CMB NON-GAUSSIANITIES: STATISTICAL METHODS AND THEIR APPLICATIONS

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OUTLINE

1. Non-Gaussianity: Brief Intro
2. Statistical Techniques
3. Pixel Statistics
4. Minkowski Functionals
5. Basic Highlights

MAIN REFERENCES

- **G. Rossi**, P. Chingangbam & C. Park (2010), MNRAS in press
- **G. Rossi**, R. K. Sheth, C. Park & C. Hernández-Monteagudo (2009), MNRAS, 399, 304-316
- P. Chingangbam, **G. Rossi** & C. Park, JCAP in prep.

Non-Gaussianity as a Probe of New Physics

Non-Gaussianity as a Probe of the Physics of the Primordial Universe and the Astrophysics of the Low Redshift Universe

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In the coming decade, *non-Gaussianity* will become an important probe of both the early and the late Universe. Specifically, it will play a leading role in furthering our understanding of two fundamental aspects of cosmology and astrophysics → NEW PHYSICS RELATED TO COSMOLOGY

- The physics of the very early universe that created the primordial seeds for large-scale structures
- The subsequent growth of structures via gravitational instability and gas physics at later times

Why is Non-Gaussianity Important?

Canonical Slow-Roll Inflation

- φ free scalar field in ground state of Bunch-Davis vacuum
- $\mathcal{R} = -[H(\phi)/\dot{\phi}_0]\varphi$ primordial curvature pert. (linear order)
- If $p(\varphi) \rightarrow$ Gaussian then $p(\mathcal{R}) \rightarrow$ Gaussian

Slow-Roll Inflation – Breaking Gaussianity

- NG $\rightarrow$ Allow interactions between scalar fields
- NG $\rightarrow$ Non-linear corrections to the relation $\mathcal{R} \rightarrow \phi$

Beyond Canonical Models

- Non-standard inflationary models (ex. $\rightarrow$ *Sasaki 2008*)
- Alternative early-universe models (ex. $\rightarrow$ *Brandenberger 2009*)

WHY NG NOW?

GAUSSIANITY

- **1980-1989** → 153 titles, 1387 abstracts
- **1990-1999** → 723 titles, 5835 abstracts
- **2000-2010** → 3466 titles, 14946 abstracts

NON-GAUSSIANITY

- **1980-1989** → 0 titles, 0 abstracts
- **1990-1999** → 31 papers titles, 85 abstracts
- **2000-2010** → 495 titles, 1266 abstracts

"VIVE LA RESOLUTION" → BOUCHET'S TALK

- **1980-1989** → No powerful observational probes
- **1990-1999** → COBE
- **2000-2010** → WMAP, PLANCK

Describing NG: the $f_{\rm NL}$ or $g_{\rm NL}$ business

$$\langle \Phi(\mathbf{k_1})\Phi(\mathbf{k_2})\Phi(\mathbf{k_3})\rangle = (2\pi)^3\delta^3(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})F(k_1,k_2,k_3)$$

$$\Phi = \phi_{\rm L} + f_{\rm NL}\cdot(\phi_{\rm L}^2 - \langle\phi_{\rm L}^2\rangle) + g_{\rm NL}\cdot\phi_{\rm L}^3 + \ldots$$

- **Multi-Field Models $\rightarrow$ Break Single-Field**
  - Source of density perturbation $\rightarrow$ second light scalar field σ

$$F(k_1,k_2,k_3) = f_{\rm NL}^{\rm local}\, 2\Delta_\Phi^2\left(\frac{1}{k_1^3k_2^3}+\frac{1}{k_1^3k_3^3}+\frac{1}{k_2^3k_3^3}\right)$$

  - Amplitude of bispectrum of *"squeezed"* triangles
  - Curvaton scenario, variable decay width model, ...

- **Single-Field Models $\rightarrow$ Break Slow-Roll**

$$F(k_1,k_2,k_3) = f_{\rm NL}^{\rm equil}\, 6\Delta_\Phi^2\left(\frac{1}{k_1k_2^2k_3^3}+\ldots\right)$$

  - Amplitude of bispectrum of *"equilateral"* configurations
  - Preheating, field-dependent variable, ...

- **Alternative Models**
  - Ekpyrotic scenario
  - String gas
  - Cosmic strings

INFLATION MODELS $\rightarrow f_{\rm NL}$ AND $g_{\rm NL}$

MODEL	$f_{NL}(\mathbf{k_1}, \mathbf{k_2})$
Canonical inflation	$\simeq 0.1$
Curvaton	$5/4r$
Modulated reheating	$-5/4 - I$
Multi-field inflation	large?

MODEL	$f_{NL}(\mathbf{k_1}, \mathbf{k_2})$
Ekpyrotic models	$-50 < f_{\rm NL} < 200$
Generalized slow-roll	$f_{\rm NL} \gg 1$
Warm inflation	typically $\simeq 0.1$
Multi-DBI inflation	...

WMAP7 (95 % CL)

- $-10 < f_{\rm NL}^{\rm local} < 74$
- $-151 < f_{\rm NL}^{\rm equil} < 253$

OTHER PROBES (95 % CL)

- $-29 < f_{\rm NL}^{\rm local} < 70$ (Slosar et al. 2009)
- $-4 < f_{\rm NL}^{\rm local} < 80$ (Smith et al. 2009)
- $-36 < f_{\rm NL}^{\rm local} < 58$ (Smidt et al. 2010)

MODEL	$g_{NL}(\mathbf{k_1}, \mathbf{k_2})$
Slow-roll inflation (including multiple fields)	$O(\epsilon, \eta)$
Curvaton scenario	$\lvert g_{\rm NL}\rvert \simeq 10^5$
Inhomogeneous reheating	$(5/3)f_{\rm NL}^2 + \ldots$
DBI inflation	$\simeq 0.1 c_s^4$
Ekpyrotic models	$\lvert g_{\rm NL}\rvert \lesssim 10^4$

SDSS + N-BODY

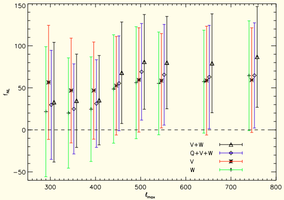
$-3.5 \times 10^5 < g_{\rm NL} < +8.2 \times 10^5$
(Desjacques & Seljak 2010)

NG: EVIDENCE? DETECTION?

Profound implications if non-Gaussianity detected

NON-GAUSSIANITY → EVIDENCE? DETECTION?

- One-point statistics (Jeong & Smoot 2007)
- Bispectrum estimator (Yadav & Wandelt 2008)



ANOMALIES OF ANY KIND

- *"The mystery of the WMAP cold spot"* (Naselsky et al. 2008)
- *"The CMB cold spot: texture, cluster or void?"* (Cruz et al. 2008)
- *"CMB cold spot: a gate to extra dimensions?"* (Cembranos et al. 2008)
- *Asymmetries, alignments* (i.e. Kim & Naselsky 2010)

NG in the CMB: Methods and Philosophy

- **Real-space methods (selection)**
  - Pixel statistics
  - Peak statistics
  - Morphology of hotspots
  - Fractal analysis
  - Minkowski functionals
- **Harmonic-space methods (selection)**
  - Bispectrum
  - Trispectrum
  - Wavelets

Modus operandi

- Choose a priori the statistics
- Select type of primordial NG
- Test statistics performance under assumed NG

Conceptual Points

- Choice of statistics $\rightarrow$ a priori!
- Type of primordial NG is unknown
- A posteriori statistics $\rightarrow$ misleading
- Concept of "optimal" $\rightarrow$ related to the type of NG
- Geometrical and topological tests

NG IN THE CMB: STRATEGY

STRATEGY

(1) Theory (2) Simulations (3) Data Analysis

MAP-MAKING PROCEDURE

Method:

$$\Delta T(\hat{n}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{n})$$

Rewrite $a_{\ell m}$ as real space integral

$$a_{\ell m} = \int dr\, r^2 \Phi_{\ell m}(r) \Delta_\ell(r)$$

$$\Phi_{\ell m}(r) \equiv \Phi^{G}_{\ell m}(r) + f_{NL}\Phi^{NG}_{\ell m}(r) + g_{NL}\Phi^{NNG}_{\ell m}(r)$$

Technique, Assumptions, Notation

Main Technique

Statistics of hot and cold pixels above threshold (excursion sets)

Assumptions

$D = T - \langle T \rangle \equiv \delta T = s + n$

Signal: homogeneous, may have spatial correlations

Noise: independent of signal, inhomogeneous, spatial correlations

Basic notation

$p(D)$: observed one-point distribution of D

$G(s)$: distribution of s

$p(\sigma_n)$: rms noise distribution

$g(n|\sigma_n)$: distribution of the noise when its rms value is σ_n

ONE- AND TWO-POINT FUNCTIONS

ONE-POINT FUNCTION

$$\begin{aligned} p(D) &= \int d\sigma_n \, p(\sigma_n) \int ds \, G(s) \, p(D - s|\sigma_n) \\ &= \int d\sigma_n \, p(\sigma_n) \, p(D|\sigma_n) \end{aligned}$$

TWO-POINT FUNCTION

$$p(D_1, D_2|\theta) = \int_0^\infty d\sigma_1 \int_0^\infty d\sigma_2 \, p(\sigma_1, \sigma_2|\theta) \, p(D_1, D_2|\sigma_1, \sigma_2, \theta)$$

- Assume PDFs to be Gaussian or non-Gaussian
- Measure $p(\sigma_n)$ and $p(\sigma_1, \sigma_2|\theta)$ from data

Number Density and Clustering

→ Merge the noise model into the two-point statistics formalism
→ Obtain the two-point function above or below threshold
→ Provide analytic formulae in the weak non-Gaussian limit

Main Goals → ND & Clustering

$$\text{Number Density} \to n_{\rm pix}(\nu) = \frac{N_{\rm pix,tot}}{4\pi} \cdot P_1, \quad (1)$$

$$\text{Clustering} \to 1 + \xi_\nu(\theta) = P_2/P_1^2, \quad (2)$$

$$P_1 = \int_\nu^\infty p(D)\mathrm{d}D \quad (3)$$

$$P_2 = \int_\nu^\infty \mathrm{d}D_1 \int_\nu^\infty \mathrm{d}D_2\, p(D_1, D_2, w) \quad (4)$$

Edgeworth Expansions

- Conceptually simple
- Use Edgeworth expansion around a Gaussian field

$$p(\mu)\mathrm{d}\mu \approx \frac{1}{\sqrt{2\pi}} e^{-\mu^2/2} \left\{ 1 + \frac{\sigma S^{(0)}}{6} \mu(\mu^2 - 3) \right\} \mathrm{d}\mu, \quad (5)$$

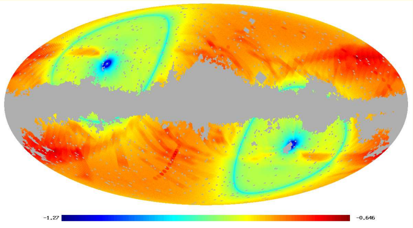
$$n_{\rm pix}^{\rm NG}(\nu) = n_{\rm pix}^{\rm G}(\nu) + n_{\rm pix}^{f_{\rm NL}}(\nu) \quad (6)$$

$$n_{\rm pix}^{f_{\rm NL}}(\nu) = \frac{N_{\rm pix,tot}}{4\pi} \left\{ \frac{\sigma S^{(0)}}{6\sqrt{2\pi}} (\nu^2 - 1) e^{-\nu^2/2} \right\}. \quad (7)$$

$$p(\mu_1, \mu_2, w)\mathrm{d}\mu_1 \mathrm{d}\mu_2 \approx \frac{1}{2\pi\sqrt{1-w^2}} \exp\left\{ -\frac{\mu_1^2 + \mu_2^2 - 2\mu_1\mu_2 w}{2(1-w^2)} \right\}$$
$$\times \left[1 + \sigma S^{(0)} \left(\frac{H_{30} + H_{03}}{6} \right) + \lambda \left(\frac{H_{21} + H_{12}}{2} \right) \right] \mathrm{d}\mu_1 \mathrm{d}\mu_2 \quad (8)$$

MAIN MOTIVATION → WMAP5 ANOMALIES

Rossi et al. (2009)



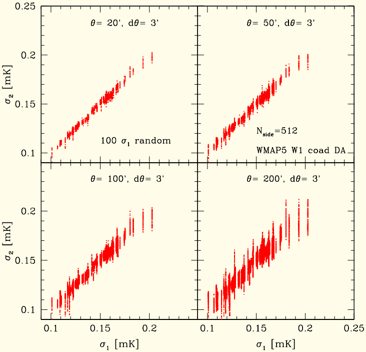
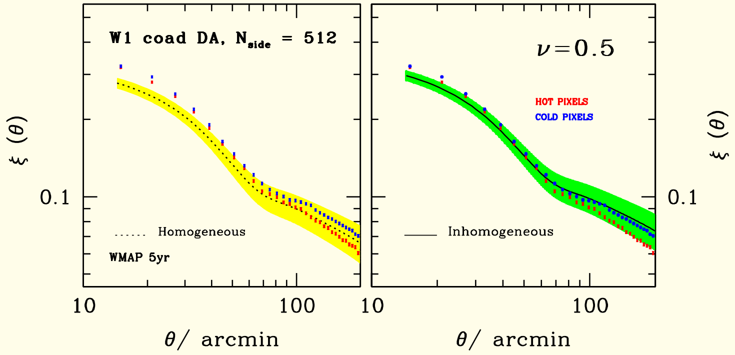


FIGURE: Joint distribution $p(\sigma_1, \sigma_2|\theta)$ at four different angular distances

- Pixel covariance matrix is diagonal
- $\sigma(p) = \sigma_0/\sqrt{N_{obs}(p)}$
- Noise is inhomogeneous

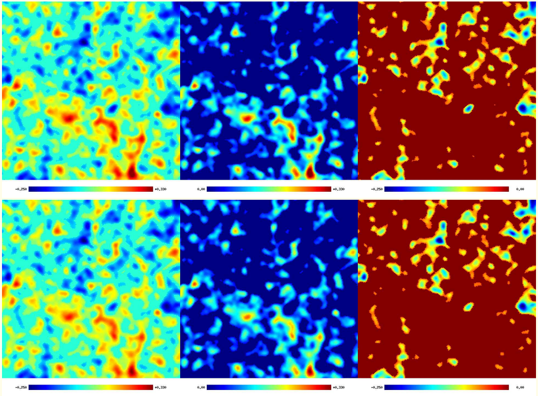
MAIN MOTIVATION → WMAP5 ANOMALIES

Rossi, Sheth, Park & Hernández-Monteagudo (2009)



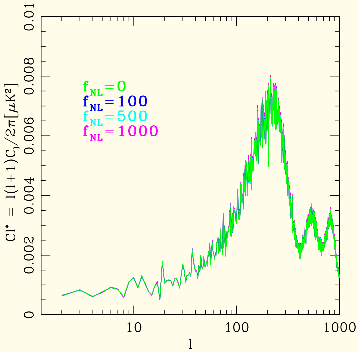
NON-GAUSSIAN SIMULATIONS: f_{NL} MOCK MAPS

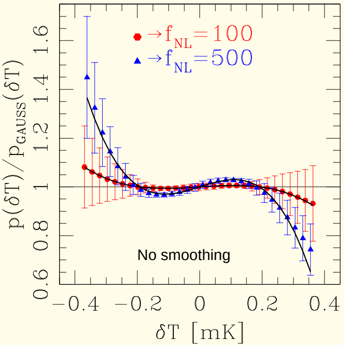
ALL HOT COLD



- $\simeq 10^\circ \times 10^\circ$
- $f_{\mathrm{NL}} = 0 \uparrow$
- $f_{\mathrm{NL}} = 500 \downarrow$
- $\nu = 0.50$
- FWHM=30'

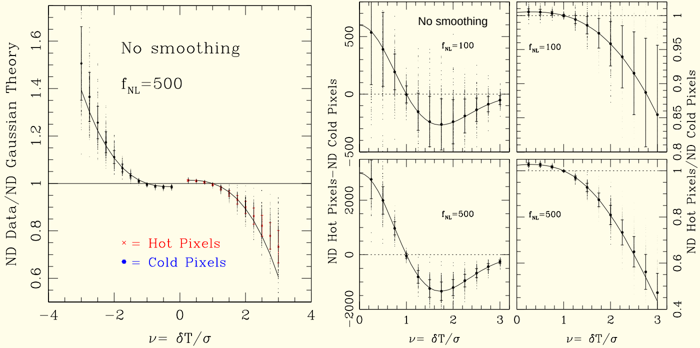
Power Spectra and Temperatures





PIXEL NUMBER DENSITY AND NG

Rossi, Chingangbam & Park (2010)

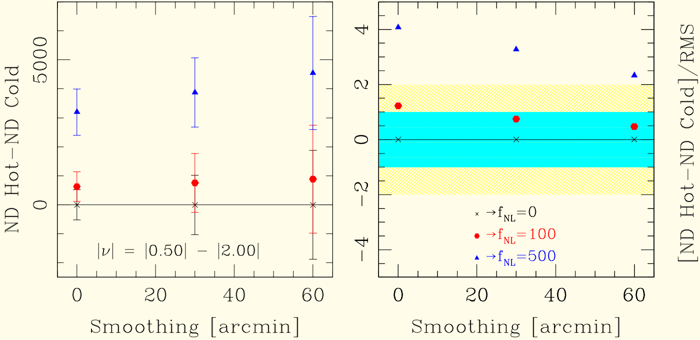


Solid lines → Theory predictions using the Edgeworth expansion

(1) Regions where NG is maximized (2) Non-optimal ν (3) $f_{\rm NL}$ and ND

Number Density and NG → A New Estimator

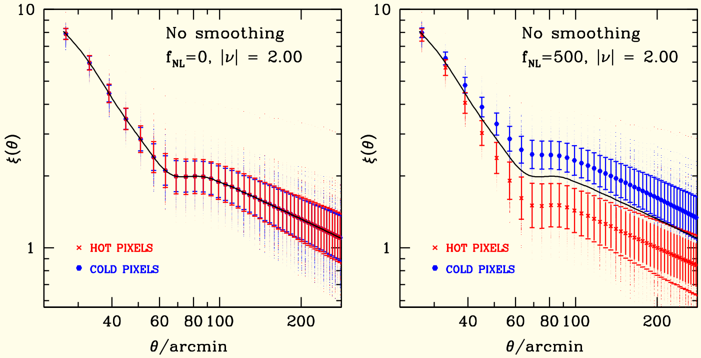
REAL UNITS **RMS UNITS**



Derived quantity which amplifies the $f_{\rm NL}$ contribution

PIXEL CLUSTERING AND NG

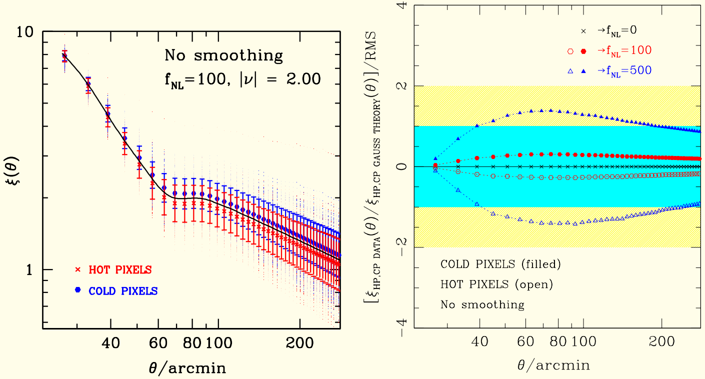
Rossi, Chingangbam & Park (2010)



$|\nu| = 2.00$, $f_{\rm NL} = 500 \rightarrow$ cold pixel clustering enhanced around $\theta \simeq 75'$

PIXEL CLUSTERING AND NG

Rossi, Chingangbam & Park (2010)

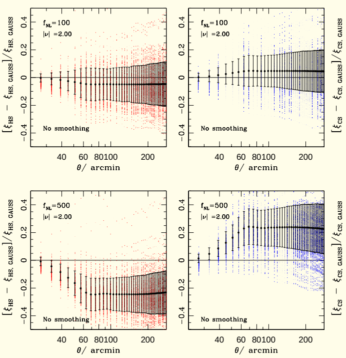


Is

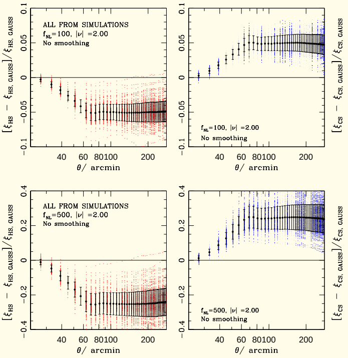
Gaussianity well-constrained? Sensitive test?

The Cosmic Variance Problem

Rossi, Chingangbam & Park (2010)



Real case



Idealized case

GAUSSIANITY
STATISTICAL TECHNIQUES
PIXEL STATISTICS
MINKOWSKI FUNCTIONALS
HIGHLIGHTS

Minkowski Functionals: Theory

Gott et al (1990)

Threshold: $\nu \equiv \frac{\Delta T}{\sigma_0}, \quad \sigma_0 = \sqrt{\langle \Delta T \Delta T \rangle}$.

- Area fraction above threshold $\sim V_0(\nu)$
- Contour length of iso-temperature contours $\sim V_1(\nu)$
- Genus $=$ number of hot spots - number of cold spots $\sim V_2(\nu)$

For Gaussian field:

$$V_k(\nu) \propto \left(\frac{\sigma_1}{\sigma_0}\right)^k \exp^{-\nu^2/2} H_{k-1}(\nu), \quad \sigma_1 = \sqrt{\langle |\nabla(\Delta T)|^2 \rangle}$$

SUMMARY

NON-GAUSSIANITY: NEW FRONTIER

- Reliable theoretical prediction of NG from models
- Extract information on non-Gaussianity from data
- Characterization of non-Gaussian confusion effects

ACHIEVEMENTS: OBSERVATIONAL SIDE

- New model for the effects of inhomogeneous noise
- Anomalies detected and plausible explanations

ACHIEVEMENTS: THEORETICAL SIDE

- Excursion set statistics extended to $f_{\rm NL}$ models
- Theoretical insights: optimal thresholds, Edgeworth approximation
- New statistical tests, in order to minimize cosmic variance

Waiting for Planck ...

Constraining/detecting NG smoking-gun for non-standard inflation models

- Planck gains a factor of 2.5 in angular resolution and up to 10 in instantaneous sensitivity with respect to WMAP
- Nearly photon noise limited in the CMB channels
- Temperature PS limited by ability to remove foregrounds
- 2 acoustic peaks above WMAP V band
- Polarization, NG, SZ clusters
- Most accurate microwave experiment to date in terms of control, reduction and correction of systematic and stochastic noise

Kamsahamnida!