Nongaussianities and features in multiple-field inflation

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Primordial features and nongaussianities

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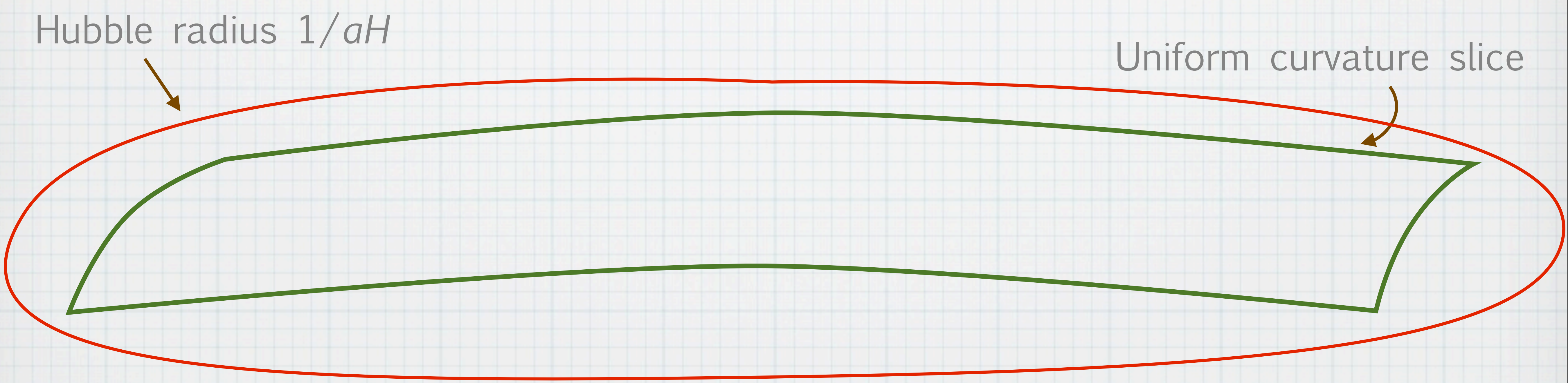
What is this all about?

- Different methods for generating nongaussianities in multiple-field inflation.
  In contrast to single-field models, this consists of nonlinear **reprocessing of existing perturbations**.
- Some **typical classes of behaviour** in these models.
  These show the effects which it is possible to get at the present state of the art.

Statistics of density perturbations



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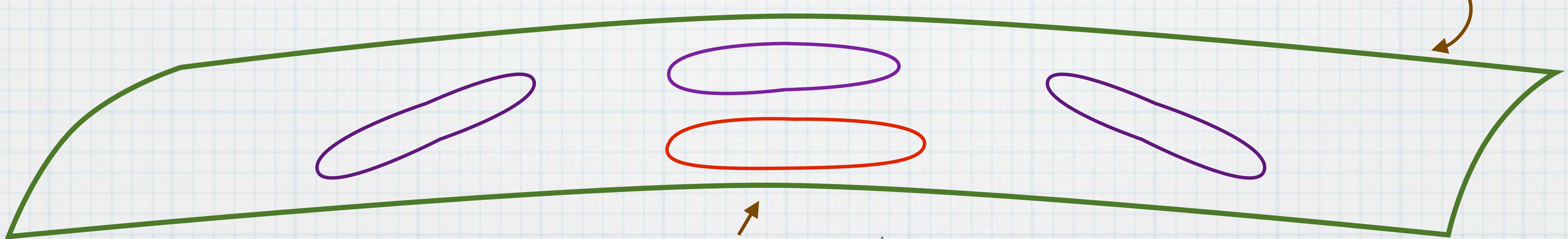

during inflation, the comoving Hubble length decreases

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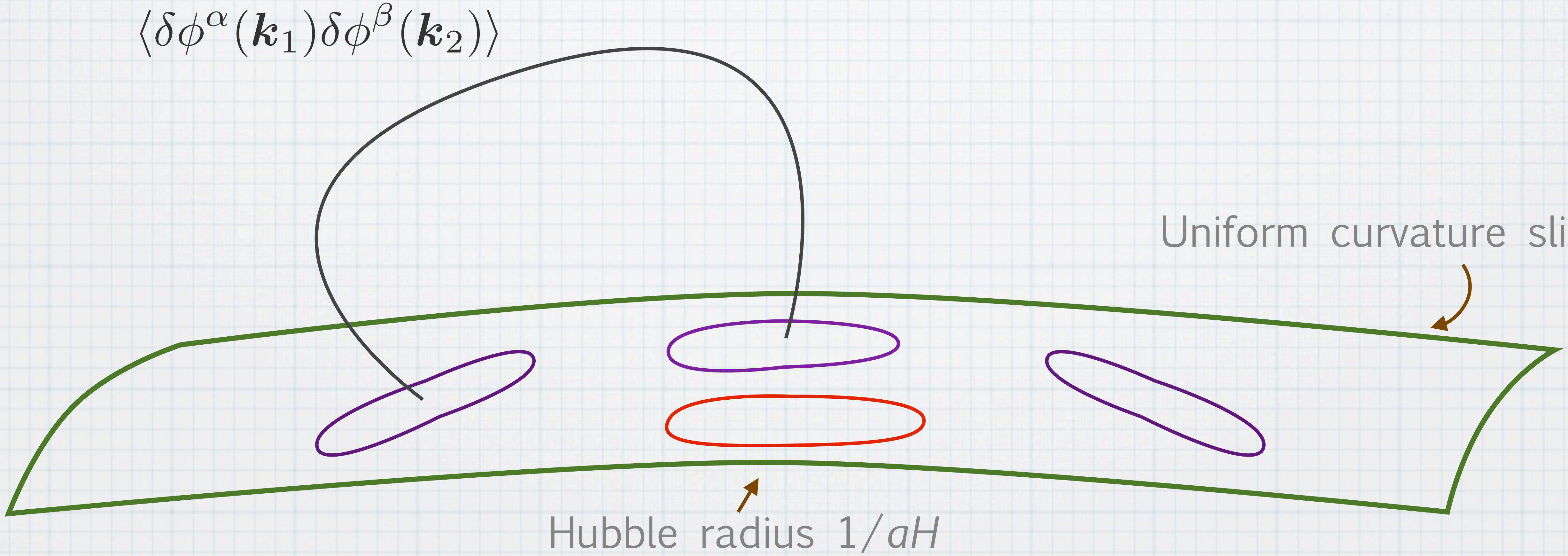

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Statistics of density perturbations

$\langle \delta\phi^\alpha(\boldsymbol{k}_1)\delta\phi^\beta(\boldsymbol{k}_2)\rangle$ two-point function

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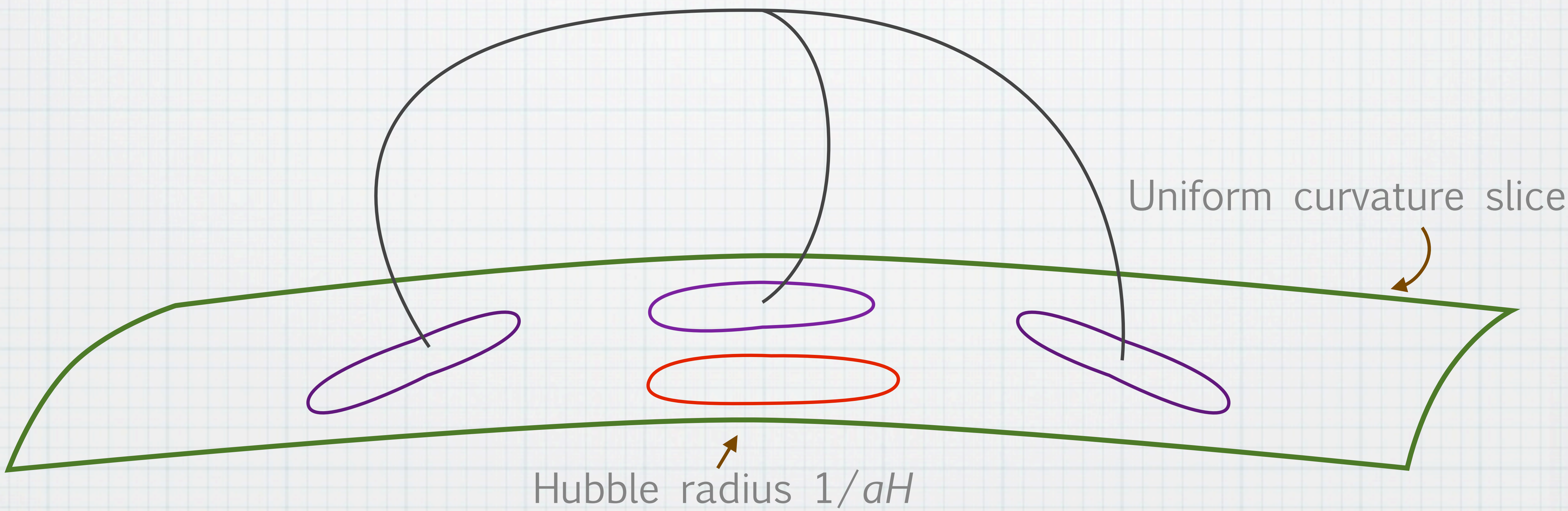

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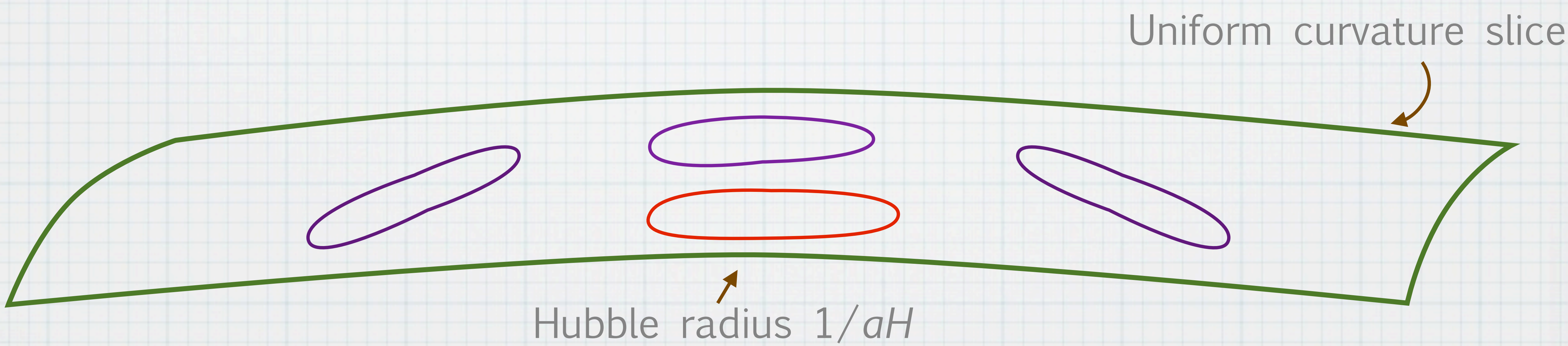

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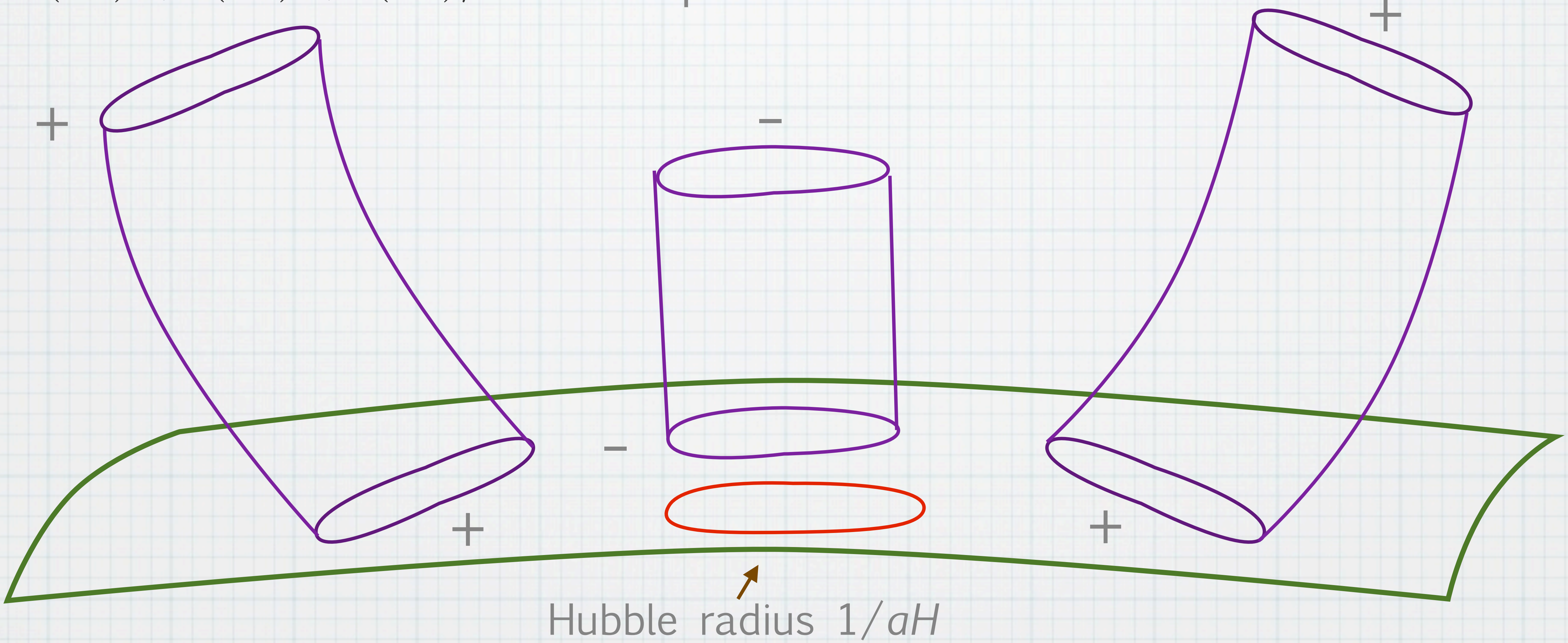

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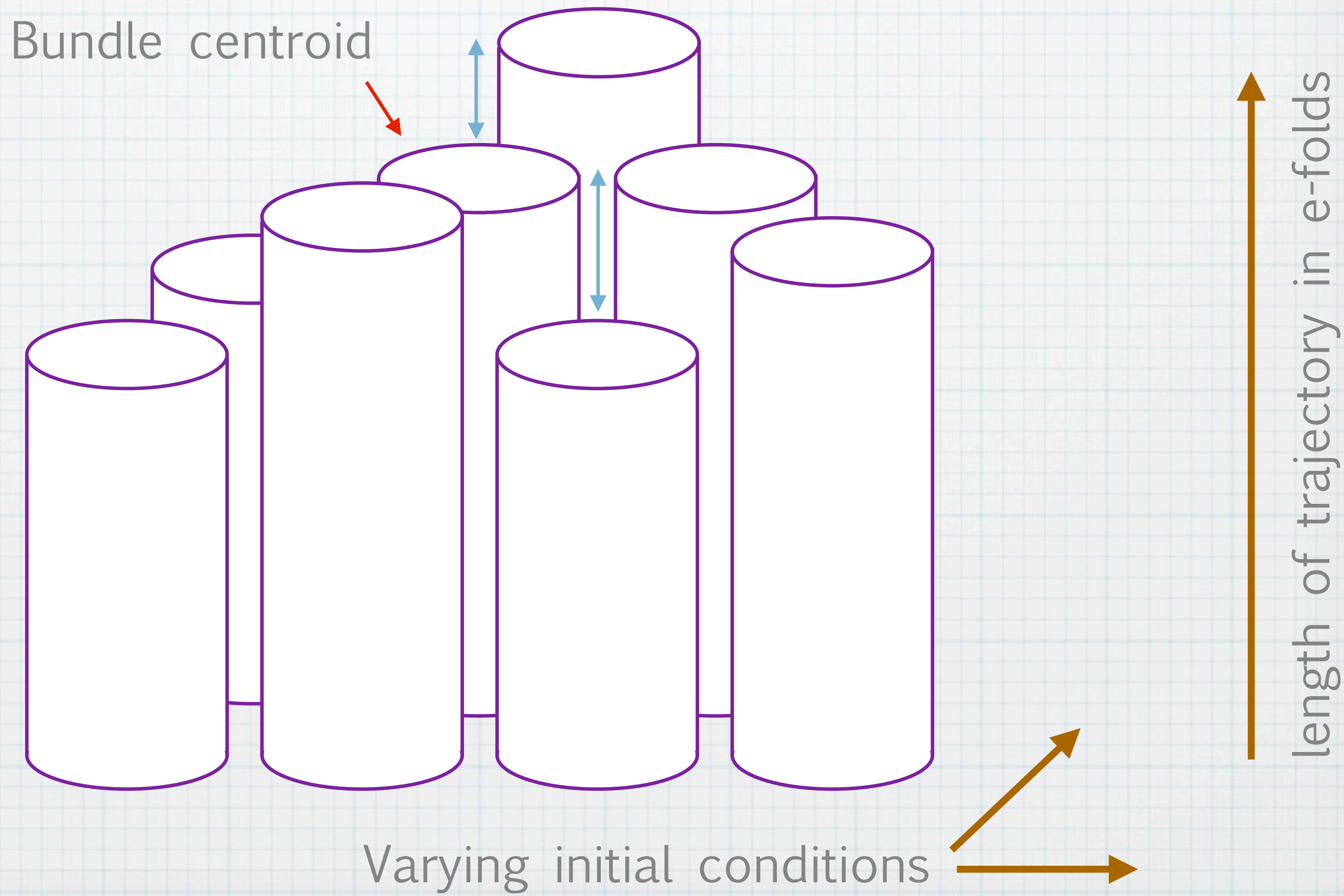
We end up with a **bundle of different trajectories**

In a continuum approximation, we can describe the bundle statistically by calculating its moments



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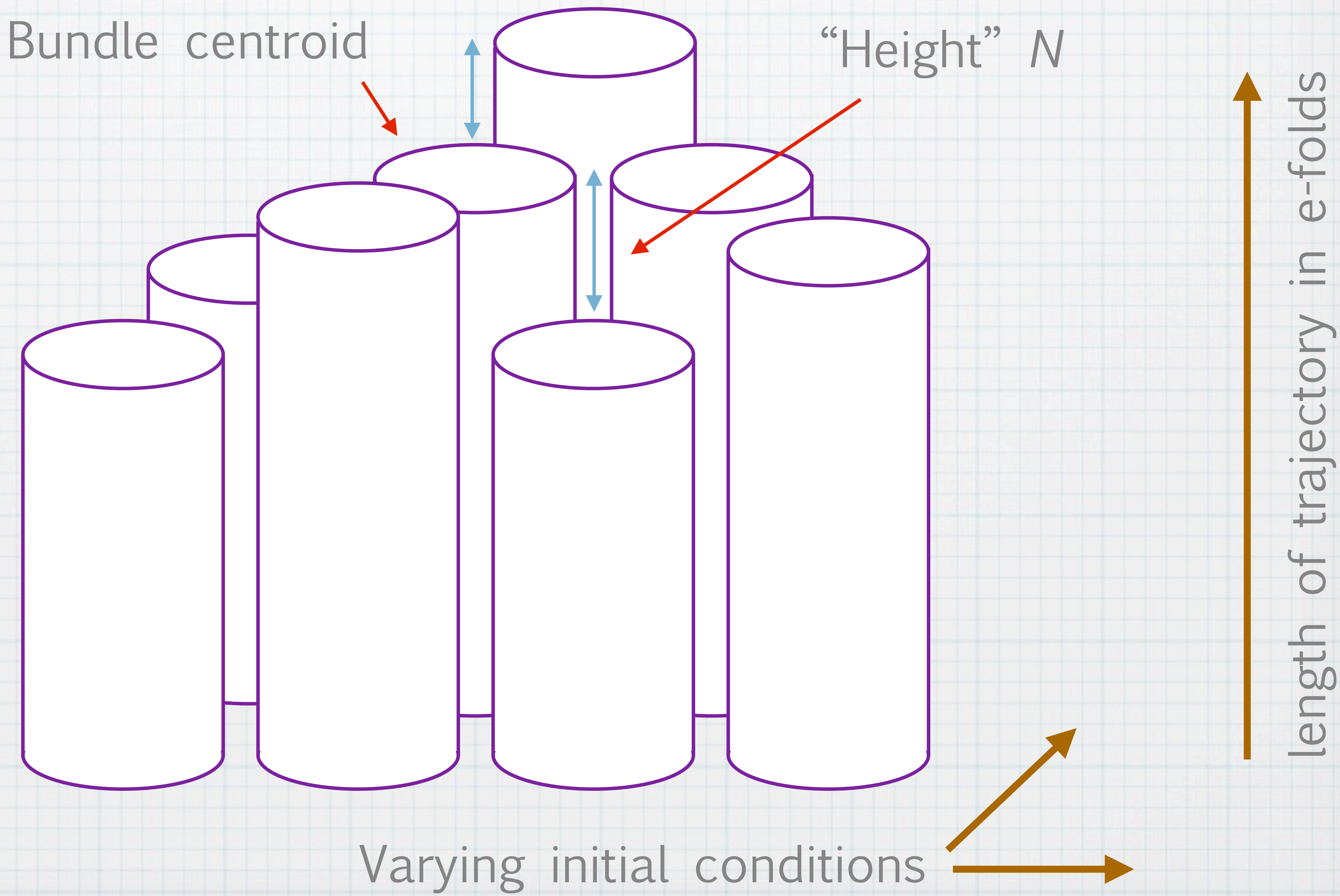
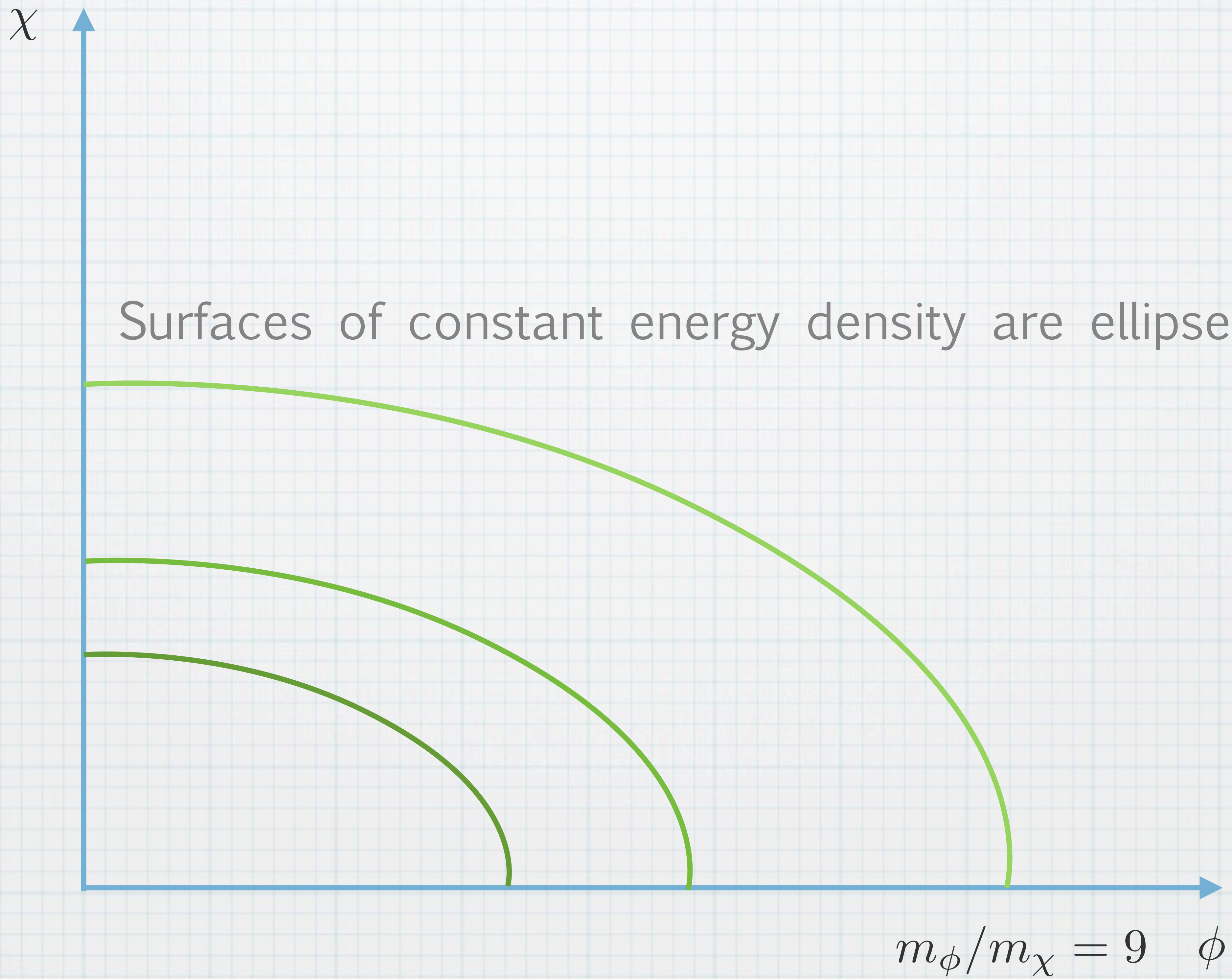


Double quadratic inflation $V(\phi,\chi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2$



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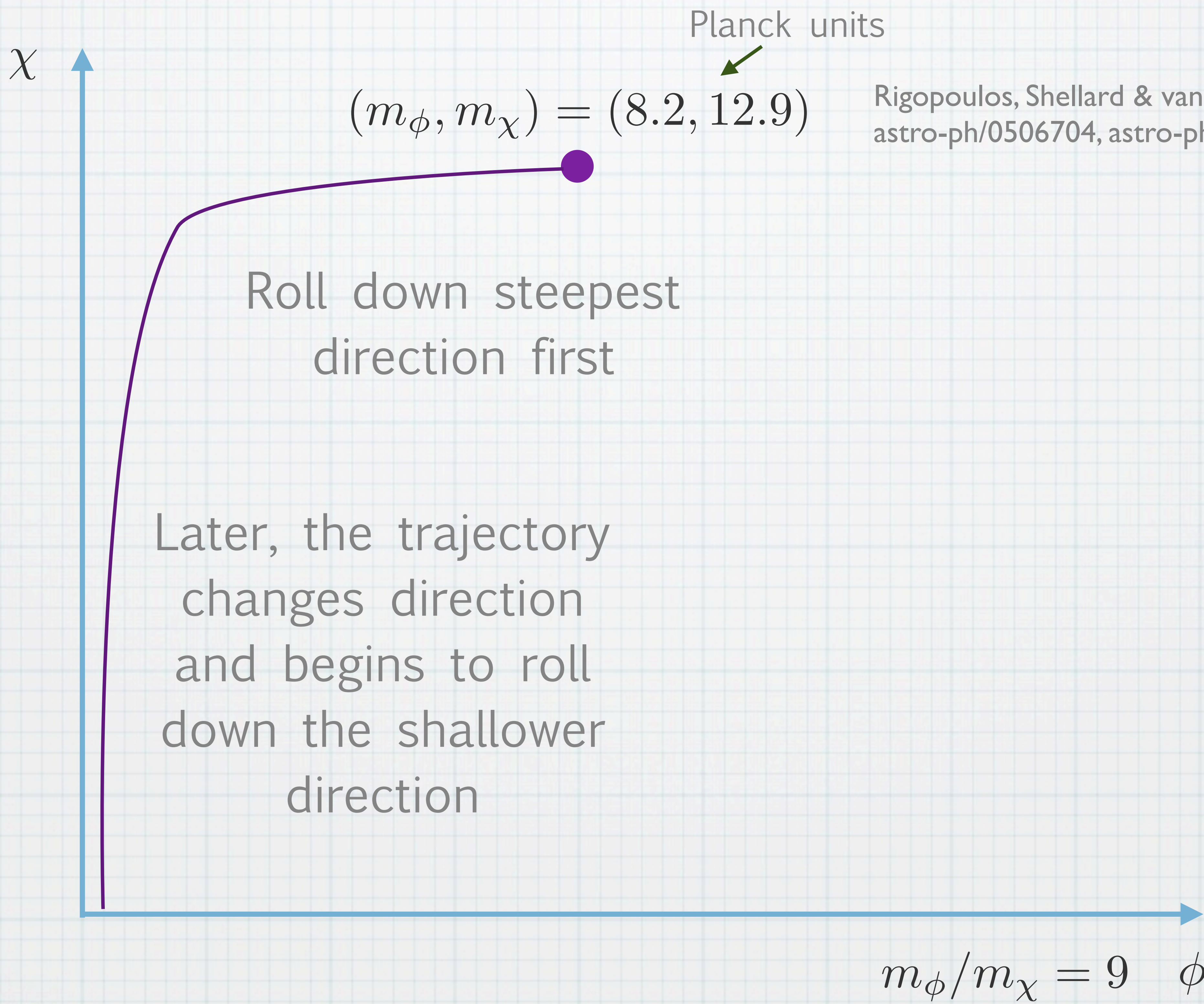
Rigopoulos, Shellard & van Tent
astro-ph/0506704, astro-ph/0511041



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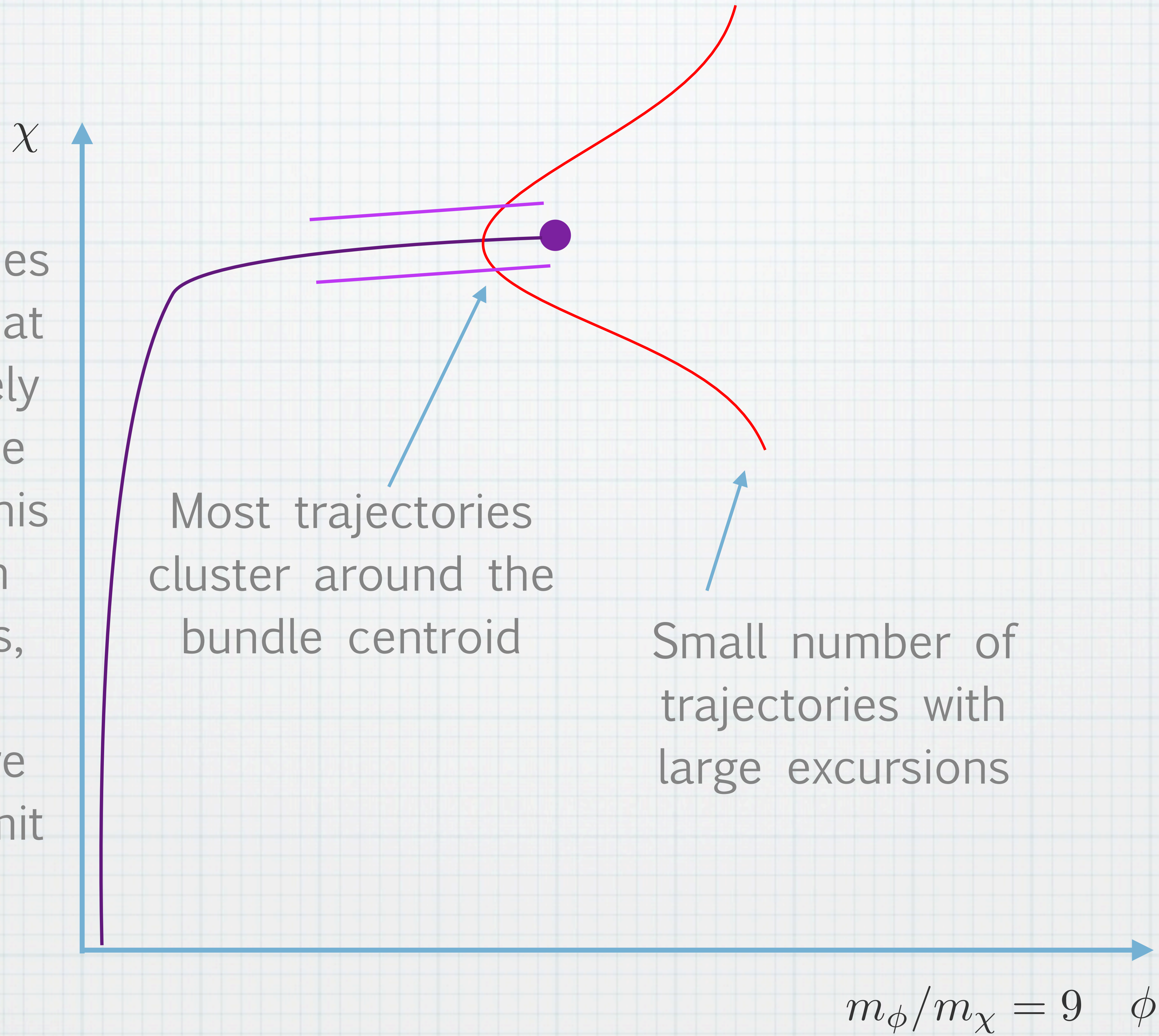
All trajectories should reheat almost surely in the same minimum. This is trivial in most cases, but might require care in the IR limit



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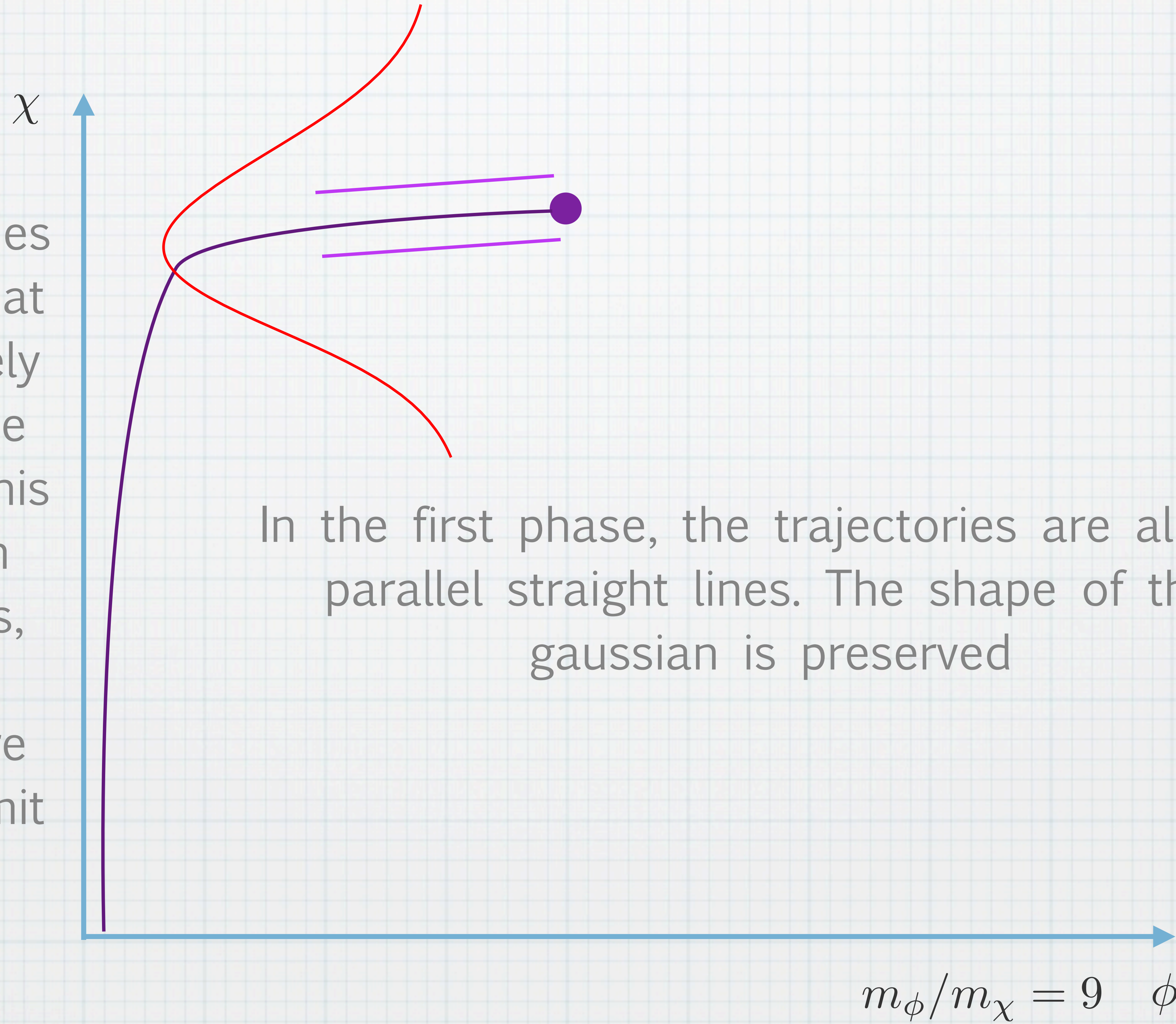
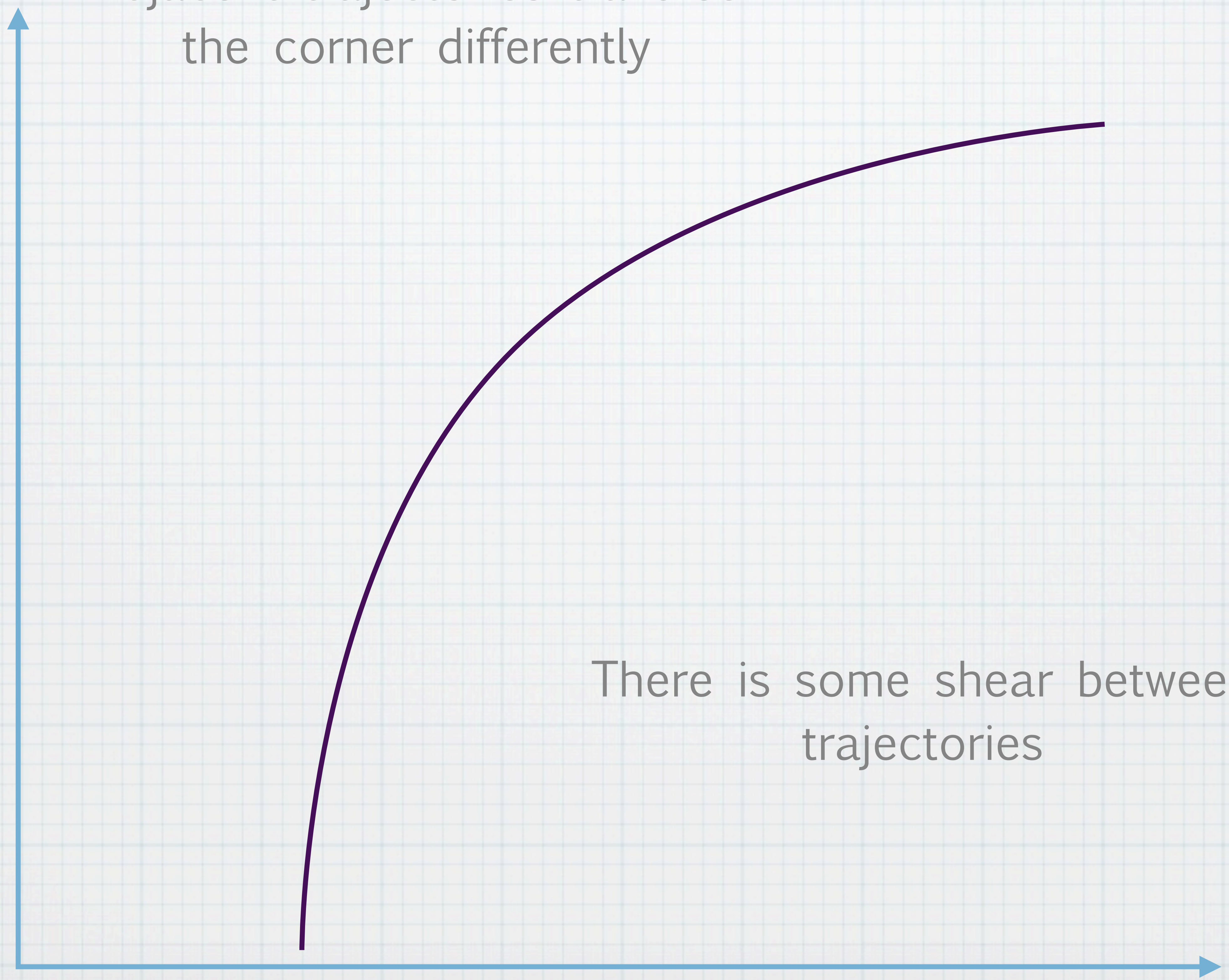


In the first phase, the trajectories are almost parallel straight lines. The shape of the gaussian is preserved

Eventually, we have to turn the corner

Gordon, Wands, Bassett & Maartens (2000)

Adjacent trajectories traverse the corner differently



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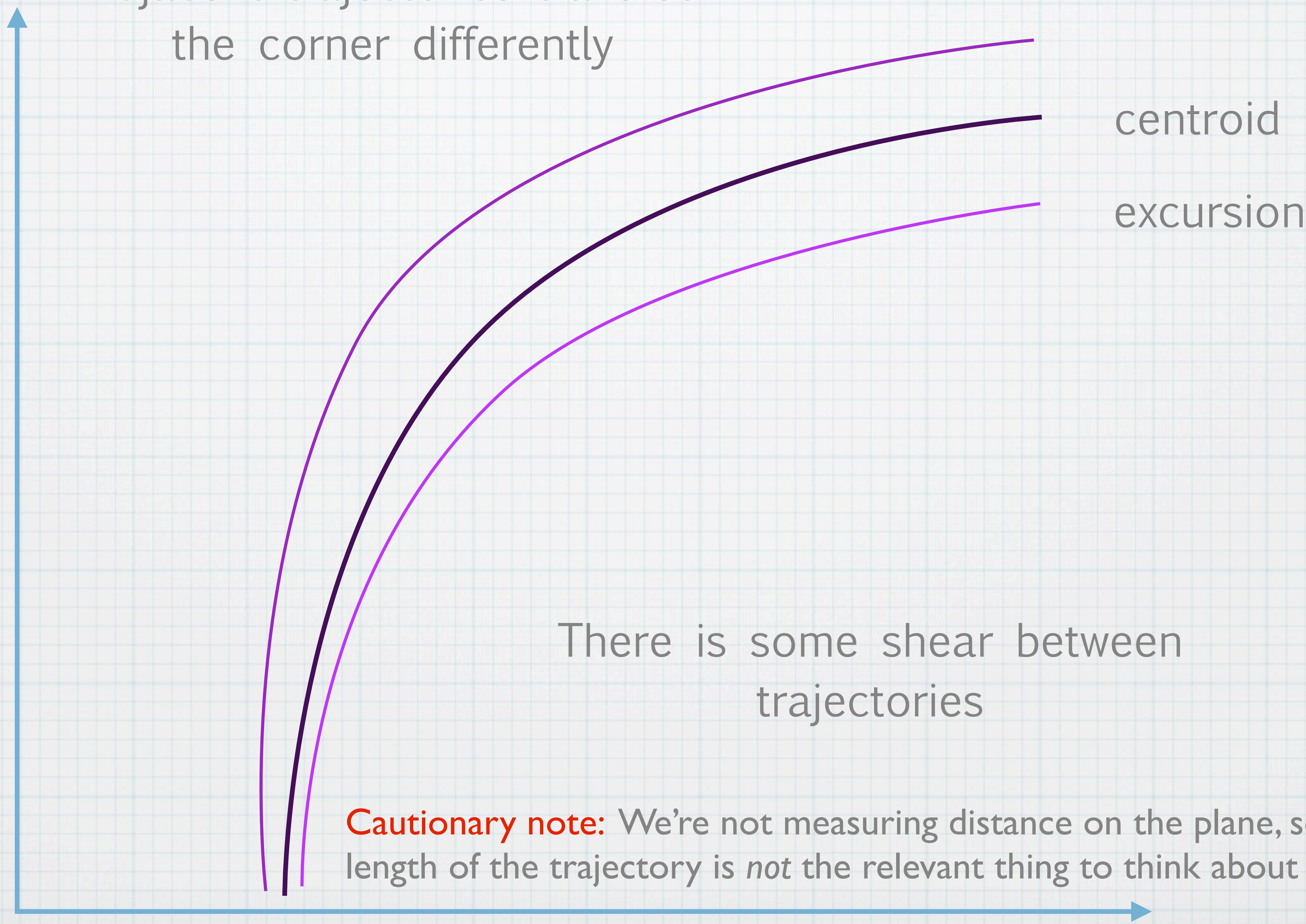

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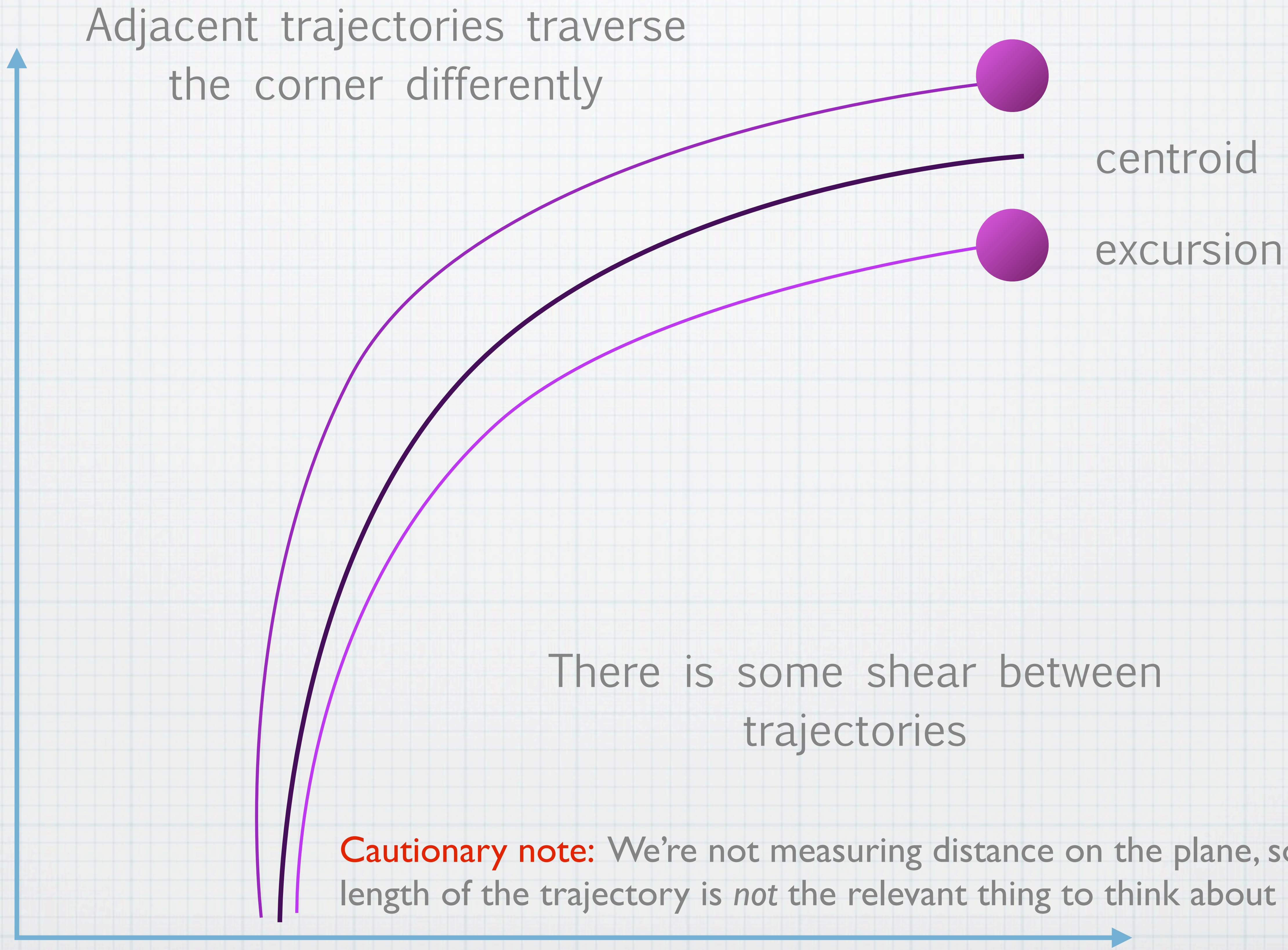

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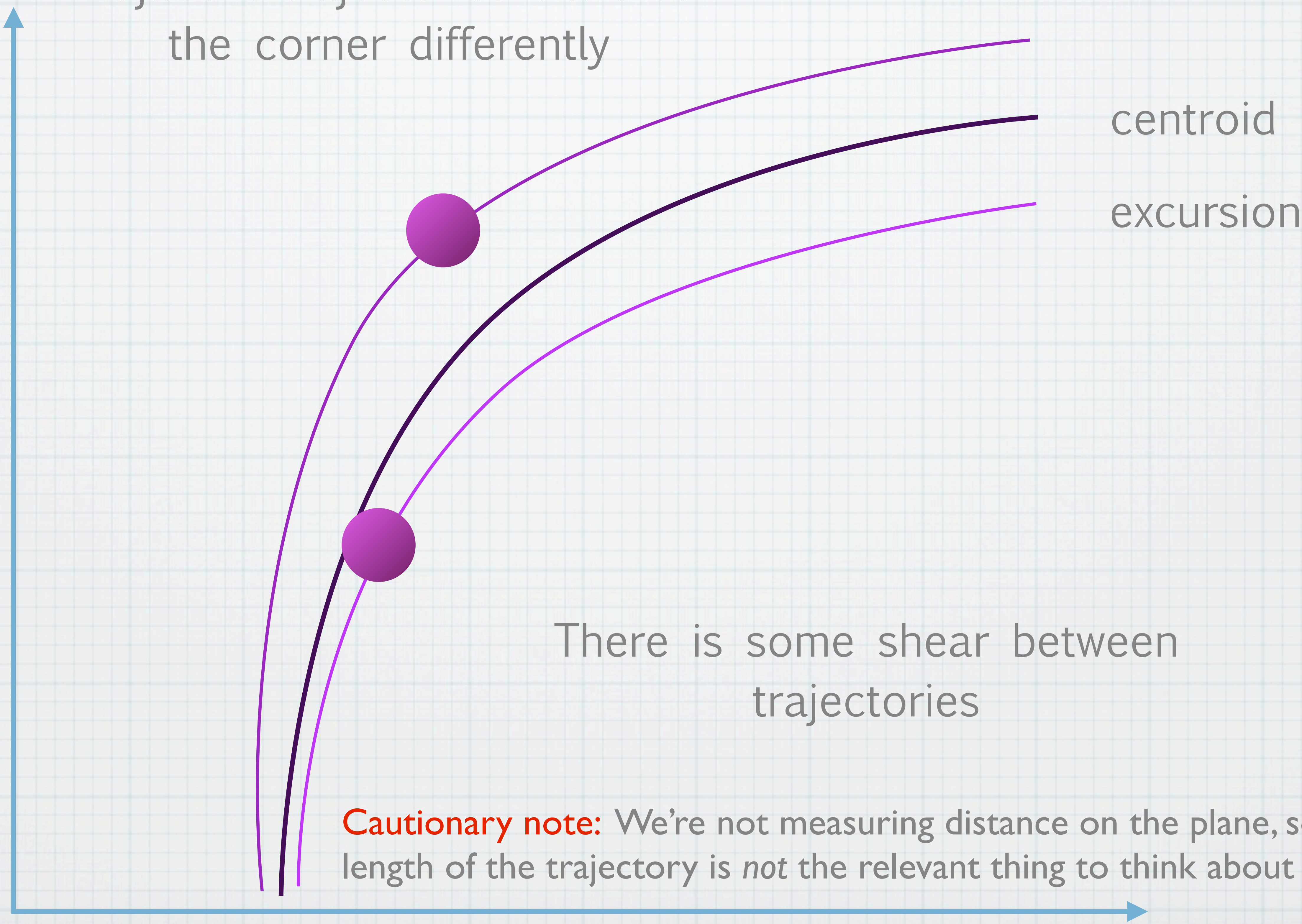

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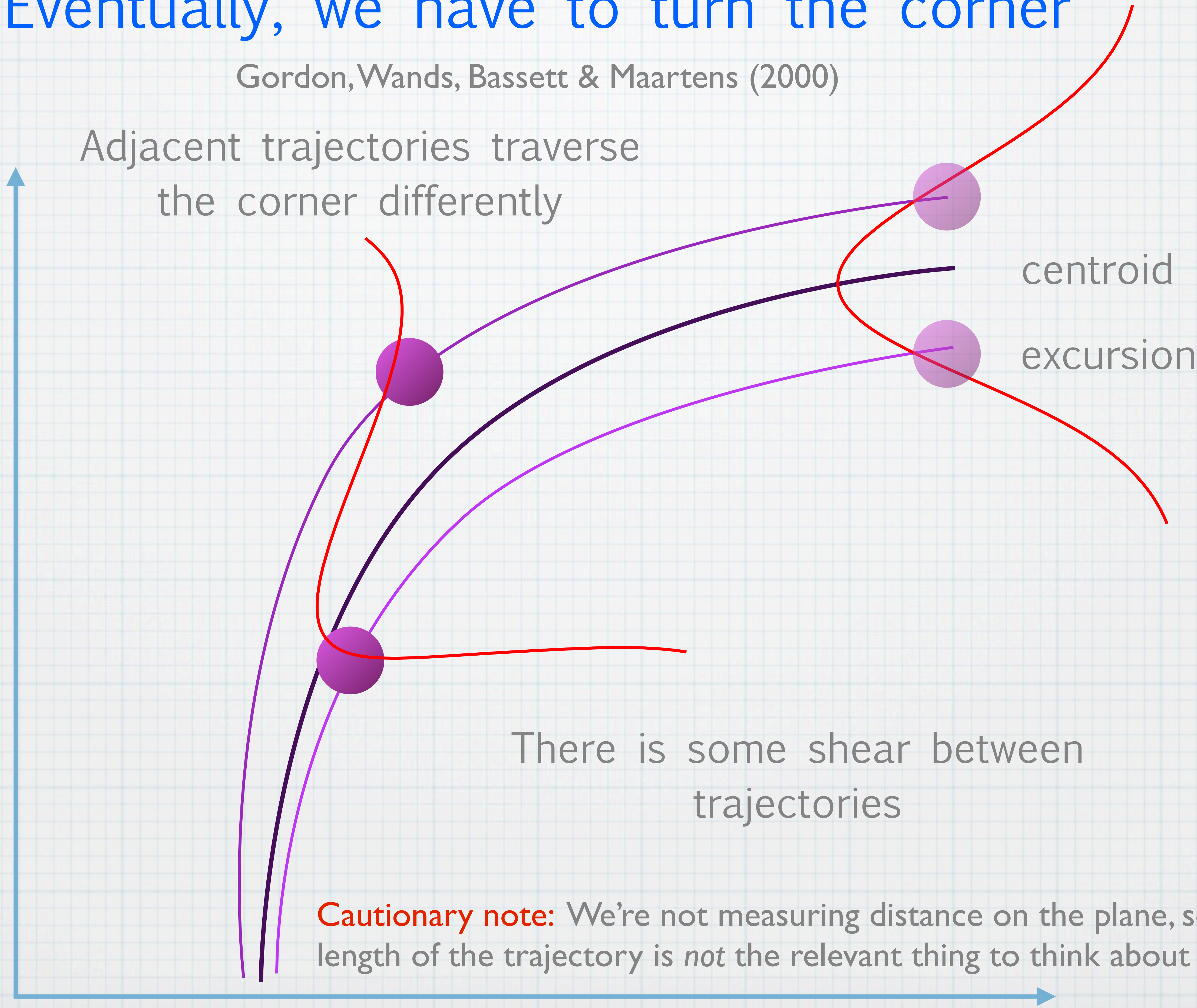

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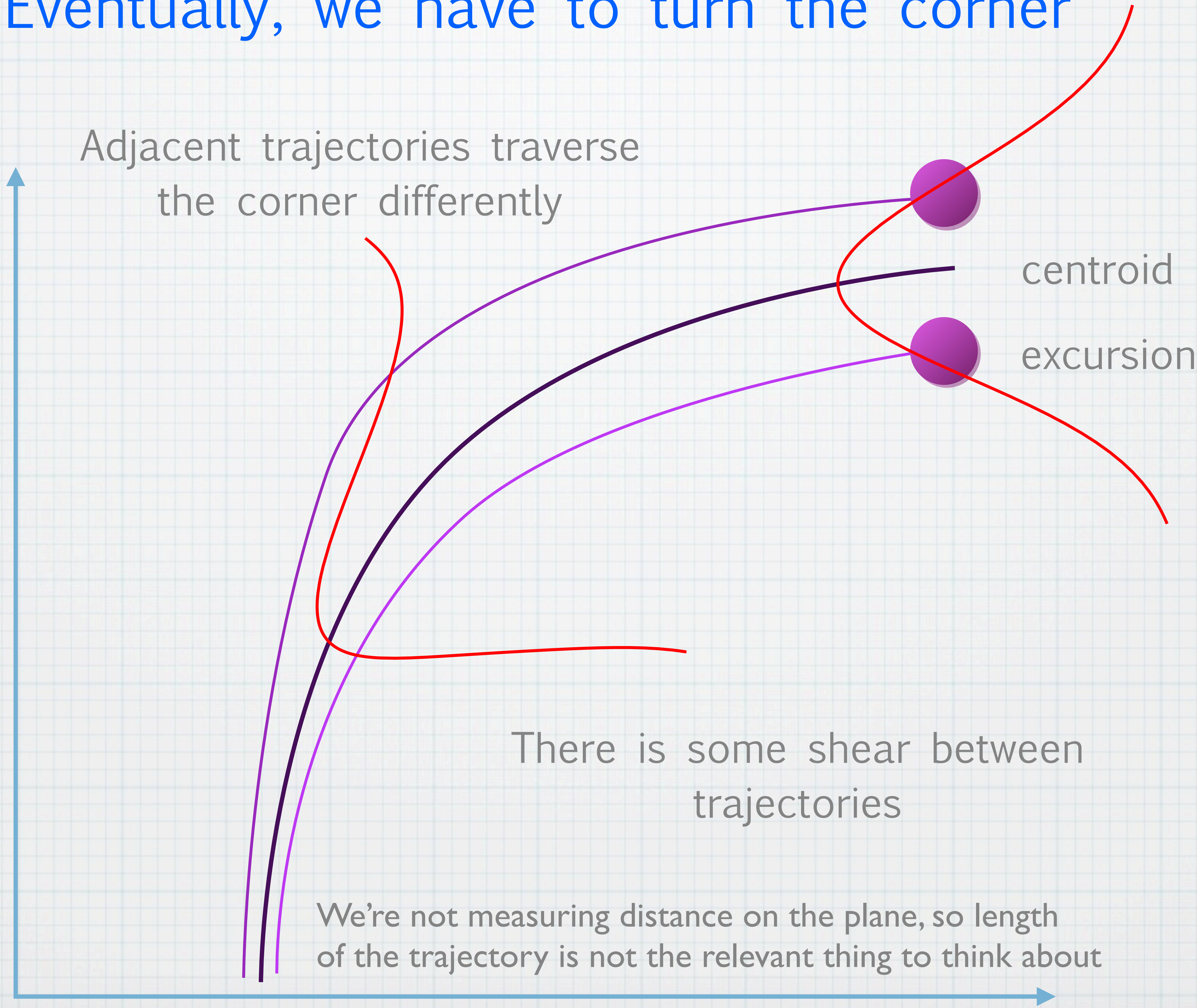

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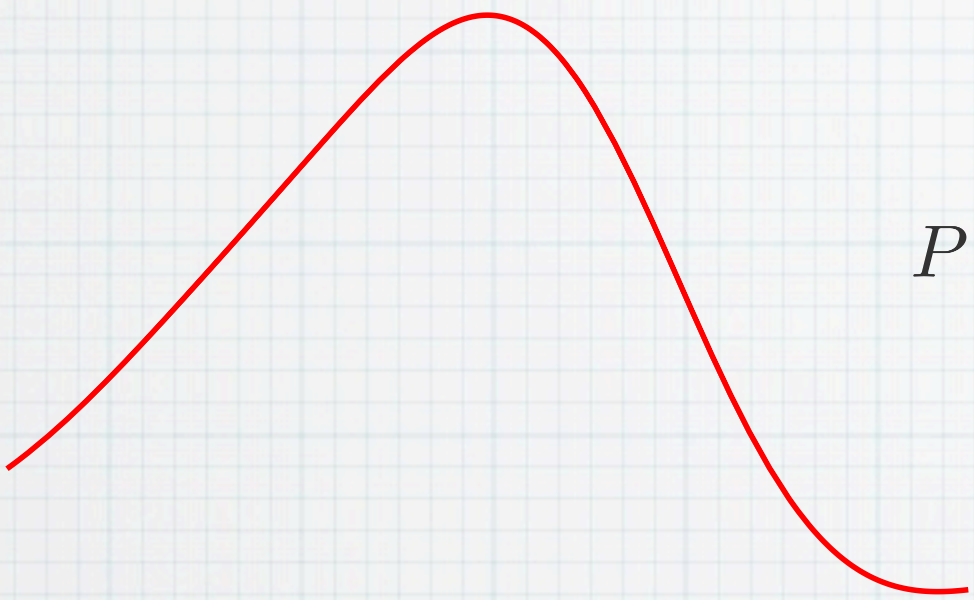


$$P(s) = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\frac{s^2}{\sigma^2}\right)\left[1 + \frac{\alpha}{6\sigma^3} H_3(s/\sigma)\right]$$

Gauss-Hermite series (“Gram-Charlier”, “Edgeworth”)

Contaldi & Magueijo (2001); Matarrese, Verde & Jimenez (2000), LoVerde et al. (2007)

This is the basic mechanism by which you generate multifield nongaussianity. It is a reprocessing effect

$$\zeta(\boldsymbol{x}) = \zeta[\zeta_g(\boldsymbol{x})] \approx \zeta_g(\boldsymbol{x}) + \frac{1}{2}\zeta'\zeta_g(\boldsymbol{x})^2 + \cdots$$

Because the reprocessing is local, the shape is local

In practice, our job is to calculate these moments

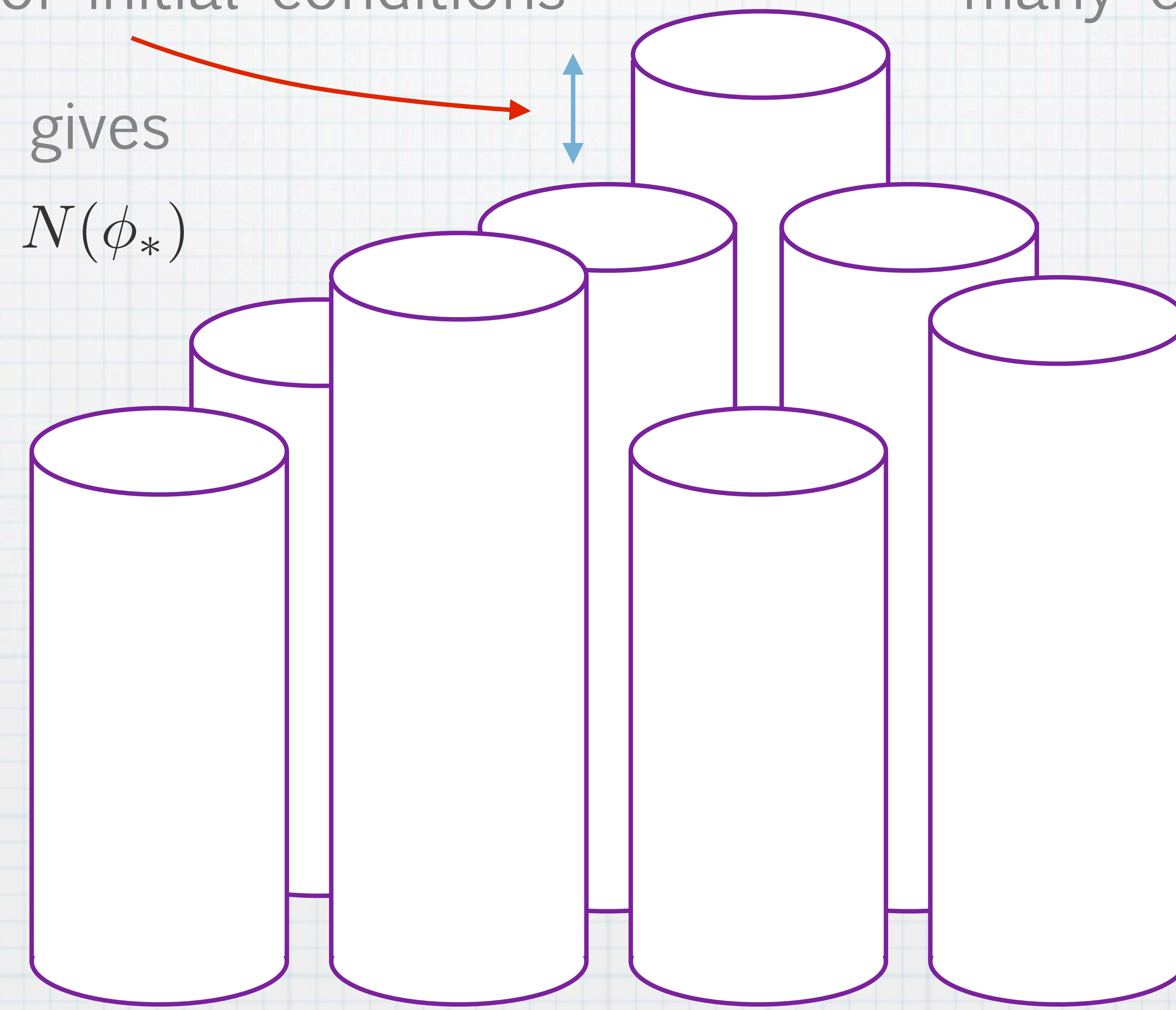
One can think of at least two methods

One is to calculate this length, for each choice of initial conditions

this gives

$N = N(\phi_*)$

Then, one works out how many columns there are of each height



label the initial conditions ϕ_*

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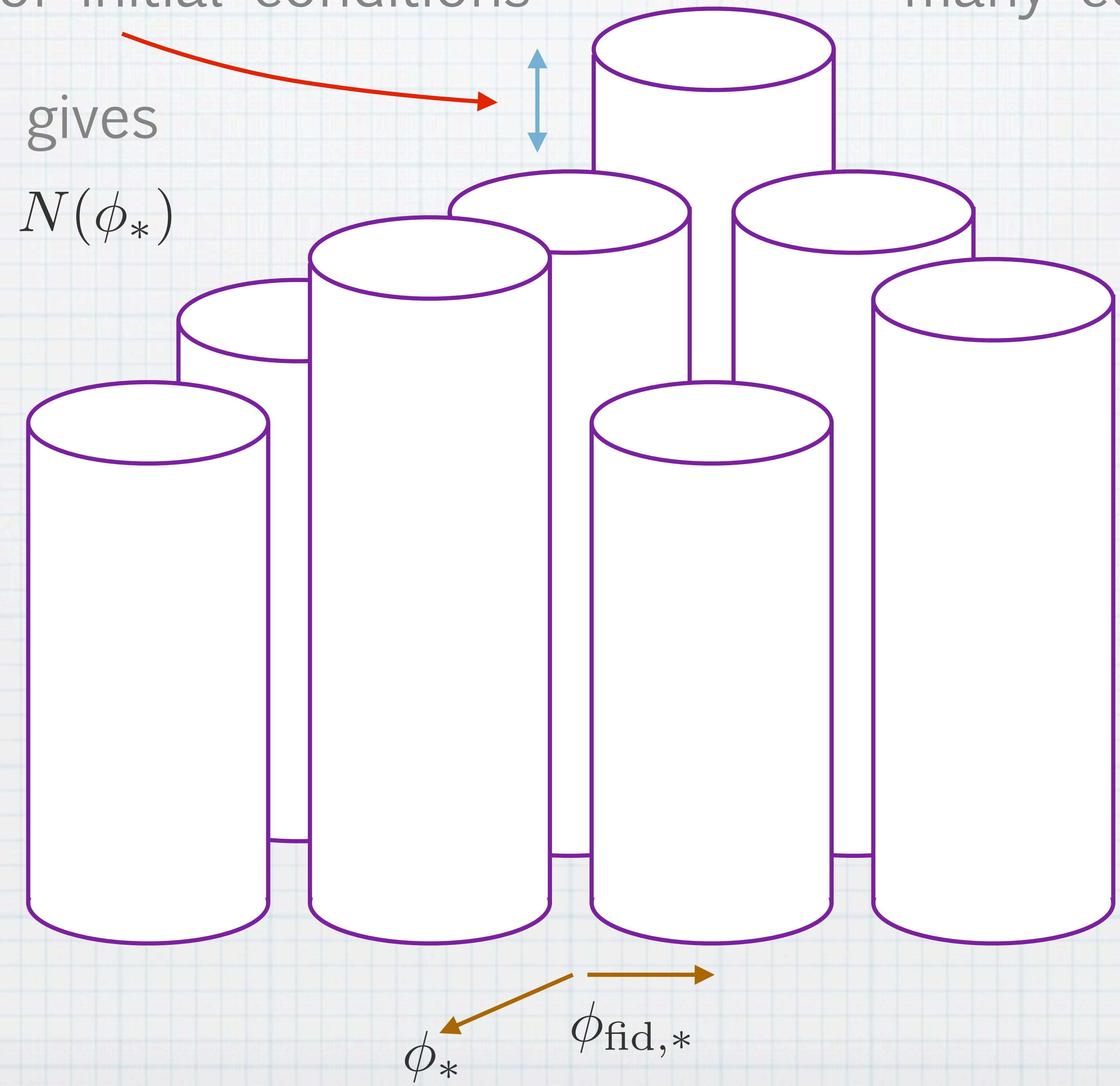

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Lyth & Rodríguez (2005)

$$\zeta(\boldsymbol{x}) = \delta N = \frac{\partial N}{\partial \phi_{i*}} \delta\phi_{i*}(\boldsymbol{x}) + \frac{1}{2} \frac{\partial^2 N}{\partial \phi_{i*} \partial \phi_{j*}} \delta\phi_{i*}(\boldsymbol{x}) \delta\phi_{j*}(\boldsymbol{x}) + \cdots$$

$$\Big\downarrow \text{ Fourier transformation}$$

$$\zeta(\boldsymbol{k}) = \delta N = \frac{\partial N}{\partial \phi_{i*}} \delta\phi_{i*}(\boldsymbol{k}) + \frac{1}{2} \frac{\partial^2 N}{\partial \phi_{i*} \partial \phi_{j*}} [\delta\phi_{i*} * \delta\phi_{j*}]_{\boldsymbol{k}} + \cdots$$

This is the “δN” method. It’s good both analytically and numerically.

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This is the "δN" method. It's good both analytically and numerically.

To make use of this, one forms correlation functions of ζ and uses Wick's theorem

$$\langle \zeta(\boldsymbol{k}_1) \zeta(\boldsymbol{k}_2) \cdots \zeta(\boldsymbol{k}_n) \rangle \supseteq (\delta N \text{ prefactors}) \times \langle \underbrace{\delta\phi_*(\boldsymbol{k}_1)(\delta\phi}_{\langle \delta\phi_* \delta\phi_* \rangle} * \underbrace{\delta\phi)_*(\boldsymbol{k}_2) \cdots \delta\phi_*}_{\langle \delta\phi_* \delta\phi_* \rangle}(\boldsymbol{k}_n) \rangle$$

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$$\langle \delta\phi_* \delta\phi_* \delta\phi_* \rangle$$

This makes ζ a sum of correlation functions of the initial conditions

In fact, δN does *two* jobs for us

surface on which we care about the answer. *Really*, last scattering (usually we stop somewhere before)

Correlation functions of ζ **here** expressed in terms of correlation functions of the initial conditions **here**

surface on which we give initial conditions

we usually calculate correlation functions here, perturbatively in H/M_P and slow-roll parameters

In fact, δN does *two* jobs for us

I'm going to briefly change subject on the next slide, but this will come back later

comoving hypersurface

surface on which we care about the answer. Really, last scattering

(usually we stop somewhere before)

$\mathrm{O}(\epsilon^2)$

Zaldarriaga (2003); DS, Lyth & Malik (2008); Mahajan & Rangarajan (2010) + talk today!

$$\langle \delta\phi(\boldsymbol{k}_1)\delta\phi(\boldsymbol{k}_2)\delta\phi(\boldsymbol{k}_3)\rangle_\eta \supseteq \xi \ln |k\eta_*|$$

cf. Renaux-Petel talk

spatially flat hypersurface

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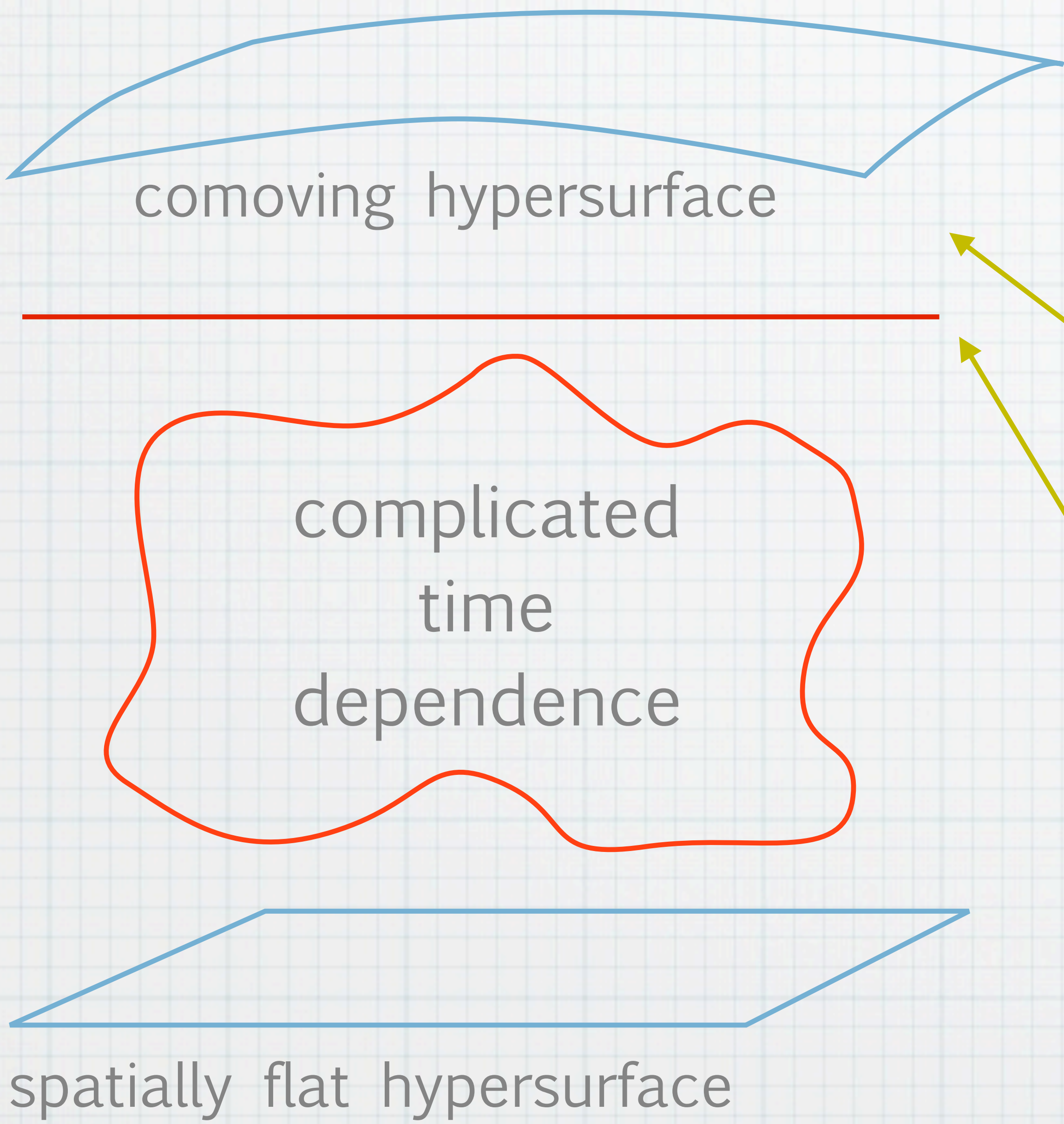


surface on which we care about the answer. Really, last scattering
(usually we stop somewhere before)

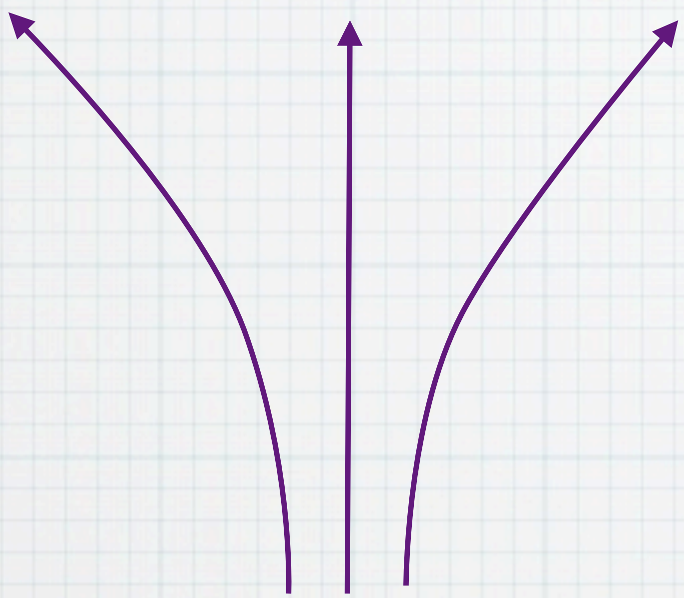
then, a gauge transformation is required

complicated time-dependent logs summed up to **here**

surface on which we give initial conditions

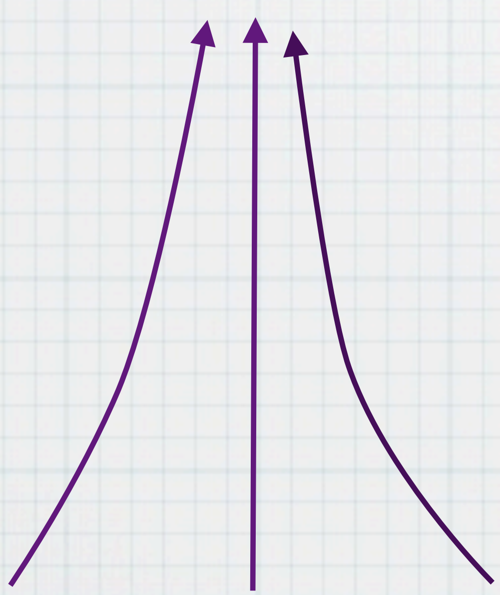
we usually calculate correlation functions here, perturbatively in H/M_P and slow-roll parameters

Alternatively, the picture of shearing trajectories implies there must be a formulation similar to geometrical optics, or a laser in a box (eg. nongaussian beams at LIGO)



Trajectories dispersing or defocusing

Variance increasing



Trajectories converging or focusing

Variance decreasing

Mulryne, DS, Wesley (2009,2010)

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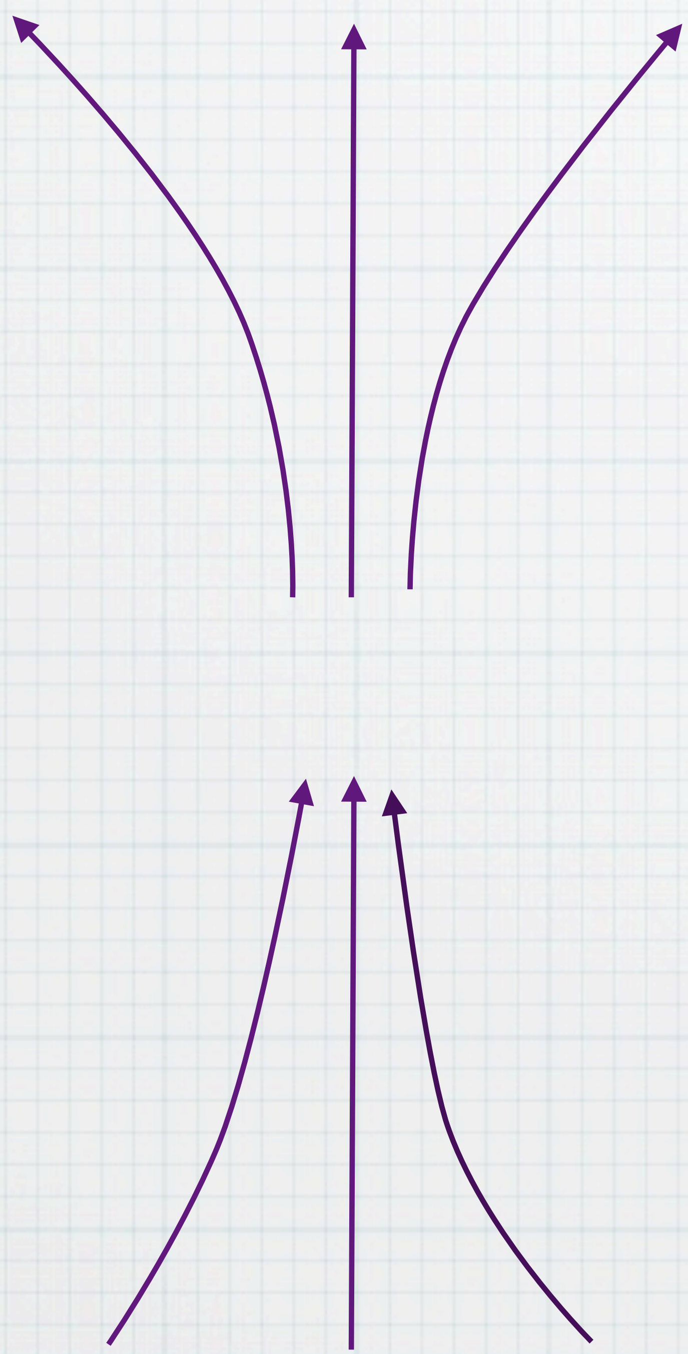


Trajectories dispersing or defocusing

Variance increasing

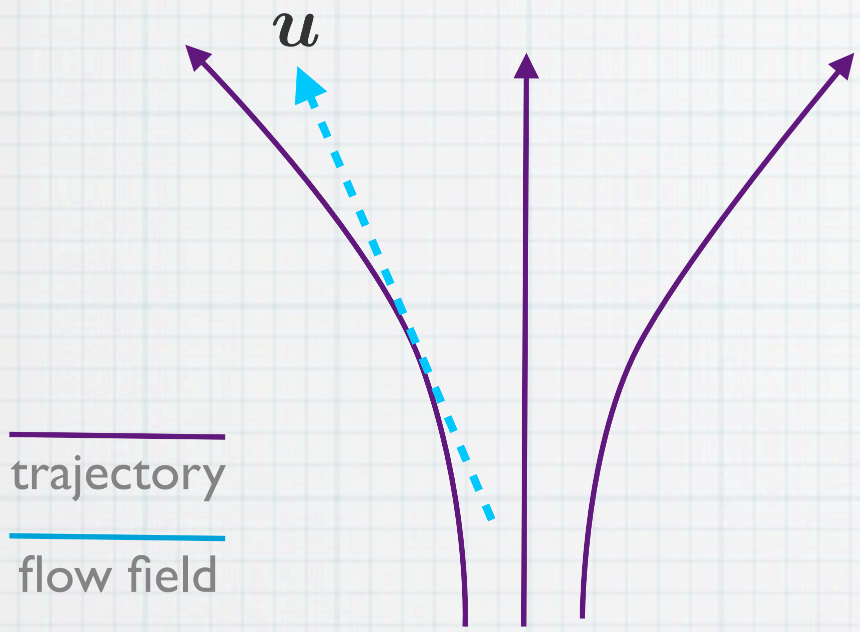
The trajectories are global properties of the flow. In other disciplines, we know this is useful in proving theorems, eg. Raychaudhuri equation and Penrose/Geroch/Hawking thms.

Trajectories converging or focusing

Variance decreasing

Mulryne, DS, Wesley (2009,2010)

Transport of the probability distribution



The trajectories are the integral curves of a flow field **u**

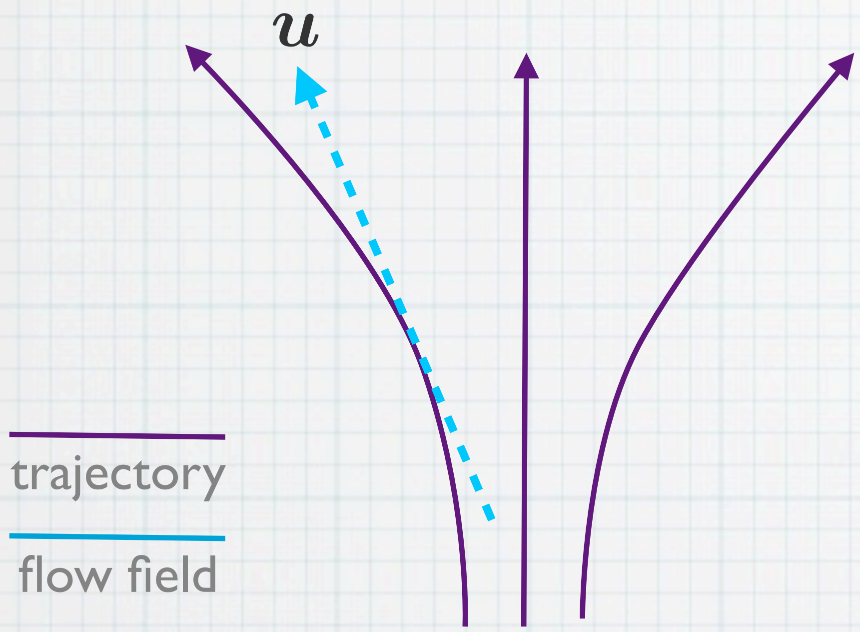
$$\boldsymbol{P} = \frac{1}{\sqrt{2\pi}\sigma} \exp\left(-\frac{1}{2}\frac{s^2}{\sigma^2}\right)\left[1 + \frac{\alpha}{6\sigma^3} H_3(s/\sigma)\right]$$

To work out how the distribution evolves under the flow, it is sufficient to impose conservation of probability

$$\frac{\mathrm{d}\boldsymbol{P}}{\mathrm{d}t} + \frac{\partial}{\partial\phi}(u\boldsymbol{P}) = 0$$

This is the continuity equation for fluid flow

Dilatation and shear-vorticity decomposition



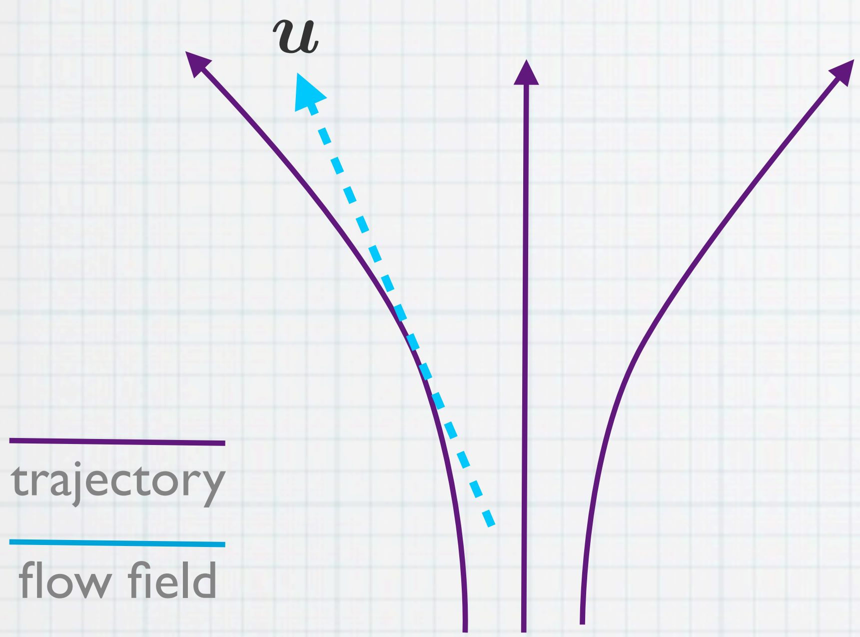
The trajectories are the integral curves of a flow field **u**

$$\partial_i u_j = \underbrace{\frac{1}{3}\theta\delta_{ij}}_{\text{dilatation}} + \underbrace{\sigma_{ij}}_{\text{shear-vorticity}}$$

Also, decompose Σ_{ij} similarly

$$\Sigma_{ij} = \underbrace{\frac{1}{3}\Sigma\delta_{ij}}_{\text{sum of variances}} + \underbrace{\Omega_{ij}}_{\text{“covariance”}}$$

Dilatation and shear-vorticity decomposition



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dilatation   shear-vorticity

$$\Sigma_{ij} = \frac{1}{3}\Sigma\delta_{ij} + \Omega_{ij}$$

sum of variances   "covariance"

It turns out there are two **transport equations** which describe focusing and shearing

$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\exp\left(-\frac{2}{3}\Theta(t)\right)\Sigma\right] = 2\exp\left(-\frac{2}{3}\Theta(t)\right)\sigma_{ij}\Omega_{ij}$$

Derivative along the flow

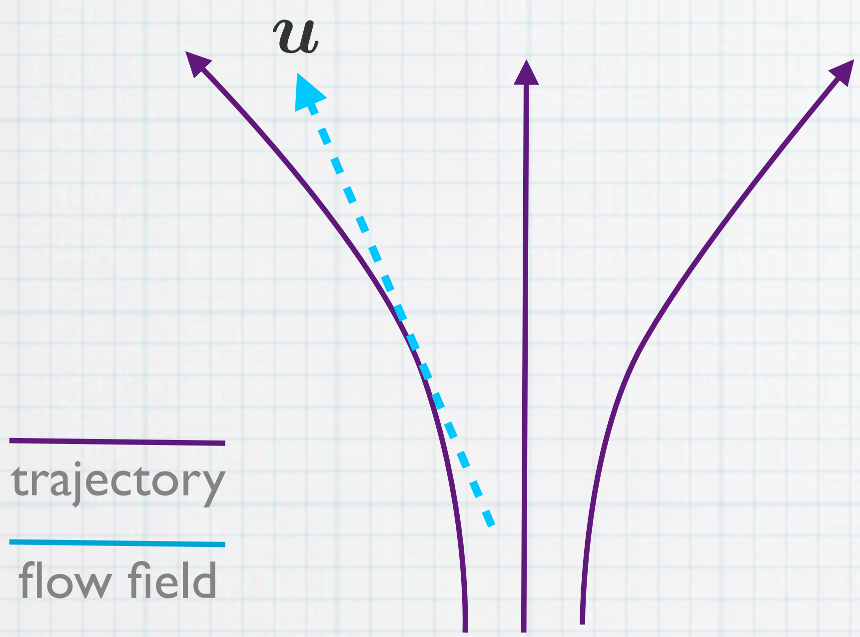
influence of shear

$$\Theta(t) = \int_{t_0}^{t}\theta(t)\,\mathrm{d}t$$

the **integrated dilatation**

Fiducial start time

Dilatation and shear-vorticity decomposition



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For example, in a shear-free flow — just isotropic expansion

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In other words, Σ depends only on the local integrated expansion of the flow

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$$+\exp\left(-\frac{2}{3}\Theta(t)\right)\sigma_{am}\Omega_{mb}\left[\delta_{ia}\delta_{jb}+\delta_{ib}\delta_{ja}-\frac{2}{3}\delta_{ab}\delta_{ij}\right]$$

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"isotropic" growth

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$$\frac{\mathrm{d}}{\mathrm{d}t}\left[\exp\left(-\frac{2}{3}\Theta(t)\right)\Omega_{ij}\right] = \frac{2}{3}\exp\left(-\frac{2}{3}\Theta(t)\right)\Sigma\sigma_{(ij)}$$

"isotropic" growth

$$+\exp\left(-\frac{2}{3}\Theta(t)\right)\sigma_{am}\Omega_{mb}\left[\delta_{ia}\delta_{jb}+\delta_{ib}\delta_{ja}-\frac{2}{3}\delta_{ab}\delta_{ij}\right]$$

anisotropic growth

None of this is particularly new, although this way of expressing it is

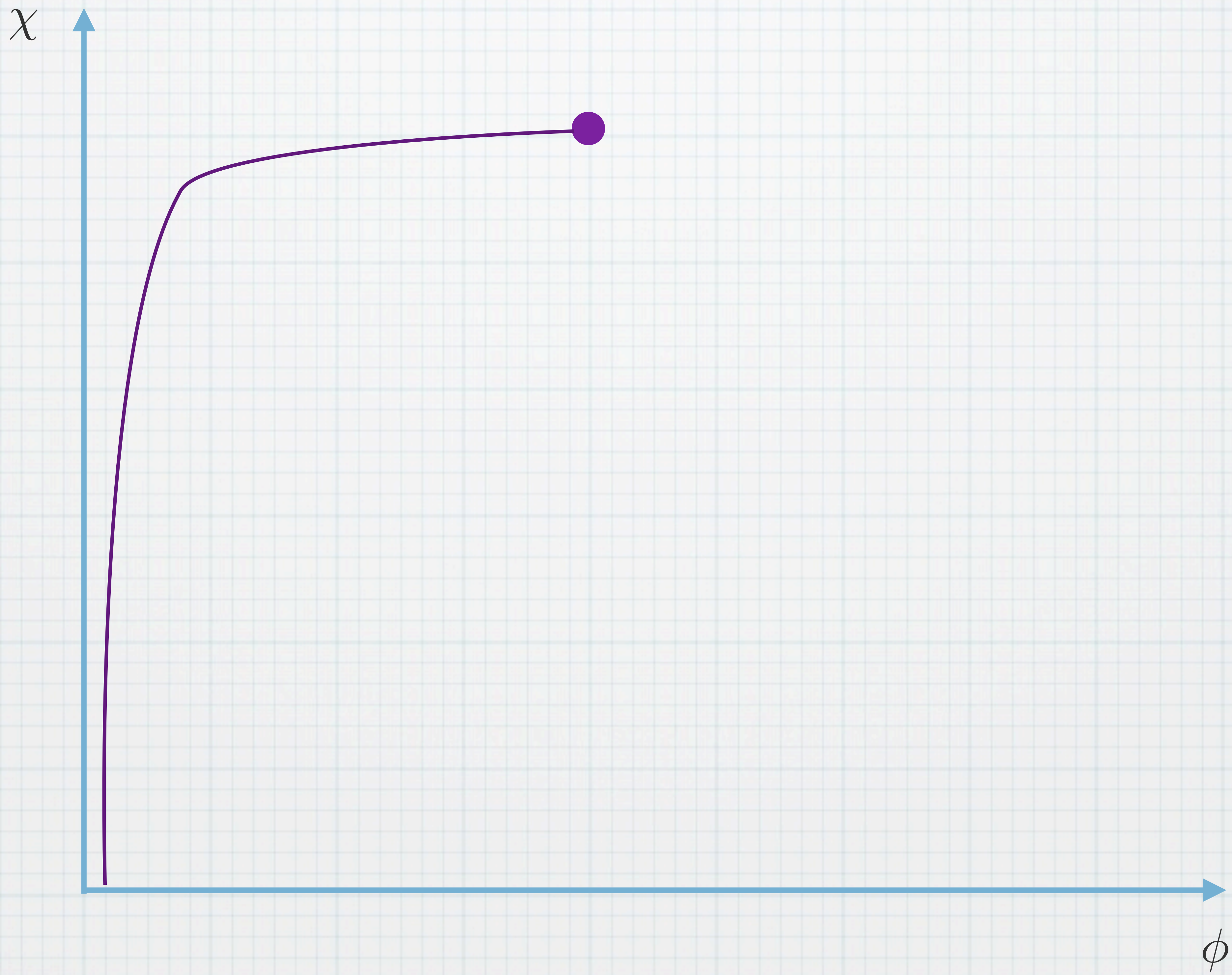
Gordon et *al.* 2001	decomposition into trajectories
Groot Nibbelink & van Tent (2001)	curved field space metric, vielbein adapted to the trajectories
Rigopoulos, Shellard & van Tent (2004, 2005)	first application of this picture to nongaussianities (as far as I know)
Byrnes, Choi & Hall (2008, 2009)	reverse engineer interesting trajectories
Peterson & Tegmark (2010a,b)	similar to N&vT, RS&vT, BC&H specialized to two fields

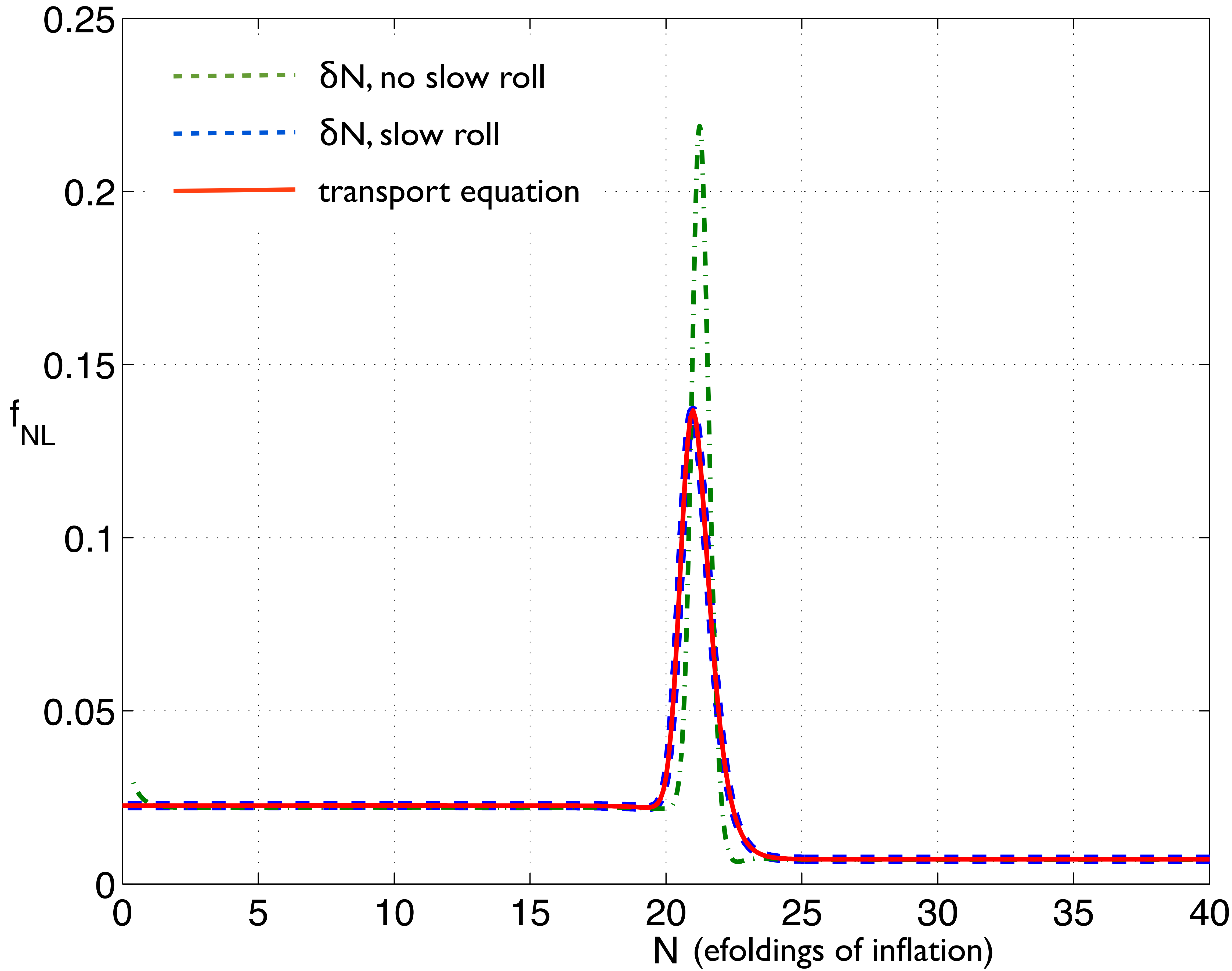
I want to compute some exciting nongaussianities! What do I do?

conventional δN (default)	Typically need initial surface at horizon crossing. Computation of $\partial\phi(\text{late})/\partial\phi(\text{early})$ is trouble Vernizzi & Wands (2006); Battefeld & Easther (2006); Meyers & Sivanandam (2010) + talk today!
Suyama, Tanaka & Yokoyama	Use transport to get ϕ(late) in terms of ϕ(early). Use δN for gauge transform Suyama, Tanaka & Yokoyama (2007a,b)
Mulryne, DS & Wesley	Either use transport to get ϕ(late) in terms of ϕ(early) and δN for gauge transform, or transform to ζ immediately and do the calculation there

Double quadratic inflation

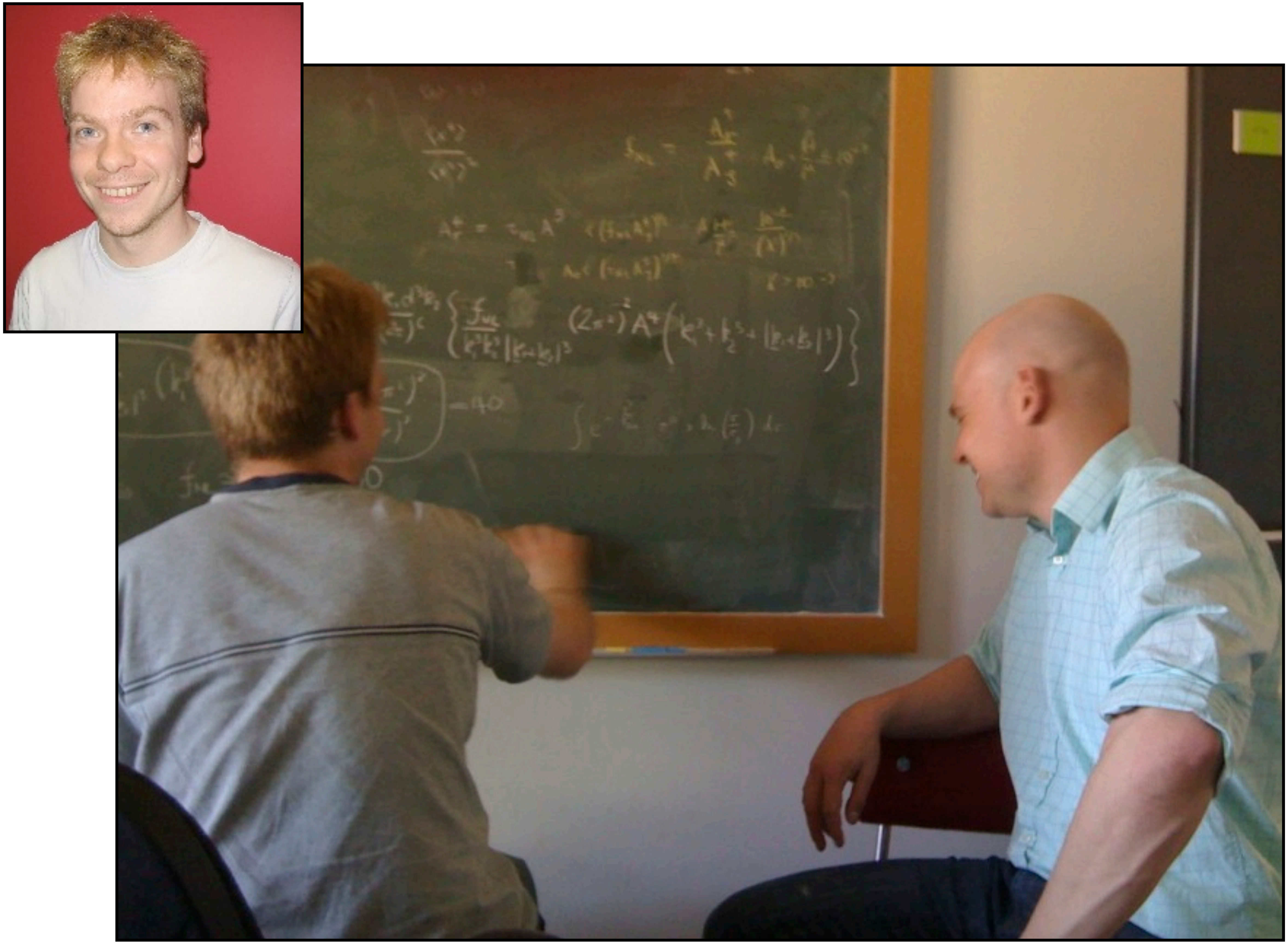
$$V(\phi,\chi) = \frac{1}{2}m_\phi^2\phi^2 + \frac{1}{2}m_\chi^2\chi^2$$





δN, no slow roll
δN, slow roll
transport equation
0.25
0.2
0.15
0.1
0.05
0
fNL
0
5
10
15
20
25
30
35
40
N (efoldings of inflation)

All these plots were made by David Mulryne (Imperial/QMUL)



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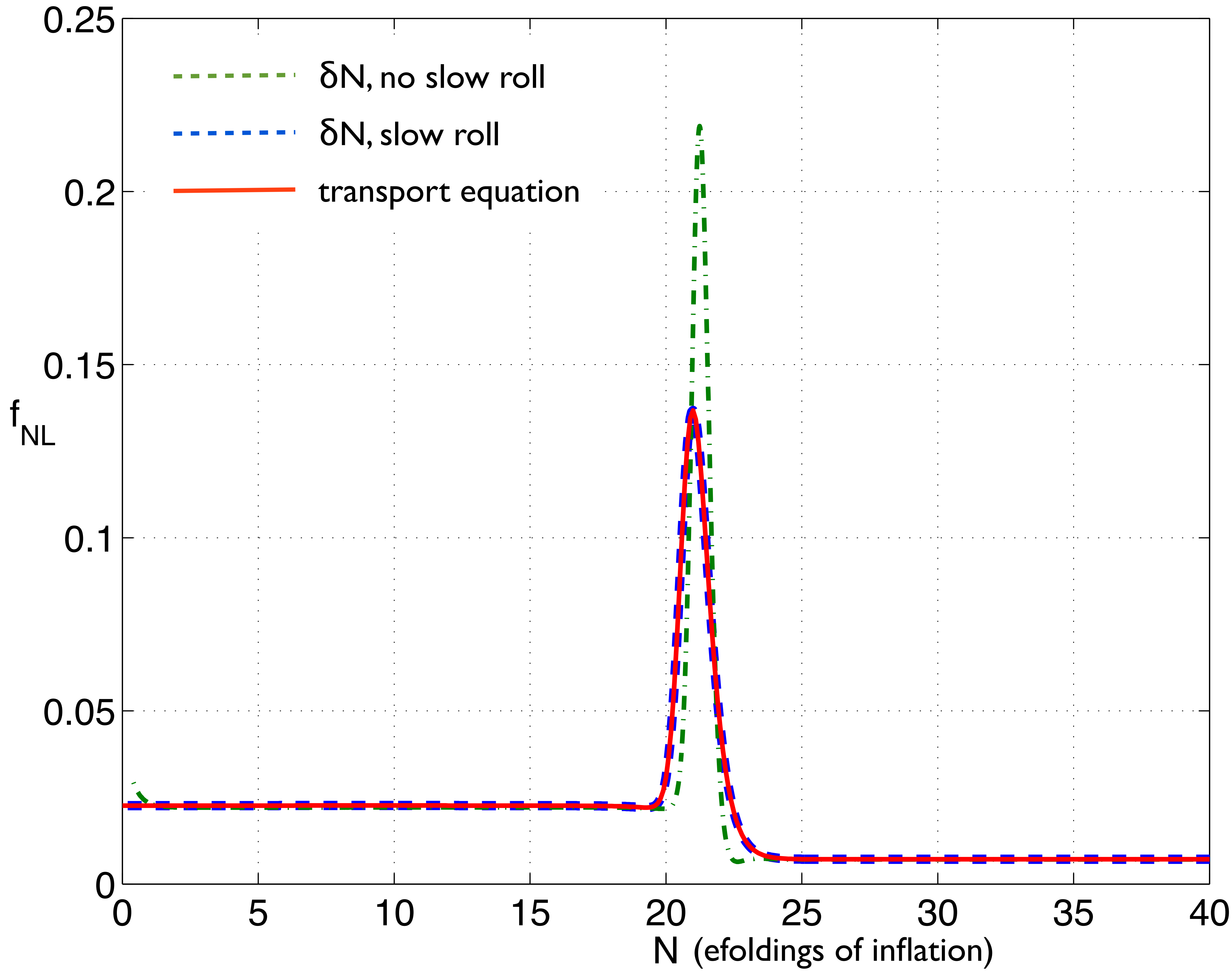
δN, no slow roll
δN, slow roll
transport equation
0.25
0.2
0.15
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0
f_{NL}
0
5
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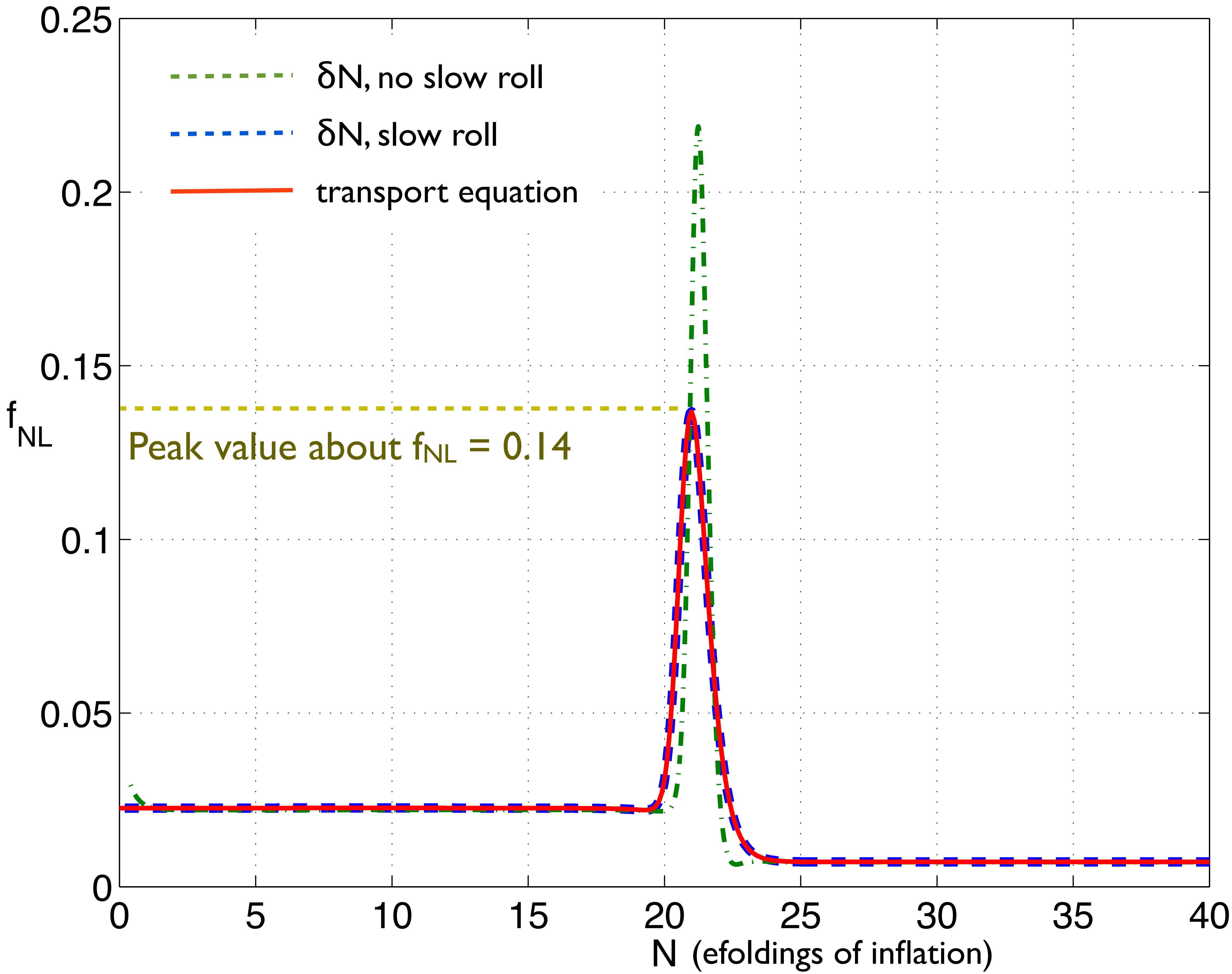
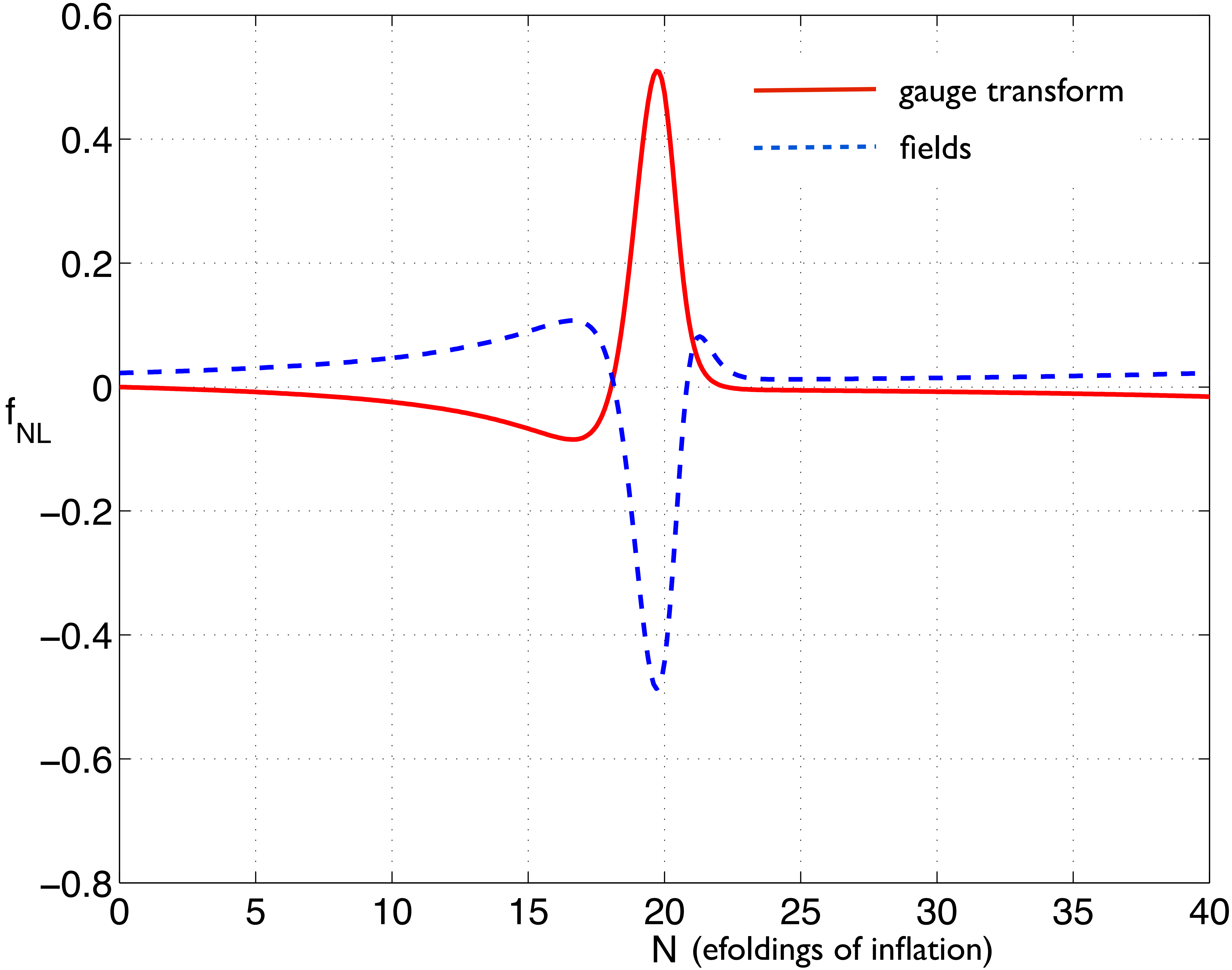
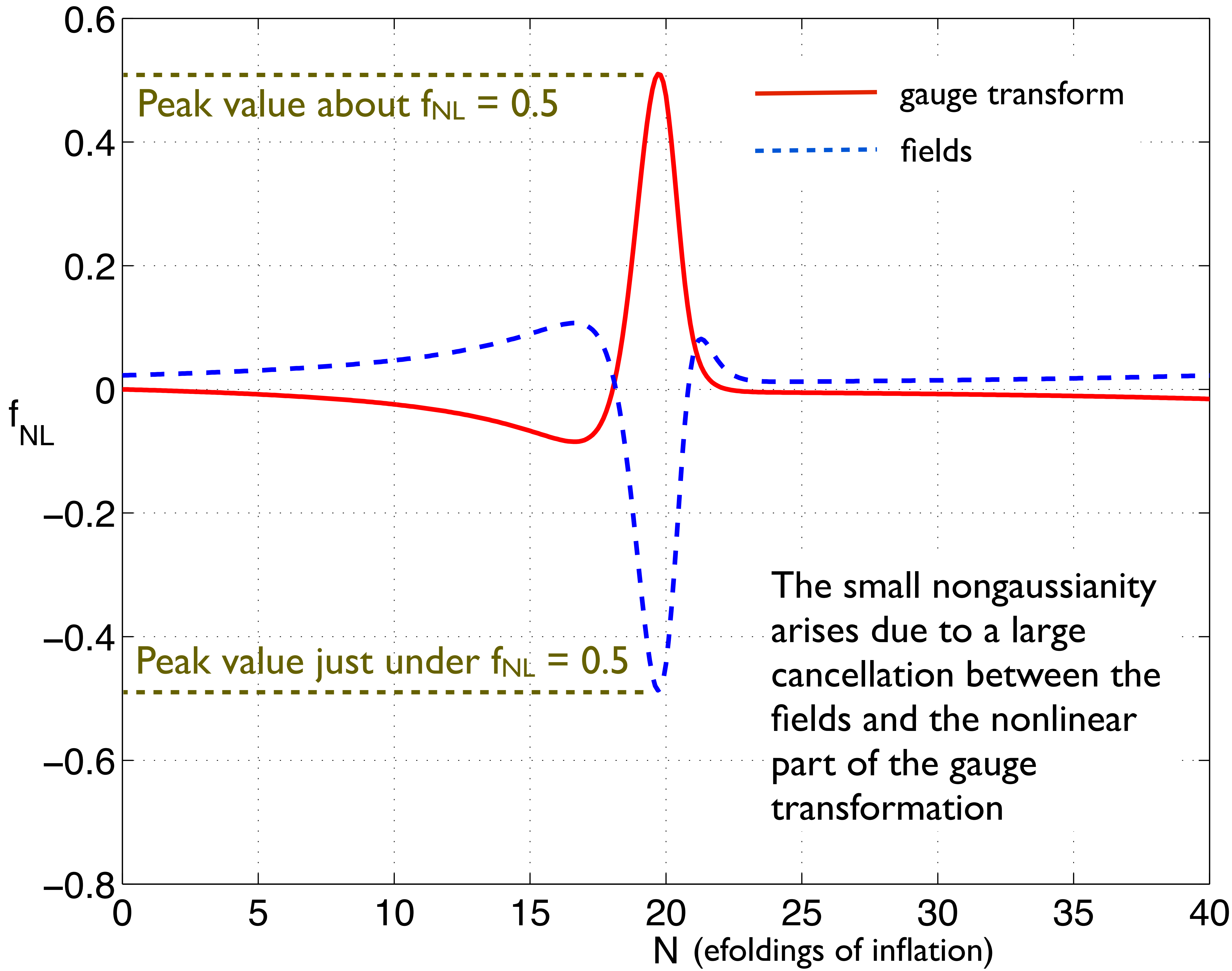
N (efoldings of inflation)


δN, no slow roll
δN, slow roll
transport equation
Peak value about f_{NL} = 0.14
f_{NL}
N (efoldings of inflation)
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
20
25
30
35
40

Double quadratic inflation: gauge/evolution split



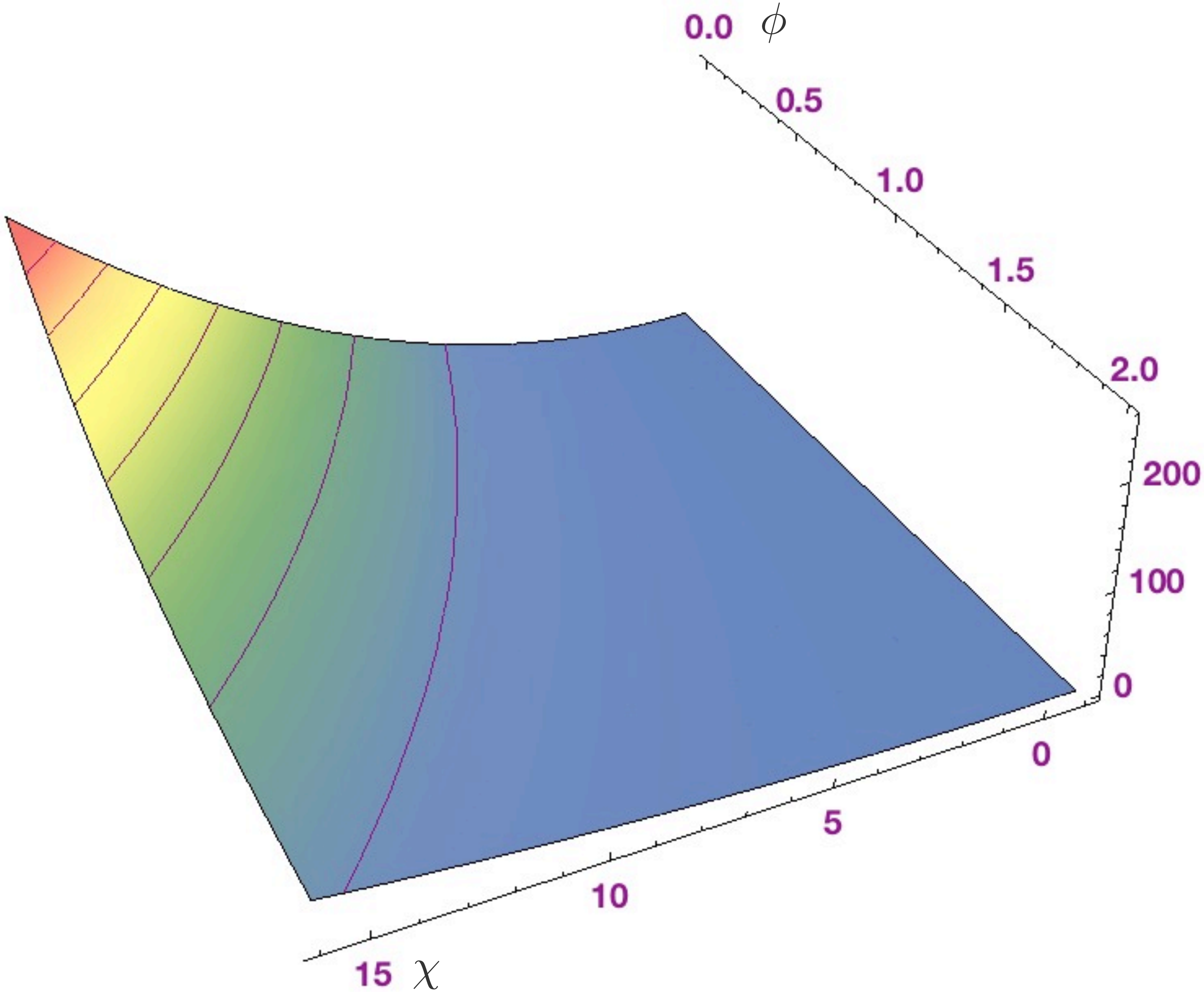
Double quadratic inflation: gauge/evolution split



The small nongaussianity arises due to a large cancellation between the fields and the nonlinear part of the gauge transformation

A model due to Byrnes, Choi & Hall

$$V = V_0 \chi^2 \exp(-\lambda\phi)$$



A model due to Byrnes, Choi & Hall

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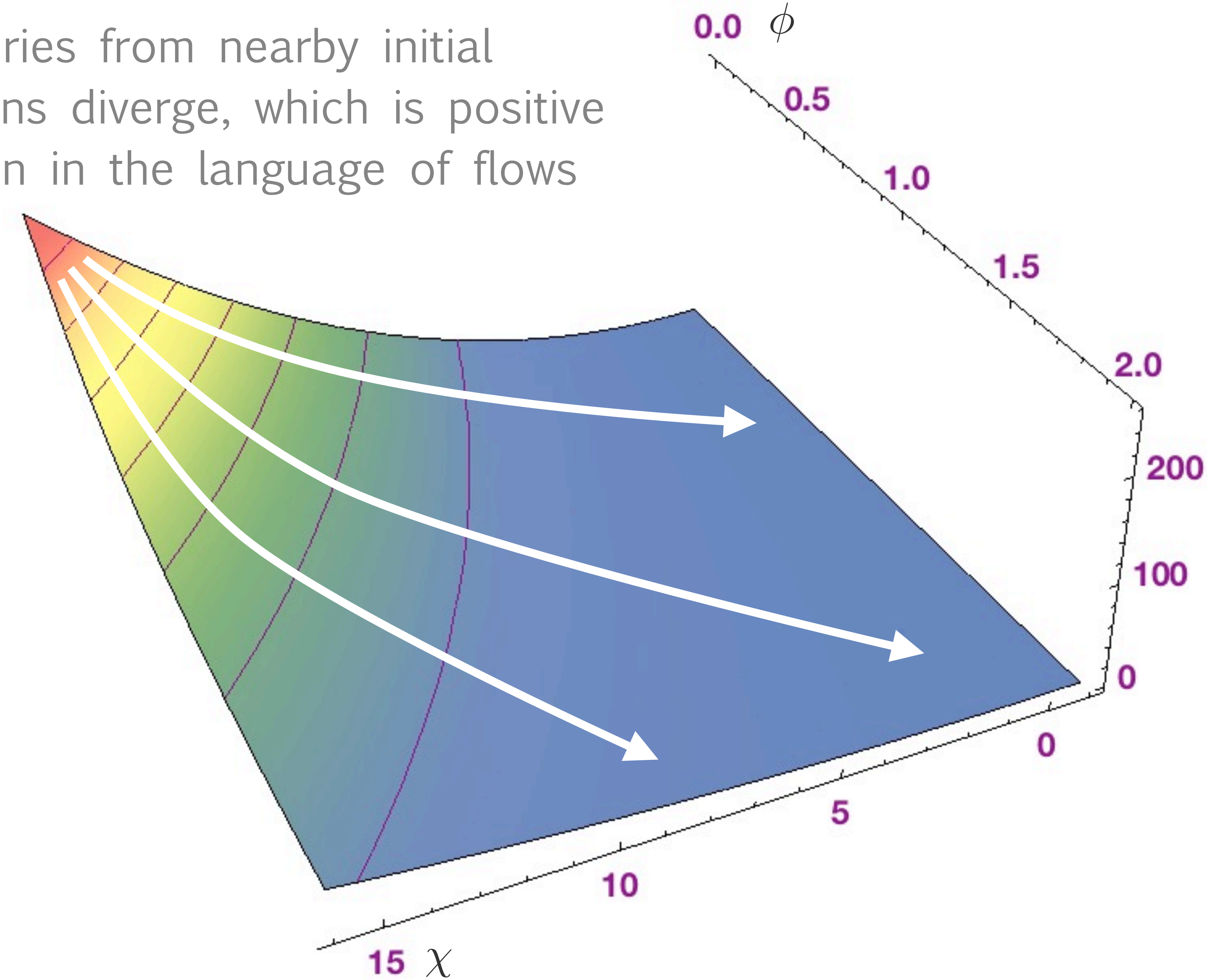
Trajectories from nearby initial conditions diverge, which is positive dilatation in the language of flows



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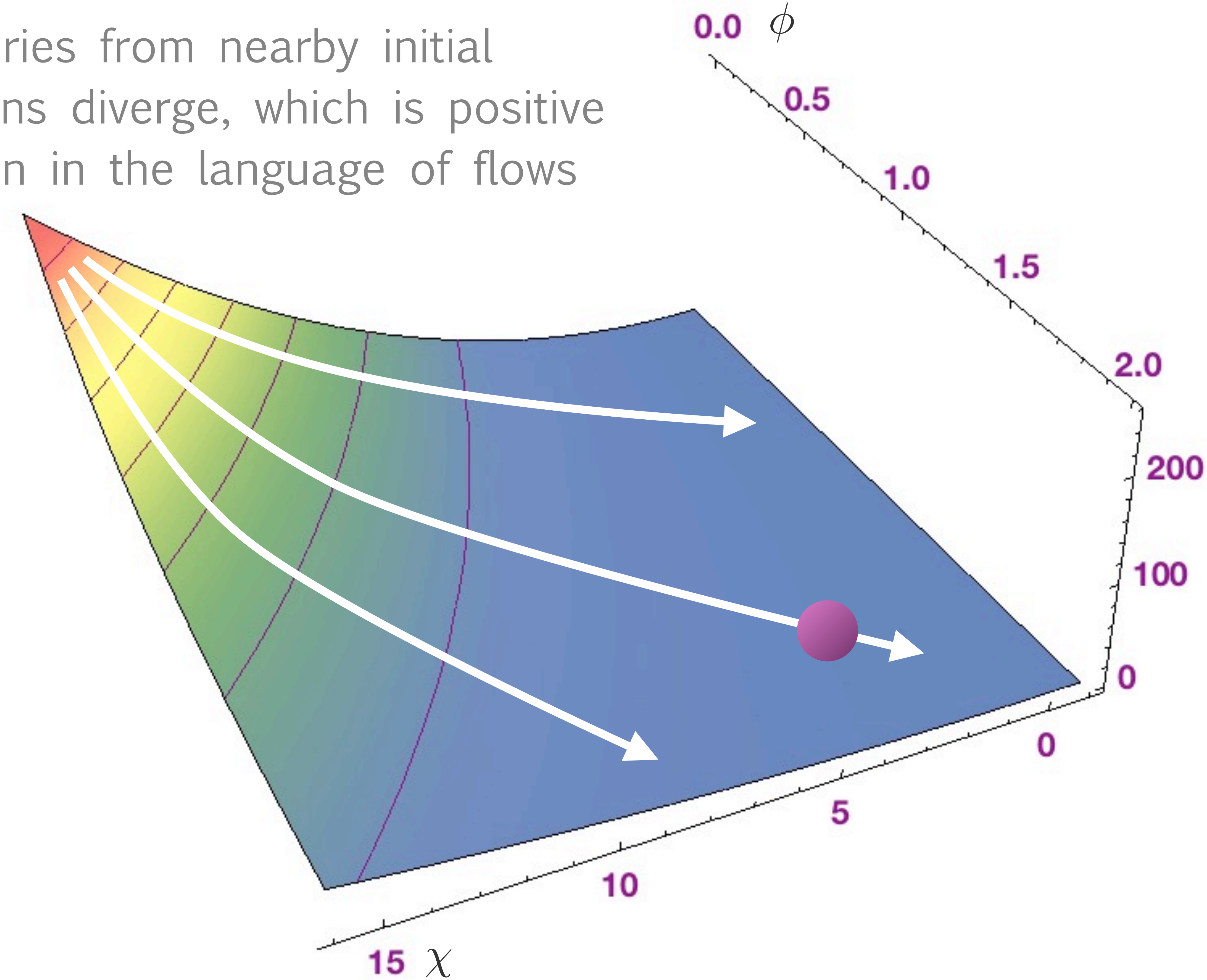
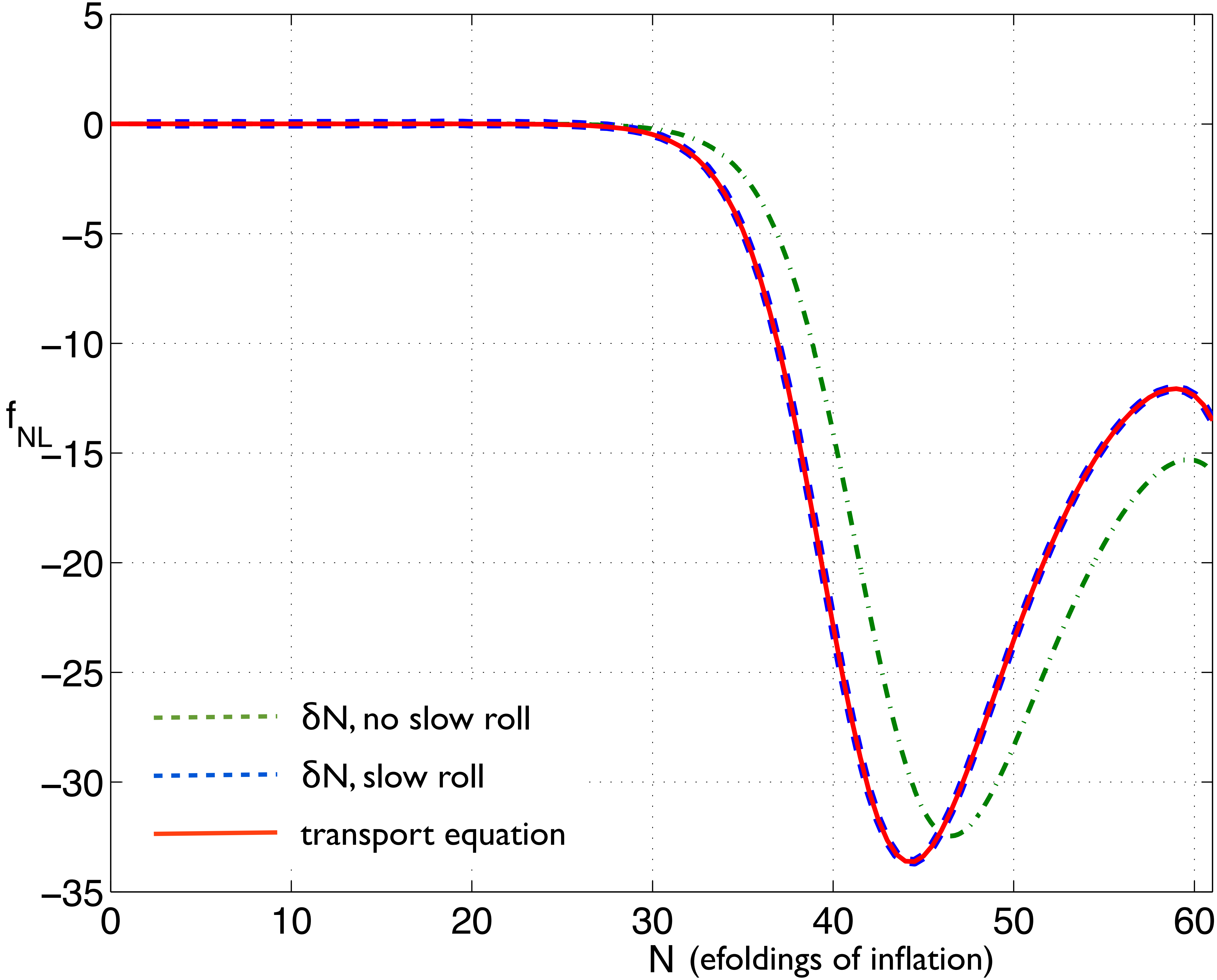
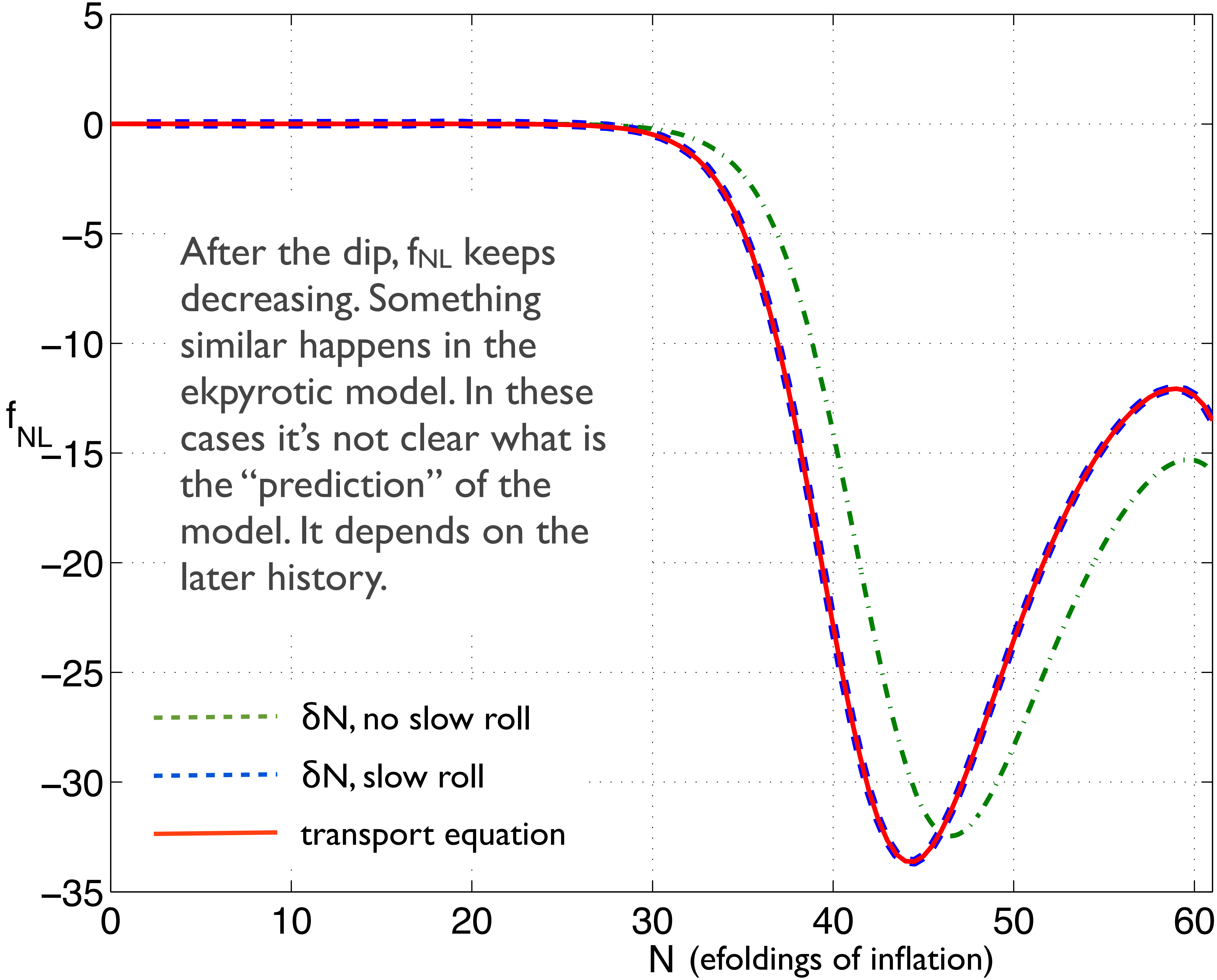
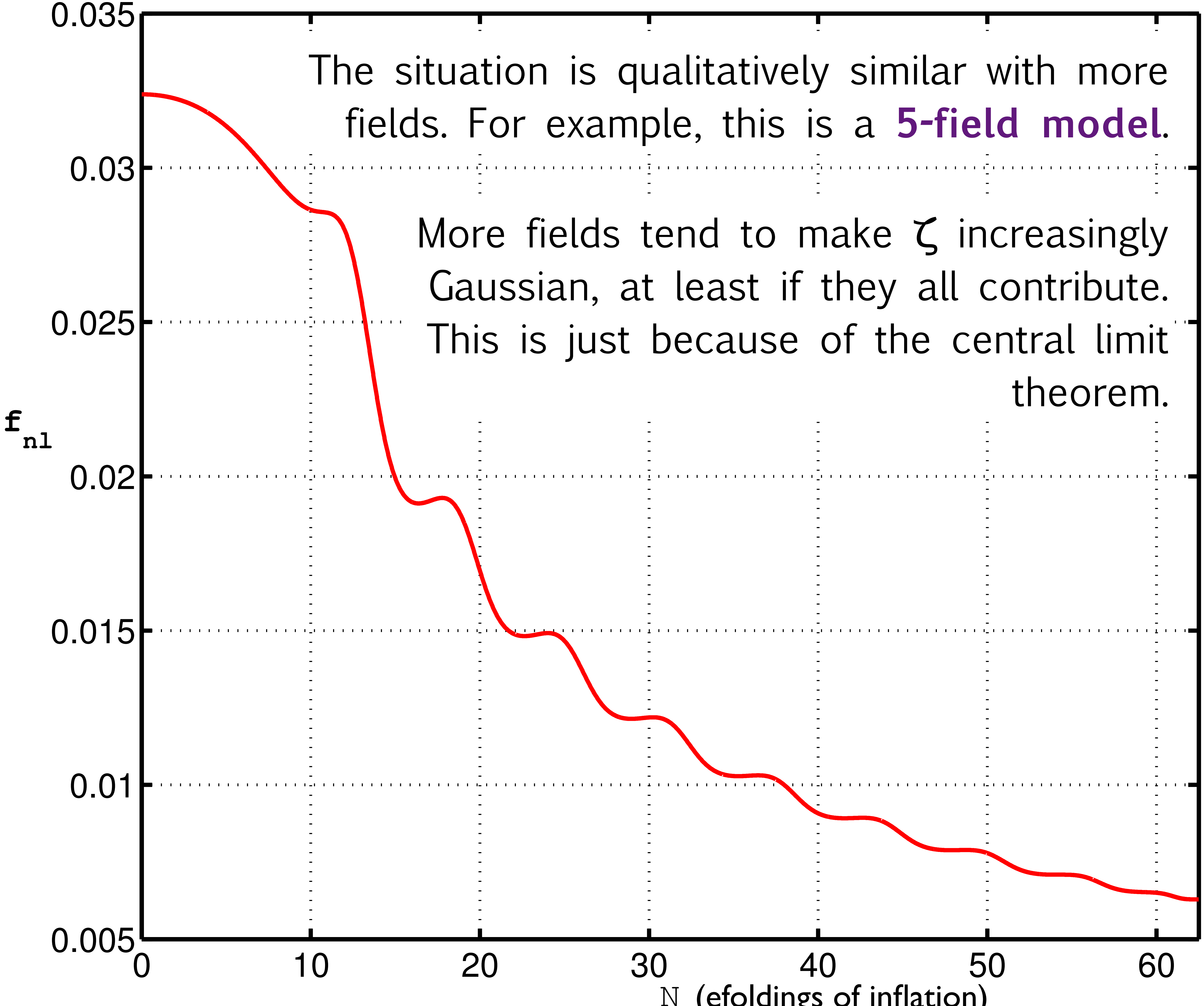


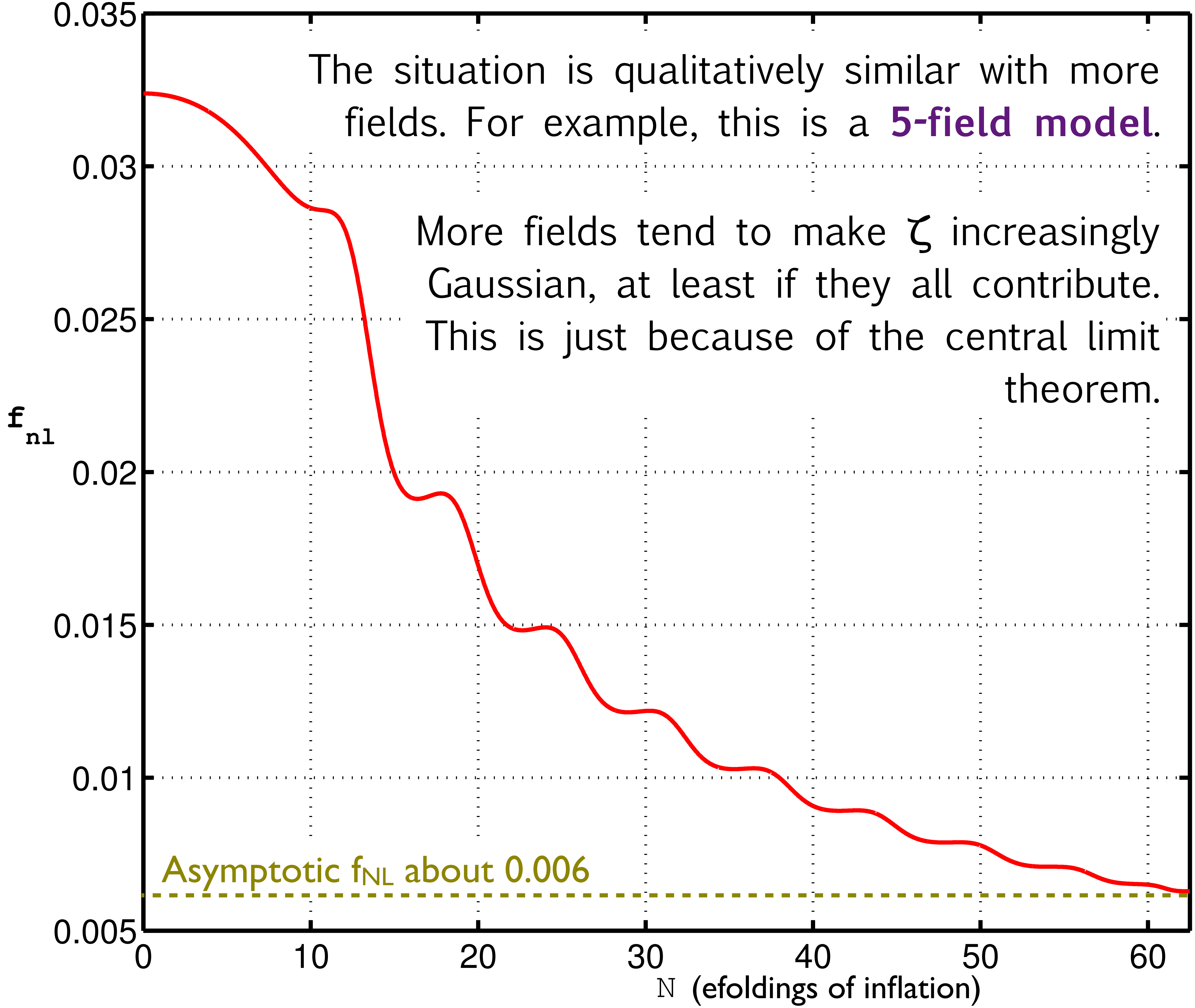


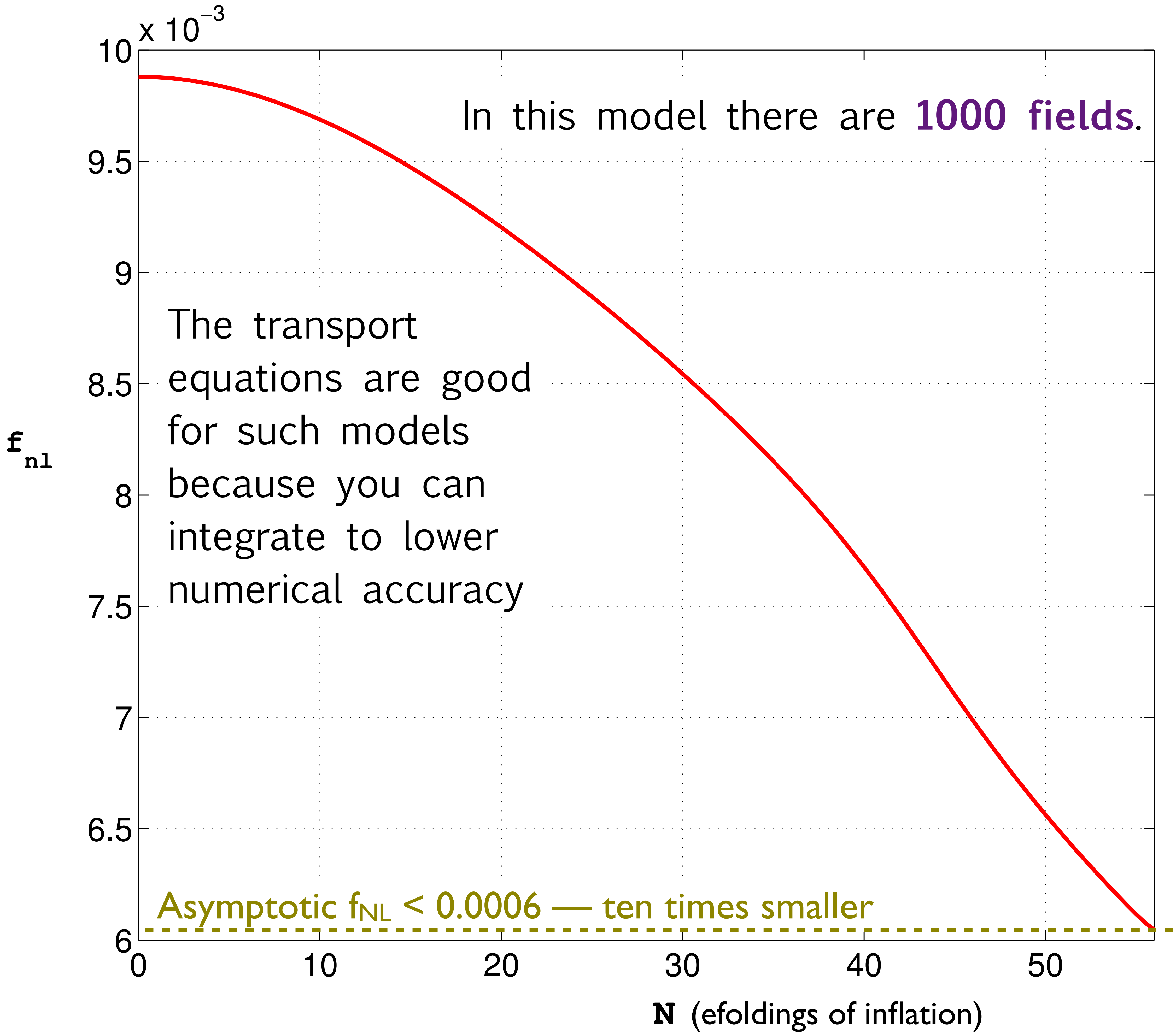
5
0
−5
−10
−15
−20
−25
−30
−35
f_NL
0
10
20
30
40
50
60
N (efoldings of inflation)
δN, no slow roll
δN, slow roll
transport equation



5
0
−5
−10
−15
−20
−25
−30
−35
f_{NL}
0
10
20
30
40
50
60
N (efoldings of inflation)
After the dip, f_{NL} keeps decreasing. Something similar happens in the ekpyrotic model. In these cases it's not clear what is the "prediction" of the model. It depends on the later history.
δN, no slow roll
δN, slow roll
transport equation


The situation is qualitatively similar with more fields. For example, this is a **5-field model**.
More fields tend to make ζ increasingly Gaussian, at least if they all contribute. This is just because of the central limit theorem.
f_nl
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
10
20
30
40
50
60
N (efoldings of inflation)


The situation is qualitatively similar with more fields. For example, this is a **5-field model**.
More fields tend to make ζ increasingly Gaussian, at least if they all contribute. This is just because of the central limit theorem.
Asymptotic f_{NL} about 0.006
f_{nl}
N (efoldings of inflation)
0.035
0.03
0.025
0.02
0.015
0.01
0.005
0
10
20
30
40
50
60


x 10^-3
10
9.5
9
8.5
8
7.5
7
6.5
6
0
10
20
30
40
50
f nl
N (efoldings of inflation)
In this model there are 1000 fields.
The transport equations are good for such models because you can integrate to lower numerical accuracy
Asymptotic fNL < 0.0006 — ten times smaller

We've seen two different ways to generate large f_{NL} during multifield inflation

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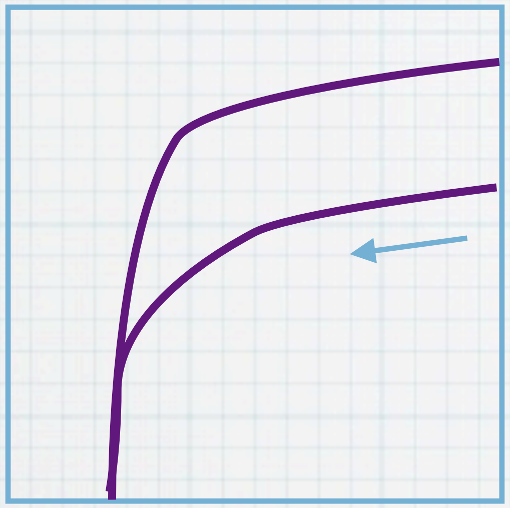

Turn a corner in field space. There is some dispersion and shear as you turn, but then refocusing, which tends to decrease f_{NL}.

Prediction: likely negligible f_{NL} unless interrupted.

(For me, this is the moral of Vernizzi & Wands)

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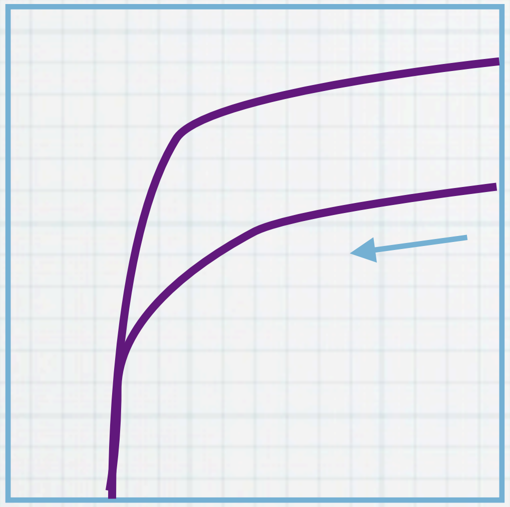

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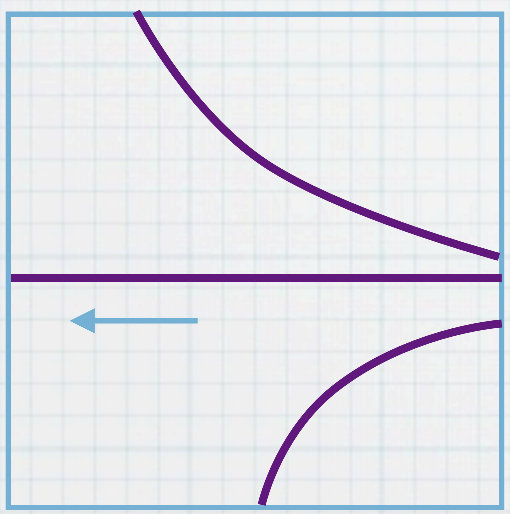

Disperse in field space. To limit the growth of f_{NL}, you have to eventually refocus eg., to an attractor like local thermal equilibrium (cf. Weinberg/Meyers)
Prediction: depends whether you refocus before inflation ends, eg., by a hybrid transition

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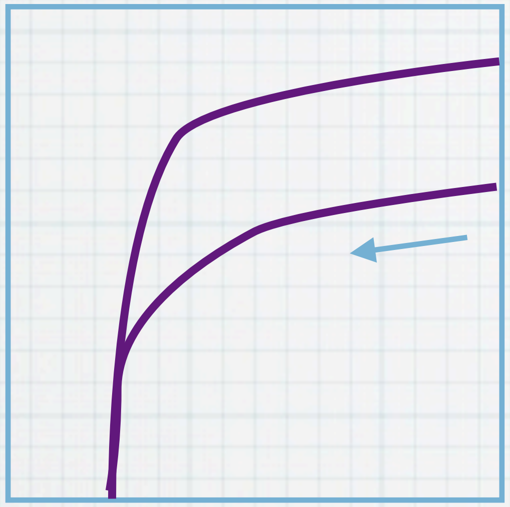

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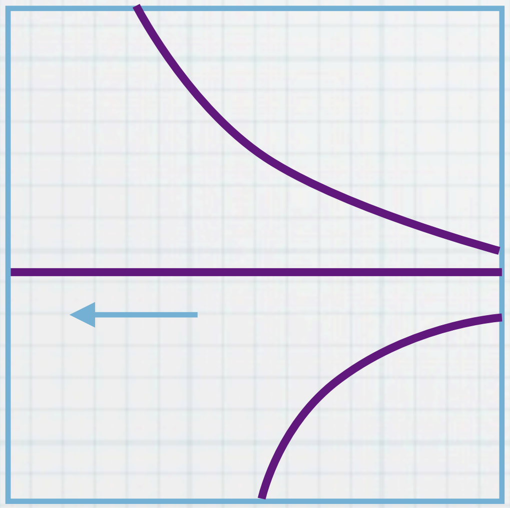

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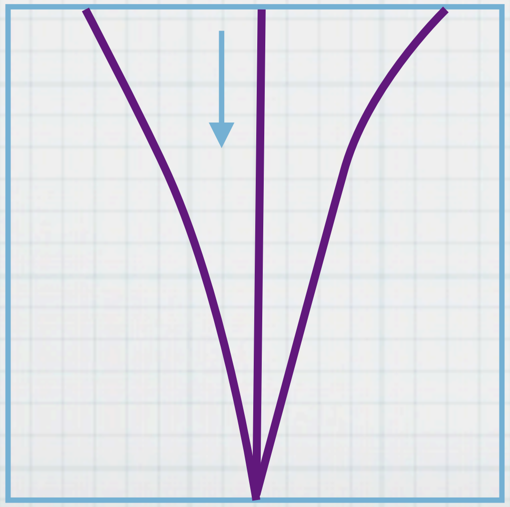


Live with the necessity of an attractor!

Do your best to change the value that f_{NL} *converges to.*

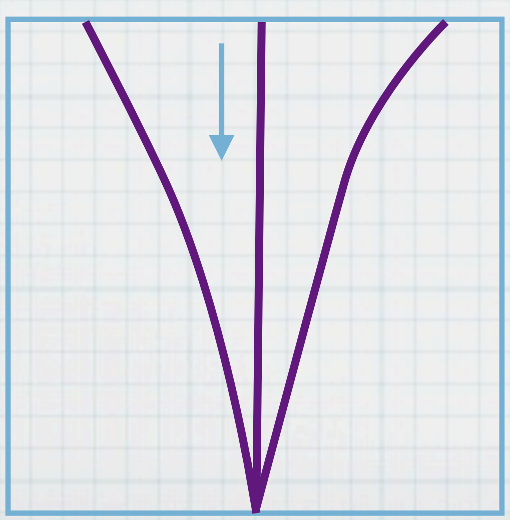
Prediction: reasonably unambiguous, but you can't make f_{NL} unboundedly large

We've seen two different ways to generate large f_{NL} during multifield inflation

In saying this, we're not really discovering anything new. The necessity has been well-understood for a long time, almost since people began to think seriously about multiple field inflation.

As has long been known, there can indeed be a large variation of ζ ... which can continue until a thermalized radiation-dominated universe has been established. Indeed, in models where one of the fields survives to the present Universe ... variation in ζ can continue right to the present. This variation is due to the presence on large scales of classical perturbations in both fields ... generated during inflation, and the effect of these must always be considered in a multi-component inflation model

Liddle, Lyth, Malik & Wands (1999) hep-ph/9912473



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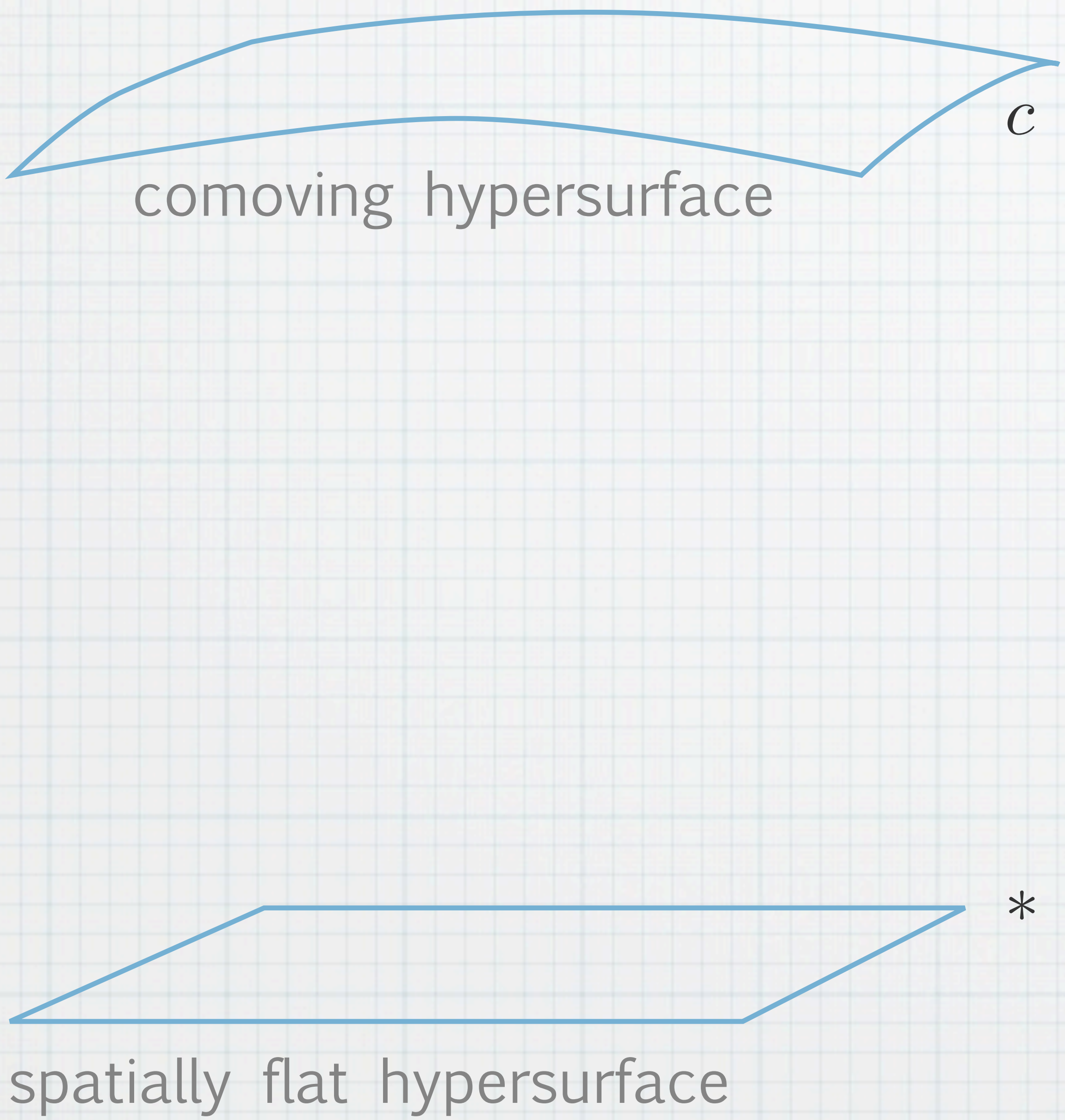
When the trajectories refocus, f_{NL} does not usually converge to zero

$$\zeta = \delta N = \delta \int_*^c H \, \mathrm{d}t$$



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$$\zeta = \delta N = \delta \int_*^c H \, \mathrm{d}t$$

$$+ \left. \frac{H}{\dot{\phi}} \right|_c \frac{\partial \phi_c}{\partial \phi_*}$$

$$- \left. \frac{H}{\dot{\phi}} \right|_*$$

When the trajectories refocus, f_{NL} does not usually converge to zero

$$\zeta = \delta N = \delta \int_*^c H \, \mathrm{d}t$$

comoving hypersurface

c

$$+ \cancel{\left.\frac{H}{\dot\phi}\right|_c \frac{\partial \phi_c}{\partial \phi_*}}$$

One definition of refocusing might be that the derivative $\partial\phi_c/\partial\phi_* \rightarrow 0$

(This is not always right)

spatially flat hypersurface

$*$

$$- \left.\frac{H}{\dot\phi}\right|_*$$

Then, only this term is left.
Often, but **not always**, it is small

Begin with an axion potential $V = \Lambda^4 \left(1 - \cos \frac{2\pi\phi}{f}\right)$

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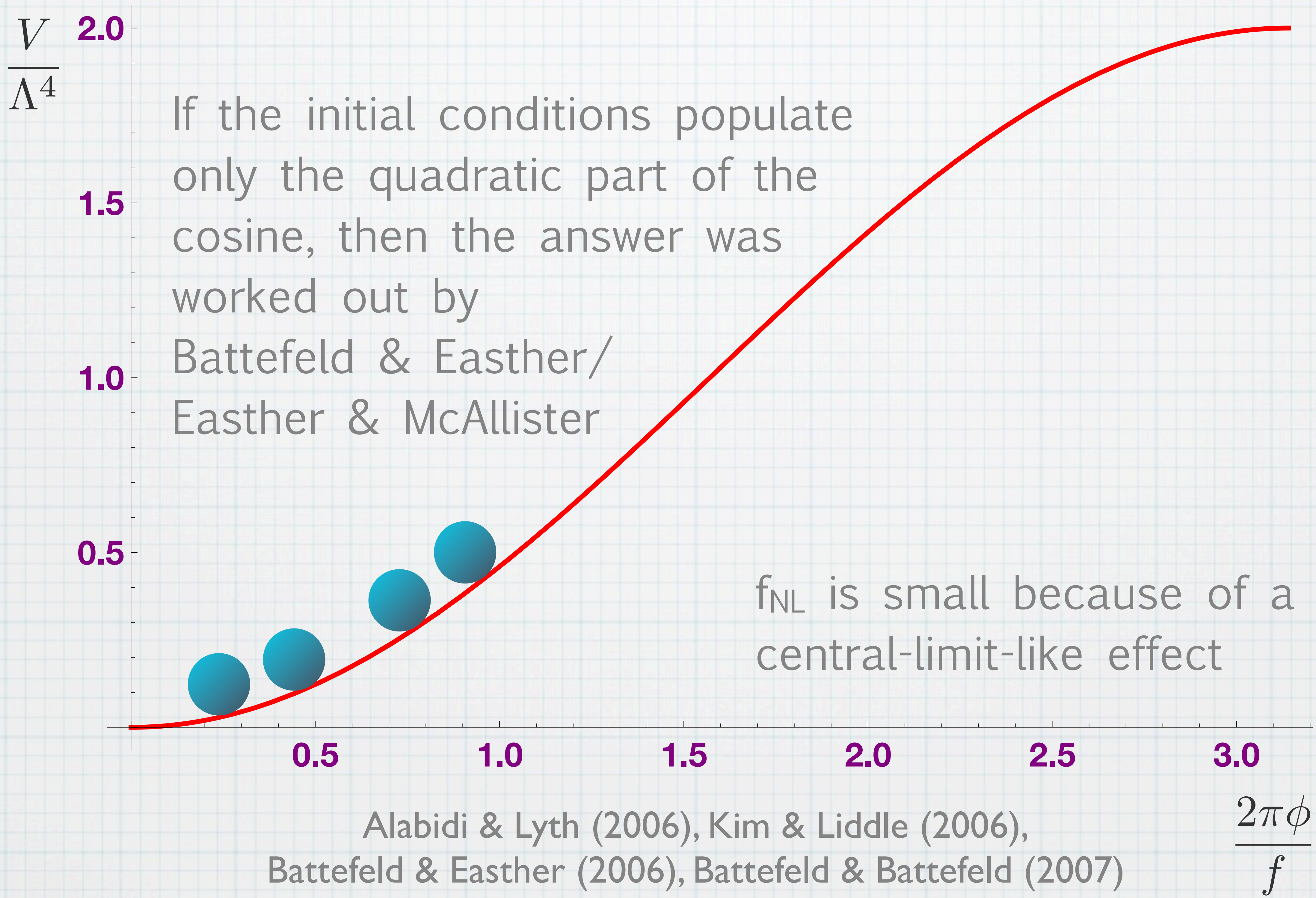

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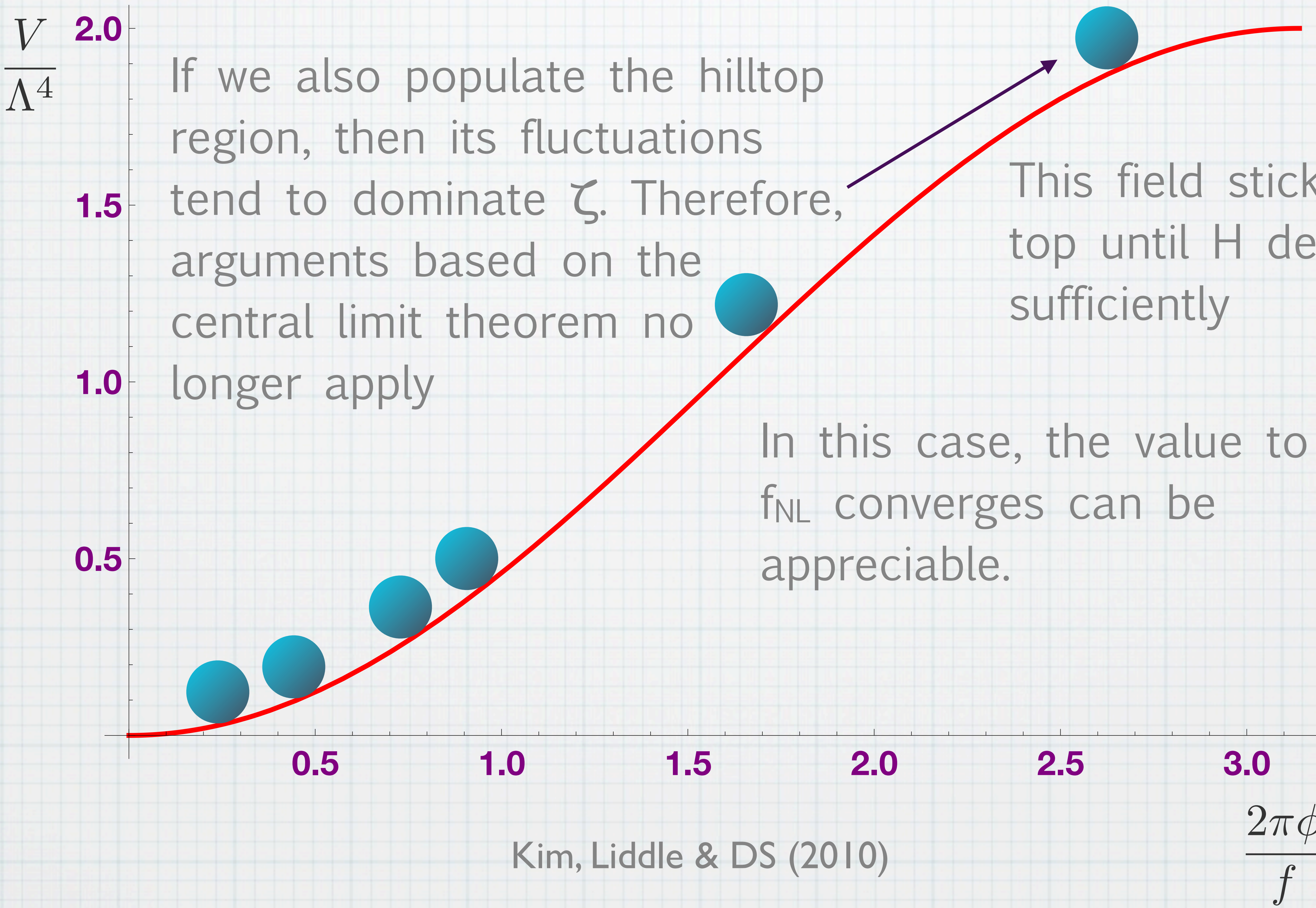

If we also populate the hilltop region, then its fluctuations tend to dominate ζ. Therefore, arguments based on the central limit theorem no longer apply

This field sticks at the top until H decays sufficiently

In this case, the value to which f_{NL} converges can be appreciable.

Kim, Liddle & DS (2010)

Now,

$$\frac{6}{5} f_{\mathrm{NL}} \to \frac{3}{2}\epsilon_* - \eta_* + \epsilon_* f(k_i)$$

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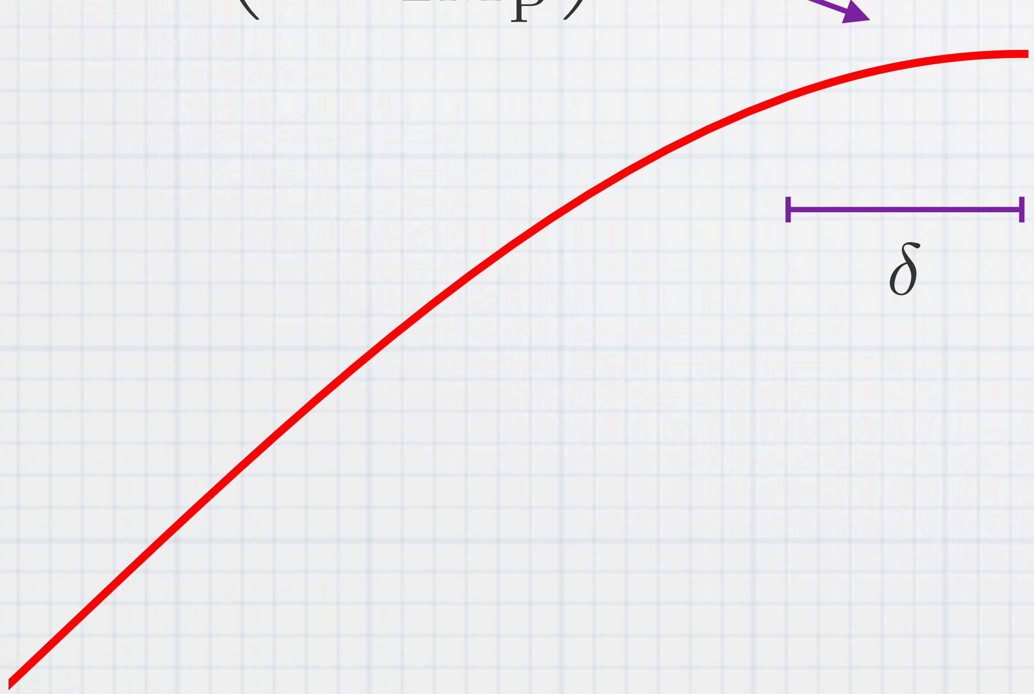
$f(k_i)$ is a complicated function of the k_i with well-defined limits, finite everywhere (Maldacena 2002)

near the hilltop

$$V \approx 2\Lambda^4 \left(1 + \frac{\eta\delta}{2M_{\mathrm{P}}^2}\right)$$



It turns out that

$$\epsilon_* \approx 0$$

$$\eta * \approx -2\pi^2 \frac{M_{\mathrm{P}}^2}{f^2}$$

Now,

$$\frac{6}{5} f_{\mathrm{NL}} \to \frac{3}{2}\epsilon_* - \eta_* + \epsilon_* f(k_i)$$

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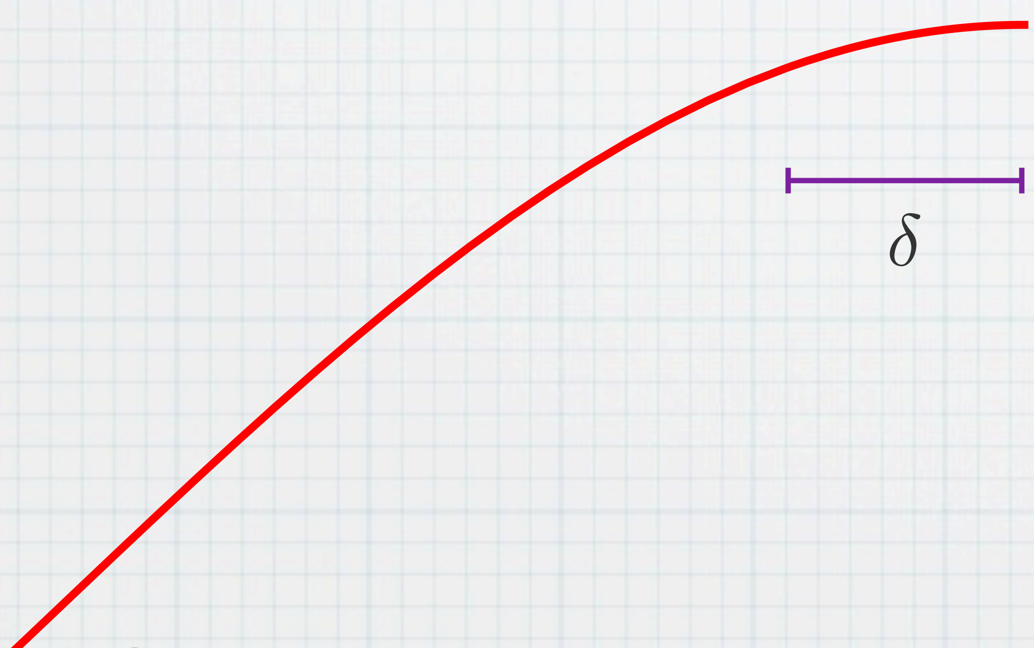


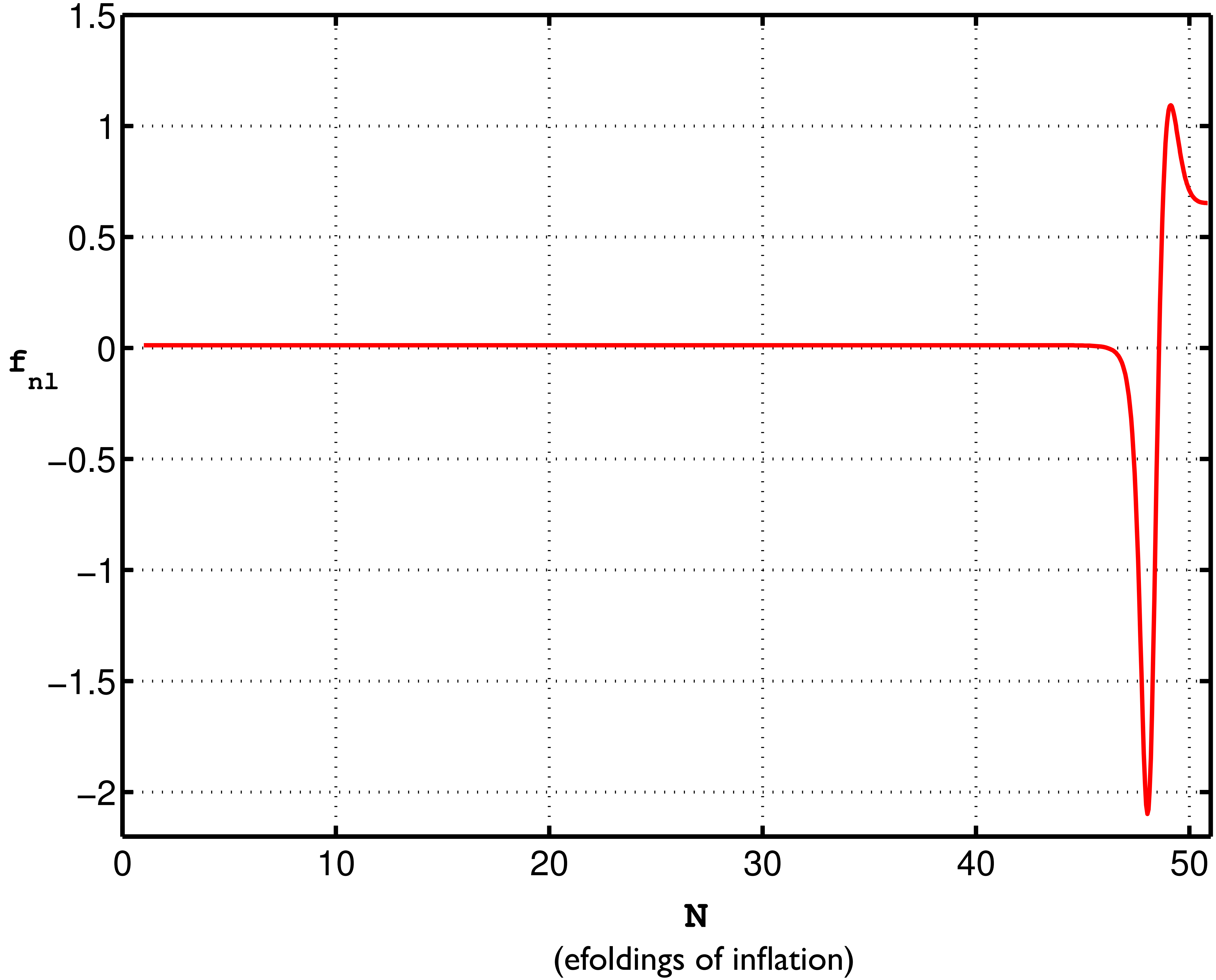
It turns out that

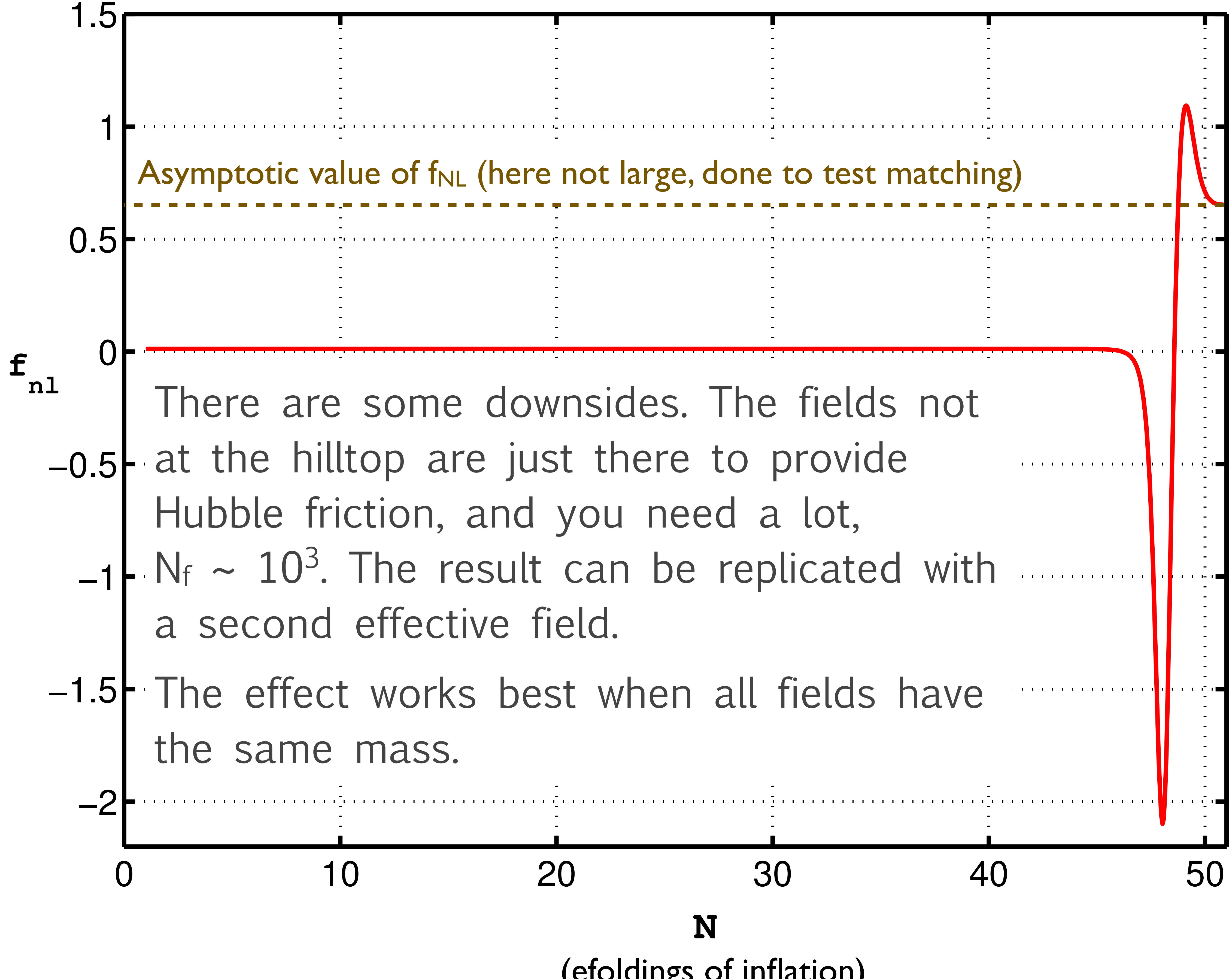
$$\epsilon_* \approx 0$$

$$\eta* \approx -2\pi^2 \frac{M_{\mathrm{P}}^2}{f^2}$$

≈ 20

So, when the attractor is reached and any f_{NL} generated by shear, divergence, focusing, etc., has decayed, f_{NL} asymptotes to a rather large number


1.5
1
0.5
0
−0.5
−1
−1.5
−2
f nl
0
10
20
30
40
50
N
(efoldings of inflation)



There are some downsides. The fields not at the hilltop are just there to provide Hubble friction, and you need a lot, $N_f \sim 10^3$. The result can be replicated with a second effective field.

The effect works best when all fields have the same mass.

Summary

- Typically, multifield inflation does not generate features in the sense they've been used so far. It generates **transient spikes, bumps and wiggles in f_{NL}**.
- f_{NL} is sourced by geometrical properties of the flow on field space. In all known examples (I believe) there is strong dispersion of trajectories (eg., BC&H models; ekpyrosis; KL&S hilltop model)
- Whether these are relevant depends on a prescription for ending inflation and what happens later. For example, **if the trajectories are continuously dispersing, you have to say what makes them refocus**.
- In some models, **any f_{NL} generated during evolution can disappear due to refocusing of the trajectories**.
- Many fields typically make ζ more gaussian unless one dominates