

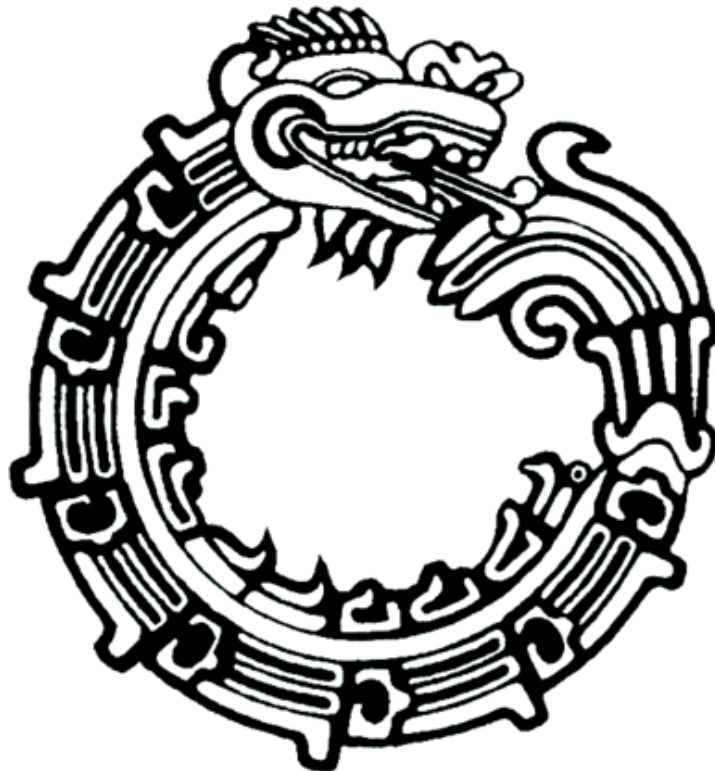
Aspects of String Cosmology

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The Serpent eats its Tail!

Physics at the longest of distance scales is intimately tied to physics at the shortest of distance scales!



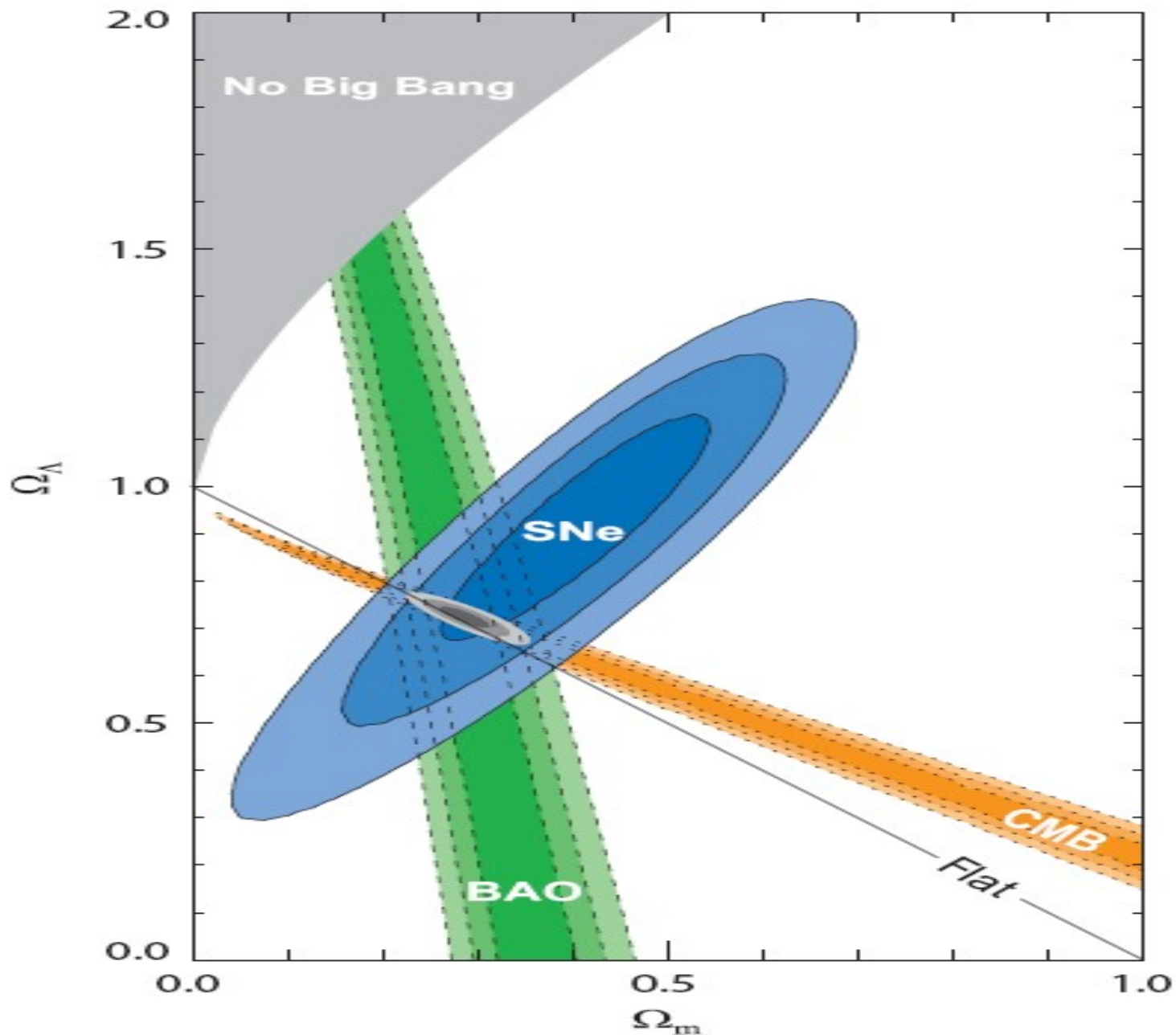
Outline

- String Theory and Dark Energy
- String Theory and Inflation
- Time Dependent Space-times with High curvature
- Conclusions

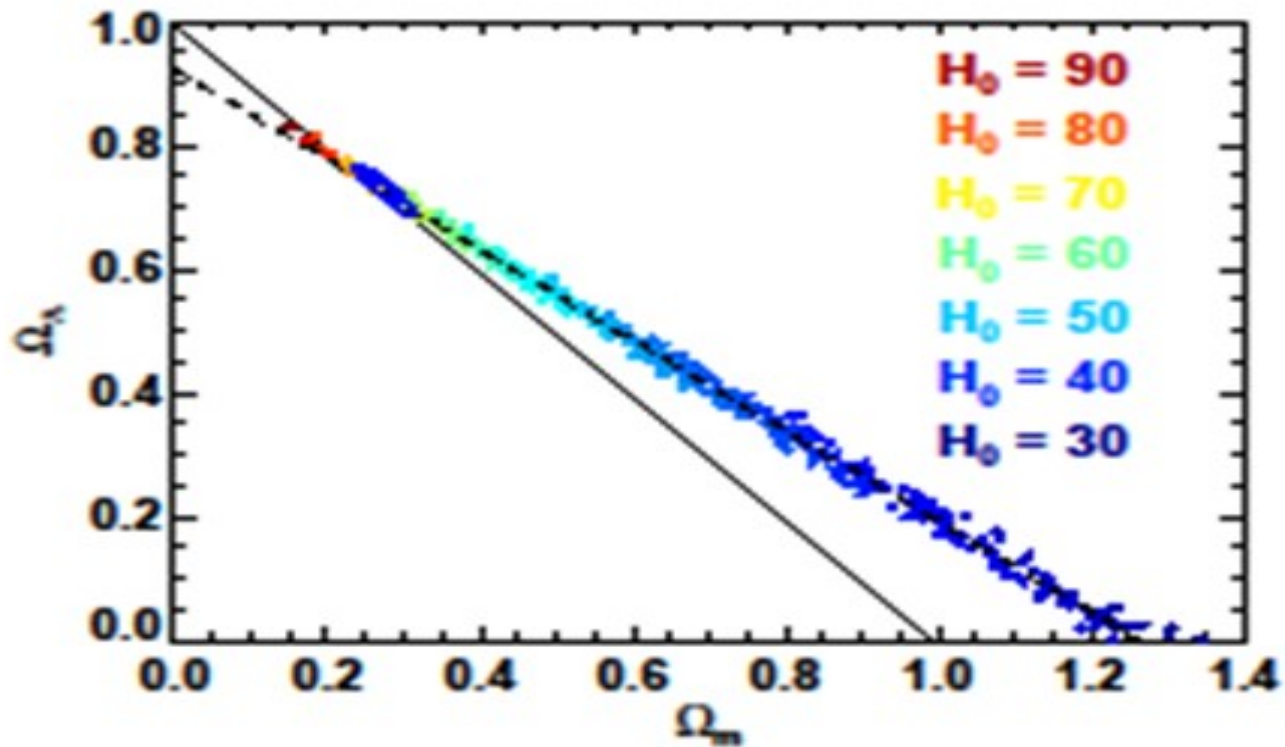
String Theory and Dark Energy

- Dark Energy:

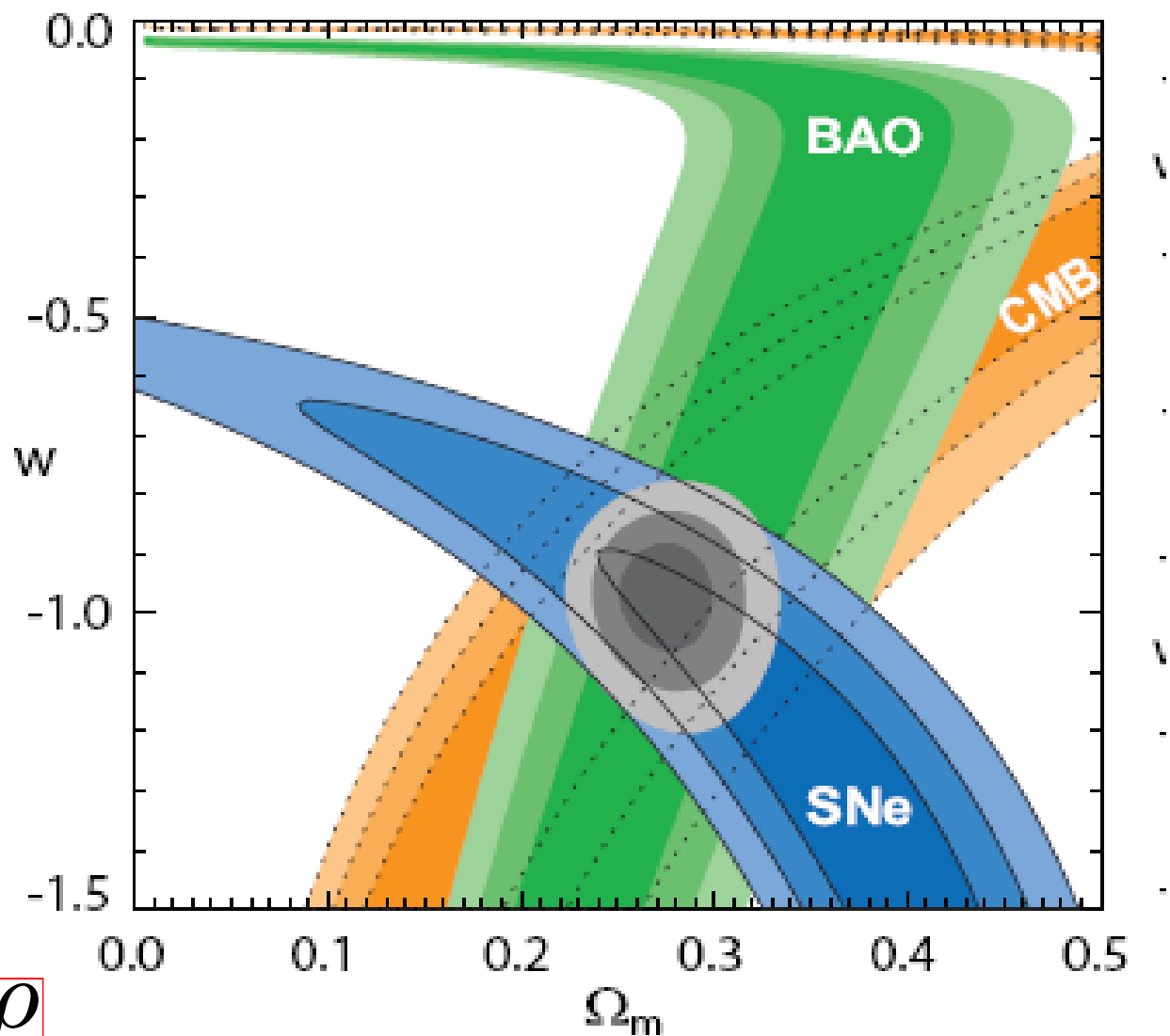
Leading Candidate: Positive
Cosmological Constant



Constraints on Ω_Λ , Ω_M Kowalski(08)



WMAP 7 Year Results



Constraints on w , Ω_M ; Blue: SNe, Orange: CMB, Green BAO (Kowalski et. Al. (2008)).

In a flat universe $\Omega_k = 0$

WMAP+BAO+H0:

$$\omega = -1.10 \pm 0.14 (68\% CL)$$

WMAP+BAO+SN:

$$\omega = -0.980 \pm 0.053 (68\% CL)$$

(not including systematic errors on SN)

(Komatsu et. Al. WMAP 7-year data)

The Cosmological Constant

- The leading theoretical candidate for Dark Energy.
- It is the ground state energy density.
- If this energy is positive it gives rise to acceleration

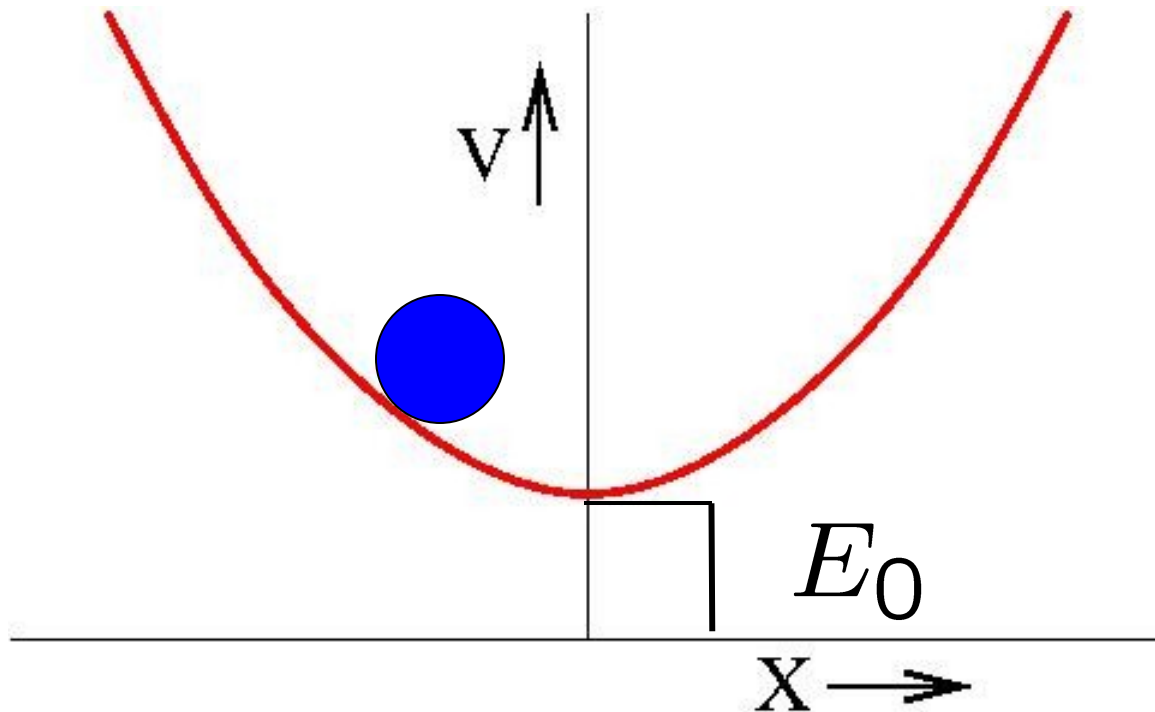
The Cosmological Constant

Can a positive cosmological constant arise in string theory?

Both gravity and quantum mechanics are needed to answer this question.

Moreover the question is very sensitive to short distance physics.

The Harmonic Oscillator



Ground State Energy

Example: Harmonic oscillator

$$E = E_0 + \frac{1}{2}\hbar\omega$$

- Nothing depends on the constant E_0 (except for gravity).
- Quantum Mechanics is essential to calculate the answer.

The Cosmological Constant and Gravity

To calculate the cosmological constant from first principles we need a theory of gravity consistent with the rules of quantum mechanics.

Enter String Theory!

“De Sitter vacua in string theory,”

S. Kachru, R. Kallosh, A. Linde,
S. P. Trivedi,

Phys. Rev. D 68, 046005 (2003)

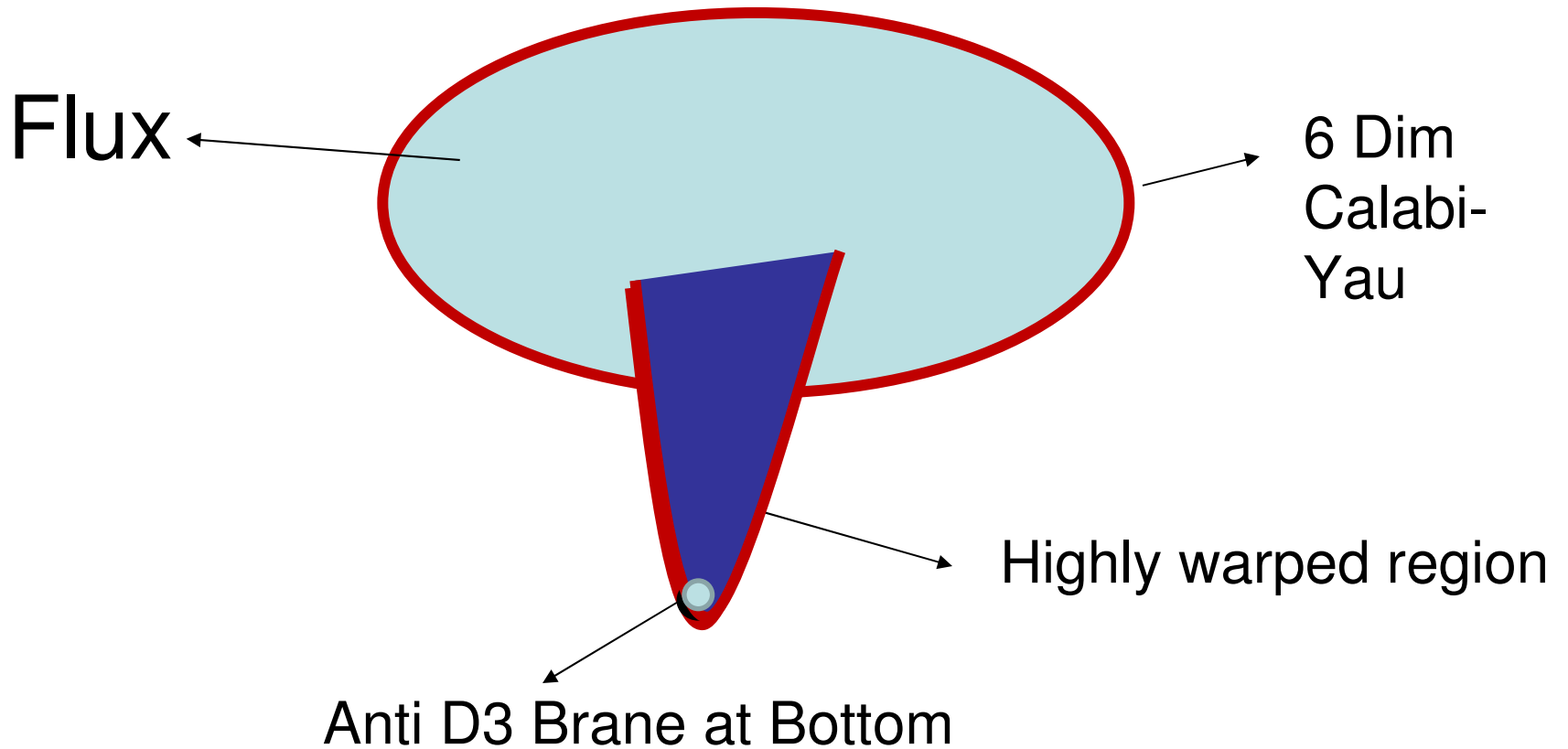
Result:

Positive Values for the Cosmological
Constant Can Indeed Arise in String
Theory.

The Construction

Has several ingredients:

- 1) An internal manifold: Calabi-Yau Manifold.
- 2) Fluxes
- 3) Non-perturbative effects
- 4) Anti-Branes.



The KKLT Construction

The Construction

- Subsequent research has focused on understanding these different ingredients in greater depth.
- What we have learnt gives greater confidence that the construction, and others like it, do make sense.

The Cosmological Constant

Other constructions using similar ideas/ingredients have also been investigated since then.

- However, the last word has not been said.
- Much more needs to be done to rigorously ensure deSitter vacua do exist in string theory.
- To be totally sure we will probably need a deeper understanding akin to what we have today for negative cosmological constant vacua.

Summary: Cosmological Constant

- String Theory can accommodate a positive cosmological constant.
- This can be viewed as a “theoretical test” of the idea.
- In turn accounting for such a cosmological constant as spurred progress in string theory which is still ongoing.

Many, Many, Universes

- Such constructions lead to many different vacua.
- The essential reason is that many different fluxes can be turned on.
- Preliminary estimate,

$$N_{vac} \sim 10^{100} - 10^{1000}$$

(Douglas, Denef,..).

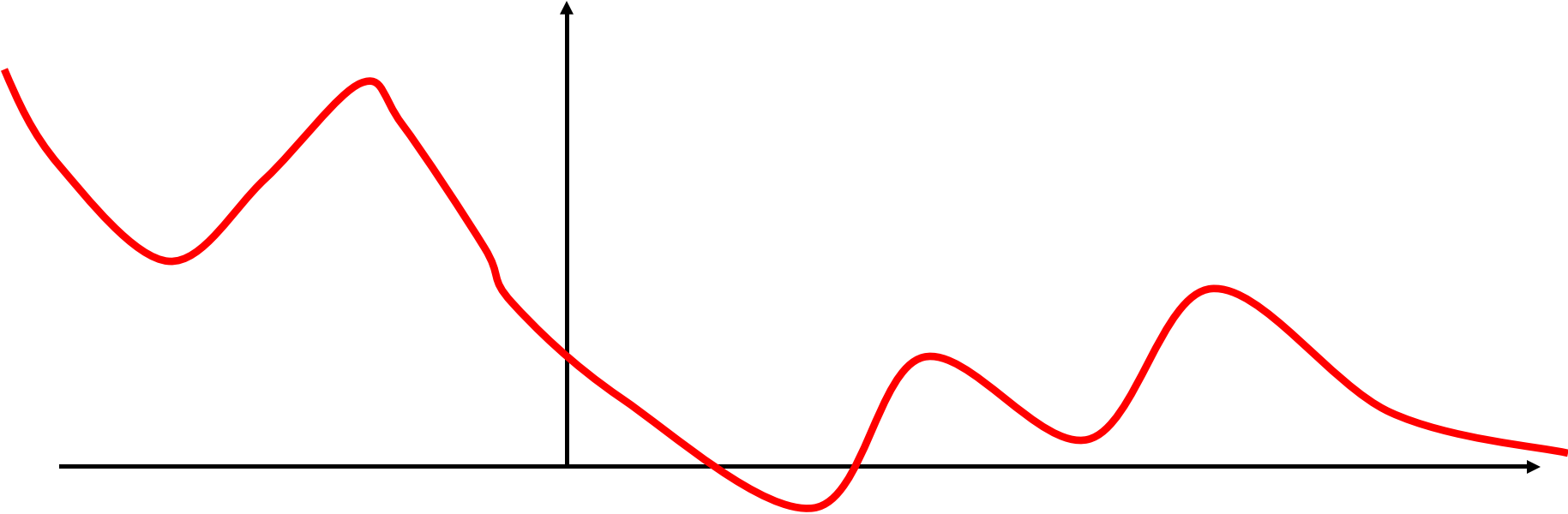
Bousso Polchinski

Susskind

Landscape

- ~ 100 Directions.
- ~ $10^{100} - 10^{1000}$ different vacua.
- Varying cosmological constants.
- Transitions among vacua possible:
Through tunneling and possibly due to the thermal fluctuation in deSitter space.

Landscape



The Cosmological Constant

Required value 2×10^{-29} gm/cc very small.

- Of order $10^{(-120)}$ In Planck units !
- This smallness is the famous Cosmological Constant problem.

The Small Value of the Cosmological Constant

Observed value is about 10^{-120} in natural units.

The Landscape suggests an “unconventional” explanation for this smallness.

An Anthropic Explanation for the Small Cosmological Constant:

- A large range of values is allowed for Λ
- But values very much bigger than the one observed would not lead to life.
- Galaxies would not form. (Weinberg 1987).

For Small Cosmological Constant, ,

$$\Lambda = 10^{-120}$$

Number of vacua, $N(\Lambda) \sim 10^{940}$

Landscape and Extracting Predictions

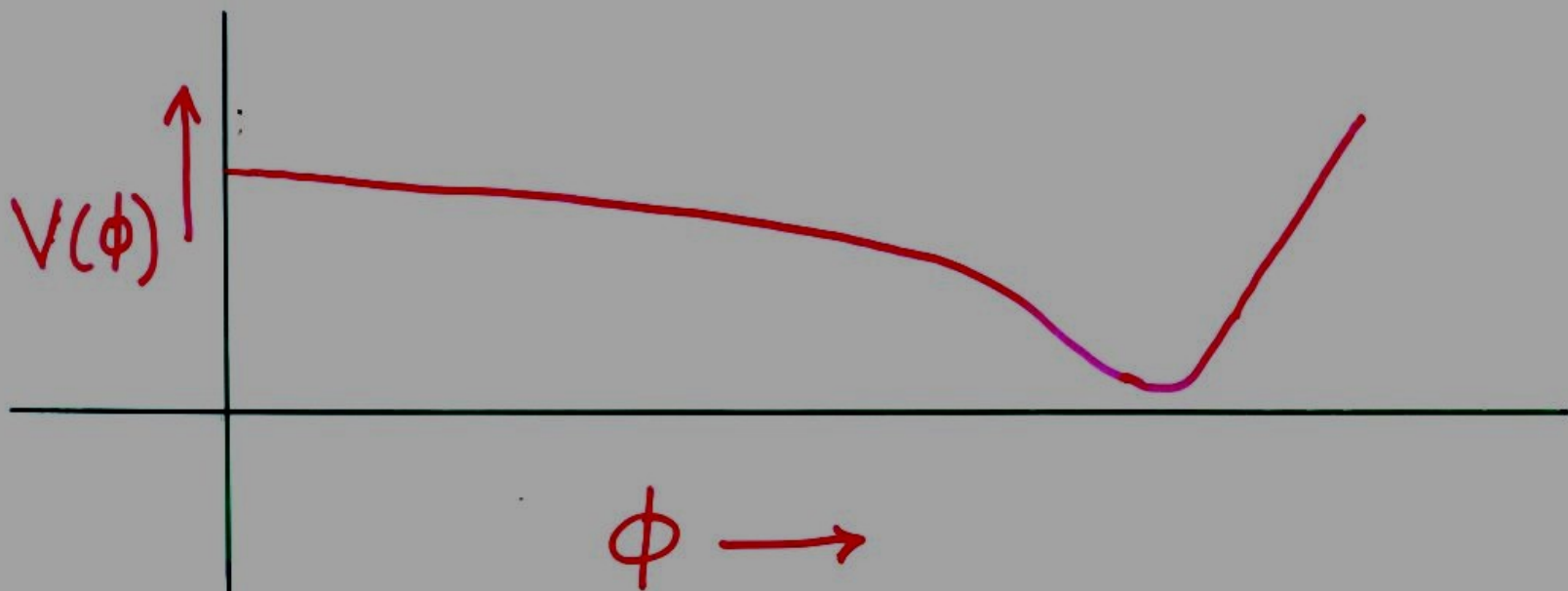
- In what sense is this an explanation?
- More generally how do we extract predictions from the landscape?
- What are the rules to assign probabilities?
- Should Anthropic considerations be included?

Landscape and Extracting Predictions

- These are topics of considerable debate in string theory.
- It is clear that string theory needs to be developed further.
- Perhaps the question of probabilities can be side-stepped to some extent.

Alternatives To The Cosmological Constant:

Quintessence: Scalar field whose energy slowly relaxes.



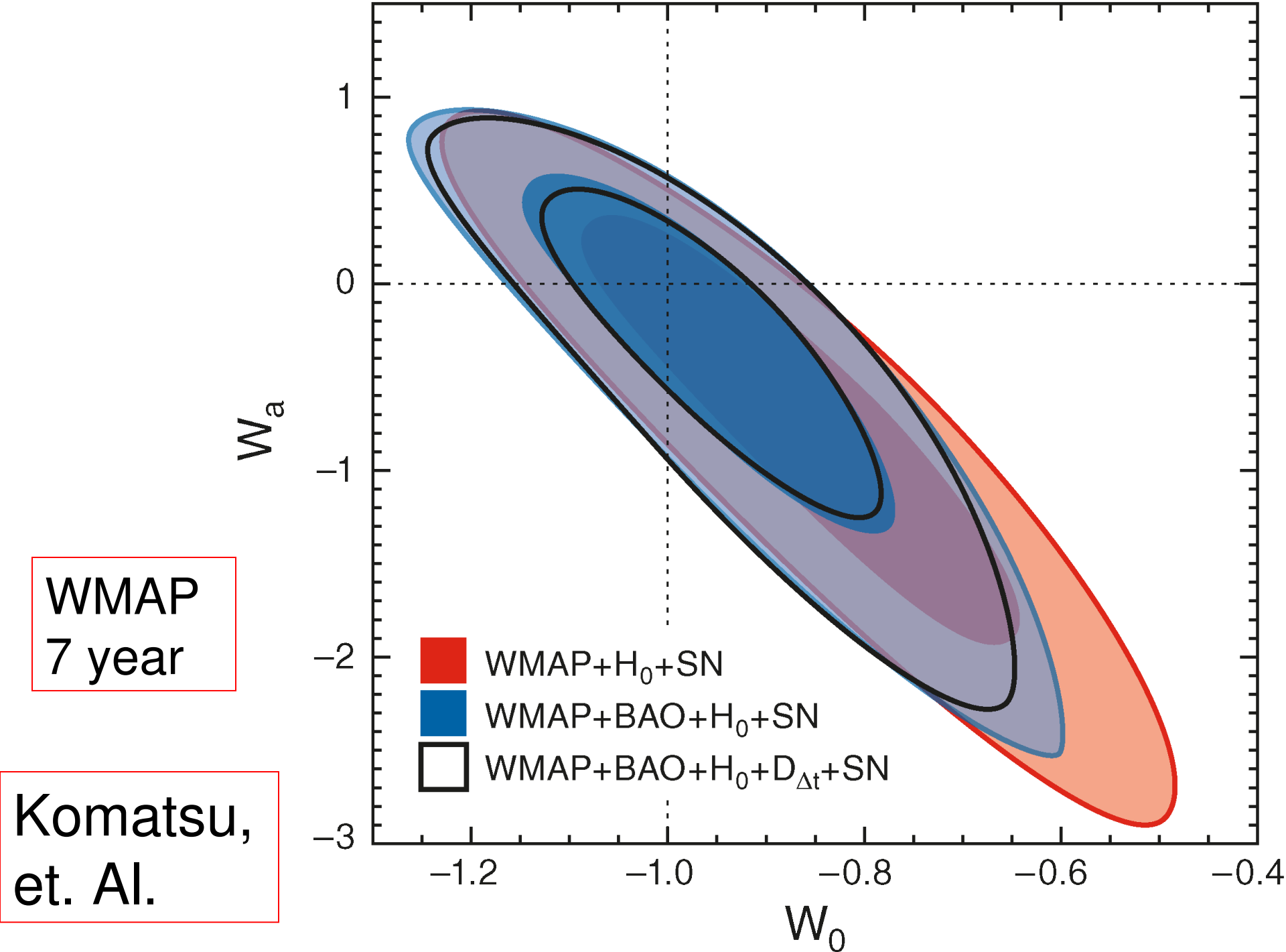


FIG. 13.— Joint two-dimensional marginalized constraint on the linear evolution model of dark energy equation of state, $w(a) = w_0 + w_a(1 - a)$. The contours show the 68% and 95% CL from WMAP+ H_0 +SN (red), WMAP+BAO+ H_0 +SN (blue), and WMAP+BAO+ H_0 + $D_{\Delta t}$ +SN (black), for a flat universe.

careful about the treatment of perturbations in dark energy when w crosses -1 . We use the “parametrized post-Friedmann” (PPF) approach, implemented in the CAMB code following Fang et al. (2008).

In Figure 13, we show the 7-year constraints on w_0 and w_a from WMAP+ H_0 +SN (red), WMAP+BAO+ H_0 +SN (blue), and WMAP+BAO+ H_0 + $D_{\Delta t}$ +SN (black). The angular diameter distances measured from BAO and $D_{\Delta t}$ help exclude models with large negative values of w_a . We find that the current data are consistent with a cosmological constant, even when w is allowed to depend on time. However, a large range of values of (w_0, w_a) are still allowed by the data: we find

Komatsu et. Al. $\omega_0 = -0.93 \pm 0.13$

$$\omega_a = -0.41^{+0.72}_{-0.71} \text{ (68\% CL)}$$

$$\omega = \omega_0 + \omega_a(1 - a)$$

Flat universe assumed $\Omega_k = 0$

Challenge: Can the potential be flat on present day cosmological scales, even though supersymmetry breaking at

$$M_{SB} \geq O(1 \text{ TeV})$$

This question is sensitive to very high energy physics (upto dimension 10 operators suppressed by $\frac{1}{M_{Pl}^6}$)

Such a model has been recently constructed in string theory: Panda, Sumitomo, S.P.T.

Based on idea of Axion Monodromy
(McAllister, Silverstein and Westphal)

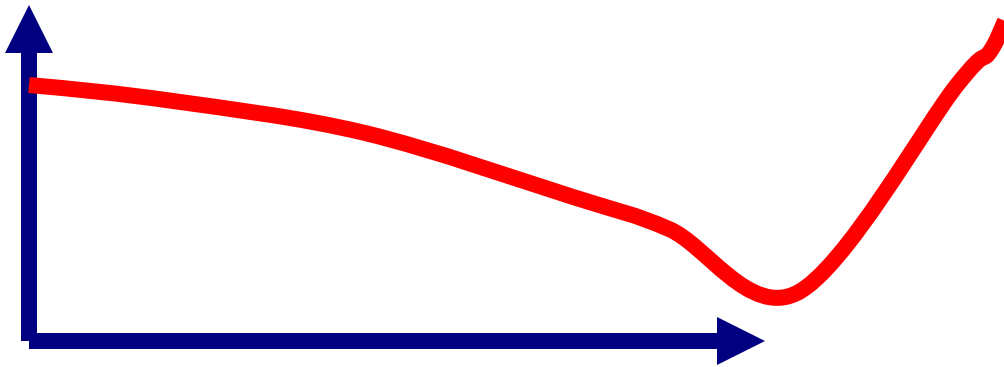
An axion plays the role of quintessence.

Its approximate shift symmetry is broken by branes placed in highly warped regions which gives rise to a slowly varying potential.

Axions as Quintessence

- Worth developing these models further.
- Perhaps in some class there are some predictions.
- If nothing else, as a foil to the cosmological constant

String Theory and Inflation:



Slowly Varying Potential: Inflation

String Theory and Inflation

Two kinds of Models:

Small Field Inflation

$$\frac{\Delta\Phi}{M_{Pl}} \ll 1$$

Large Field Inflation

$$\frac{\Delta\Phi}{M_{Pl}} \geq O(1)$$

Observational Difference:

Gravity waves (Tensor perturbations)
significant only for large field case.

(Lyth)

$$r = \frac{\mathcal{P}_{tensor}}{\mathcal{P}_{scalar}}$$

$$r = 6.2 \left(\frac{\Delta\phi}{M_{pl}} \right)^2 \frac{1}{N_e^2}$$

$$\epsilon = \frac{M_{Pl}^2}{2} \left(\frac{V'}{V} \right)^2$$

$$r = \frac{\mathcal{P}_{tensor}}{\mathcal{P}_{scalar}} = 12.4\epsilon$$

$$\sqrt{\epsilon} = \frac{1}{\sqrt{2}} \left(\frac{\Delta\phi}{M_{Pl} N_e} \right)$$

$$r = 6.2 \left(\frac{\Delta\phi}{M_{pl}} \right)^2 \frac{1}{N_e^2}$$

- Model building:

- Slow Roll conditions:

$$\epsilon = \frac{M_{Pl}^2}{2} \left(\frac{V'}{V} \right)^2 < \sim O(10^{-2})$$

$$\eta = \frac{V''}{V} M_{Pl}^2 < \sim O(10^{-2})$$

- Require a flat potential.

- Ensuring this when $\frac{\Delta\Phi}{M_{Pl}} \geq O(1)$ more
challenging at least in effective
field theory

- The potential has to be known for a very large range in field space.
- A term suppressed by a high power of M_{Pl} would still be important.

$$\delta V = C_1 \frac{(\Delta\phi)^p}{M_{Pl}^{p-4}}$$

- For this sort of reason the first models built in string theory of inflation were of the ``small field'' kind.
 - (e.g. Brane inflation)
- More recently, using an axion as the inflaton, models of large field inflation have been built (Silverstein, Westphal, McAllister ...).

String Theory and Inflation

Key issue: The direction of steepest descent must still be slowly enough varying.

Not easy to ensure because typically many directions in field space (moduli).

Small field Inflation

There is still sensitivity to Planck scale physics through dim. 6 operators.

$$\delta V = V\left(\frac{\phi^2}{M_{Pl}^2}\right)$$

Results in:

$$\eta \sim O(1)$$

Or:

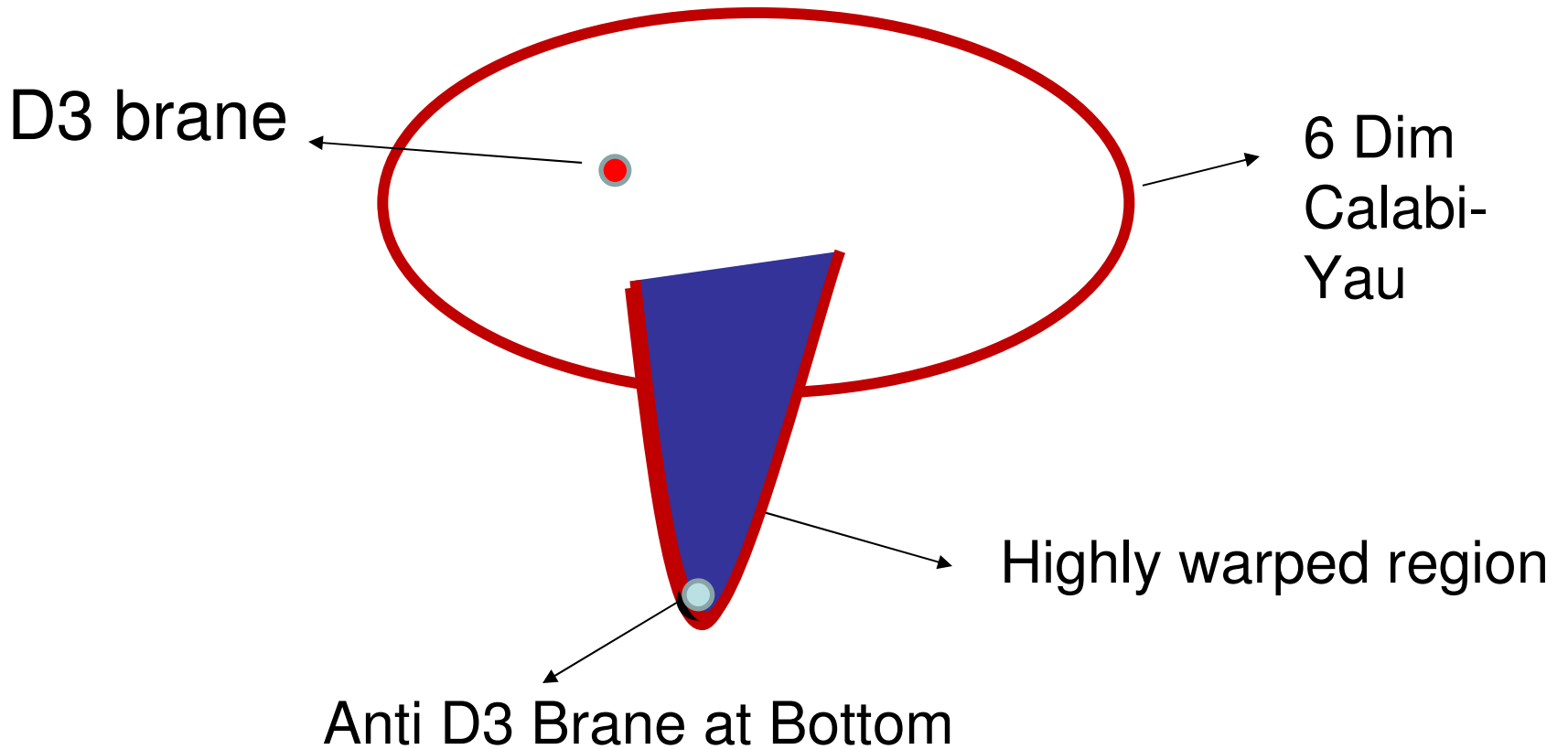
$$\delta L = c\mathcal{R}\phi^2$$

Symmetries cannot forbid these terms.

Brane Inflation:
(Kachru, Kallosh, Maldacena,
McAllister, Linde and S.P.T.)

Based on Dvali and Tye

Inflaton : Brane-Anti-brane
seperation.



The KKLT Model

Gives rise to a flat potential:

$$V(\phi) = \frac{4\pi^2 \phi_0^4}{N} \left(1 - \frac{1}{N} \frac{\phi_0^4}{\phi^4}\right)$$

$$\phi_0 \sim e^{\frac{-2\pi K}{3M g_s}}$$

Can be made very small due to warp factor.

However closer analysis of interplay between moduli stabilisation and brane-anti brane system showed additional contribution:

$$\delta L = \frac{1}{6} \mathcal{R} \phi^2$$

Unacceptable.

- Since then considerable effort has gone into trying to see if the effects of this unwanted term can be suppressed or “undone” by additional terms in potential.

Idea 1:

The superpotential that stabilises the volume can itself depend on the inflaton.

- Happens if superpotential arises due to gauge dynamics on 7-branes.
- This idea can work but requires tuning.

Baumann, Dymarksy, Klebanov, McAllister;
Berg, Haack, Kors;
Giddings, Maharana;
Baumann, Dymarksy, Klebanov,
Maldacena, McAllister and Murugan;
Baumann, Dymarksy, Klebanov, McAllister,
Steinhardt;
Ali, Deshmukhya, Panda and Sami;

.....

Idea 2:

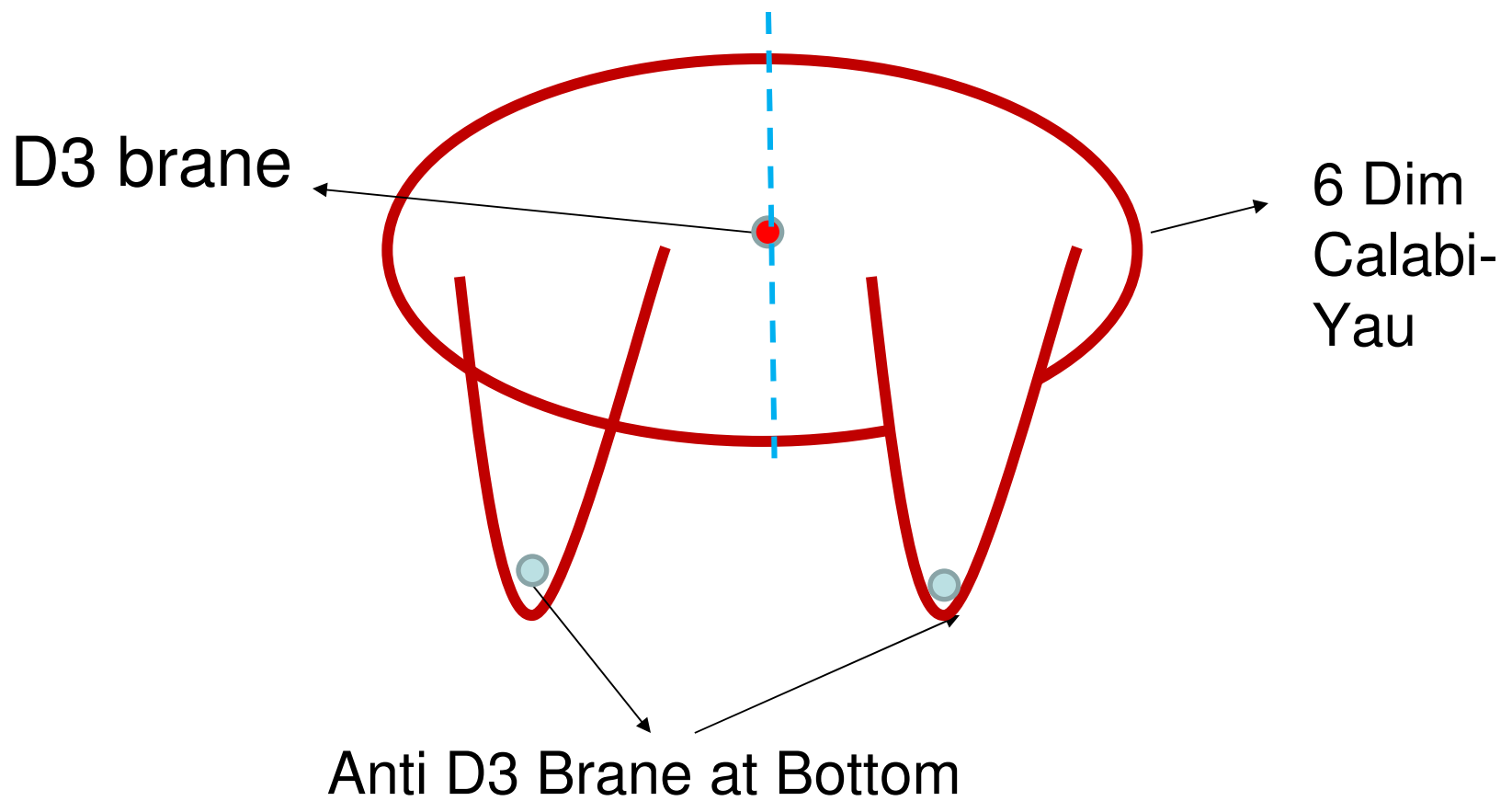
Use DBI action (different Kinetic energy term) to flatten out potential.

(Silverstein and Tong; ...)

Idea 3:

Use a symmetry to guarantee a maximum. Then dial parameters to make maximum flat enough. (Iizuka and Trivedi, '04)

$$V = -\frac{1}{2}m^2\phi^2$$



Z_2

Symmetric Model

One Observationally Interesting
Consequence:

Cosmic Superstrings often produced at end
of inflation.

These have tension $G\mu \sim 10^{-9}$

Might lead to detection in LIGO, etc.

(Jones, Stoica, Tye; Sarangi, Tye;
Copeland, Myers, Polchinski)

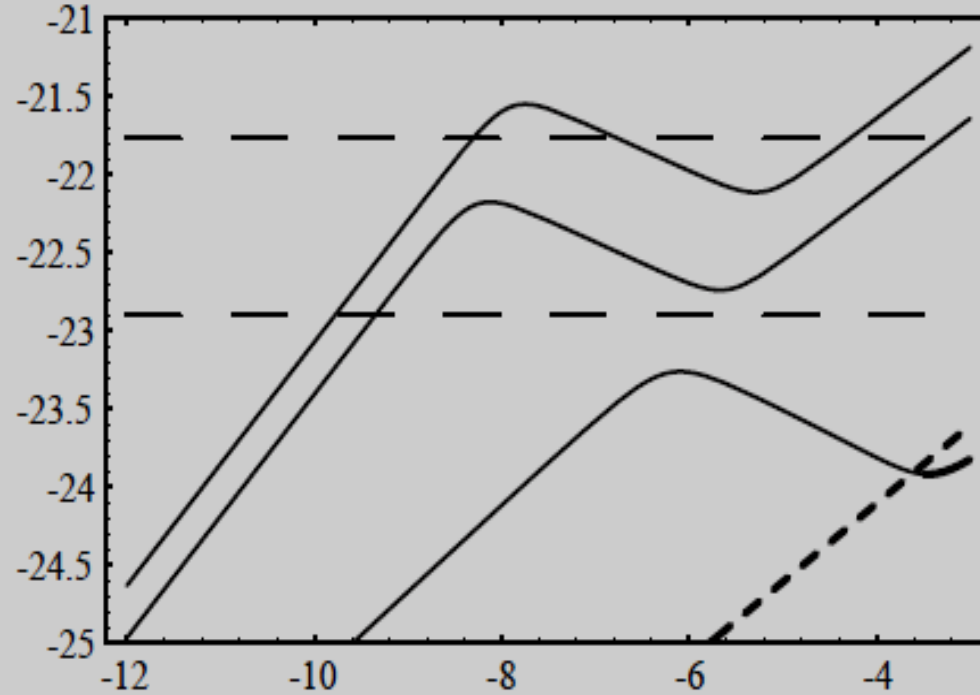


Figure 3: Gravitational wave cusp signals, taken from Damour and Vilenkin [65]. The horizontal axis is $\log_{10} \alpha$ where $\alpha = 50G\mu$. Thus the brane inflation range $10^{-12} \lesssim G\mu \lesssim 10^{-6}$ becomes $-10.3 < \log_{10} \alpha < -4.3$. The vertical axis is $\log_{10} h$ where h is the gravitational strain in the LIGO frequency band. The upper and lower dashed horizontals are the sensitivities of LIGO I and Advanced LIGO at one event per year. The upper two curves are the cusp signal under optimistic and pessimistic network assumptions. The lowest solid curve is the signal from kinks, which form whenever strings reconnect. The dashed curve is the stochastic signal.

Superstrings vs Gauge Strings

Gauge Strings: $P \sim O(1)$

F strings $P \sim g_s^2 \sim 10^{-2}$

D strings: $P \sim 10^{-1} \leq P < 1$

Also (p,q) string networks can form which behave differently.

$$\mu = \mu_0 \sqrt{p^2 + \frac{q^2}{g_s^2}}$$

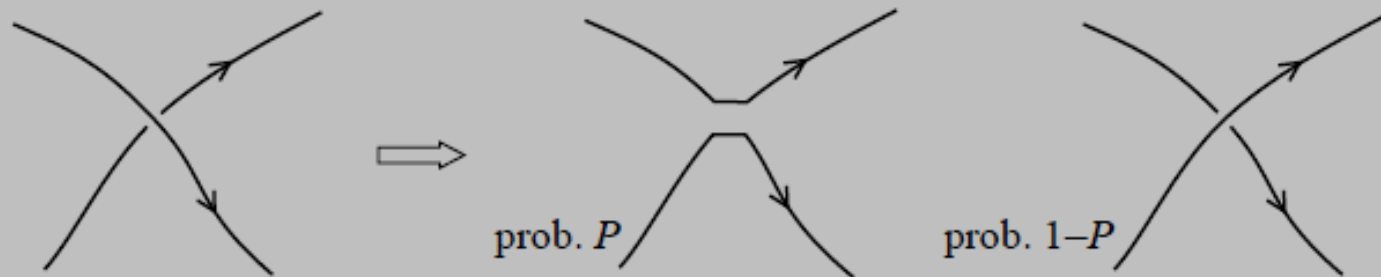


Figure 1: When two strings of the same type collide, they either reconnect, with probability P , or pass through each other, with probability $1 - P$. For classical solitons the process is deterministic, and $P = 1$ for the velocities relevant to the string network.

Polchinski Cargese Lectures

Determination of string network properties by fitting to data can allow us to distinguish the two.

Large Field Inflation

Can be implemented using the ideas of axion monodromy.

Silverstein, Westphal;
McAllister, Silverstein, Westphal;
Flaugher et. Al. JCAP 1006.

Time Dependent Cosmologies with High curvature

(with S. Das, K. Narayan, A. Awad, A. Ghosh, J. Kim, K. Narayan, S. Nampuri)

Related Work Includes:

- Craps, Sethi, Verlinde
- Das, Michelson
- Chu, Ho
- Kodama, Ohta
- Horowitz, Lawrence, Silverstein

Main Idea

- AdS/CFT provides a non-perturbative formulation of quantum gravity.
- Can it teach us something about Cosmological Singularities?

Future? With smooth spacetime?

$t = 0$

What
happens?

$\mathcal{N} = 4$ SYM

$AdS_5 \times S^5$

More on AdS/CFT

Gravity

Corrections:

3) α' Corrections
corrections

$$\frac{R_{AdS}^4}{l_s^4} \sim g_s N$$

Gauge Theory

Planar

λ

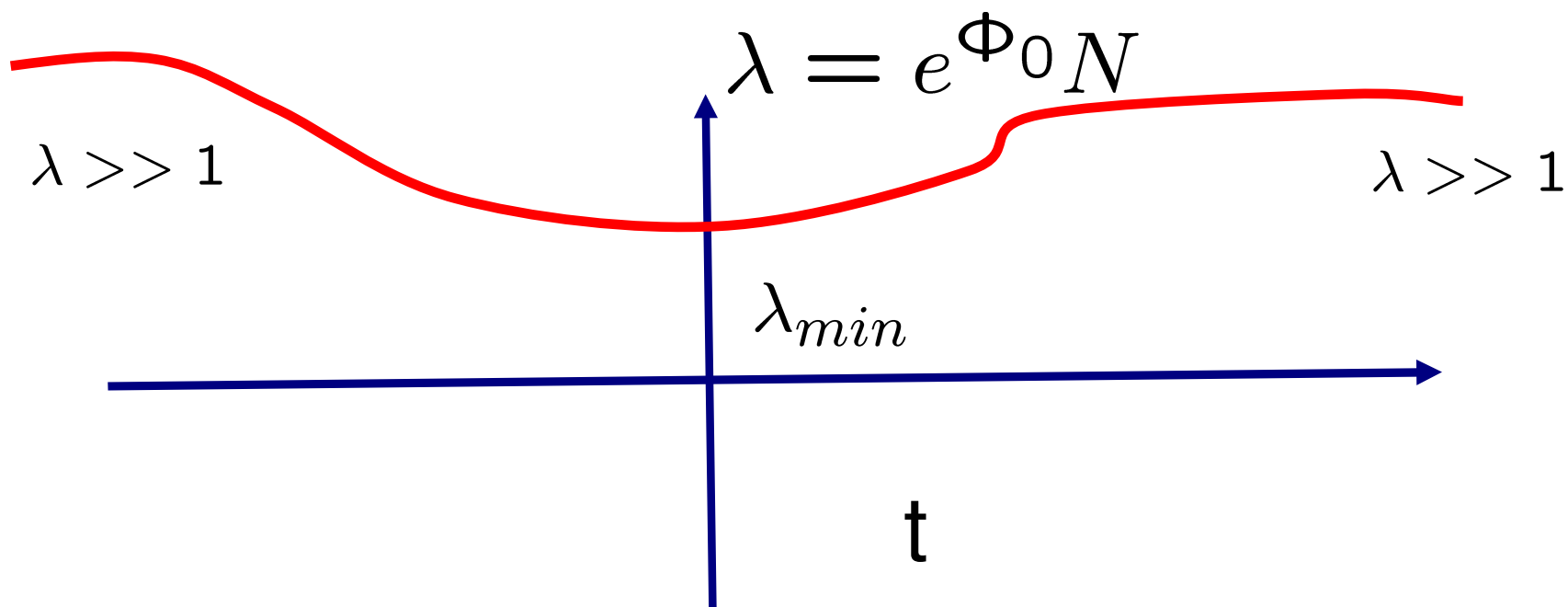
't Hooft

$$\lambda = g_{YM}^2 N$$

Work in Large N limit.

Vary boundary value of dilaton.

Ask what happened in bulk.



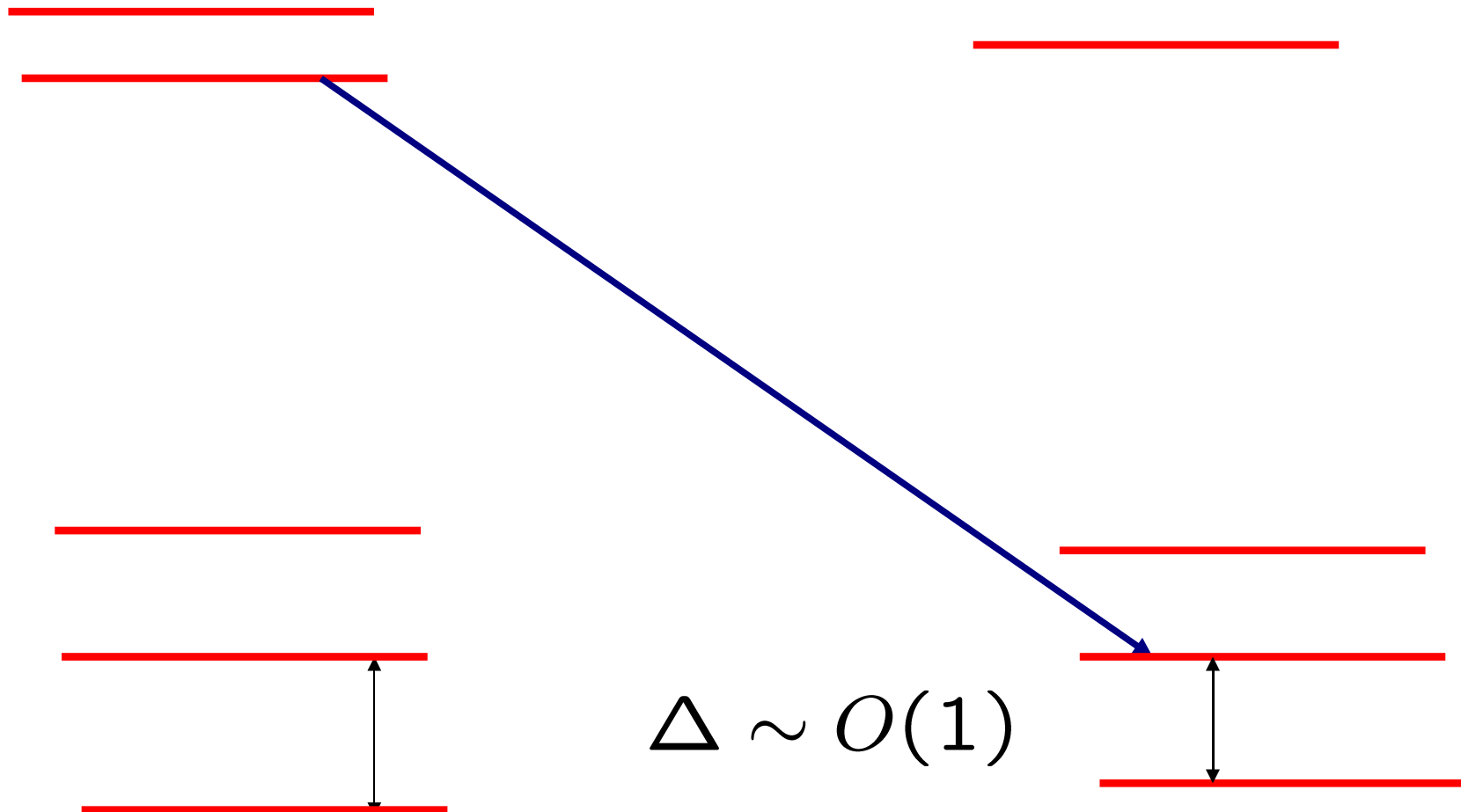
$$\frac{R_{AdS}^4}{l_s^4} \sim \lambda$$

- Convenient to put the YM theory on S^3
- This introduces a scale R – radius of S^3
- By taking the dilaton to vary slowly in units of R we get a small parameter, ϵ

- When $N\epsilon \ll 1$, so that total time taken $t \sim N$:

Quantum adiabatic perturbation theory ensures that the system in the far future is AdS space, with no extra excitations, upto corrections which are exponentially suppressed.

Spectrum: Gap is Robust



Slow Enough:

$$\left| \left\langle n \left| \frac{\partial H}{\partial t} \right| 0 \right\rangle \right| \ll (\Delta E)^2$$

Here we get:

$$N\epsilon \ll 1$$

More on conditions for Adiabatic Approximation

$$H \sim \frac{\text{Tr}(F^2)}{g_{YM}^2}$$

$$\frac{\partial H}{\partial t} \sim \dot{\Phi} \frac{\text{Tr}(F^2)}{g_{YM}^2} \sim N \epsilon \frac{\text{Tr}(F^2)}{\lambda}$$

More on conditions for Adiabatic Approximation:

Now $\frac{Tr(F^2)}{\lambda}$ creates states out of vacuum with unit weight.

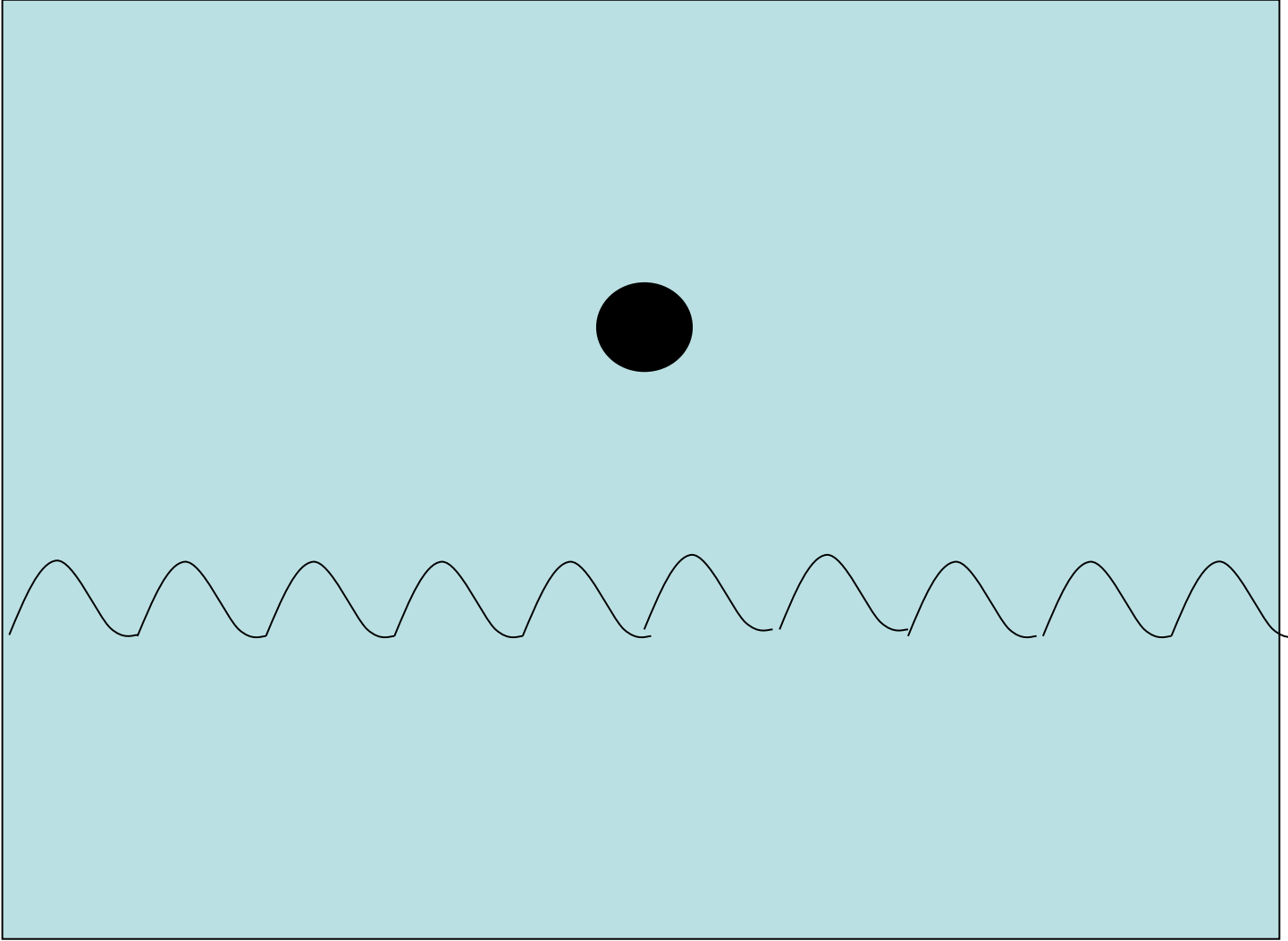
Thus condition is $N\epsilon \ll 1$

•When $\epsilon \ll 1, N\epsilon \geq O(1)$

Two possibilities:

1) Again match on to smooth AdS in the far future

2) Match to AdS with a small black hole which then decays $t \sim R_{AdS} N^2$



In both cases spacetime mostly smooth.

Conclusions:

In some cases smooth geometries can evolve to highly curved one and then in the far future again became smooth.

Whether this can happen for cosmologically interesting cases remains to be seen.

Conclusions of Talk

- Progress in cosmology has spurred progress in string theory.
- Progress in string theory has suggested scenarios of interest in cosmology.

Conclusions of Talk

- As time dependent string theory comes of age, the connection between string theory and cosmology should get even more interesting!



c
a

Facts To Remember

- Age 13.7 Billion Years
- Last Scattering 380,000 years after big bang

$$M_{pl}^2 = \frac{\hbar c}{G_N}$$

Lambda CDM Model

Six Parameters:

$$\Omega_b h^2, \Omega_c h^2, \Omega_\Lambda$$

$$\Delta_R^2, n_s$$

$$\tau \quad (\text{reionisation depth})$$

$$H = 100h \text{ Km/s/Mpc}$$