Oscillations and features in the primordial power spectra

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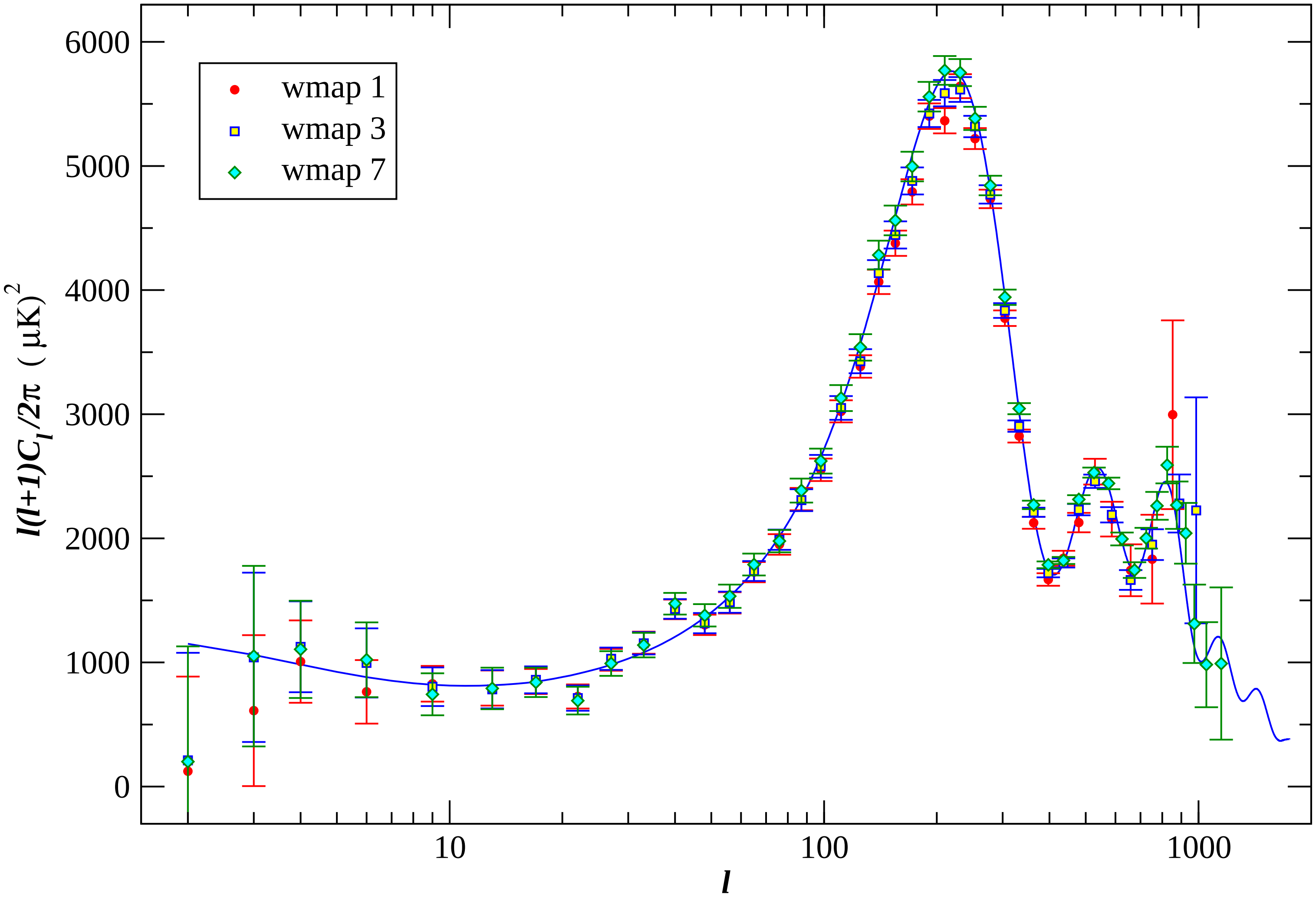
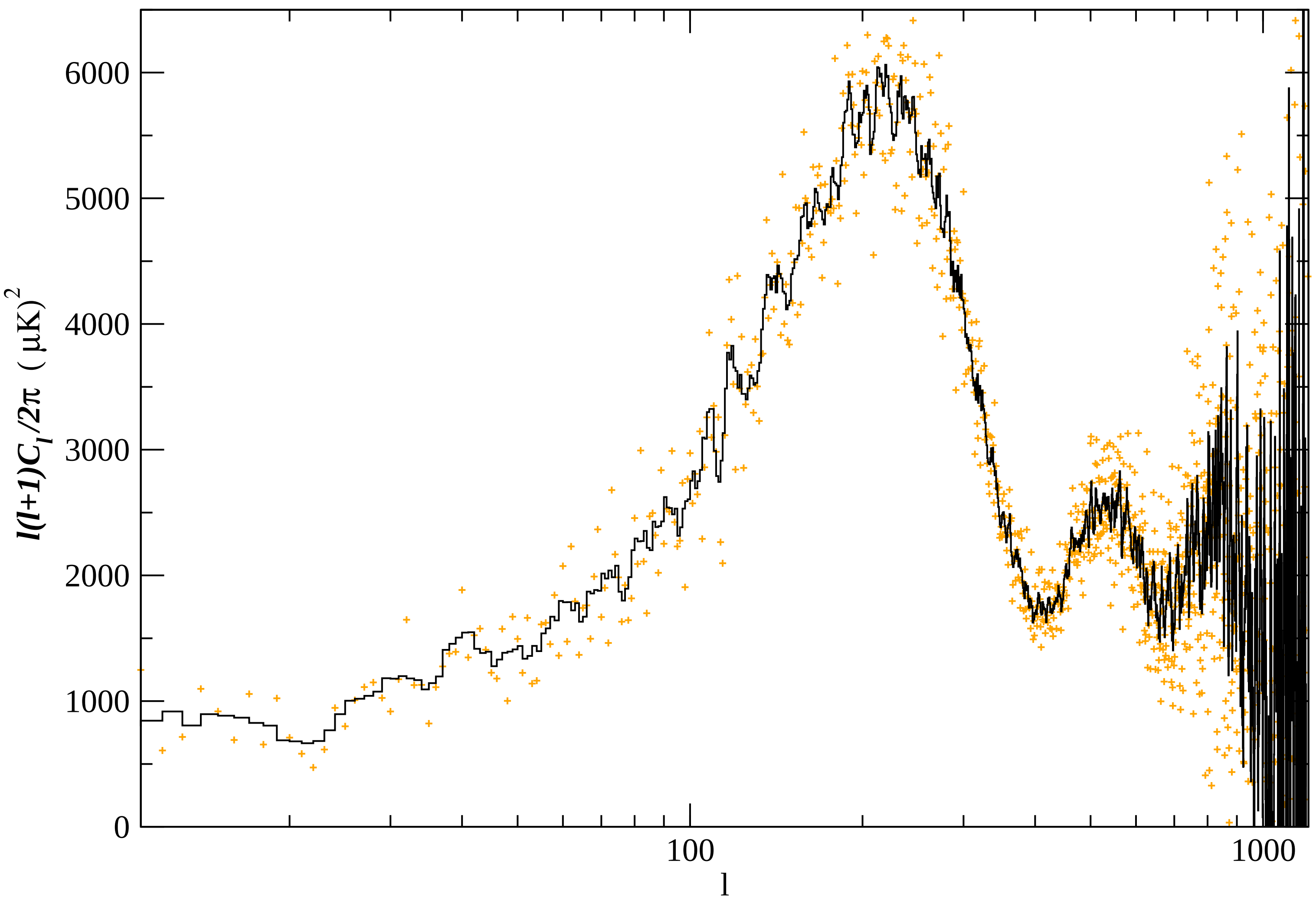
Outline

- Angular power spectra outliers?



WMAP7 data

- Unbinned + 5 pts average (TT data)



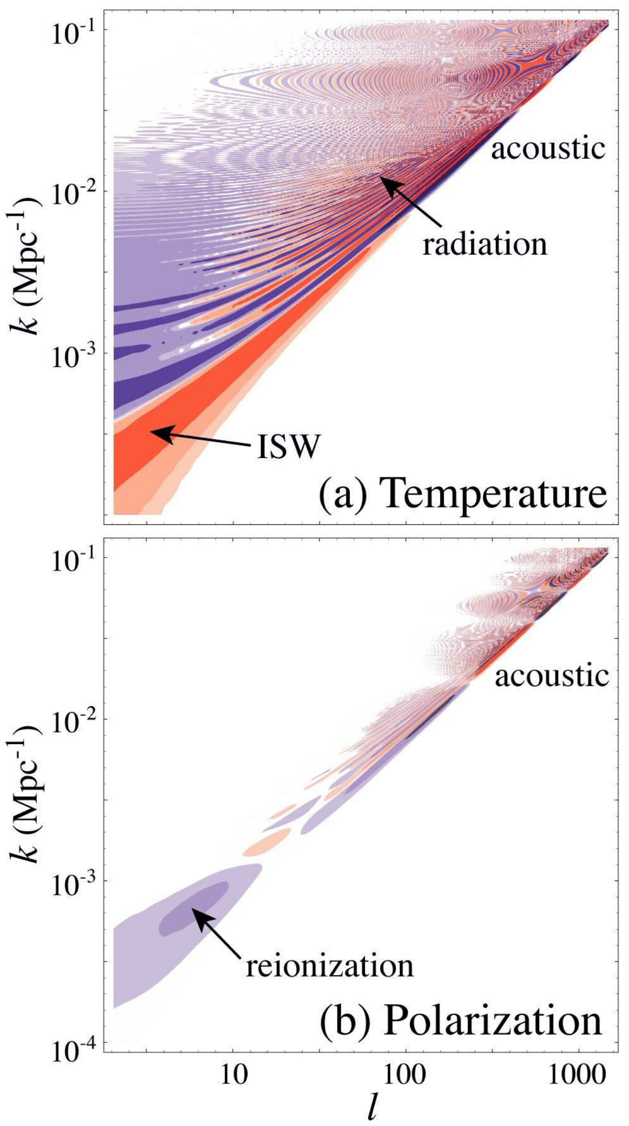
Primordial versus astrophysical features

- Oscillating transfer functions [Hu 04]

$$\ell(\ell+1)C_\ell^{XY} \propto \int \mathcal{T}_\ell^X(k)\mathcal{T}_\ell^Y(k)\mathcal{P}(k)\,\mathrm{d}(\ln k)$$

- Features in C_ℓ can either come from transfer functions or $\mathcal{P}(k)$.
- Transfer effects are unlikely to affect all scales
- Disambiguation is possible by using several observables [Hamann 08, Mortonson 09]

$$C_\ell^{EE}, \quad C_\ell^{TT}, \quad C_\ell^{TE}, \quad P_\mathrm{m}(k), \quad \text{21cm}$$

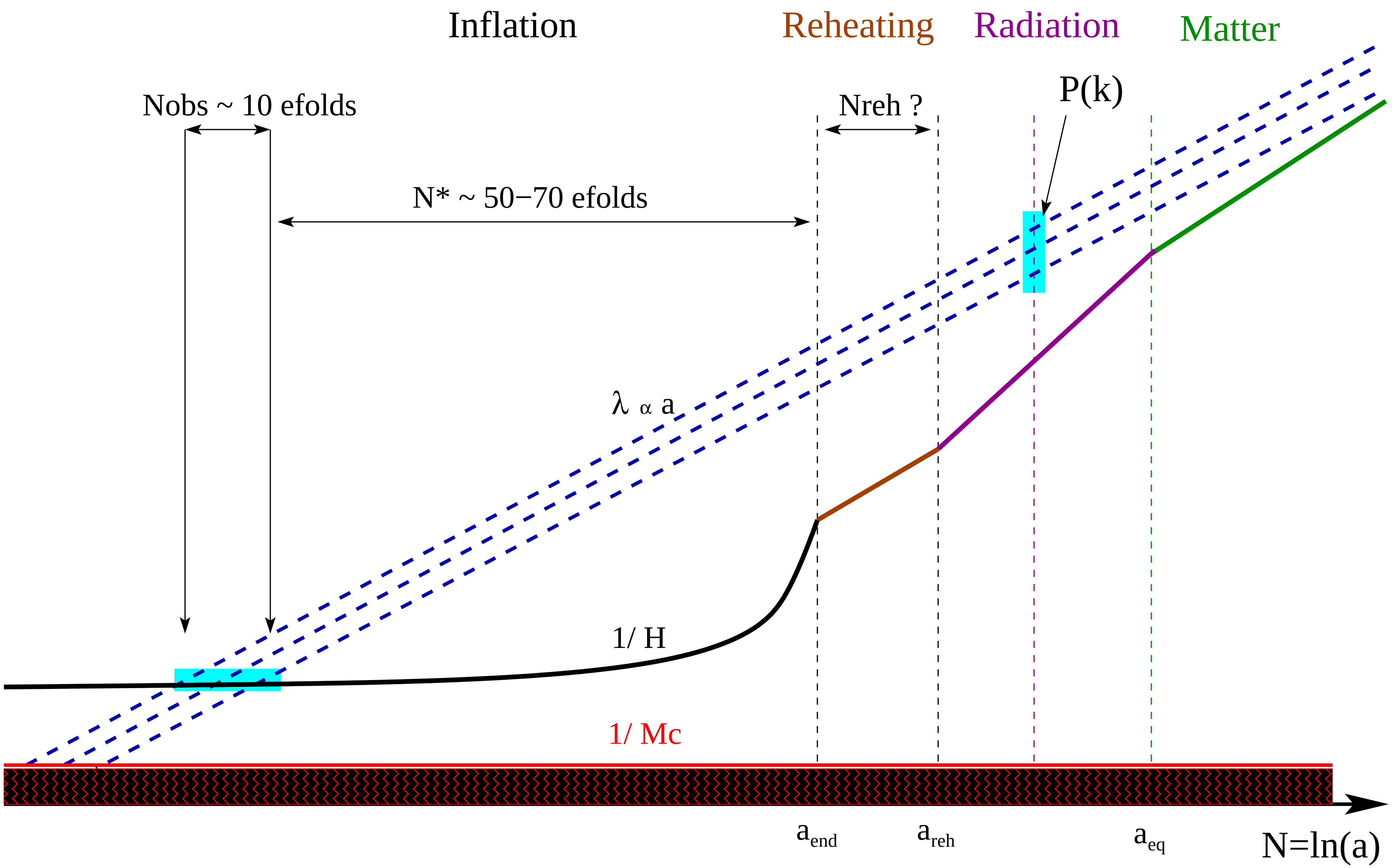


Theoretical proposals

- Background induced features
  - Inflation + steps [Starobinsky 92, Covi 06, Joy 08, Hazra 10]
  - Multiple inflation [Barriga 01, Hunt 04]
  - Oscillating pot. [Wang 05]
  - Variable mass fields [Langlois 05]
  - Multifieds inflation [Achucarro 10]
  - Bouncing universe models [Martin 03b, Falciano 08, Brandenberger 09]
  - Warm inflation [Barnaby 09]
  - Monodromy inflation [Flauger 10]
  - Cyclic inflation [Biswas 10]
- Non-standard perturbations
  - Non-vacuum initial state [Martin 00]
  - Modified dispersions [Corley 96, Brandenberger 01, Niemeyer 01]
  - Quantum deformations [Kempf 01, Easther 01, Hassan 03, Sriramkumar 06]
  - Non-commutative geometry [Lizzi 02, Tsujikawa 03]
  - Decaying modes [Amendola 05]
  - Higher order op. [Armendariz-Picon 09]
  - WKB violations [Kinney 08, Lorenz 08]
  - Defects during inflation [Tseng 09]
- Minimal trans-Planckian effects: non-standard initial conditions [Danielsson 02, Niemeyer 02, Easther 02, Martin 03a, Kaloper 03, Brandenberger 05, Greene 05]

Cosmological perturbations from inflation

- Sub-Hubble modes are also of sub-Planckian wavelength?



Perturbations of quantum mechanical origin

- Linear perturbations in the field $\delta\phi$ and metric $g_{\mu\nu}$

$$\mathrm{d}s^2 = -a^2(1+2\phi)\mathrm{d}\eta^2 + a^2\left[(1-2\Psi)\delta_{ij} + h_{ij}\right]\mathrm{d}x^i\mathrm{d}x^j$$

- Dynamics: perturbed KG + Einstein equations

$$\left.\begin{array}{l} \mu_\mathrm{T} \propto ah \\ \mu_\mathrm{S} \propto a(\delta\phi + \sqrt{2\epsilon_1}\Psi) \\ \epsilon_1 \equiv 1 - \mathcal{H}'/\mathcal{H}^2 \end{array}\right\} \Rightarrow \mu''_\mathrm{TS} + \underbrace{\left[k^2 - \frac{(a\sqrt{\epsilon_1})''}{a\sqrt{\epsilon_1}}\right]}_{\omega^2(\eta)} \mu_\mathrm{TS} = 0$$

- Canonical quantization [Mukhanov 81, Starobinsky 82]

$$\hat{\mu}(\eta, \boldsymbol{x}) = \int \frac{\mathrm{d}^3\boldsymbol{k}}{(2\pi)^{3/2}}\left[\hat{c}_{\boldsymbol{k}}(\eta_0)\mu_k^*(\eta)e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{c}_{\boldsymbol{k}}^\dagger(\eta_0)\mu_k(\eta)e^{-i\boldsymbol{k}\cdot\boldsymbol{x}}\right]$$

$$\hat{\mu}(\eta, \boldsymbol{k}) = \hat{c}_{\boldsymbol{k}}(\eta_0)\mu_k^*(\eta) + \hat{c}_{-\boldsymbol{k}}(\eta_0)\mu_k(\eta) \quad \Rightarrow \quad \mathcal{P} \propto k^3\left|\frac{\mu_{k_\mathrm{ST}}}{a\sqrt{\epsilon_1}}\right|^2$$

Standard initial conditions in slow-roll inflation

- Minkowski vacuum at decoupling $k \gg \mathcal{H}(\eta)$: positive energy waves

$$\omega(k) \to k \quad \Rightarrow \quad \lim_{k\eta \to -\infty} \bar{\mu}_k(\eta) = \frac{e^{-ik(\eta-\eta_0)}}{\sqrt{2k}} \quad \Leftrightarrow \quad \begin{cases} \bar{\mu}_k(\eta_0) = \dfrac{1}{\sqrt{2k}} \\ \bar{\mu}'_k(\eta_0) = -i\sqrt{\dfrac{k}{2}} \end{cases}$$

- Single field inflation: $\omega^2(k) = k^2 - \dfrac{\nu^2(\eta) - 1/4}{\eta^2}$
  - Slow-roll: $\nu_k^2 = 9/4 + 3\epsilon_{1k} + 3/2\epsilon_{2k} + \mathcal{O}\left(\epsilon^2\right)$ where $k\mathcal{H}(\eta_k) = 1$

$$\bar{\mu}_k(\eta) = \frac{\sqrt{\pi}}{2} e^{i(\pi/4+\nu_k/2)} \sqrt{-\eta} H^{(1)}_{\nu_k}(-k\eta)$$

- Super-Hubble spectra $(k\eta \ll 1)$ + pivot expansion: $k_* \mathcal{H}(\eta_*) = 1$

$$\bar{\mathcal{P}}_\zeta(k) = \frac{H_*^2}{8\pi^2\epsilon_{1*}} \left[1 - 2\left(C+1\right)\epsilon_{1*} - C\epsilon_{2*} - \left(2\epsilon_{1*} + \epsilon_{2*}\right)\ln\frac{k}{k_*}\right]$$

Non-standard initial conditions

- Both initial time and state may be altered
  - Mode creation at a constant physical length: $k/a(\eta_{0k}) = M_\mathrm{c}$
  - $k\eta \to -\infty$ does not apply: α-vacua $|c_k|^2 - |d_k|^2 = 1$ [Allen 85]

$$\mu_k(\eta_{0k}) = \frac{c_k + d_k}{\sqrt{2\omega(k,\eta_{0k})}}, \qquad \mu_k'(\eta_{0k}) = -i\sqrt{\frac{\omega(k,\eta_{0k})}{2}}\,(c_k - d_k)$$

- At any ulterior time: $\mu_k(\eta) = \alpha_k\bar{\mu}_k(\eta) + \beta_k\bar{\mu}_k^*(\eta)$

$$|\mu_k|^2 = \left(|\alpha_k|^2 + |\beta_k|^2\right)|\bar{\mu}_k|^2 + 2\Re\left\{\alpha_k\beta_k^*\bar{\mu}_k^2\right\}$$

- Instantaneous Minkowski vacuum: $c_k = 1$, $d_k = 0$ at $\eta = \eta_{0k}$ [Martin 03a]
  - Slow-roll, pivot and $\sigma_0 = \dfrac{H(\eta_0)}{M_\mathrm{c}}$ expansions: $\dfrac{k_0}{a(\eta_0)} = M_\mathrm{c}$

$$\frac{\mathcal{P}_\zeta(k)}{\bar{\mathcal{P}}_\zeta(k)} = 1 + \sigma_0^3\left[1 + \mathcal{O}(\epsilon_*)\right]\sin\left[\frac{2}{\sigma_0}\left(1 + \epsilon_{1*} + \epsilon_{1*}\ln\frac{k}{k_0}\right)\right] + \mathcal{O}(\epsilon_*)\,\sigma_0^3\cos\left[\ \right]$$

Decoupling amplitude and frequency

- Deviations from an initial instantaneous Minkowski state (η_{0k})

$$\begin{cases} c_k = 1 + y_k \sigma_0 + \dots \\ d_k = x_k \sigma_0 + \dots \end{cases} \quad \text{assuming} \quad \begin{cases} x_k \simeq |x| e^{i\varphi} \\ y_k \simeq y \quad (y + y^* = 0) \end{cases}$$

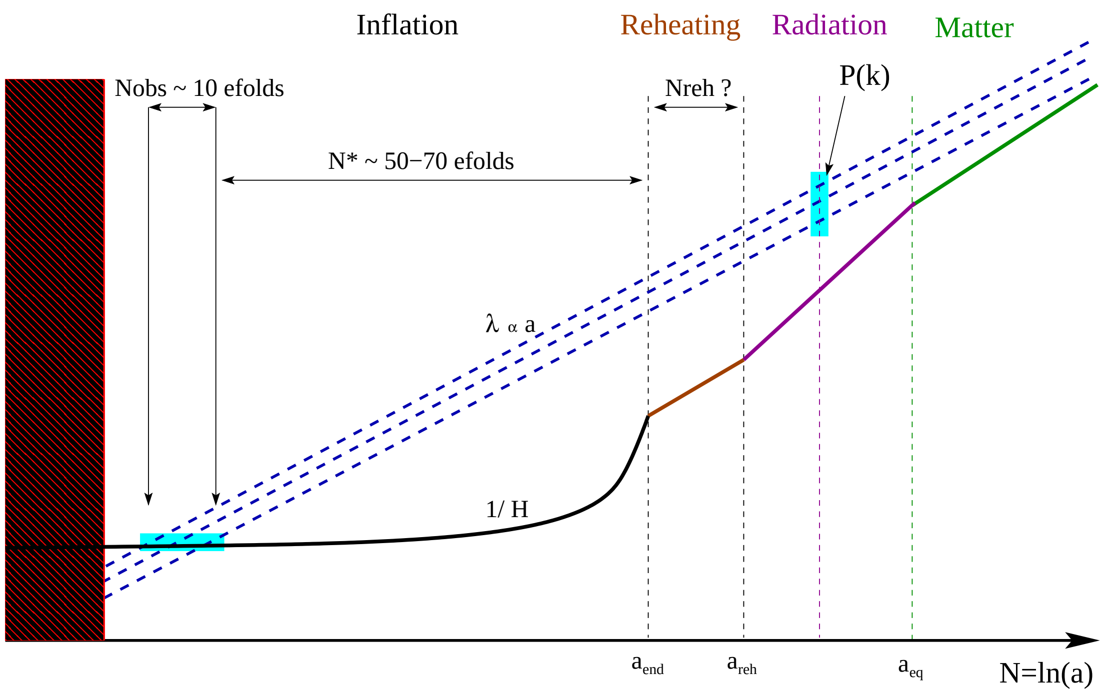
- Superimposed oscillations in the power spectra

$$\mathcal{P}_\zeta = \bar{\mathcal{P}}_\zeta \left\{ 1 - 2|x|\sigma_0 \cos\left[\frac{2}{\sigma_0}\left(1 + \epsilon_{1*} + \epsilon_{1*} \ln\frac{k}{k_0}\right) + \varphi\right]\right\} - \frac{H_*^2}{8\pi\epsilon_{1*}}(2\epsilon_{1*} + \epsilon_{2*})|x|\sigma_0 \sin\left[\frac{2}{\sigma_0}\left(1 + \epsilon_{1*} + \epsilon_{1*} \ln\frac{k}{k_0}\right) + \varphi\right]$$

$$\mathcal{P}_h = \bar{\mathcal{P}}_h \left\{ 1 - 2|x|\sigma_0 \cos\left[\frac{2}{\sigma_0}\left(1 + \epsilon_1 + \epsilon_1 \ln\frac{k}{a_0 M_c}\right) + \varphi\right]\right\} - \frac{4H_*^2}{\pi}\epsilon_{1*}|x|\sigma_0 \sin\left[\frac{2}{\sigma_0}\left(1 + \epsilon_{1*} + \epsilon_{1*} \ln\frac{k}{k_0}\right) + \varphi\right]$$

Other initial condition choices

- IC fixed at a constant initial time: oscillations in $\cos(2k/\sigma_0)$

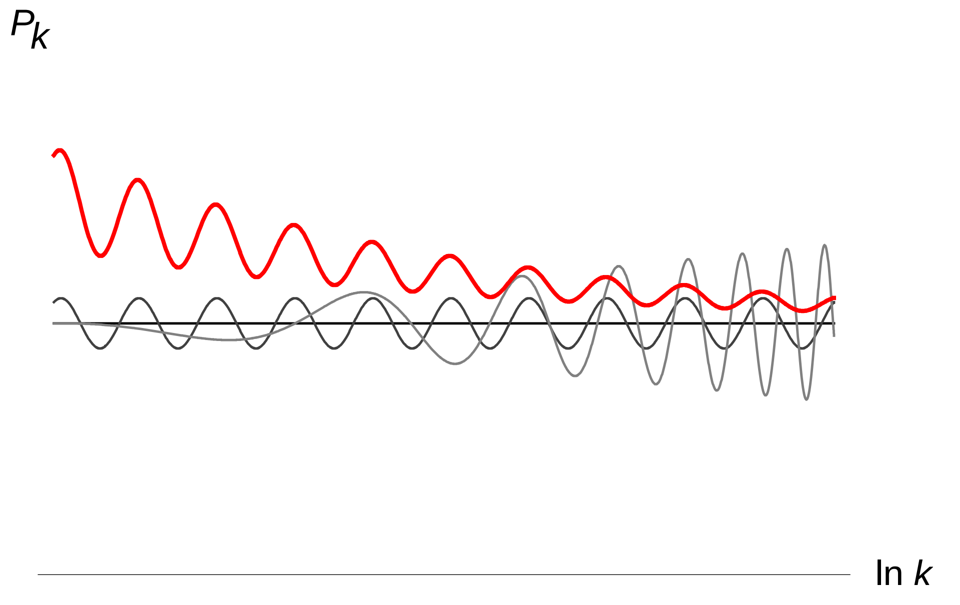


- Initial states: boundary field theories [Porrati 04, Greene 05, Schalm 05, Collins 06]
    - High energy physics encoded into new operators
    - Allows derivations of back-reaction + (loop corrections?)

Oscillations are generic

- But can be of $\neq$ shape and of $\neq$ amplitudes!
  - Integrating out massive extra degrees of freedom (in-in) [Jackson 10]

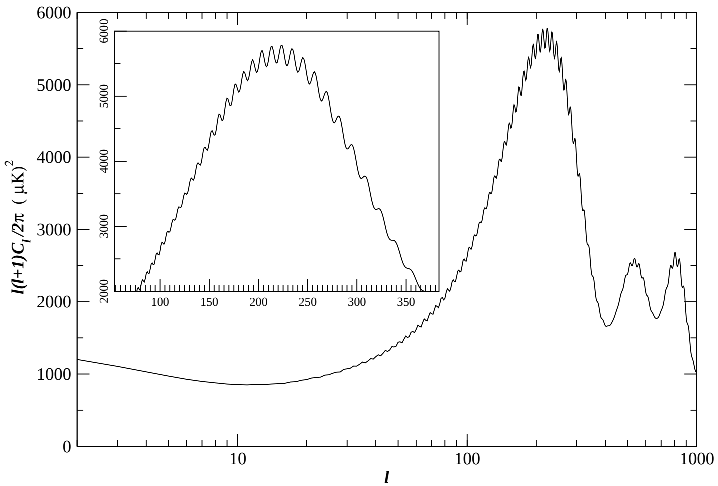
$$\mathcal{P} = \lim_{k\eta\to 0} \frac{k^3}{2\pi^2} \langle 0(\eta_0)| U^\dagger(\eta_0,\eta)\, \hat{\mu}_k^\dagger \hat{\mu}_k \, U(\eta_0,\eta)|0(\eta_0)\rangle$$

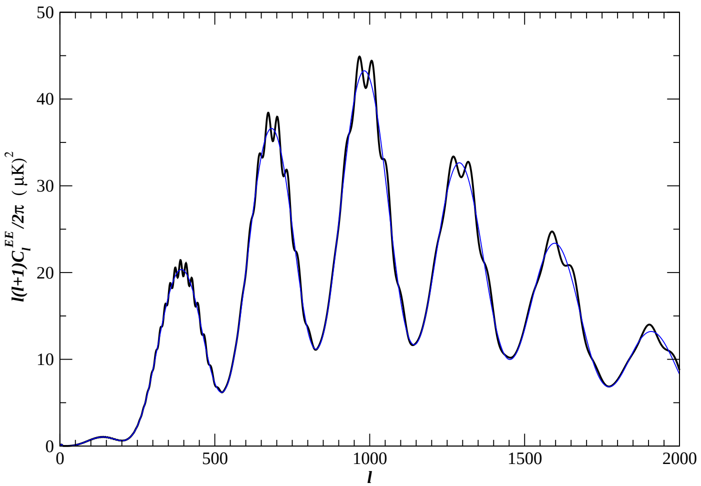


- What are the observable consequences?

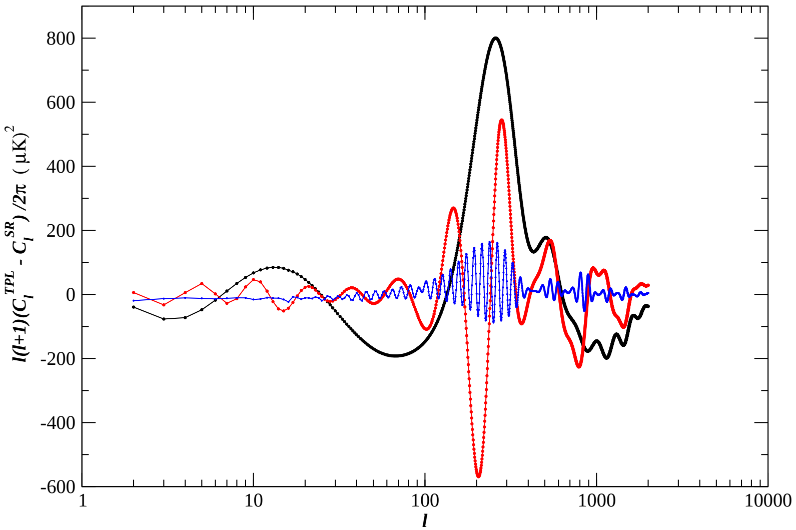
Super-imposed oscillations in the CMB spectra

- Minimal trans-Planckian: $\epsilon_{1*}/\sigma_0 \simeq 10^{1.8}$, $2|x|\sigma_0 \simeq 0.2$





- Amplitude damped for large scales and high-frequencies



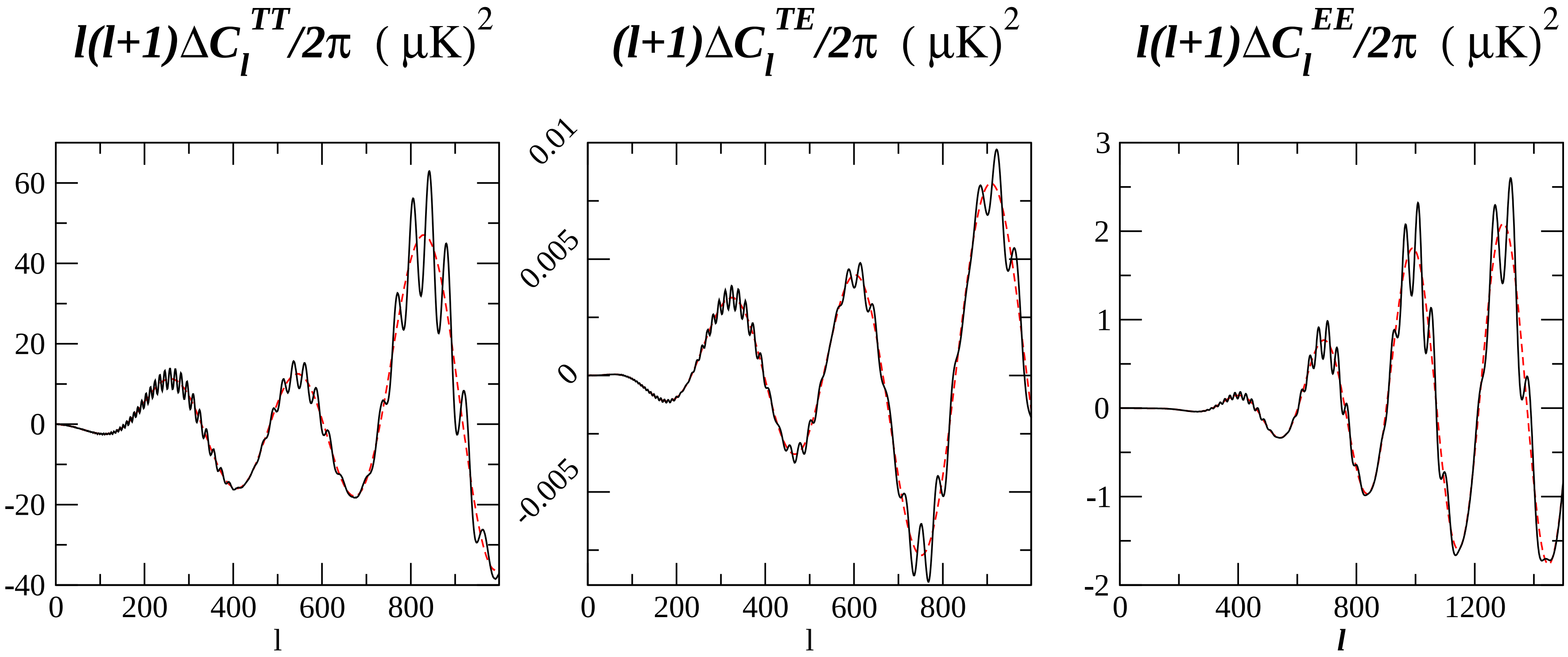
- Large scale with $\epsilon_{1*}/\sigma_0 \gg \ell$

$$\ell(\ell+1)C_\ell \simeq \frac{H_*^2}{100\pi^2\epsilon_{1*}} \times \left[1 + \sqrt{\pi}\frac{|x|\sigma_0\ell(\ell+1)}{(\epsilon_{1*}/\sigma_0)^{5/2}}\cos\left(\pi\ell+\psi\right)\right]$$

Not washed out by lensing

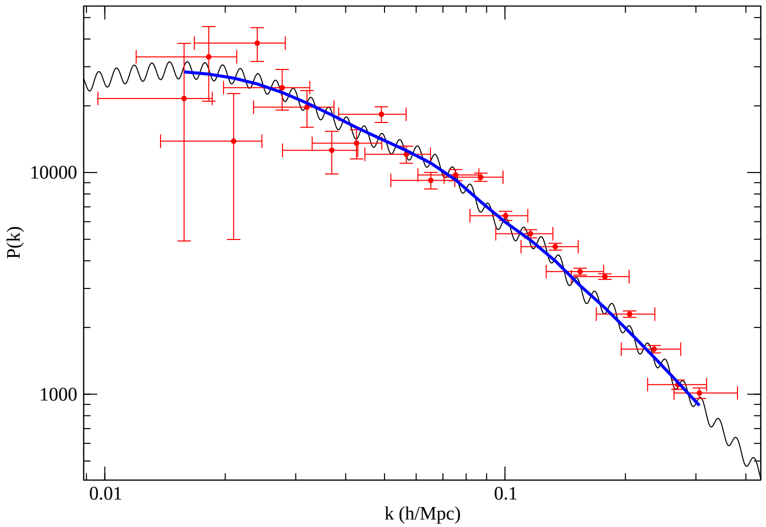
- Linear terms dominate [Hu 00]

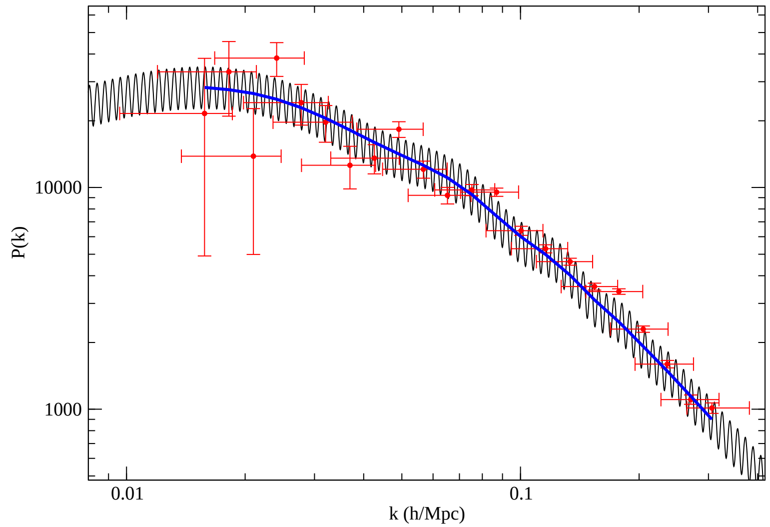
$$\widetilde{C}_l \simeq (1 - l^2 R) C_l + \int C_{l'-l} F(l', l - l')\, \mathrm{d}l'$$



Matter power spectrum

- Direct window on the primordial spectra
  - High frequency killed in CMB ⇒ large oscillations in $P_{\mathrm{m}}(k)$?



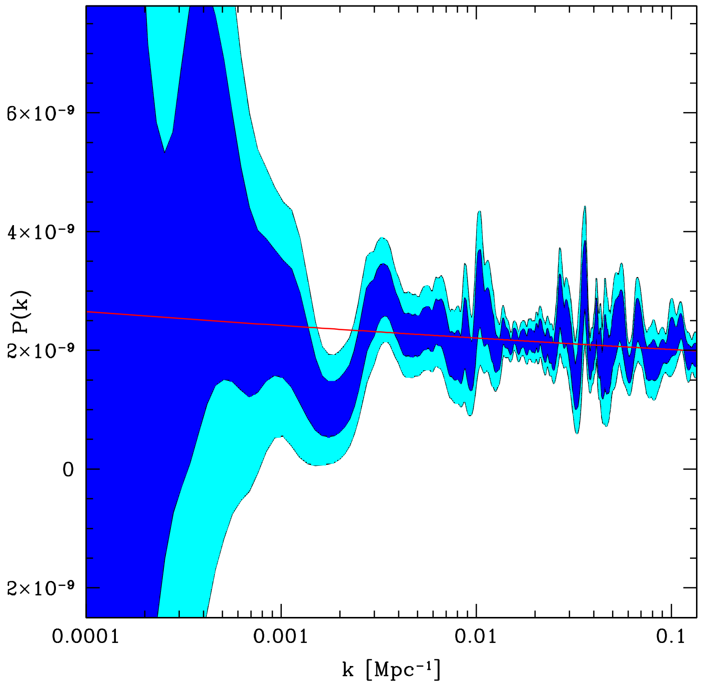


  - Linear regime: oscillations are smoothed out by the window functions width (example SDSS)
- Non-linear regime? Small halos distribution affected [Felippe S. Rodrigues 10]

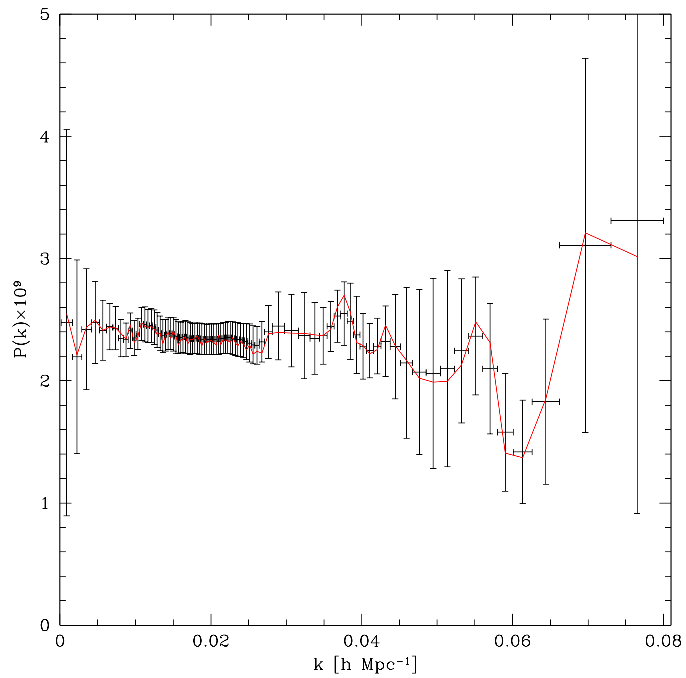
Reconstructing primordial power spectra from CMB data

- Power spectra cut into band powers and invert C_ℓ data with your favorite tool (deconvolution) [Wang 99, Bridle 03, Tocchini-Valentini 05, Shafieloo 07, Peiris 10]
  - There are as many extra-parameters as band powers
  - WMAP5 + SN1a + HST+ BBN data

Fixed cosmo, WMAP only [Nicholson 09]



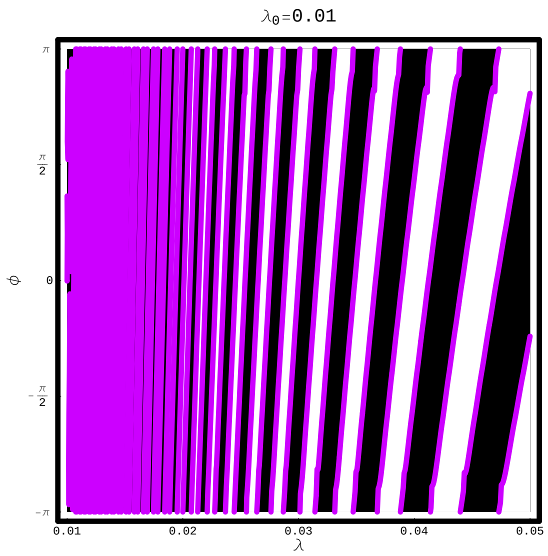
All data [Nicholson 10]



- Statistical significance is difficult to assess

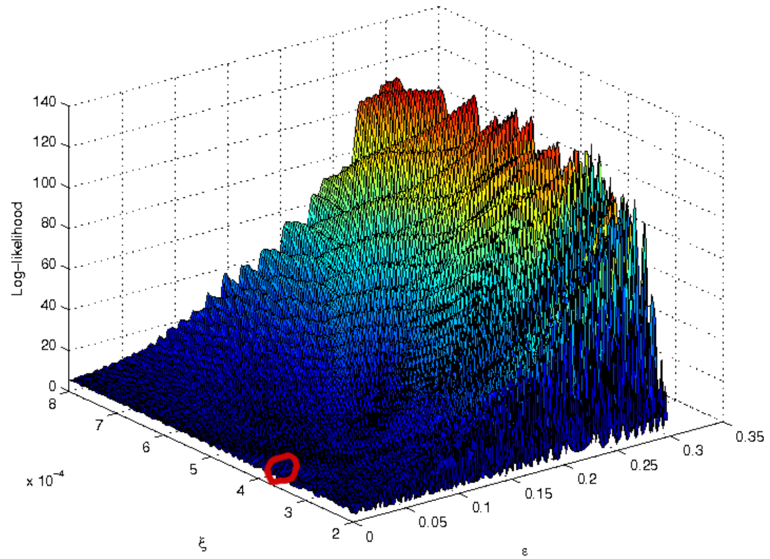
Looking for wiggles directly into CMB data

- Multi-parameter space: cosmo + astro + oscillations
  - Most approaches have been focused on $|x| = 1$ (no detection)
  - Griding methods, Fisher matrix analysis [Bergstrom 02, Elgaroy 03, Okamoto 04]
  - Forecast depends on $\epsilon_{1*} \gtrsim 10^{-3}$: at best $\sigma_0 \sim 10^{-4}$ [Hamann 08]
- Hedgehog likelihood $-2\ln\mathcal{L}^{TT} = \sum_\ell (2\ell+1)\left(\ln\dfrac{C_\ell}{\hat{C}_\ell} + \dfrac{C_\ell}{\hat{C}_\ell} - 1\right)$



[Easther 05]

[Groeneboom 08]



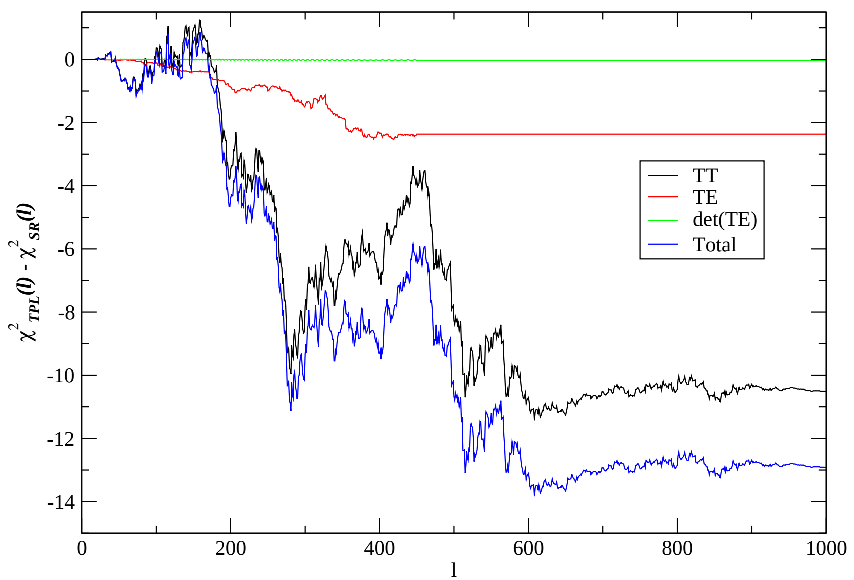
- Renders parameter estimations hard but that's a signature [Easther 05]

Independent amplitude and frequency

- 3 oscillation parameters [Martin 04b, Martin 04a, Martin 05]

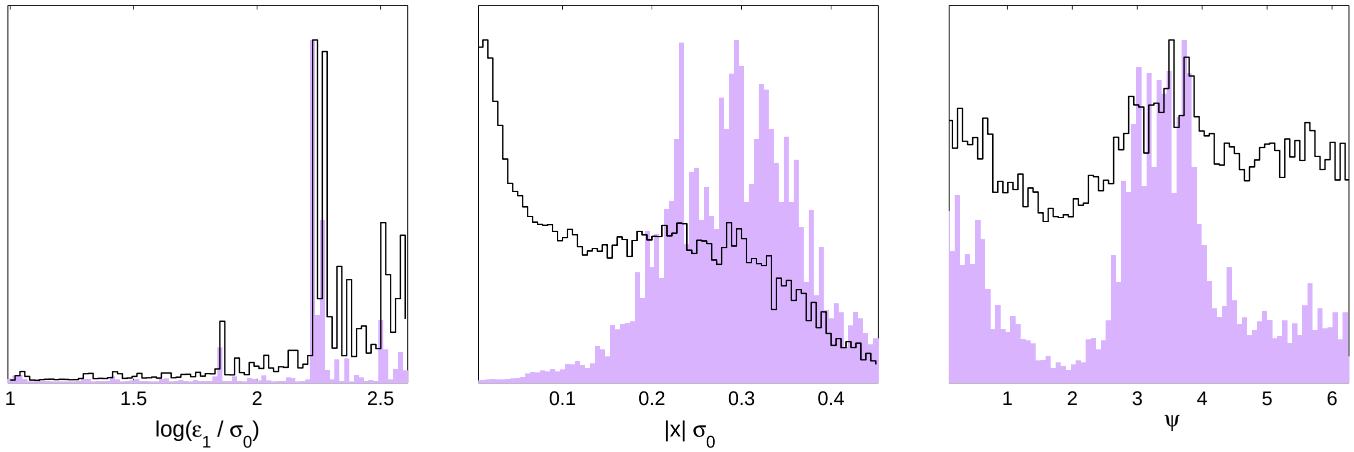
$$|x|\sigma_0, \qquad \frac{\epsilon_{1*}}{\sigma_0}, \qquad \psi \equiv \frac{2}{\sigma_0}(1+\epsilon_{1*}) + \varphi$$

- Best fit for WMAP3 had $\Delta\chi^2 \simeq -13$
  - Large high-frequency oscillations: $\log \dfrac{\epsilon_{1*}}{\sigma_0} \simeq 2.23$, $|x|\sigma_0 \simeq 0.27$
  - Fitted small-scale outliers

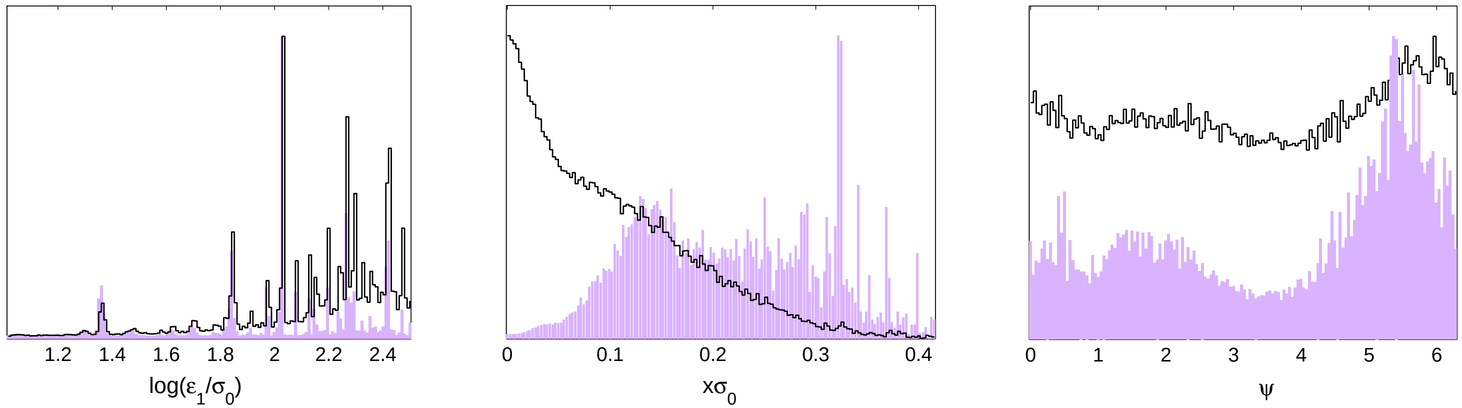


Posterior probabilities and mean likelihood

- WMAP3: $|x|\sigma_0 < 0.38$ at 95% CL [Martin 06]



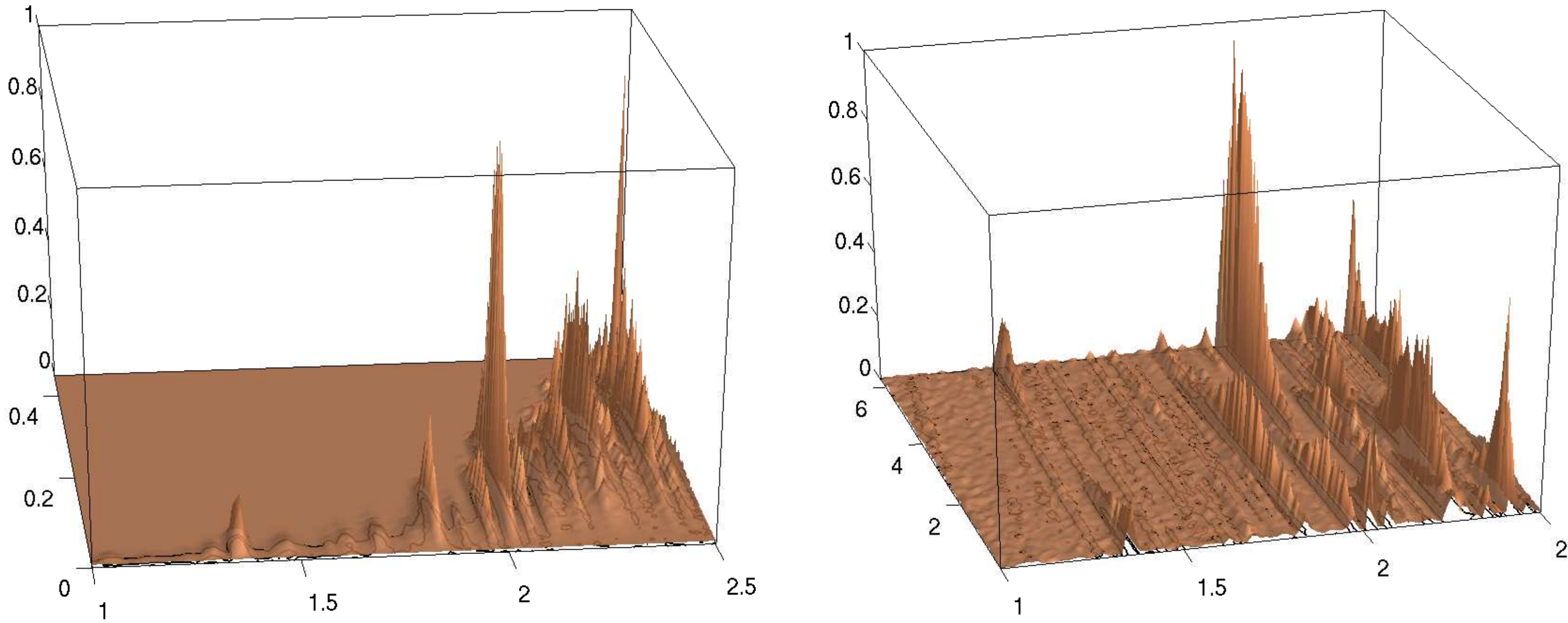
- Current WMAP7 data: $|x|\sigma_0 < 0.26$ (95%); best fit $\Delta\chi^2 \simeq -11$



- Oscillations = better fit but not favoured (prior space killed); but. . .

WMAP7 likelihood shape

- In the plane {frequency-amplitude} and {frequency-phase}

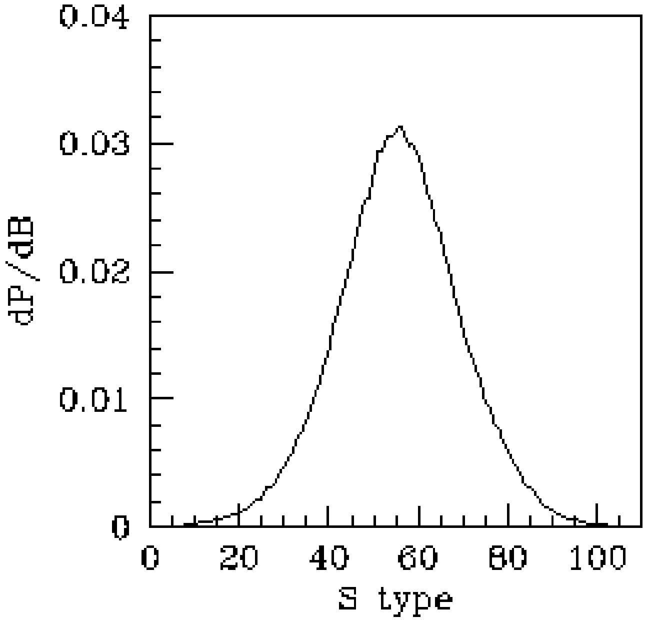


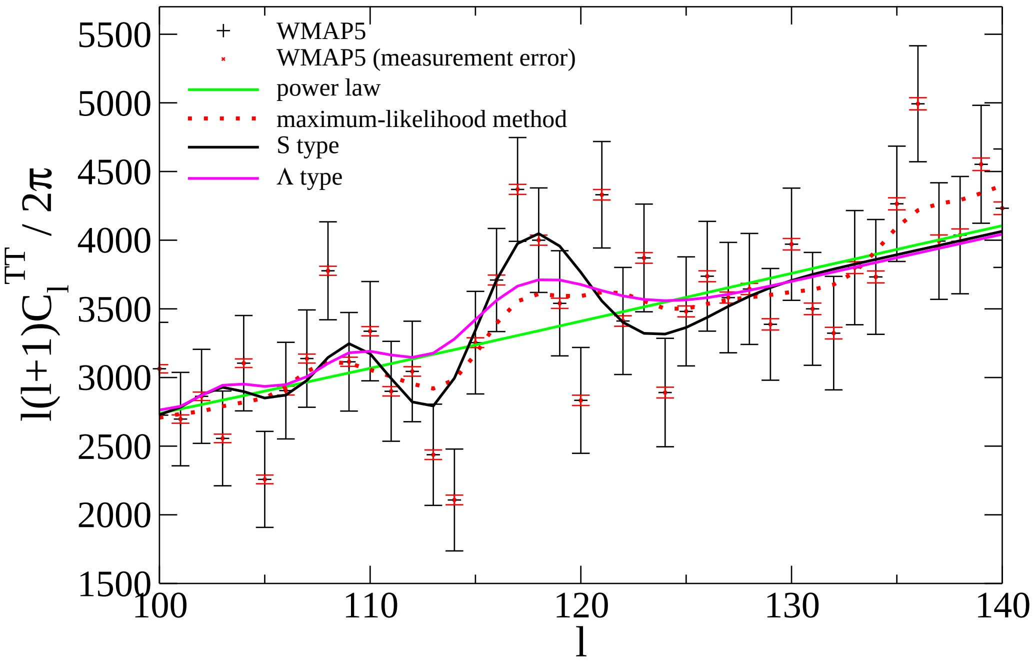
- This can be interpreted as oscillations signature

Bayesian detection of a modulated feature

- 3 extra-parameters + WMAP5: 4σ non-vanishing B [Nagata 09, Ichiki 10]

$$\frac{\mathcal{P}(k)}{\bar{\mathcal{P}}(k)} = B \exp\left[-(k-k_0)^2/\kappa^2\right] \cos\left[\pi(k-k_0)/\kappa\right]$$





Conclusion

- Oscillations and features in the CMB
  - Remember: may not necessarily come from $\mathcal{P}(k)$
  - Primordial effects reflect
    - Non-standard inflationary background evolution
    - Perturbation dynamics and/or IC are changed
  - Seems to be present in the current data
- Disambiguation needs other observables
  - Polarization, matter power spectrum, 21cm...
  - Non-Gaussianities (see other talks)

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