Inflation, NG, & UV physics

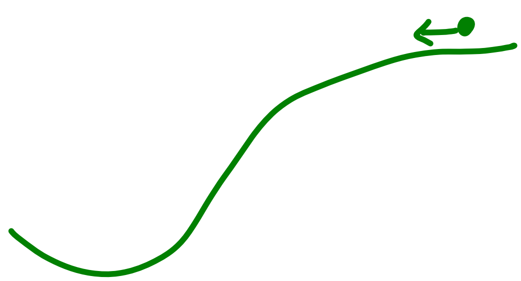
Including works & w.i.p. with

Alishahiha Dong Green Horn
McAllister Polchinski Senatore Tong
Westphal Zaldarriaga

Inflation:

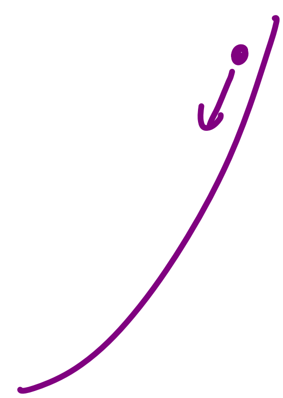
Model-independently, a source of $w \approx -1$ that dilutes slowly.

e.g.



flat potential (Slow Roll)

(can be NG with oscillations...)



steep potential with interactions slowing the field

$\rightarrow$ NG

$$ds^2 = -N^2 dt^2 + h_{ij}\left(dx^i + N^i dt\right)\left(dx^j + N^j dt\right)$$

$$h_{ij} = a^2(t)\left[e^{2\zeta}\delta_{ij} + \gamma_{ij}\right]$$

$a^2(t) \simeq e^{2Ht}$

scalar perturbation

$(\delta\phi = 0$ gauge$)$

remains constant outside horizon

$\partial_i \gamma_{ij} = 0 = \gamma_{ii}$

tensor perturbations

B modes

Seljak Zaldarriaga ...

Kamionkowski et al

BICEP/SPUD, Spider, ...

Basic Observables

tilt n_s : $P_\zeta = \dfrac{\text{const}}{k^{3+(1-n_s)}}$

tensor/scalar ratio r : $r = \dfrac{P_\gamma}{P_\zeta}$

n_s, r will be observed or constrained at $\simeq 0.01$ level

Non-Gaussianity : $\langle \zeta_{\vec{k}_1} \zeta_{\vec{k}_2} \zeta_{-\vec{k}_1-\vec{k}_2} \rangle$

Komatsu/Spergel, Maldacena ...

Inflation can be modeled in quantum field theory and general relativity.

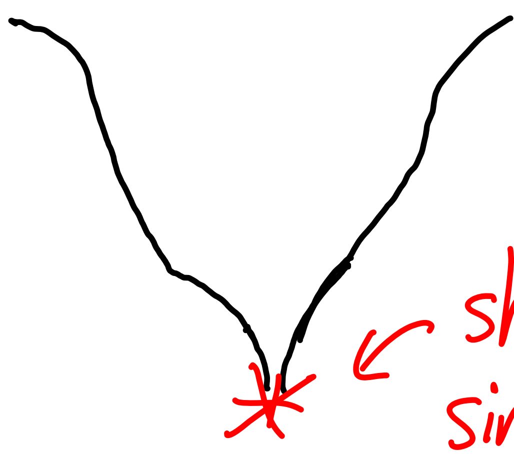
However, these do not provide a complete description. To see this, first note that the effective interaction strength of gravity <u>increases</u> with energy:

$$\lambda_G \sim G_N E^2 \sim \left(\frac{E}{M_p}\right)^2 \quad M_p \sim 10^{19}\,\text{GeV}$$

At short distances, quantum effects become important, along with any new degrees of freedom involved in a "UV completion" of the theory. This affects cosmology in several ways.

1)

Far past:

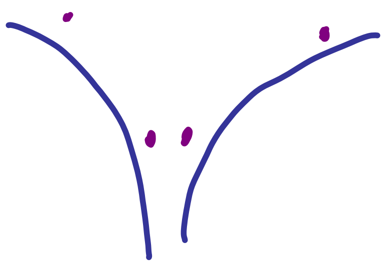


short-distance singularity

and/or "eternal inflation"

There is much more to do to understand initial conditions for cosmology. This is conceptually interesting, and may ultimately provide some sort of probability distribution for late time physics.

However, if it occurred, inflation diluted most relics of this early time



, so let us move on to inflation + in string theory + data

General Relativity describes gravity accurately at long distances

$$S = \int d^4x \sqrt{g}\, \frac{R}{G_N} + S_{matter} \rightarrow R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G_N T_{\mu\nu}$$

GR breaks down for $\lambda_G \rightarrow 1$ (or before)

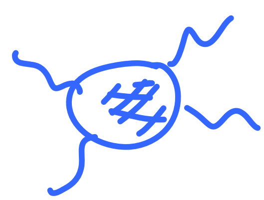
Quantum fluctuations $\rightarrow$

$$S = \int \left(\frac{R}{G_N} - V(\phi)\right)\left(1 + R\left(\frac{c_1}{M_*^2} + \tilde{c}_1 G_N\right) + \cdots\right)$$

$$+ \int (\partial\phi)^2 + k_1 \frac{(\partial\phi)^4}{M_*^2} + \cdots$$

$\leftarrow$ scale of "new physics"

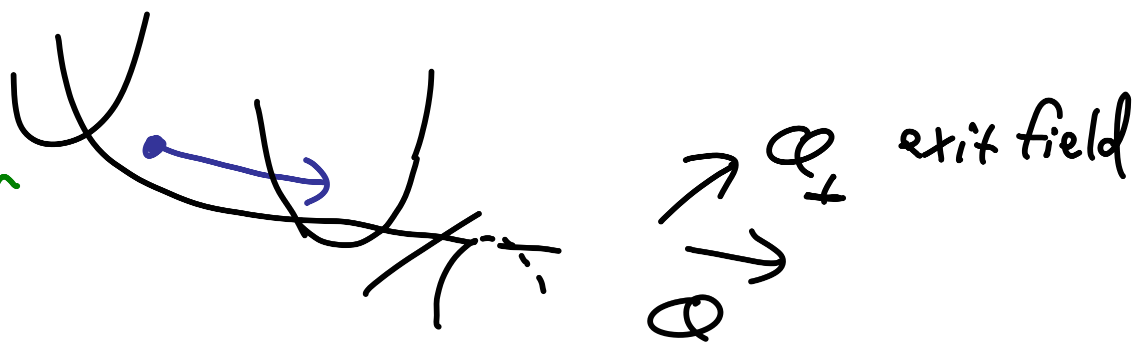
with corrections sensitive to short-distance physics



2) These corrections <u>matter</u> for inflation

e.g. A seemingly simple way to obtain inflation is to postulate a very flat potential for the inflaton $\phi(x)$.

Linde '93
hybrid inflation

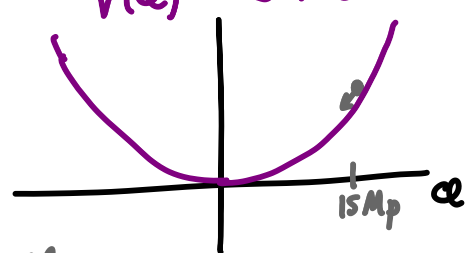


$$\varepsilon \equiv \frac{M_p^2}{2}\left(\frac{V'}{V}\right)^2 << 1 \qquad \eta \equiv M_p^2\left|\frac{V''}{V}\right| << 1$$

However, corrections from the UV physics can generate substructure in $\mathcal{L}(\phi, \partial\phi)$: $\frac{V(\phi-\phi_0)^2}{M_p^2} \to \Delta\eta \sim 1$

This UV Sensitivity is greatest in the case of "chaotic inflation" A. Linde '83 where the inflaton ϕ ranges over more than a distance M_p

e.g. $V(\phi) = \frac{1}{2}m^2\phi^2$



$$\left\{\begin{array}{l} \varepsilon = \frac{1}{2}\left(\frac{V'}{V}M_p\right)^2 \\ \eta = M_p^2\left|\frac{V''}{V}\right| \end{array}\right\} \sim \left(\frac{M_p}{\phi}\right)^2 \Rightarrow \phi \sim 15\, M_p$$

In General:

★ "Lyth Bound" : $\dfrac{\Delta\phi}{M_p} \sim \left(\dfrac{r}{0.01}\right)^{\frac{1}{2}}$

UV sensitive if $\gtrsim 1$ — observable

→ Control with approximate shift symmetry (Wilsonian 'natural')

UV Sensitivity of Inflation

① Terms of order

$$V \cdot \frac{(\varphi - \varphi_0)^2}{M_p^2}$$ (dimension 6)

in the effective action can ruin inflation

② $$\frac{\Delta\varphi}{M_p} \simeq r^{\frac{1}{2}} \frac{N_e}{\sqrt{24}}$$ (Lyth)

GUT-scale inflation (with observable tensor modes) $\Leftrightarrow$ $\Delta\varphi > M_p$

③ General Single-field inflation involves higher derivative terms which affect solution & perturbations

cf Non-Gaussianity

④ $g^2\varphi^2\chi^2$ couplings $\Rightarrow$ temporarily light fields affect evolution. . . .

5

Heavy fields (mass > H) affect results in a different way: they adjust in response to inflationary potential energy.

QFT toy model

$$V(\varphi_L, \varphi_H) = g^2 \varphi_L^2 \varphi_H^2 + m^2 (\varphi_H - \varphi_0)^2$$

$$\frac{\partial V}{\partial \varphi_H} \equiv 0 \Rightarrow V = \frac{g^2 \varphi_L^2}{g^2 \varphi_L^2 + m^2} m^2 \varphi_0^2$$

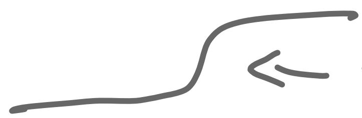
($\dot{\varphi}_H^2$ term subdominant)

flatter: energetically favorable.

Note: not universal –

a) Restricted couplings

$$V = g^2 \varphi_L^2 \varphi_H^2 + m^2 (\varphi_H - \varphi_0 - \varphi_L)^2$$



$\leftarrow$ intermediate steepening

b) Kinetic effects

$$S_{kin} = \int d^4x \sqrt{-g} \left\{ G_{LL}(\varphi_H) \dot{\varphi}_L^2 + G_{HH} \dot{\varphi}_H^2 + \cdots \right\}$$

– Can produce interesting effects,

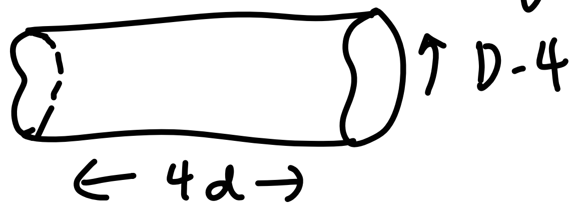
e.g. k-inflation: $\varphi_{H*} = \varphi_H(\dot{\varphi}_L^2, \varphi_L)$

Tolley/Wyman

Will use slow roll $\dot{\varphi}_L^2 \ll V$ to bound this.

– $\dot{\varphi}_H^2$ small if $\left| \frac{d\varphi_L}{d\varphi_H} \right| \ll \sqrt{\frac{G_{LL}}{G_{HH}}}$

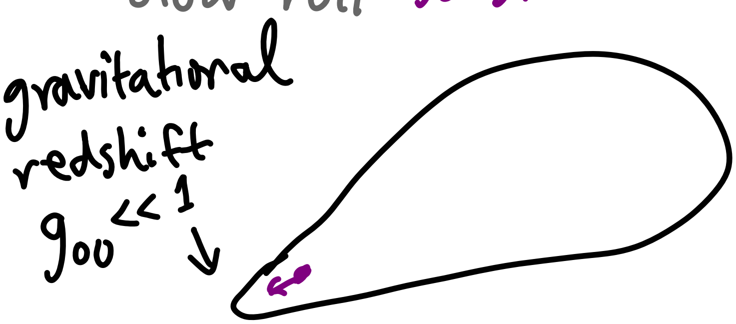
→ Model inflation in string theory.



- Many scalar field "moduli" with generically steep potential $U_{mod}(L, g, \ldots)$
  size ↗ ↖ coupling

- Many angular moduli θ such as axions, certain brane positions, etc. with more shallow potential at weak coupling → Natural candidate inflatons

e.g. KKLMMT
slow-roll D3-brane inflation



1% tune

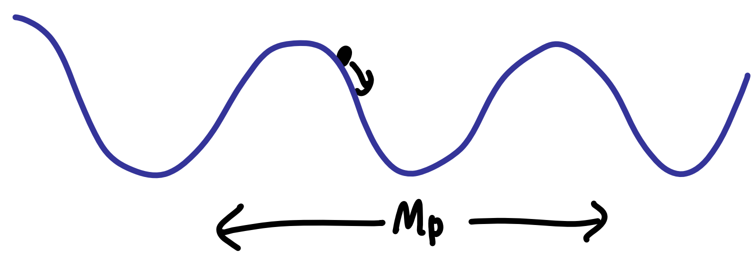
Liam's talk

Freese, Frieman, Olinto '90 ; +Adams, Bond '93

Axions naturally respect an (approximate) shift symmetry $\phi \to \phi + \alpha$

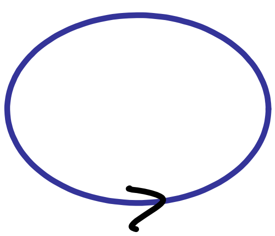
(couple via their derivatives)

→ "Natural Inflation"



$\Lambda^4 \sim e^{-\frac{\text{const}}{g^2}}$

Naturally small



$a \simeq a + (2\pi)^2$

$\phi_a = f_a a$ — canonical scalar field

→ Does $\frac{\Delta \phi}{M_p} \gtrsim 1$, protected by shift symmetry, arise in string theory?

* Basic period small compared to M_p

Banks et al ...

In string theory, the basic period $f_\theta (2\pi)^2$ a priori turns out $\ll M_p$ at weak curvature + coupling

Banks/Dine/Fox/Gorbatov
Svrcek/Witten

cf Arkani-Hamed et al

e.g. Axions

$$a = \int_{\underbrace{\Sigma_p}_{\substack{p\text{-dim'l} \\ \text{closed submanifold}}}} \underbrace{A_{i_1 \cdots i_p}}_{\substack{\text{potential field} \\ \text{(higher-dim'l analogue} \\ \text{of Maxwell } A_\mu)}} dx^{i_1} \wedge \cdots \wedge dx^{i_p}$$

f_a comes from kinetic term:

$$\int d^D x \sqrt{G_{(D)}}\, F_{i_1 \cdots i_{p+1}} G_{(D)}^{i_1 i_1'} \cdots G_{(D)}^{i_p i_p'} F_{i_1' \cdots i_{p+1}'}$$

$$= \int d^4 x \sqrt{g_4}\, f_a^2 (\partial a)^2 = \int d^4 x \sqrt{g_4}\, (\partial \theta_a)^2$$

$\Rightarrow$ for all sizes $\sim R$, this yields

$$f_a \sim M_p \left(\frac{\sqrt{\alpha'}}{R}\right)^p \ll M_p$$

$\sqrt{\alpha'}$ = string length

Note: this is an example of the fact that not "anything goes" in the landscape. (In same regime $L \gg \sqrt{\alpha'}$, $g_s \ll 1$ where we control moduli stabilization & see multiple vacua, $f_{axion} \ll 1$.)

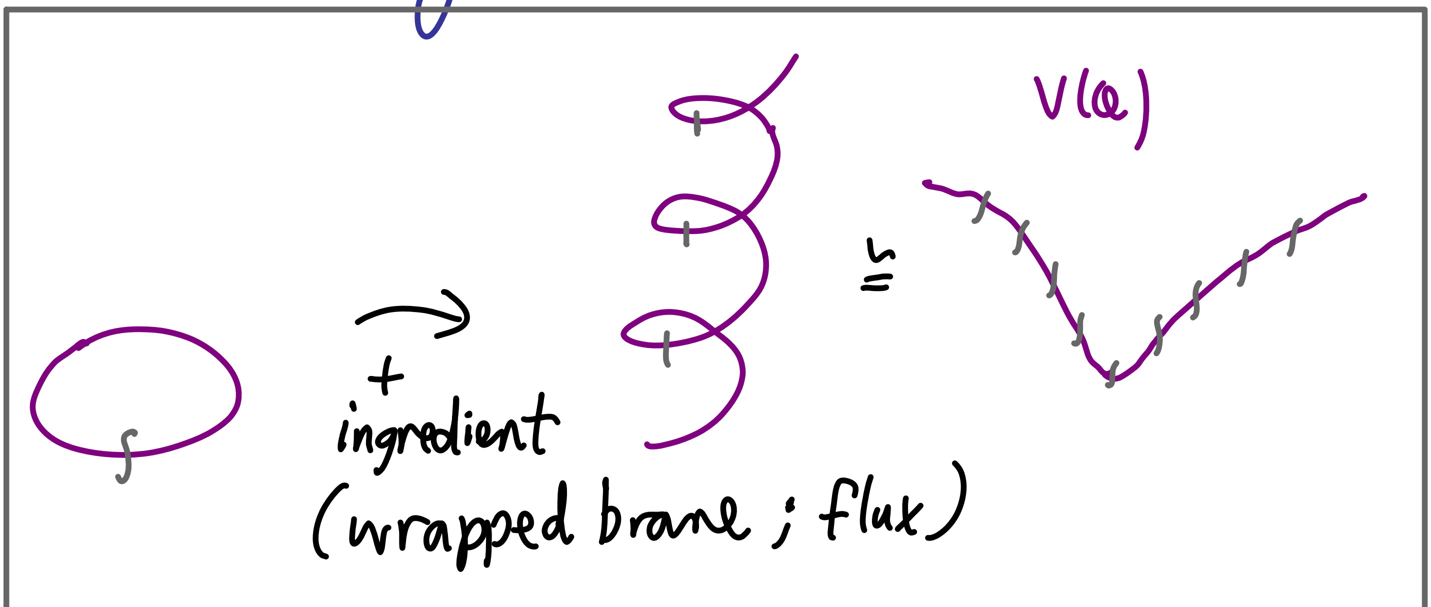
- Many axions → potential window for models

Nflation DKMcGW EMcA

$$\frac{\phi^2}{M_p^2} \sim \frac{N}{M_{p,0}^2 + N M_*^2}$$

Anyway must take into account monodromy in string compactifications

ES, Westphal '08
McAllister, ES, AW '08
cf Kaloper Lawrence Sorbo



unwraps the would-be periodic direction. → Large field range with distinctive potential, with

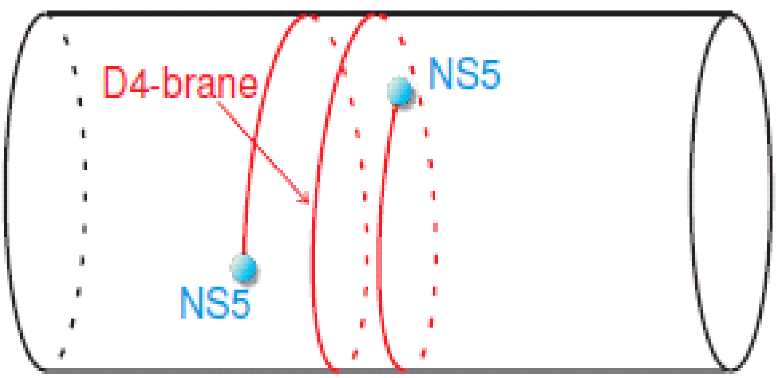
$$V(\varphi > M_p) \sim \begin{cases} \varphi^{2/3} & \text{twisted torus} \\ \varphi & \text{axions} \end{cases}$$

ES, AW '08

LMcA, ES, AW '08

the so far worked out examples.

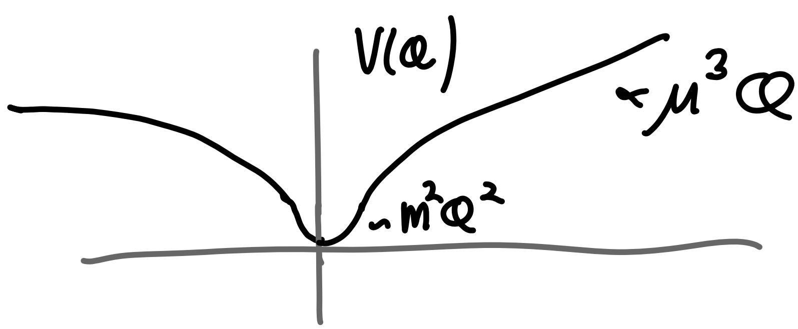
The basic mechanism is very simple :



["T-dual" to axions]

- "NS5" branes position periodic on this circle, until add stretched "D4" brane

→ Novel prediction for inflaton potential

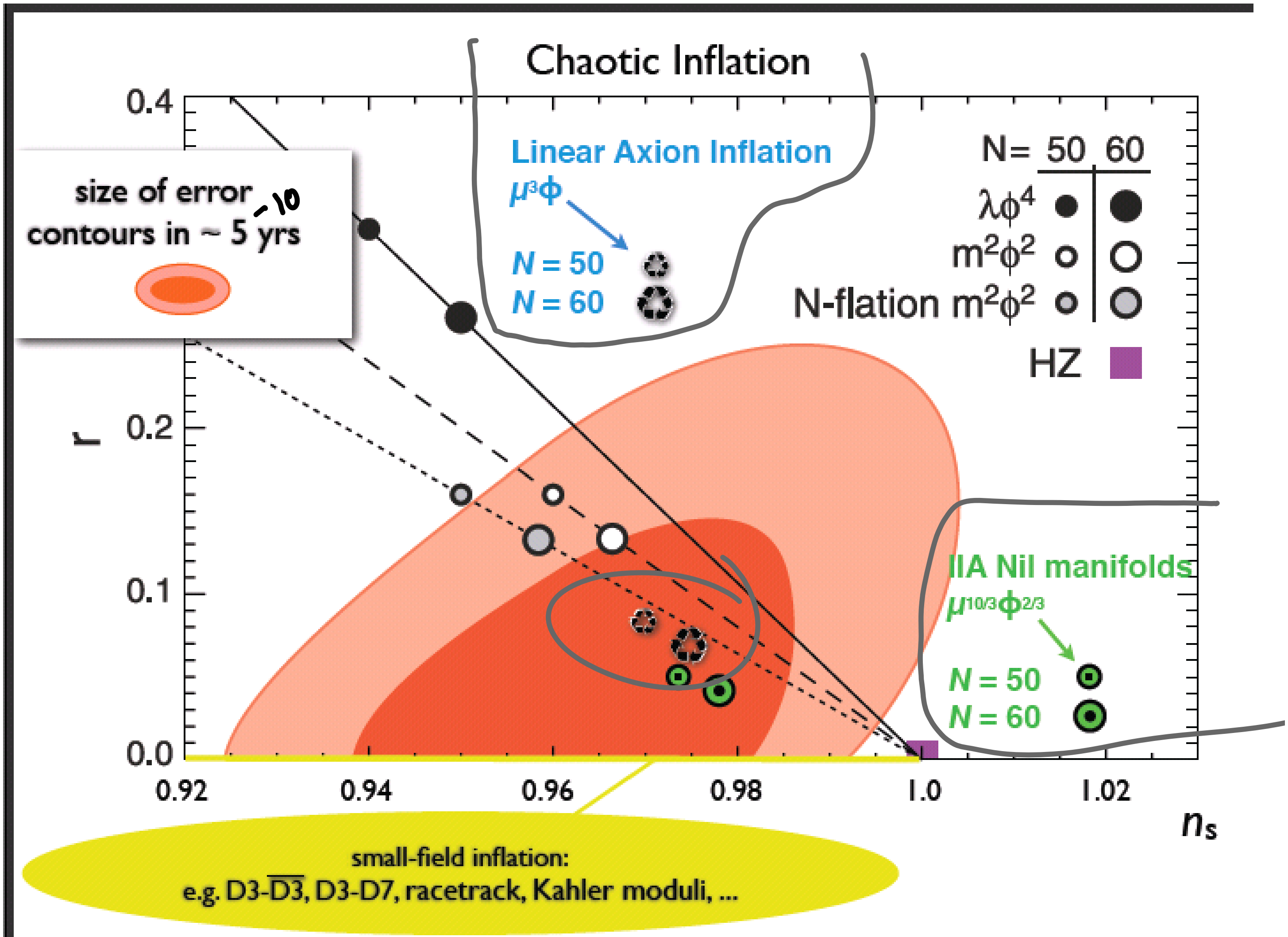


$+$

$\left(e^{-\frac{8\pi^2}{g^2}} \cos\frac{\phi}{f}\right.$

Result :

WMAP + (L. Page, D. Spergel, ... cf Komatsu talk)



$$r = 0.07 \qquad V(\varphi) \simeq \mu^3\varphi + \Lambda^4 \cos\left(\frac{\varphi}{2\pi f}\right)$$
$$n_s \simeq 0.98$$

Because of the symmetry, and oscillating nature of the (instanton-suppressed) corrections, these predictions are robust ⇒ falsifiable

Encouraging ... can we understand this effect more systematically?

Yes, a simple potential-flattening effect arises from adjustments of heavy fields:

Dong Horn ES Westphal

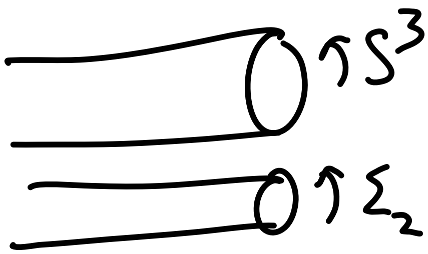
$$S_{\substack{\text{string}\\ \text{theory}}} \supset \int d^{10}x \sqrt{-G} \left\{ |dB|^2 + \big|\overbrace{dC_2 \wedge B}^{\text{Looks like } m^2\phi^2 \ldots} + \underbrace{dC_4}_{0}\big|^2 + \ldots \right\}$$

axion $\quad b = \int_{\text{submanifold } \Sigma_2} B_{ij}\, dx^i \wedge dx^j$

... but the potential energy contained in $|dC_2 \wedge B|^2$ term backreacts on geometry and fluxes:

Backreaction on geometry:

$$g_s \tilde{N}_{eff} = b \int F_3 \sim \underbrace{\frac{R^4}{l_s^4}}_{\substack{\text{Size of geometry} \\ \text{in units of} \\ \text{string tension}}}$$



Plugging this back into $S_{\text{string theory}}$

$$\rightarrow S_{\text{str theory}} \sim Vol_{4d} \frac{\tilde{N}^2}{R^{10}} \times R^6 \sim \underbrace{\frac{\tilde{N}}{g_s}}_{\text{Linear in axion}} \times Vol_{4d}$$

In general, when slow-roll inflation applies, any heavy ($m > H$) fields will adjust in an energetically favorable way: can naturally flatten V relative to $m^2\varphi^2$.

Another example:

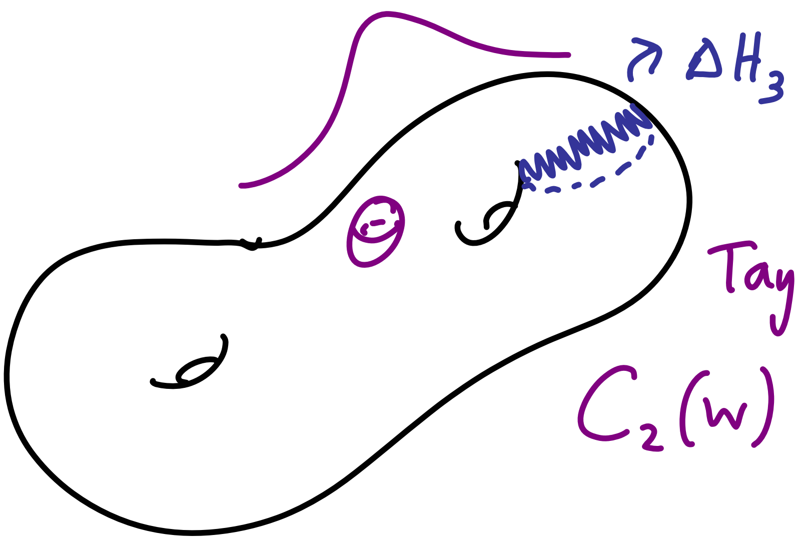
$$V = \dots + |C_2 \wedge H_3|^2 + |F_3|^2 + |H_3|^2$$

flux H_3 sloshes around to $\downarrow |C_2 \wedge H|^2$ at cost of $\uparrow |H_3|^2 \rightarrow$ new equilibrium with $V(\varphi) \propto \varphi^{p<2}$.

More detail :

$$\frac{1}{g_s^2}\left|\underset{\Sigma_3}{\underset{\downarrow}{H_3}}+\underset{\Delta\Sigma_3}{\underset{\downarrow}{\Delta H_3}}\right|^2+\left|\underset{\Sigma_2}{\underset{\downarrow}{C_2}}\wedge\left(\underset{\Sigma_3}{\underset{\downarrow}{H_3}}+\underset{\Delta\Sigma_3}{\underset{\downarrow}{\Delta H_3}}\right)\right|^2+\left|F_3\right|^2$$

To illustrate the effect, let ΔH_3 slosh at fixed $C_2 = c\,\omega_2$



$$\left(\int_{\Sigma_2}\omega_2 = 1\right)$$

Taylor expand

$$C_2(w) = \frac{c}{L^2}\left(1+\gamma\frac{w^2}{L^2\alpha'}+\dots\right)$$

around maximum of ΔH_3 support

$$\sqrt{\alpha'}\,\Delta H_3 \sim \frac{\Delta N}{\tilde{L}^3}\,e^{-w^2/\tilde{L}^2\alpha'}$$

→ the relevant terms in the potential are proportional to

$$\frac{1}{g_s^2}\left[\Delta N^2\left(\frac{L}{L^n}\right)^3 + \gamma N \Delta N\left(\frac{L^n}{L}\right)^2 \frac{Q_c^2}{M_p^2}\right]$$

Minimizing as a function of L/L^n → $V \propto Q_c^{6/5}$

(We also bounded kinetic effects in the paper.)

Another example involving kinetic effects is :

$$b = \int_{\Sigma_2} B$$

take B localized in region of size L'

size $L' \rightarrow \Sigma_2$

$$\frac{Q_b}{M_p} \sim \left(\frac{b}{L'^2}\right)\frac{L'^3}{L^3} \sim \frac{bL'}{L^3}$$

$|B \wedge F|^2$ terms $\longrightarrow$ $L'(b) \sim b^{\gamma > 0}$

e.g. $B \wedge F_3 \rightarrow L'(b) \sim b^{\frac{1}{4}}$

$\rightarrow Q_b \propto b^{5/4}$

$\rightarrow V \propto b \propto Q_b^{4/5}$

- Altogether, from this point of view, $m^2\phi^2$ inflation would be surprising (constraint on heavy fields & coupling to inflaton).

- Interesting recent application to quintessence   Panda, Sumitomo, Trivedi

Further developments $\leftrightarrow$ NG

- Model-dependent signatures
  oscillations & resonant non-Gaussianity

  Chen Easther Lim

  LMcA, ES, AW ; Flauger, McAllister, Pajer, Westpal, ...

$$V(\varphi) = \mu^3 \varphi + \Lambda \cos \frac{\varphi}{f}$$

.

$$\langle \mathcal{R}(\mathbf{k}_1, t)\mathcal{R}(\mathbf{k}_2, t)\mathcal{R}(\mathbf{k}_3, t)\rangle = (2\pi)^7 \Delta_{\mathcal{R}}^4 \frac{1}{k_1^2 k_2^2 k_3^2} \delta^3(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3)$$
$$\times f^{\mathrm{res}} \left[\sin\left(\frac{\sqrt{2\epsilon_*}}{f} \ln K/k_*\right) + \frac{f}{\sqrt{2\epsilon_*}} \sum_{i \neq j} \frac{k_i}{k_j} \cos\left(\frac{\sqrt{2\epsilon_*}}{f} \ln K/k_*\right) + \dots \right] .$$

with

$$f^{\mathrm{res}} = \frac{3b_* \sqrt{2\pi}}{8} \left(\frac{\sqrt{2\epsilon_*}}{f}\right)^{3/2} ,$$

Particle/String Production

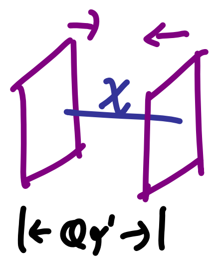
The inflaton φ generically couples to other degrees of freedom (e.g. for reheating)

(Kofman + many)

For example,

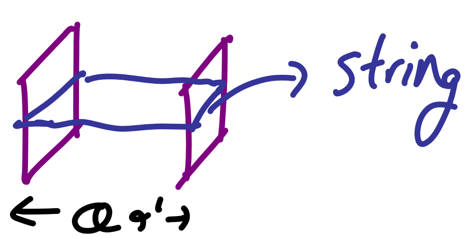
(1) $\delta\mathcal{L} = g^2 \varphi^2 \chi^2 \rightarrow m_\chi^2 = m_0^2 + \varphi^2(t,\vec{x})$

$\Rightarrow$ particle production

(brane picture : 

)

(2) $T_{string}^2 = T_{min}^2 + \varphi^2 M_0^2$

(brane picture : 

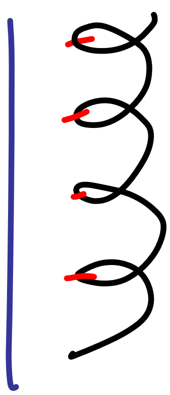
)

These extra excitations could produce interesting signatures
- NG
- GWs?

Produced particles, strings dilute in a Hubble time. However,

★ In e.g. monodromy, any production events are repeated many times

Axion monodromy → string production



cf Trapped Infl

Green Horn Senatore ES

Kofman, Linde
Kim

String production is an interesting theoretical problem in its own right.

$$S_{string} = \int d^2\sigma \, \Upsilon(X^0) \left(\partial_\alpha X^M \partial^\alpha X^N G_{MN} \right)$$

nonlinear worldsheet action

- Naive generalization of particle production is inadequate a priori:

Υ can increase so rapidly that string cannot causally track its naive oscillator spectrum $m^2_{naive} \sim N_{osc} \Upsilon^{\frac{1}{2}}$

To start, review particle production in particle case. $\Delta\mathcal{L} = g^2\varphi^2\chi^2$

$\varphi \simeq vt$ during production

$$\omega_\chi^2(t) \simeq v^2t^2 + k^2$$

→ Heisenberg equation of motion for the field is

$$\left\{ -\partial_t^2 - \omega^2(t) \right\} \psi = 0$$

WKB $\psi \xrightarrow[t\to-\infty]{} \frac{1}{\sqrt{2\omega}} e^{-i\int^t dt'\omega(t')}$

$|t| \to \infty$

$$\psi \to \frac{\alpha}{\sqrt{2\omega}} e^{-i\int^t \omega} + \frac{\beta}{\sqrt{2\omega}} e^{+i\int^t \omega}$$

→ number density $n_p = \int |\beta|^2$

$$|\beta_k|^2 = e^{-\frac{2\pi k^2}{gv}}$$

To anticipate string case, derive from world line action

$$S = \int d\lambda \sqrt{g_{\lambda\lambda}} \left(\left(\frac{dX^0}{d\lambda}\right)^2 g^{\lambda\lambda} - (v^2 X^{0^2} + k^2) \right)$$

$$\Pi_{X^0} = g^{\lambda\lambda} \partial_\lambda X^0 \longrightarrow \frac{\partial}{\partial X^0}$$

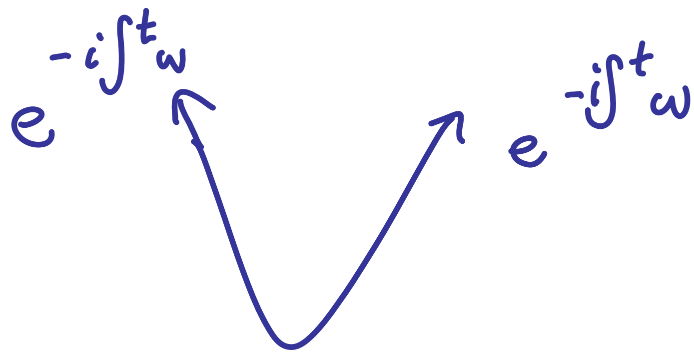
Hamiltonian constraint is

$$(\partial_\lambda X^0)^2 = v^2 X^{0^2} + k^2$$

equivalently

$$\left\{ -\frac{\partial^2}{\partial X^{0^2}} - (v^2 X^{0^2} + k^2) \right\} \psi = 0$$

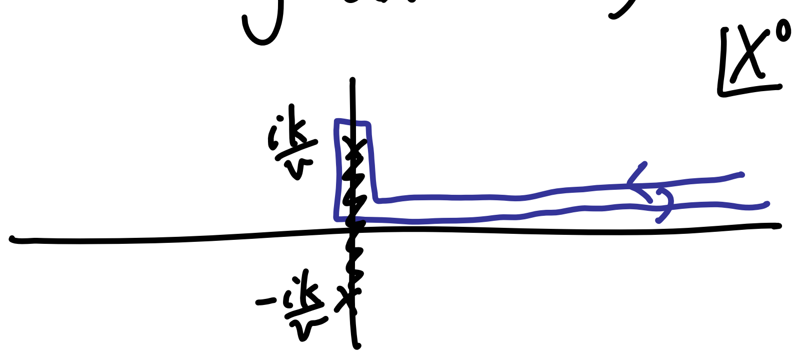
One can compute the Bogolinbov coefficient β as a propagator



$$S = \int d\lambda \left(-\left(\frac{dX^0}{d\lambda}\right)^2 - \omega^2(X^0) \right)$$

$$= -2 \int d\lambda \left(\frac{dX^0}{d\lambda}\right)^2 = -2 \int^t dX^0 \, \frac{dX^0}{d\lambda}$$

$$= -2 \int^t dX^0 \, \omega(X^0)$$



Can also compute $\mathrm{Im}\left(Z_{1\text{-loop}}\right)$

Generalization to strings: $\gamma^2 = \gamma_{min}^2 + b^2 {X^0}^2$

Hamiltonian constraint (circular, size $L \sim r$)

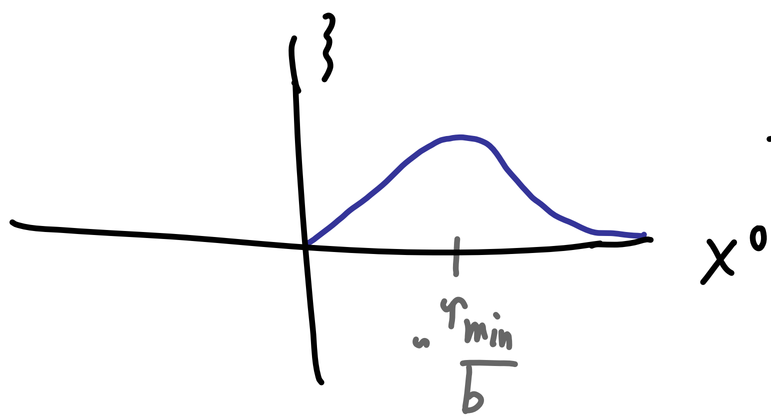
$$\left(\frac{dX^0}{d\tau}\right)^2 = \left(\frac{dr}{d\tau}\right)^2 + r^2 \gamma^2$$

i.e. $$\left(-\partial_{X^0}^2 + \partial_r^2 - r^2 \gamma^2\right) \Psi = 0$$

(1) Warmup: adiabatic regime

$$\xi \equiv L \frac{\dot\gamma}{\gamma} < 1$$

(small loops, or $X^0 \to \infty$ long loops)



$$\to \quad \mathrm{Im}\, S \sim N_{osc}^{\frac{1}{2}}\, \gamma^{3/2} / \dot\gamma$$

(2) Long loops $r \gg X^0$:

↑ Tension too fast for string to relax

$$\left(-\partial_{X^0}^2 + \partial_r^2 - r^2\left(b^2 X^{0\,2} + \Upsilon_{min}^2 + k^2\right)\right)\Psi = 0$$

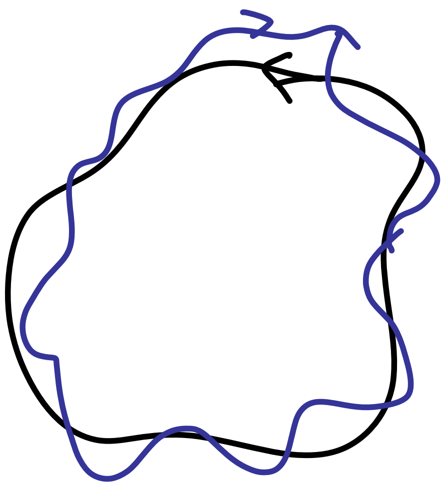
↳ find $\partial_r^2 \ll \partial_{X^0}^2$ $\left(\frac{1}{r} \ll \frac{1}{X^0}\right)$

$|\dot{r}| \ll |\dot{X}^0|$

↳ $$|\beta|^2 = e^{-\frac{2\pi}{b}\left(\frac{k^2}{r} + \Upsilon_{min}^2 r\right)}$$

Need to integrate over phase space (all shapes) and understand r-dependence here.

At face value: long loops formed for sufficiently small Υ_{min}.



- produce large pairs
- can split and join
  ↳ at high density, 1 long string preferred statistically

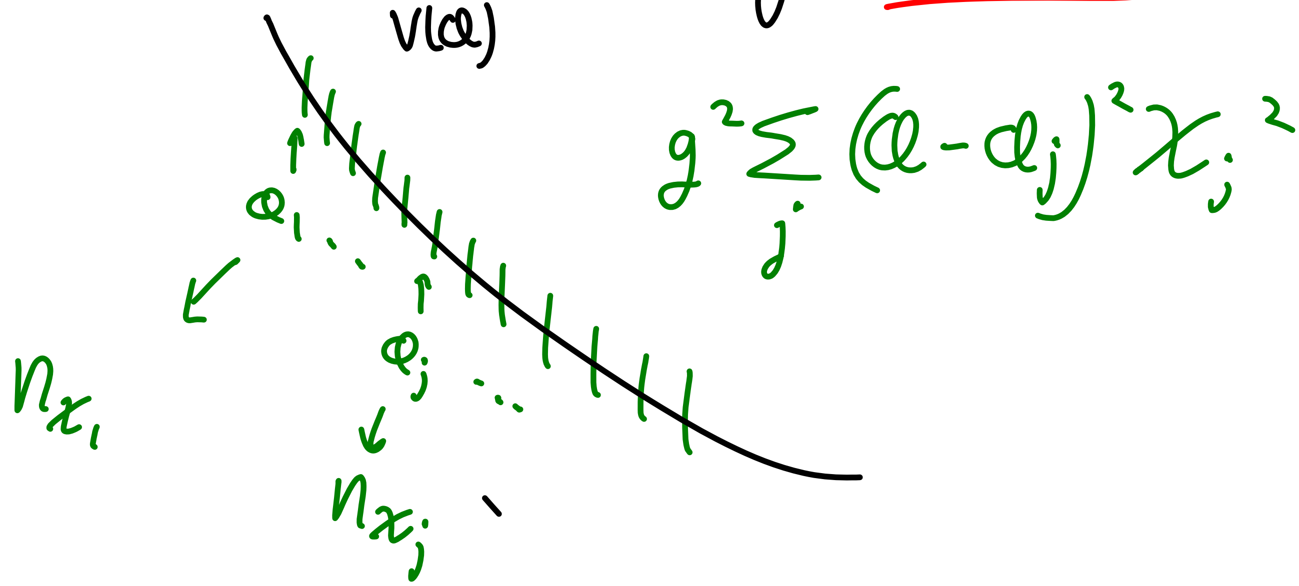
Phenomenological upshot of all this still in progress...

NG likely (as in trapped infl.)

GW signature potentially, but requires sufficiently low-frequency sources.

Particle case: Trapped Inflation

At first sight, using particle production to slow down the inflaton looks very ad hoc

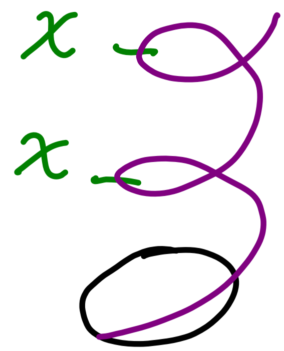


Need many events since χ's dilute

$$\Delta \equiv |\phi_j - \phi_{j+1}| \ll M_p$$

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) + \int^t \frac{g^{5/2}}{\Delta (2\pi)^3} \left(\dot{\phi}(t')\right)^{5/2} \frac{a(t')^3}{a(t)^3} dt' = 0$$

- But in θ direction, the particle production event is automatically repeated, with $\Delta \ll M_p$!



$\rightarrow$ Motivated (falls within reasonable "taste bounds")

Perturbations

$$\ddot{\varphi} + \frac{k^2}{a^2}\varphi + 3H\dot{\varphi} + V'(\phi+\varphi) + \int^{t-\delta t'} \frac{g^{\frac{5}{2}}}{\Delta(2\pi)^3}(\dot{\phi}(t'+\delta t') + \dot{\varphi}(t'+\delta t'))^{\frac{5}{2}} \frac{a(t'+\delta t')^3}{a(t)^3} dt'$$
$$= -g^2 \sum_j \left[\underbrace{(\chi_j^2 - \langle \chi_j^2 \rangle)}(\phi + \varphi - \phi_j) \right]_k , \quad (3.26)$$

leading effect : Δn_χ variance in # of produced χ's

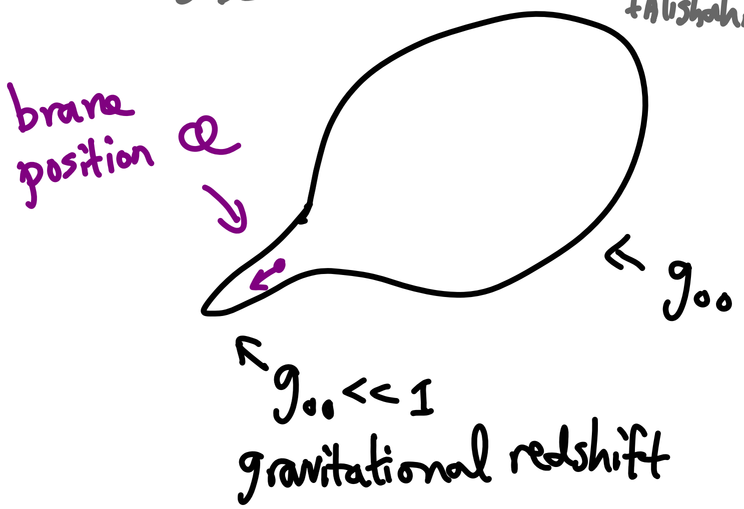
$$\delta\varphi \approx \int G(\tau,\tau')\, \Delta n(\tau')$$

- Strong non-Gaussian correction

Another example, inspired by String theory / extra dimensions

DBI Inflation

ES Tong '03
+Alishahiha '04



strong-coupling version of Trapped Inflation

relativistic motion constrained by

$$\dot{x}^2 \equiv v^2 = \frac{\lambda \dot{\phi}^2}{\phi^4} < 1$$

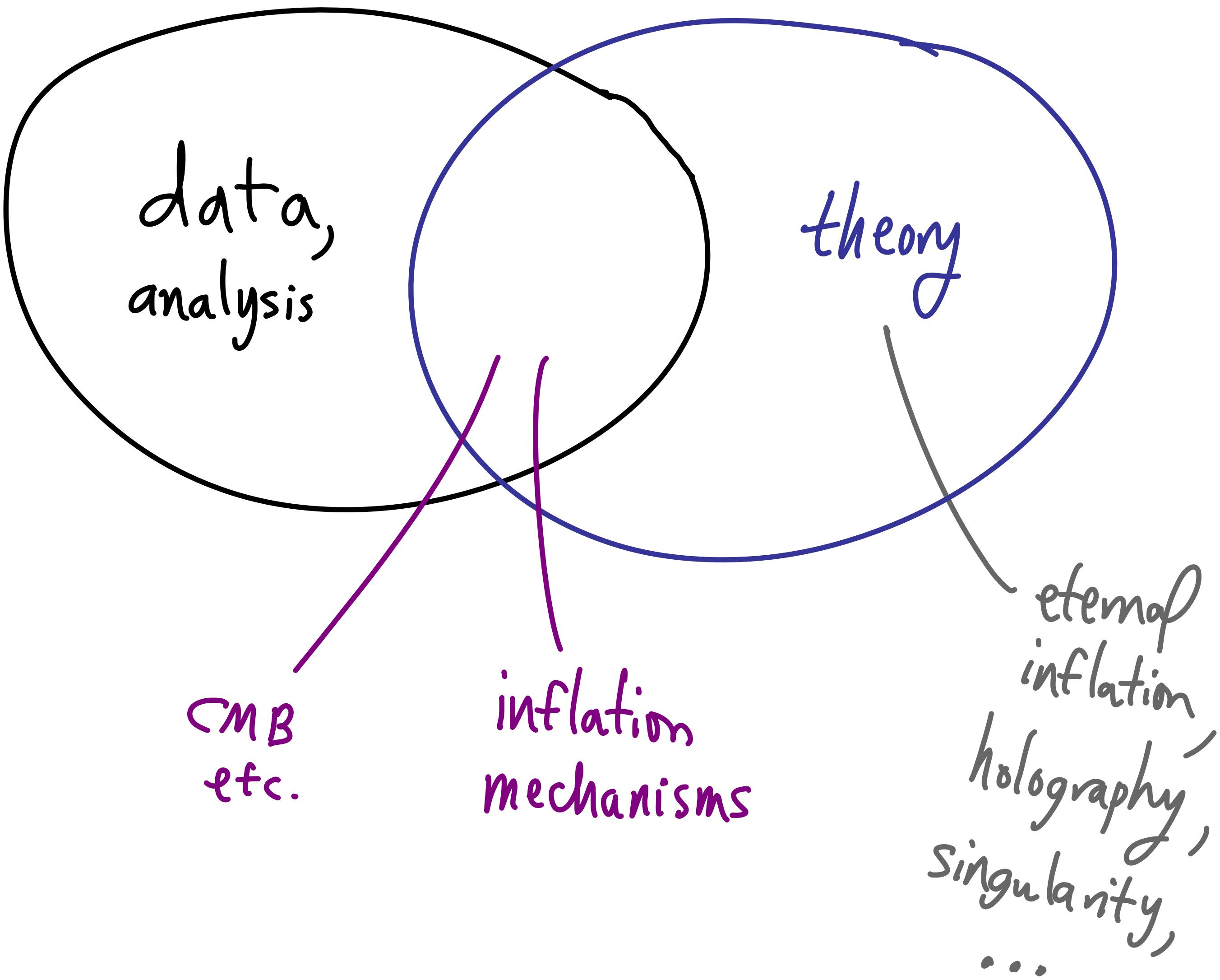
$$\mathcal{L} \propto -\sqrt{1-\dot{x}^2}$$

interactions → non-Gaussianity

- Stimulated more systematic analysis of inflation & signatures in QFT
- Full string theory embedding will be motivated if signature (equilateral NG) is seen:
  - small-N throats (Polchinski, ES)
  - Tilley/Wyman "gelaton"

Whatever its source, an observation of non-Gaussianity would be spectacular — fundamental interactions in the sky!

So far: The simplest-looking models in QFT (e.g. $V(\varphi) = \frac{1}{2} m^2 \varphi^2$) are distinct from those of string theory (e.g. $V(\varphi) = \sqrt{A + B\varphi^2}$), and are distinguishable to some extent using CMB & other observables.



Primordial Cosmology KITP Program

following/during Planck release

Coordinators: C. Hirata, ES, M. Zaldarriaga