, k_i$, k_i APPROX EQUAL

CALCULATION OF THE 3-POINT FUNCTION OF ζ(k) AND f_{NL}

$$\zeta(\mathbf{k},t) = -\frac{1}{\sqrt{2\epsilon}}\varphi(\mathbf{k},t)$$

$$+\frac{1}{2}\left(1-\frac{\eta}{2\epsilon}\right)\int \frac{d^3q}{(2\pi)^3}\,\varphi(\mathbf{k}_1-\mathbf{q},t)\varphi(\mathbf{q},t)+\cdots$$

[GAUGE TRANSFORMATION - MALDACENA 2003]

$$\langle\zeta(\mathbf{k_1},t,)\zeta(\mathbf{k_2},t)\zeta(\mathbf{k_3},t)\rangle = -\frac{1}{(2\epsilon)^{\frac{3}{2}}}\langle\varphi(\mathbf{k_1},t)\varphi(\mathbf{k_2},t)\varphi(\mathbf{k_3},t)\rangle$$

$$+\frac{1}{2\epsilon}\frac{1}{2}\left(1-\frac{\eta}{2\epsilon}\right)\langle\varphi(\mathbf{k}_1,t)\varphi(\mathbf{k}_2,t)\int \frac{d^3q}{2\pi^3}\varphi(\mathbf{k}_3-\mathbf{q},t)\varphi(\mathbf{q},t)+\mathrm{perm}\,\rangle$$

CALCULATION OF THE 3-POINT FUNCTION OF ζ(k) AND f_{NL}

$$\langle \zeta(\mathbf{k}_1, t)\zeta(\mathbf{k}_2, t)\zeta(\mathbf{k}_3, t)\rangle \equiv (2\pi)^3 \delta(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)\frac{6}{5} f_{NL} \sum_{i<j} P_\zeta(k_i) P_\zeta(k_j)$$

$$i, j = 1, 2, 3, \;\; k_i = |\mathbf{k}_i|$$

CALCULATION OF THE 3-POINT FUNCTION OF ζ(k) AND f_{NL}

$$\frac{6}{5} f_{\rm NL}(k_1,k_2,k_3,t) = \underline{\xi}\left[\frac{1}{3} + \gamma - \underline{\mathbf{N_e}} + \frac{3}{\sum_i k_i^3}\left(k_t \sum_{i<j} k_i k_j - \frac{4}{9} k_t^3\right)\right]$$

$$+\frac{3}{2}\epsilon - \underline{\eta} + \frac{\epsilon}{\sum_i k_i^3}\left(\frac{4}{k_t}\sum_{i<j} k_i^2 k_j^2 + \frac{1}{2}\sum_{i\neq j} k_i k_j^2\right)$$

$k_t = \Sigma\, k_i$, k_i APPROX EQUAL

FOR $n_s = 0.96$, **$\eta = - 0.02$**. $\xi = 0.5\, \eta^2$. **$\xi\, N_e = 0.012$, $\varepsilon \ll \xi\, N_e$**

SO THE SELF INTERACTION CONTRIBUTION ~ ξ SHOULD NOT BE IGNORED OUTRIGHT

$d\,f_{NL}/dt$

- f_{NL} IS A FUNCTION OF ε(t), η(t), ξ(t) AND $N_e = H\,(t - t_{exit})$. NOW

$$d\epsilon/dt \simeq \left[\,4\epsilon^2 - 2\eta\epsilon\,\right] H$$

$$d\eta/dt \simeq \left[\,2\epsilon\eta - \xi\,\right] H$$

$$d\xi/dt \simeq \left[\,4\epsilon\xi - \eta\xi\,\right] H$$

$$df_{\rm NL}/dt \approx (5/6)\, d[-\xi N_e - \eta]/dt$$

$d\, f_{NL}/dt$

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$$df_{\rm NL}/dt \approx (5/6)\,d[-\xi N_e - \eta]/dt = (5/6)\,[-\xi H + \xi H]\; = 0$$

SIMILAR TO SEERY ET AL (2008)

$d\, f_{NL}/dt = 0$

TIME EVOLUTION OF f_{NL} DUE TO SELF INTERACTIONS IS CANCELLED BY CONTRIBUTION OF OTHER TERMS FROM COSMOLOGICAL PERT. THEORY.

THUS THE TOTAL 3-POINT FUNCTION OF ζ DOES NOT GROW OUTSIDE THE HORIZON.

CONCLUSION

- THE ARGUMENT THAT ONE SHOULD IGNORE THE CONTRIBUTION OF SELF INTERACTIONS TO f_{NL} HAS BEEN RE-EXAMINED

- GROWS OUTSIDE THE HORIZON ~ N_e

- WE CALCULATE $< \zeta^3 (t) >$ TO STUDY SUPERHORIZON EVOLUTION

- GROWTH DUE TO SELF INTERACTIONS IS CANCELLED BY GROWTH IN OTHER TERMS IN f_{NL}