Measurements of Primordial Non-Gaussianity and Gravitational Lensing in WMAP CMB Data

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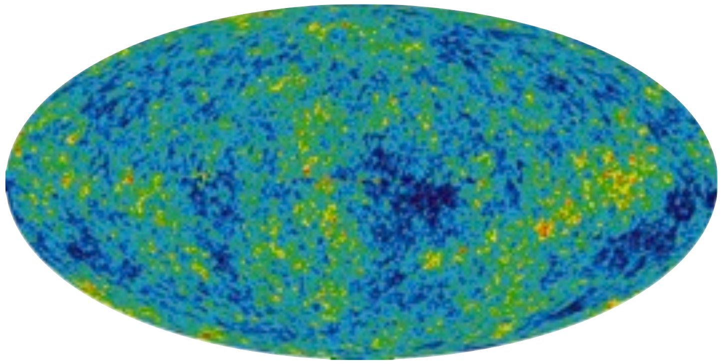
Summary

- Review Inflation and non-Gaussianity
- Introduce New Estimators
- Discuss Constraints Using Estimators
- Implications for future.
  (Fisher estimates, etc...)
- Show how we can measure lensing in the CMB using the same methods with results.

Gaussianity And The CMB



(credit : WMAP Team)

- WMAP measures differences in temperature.

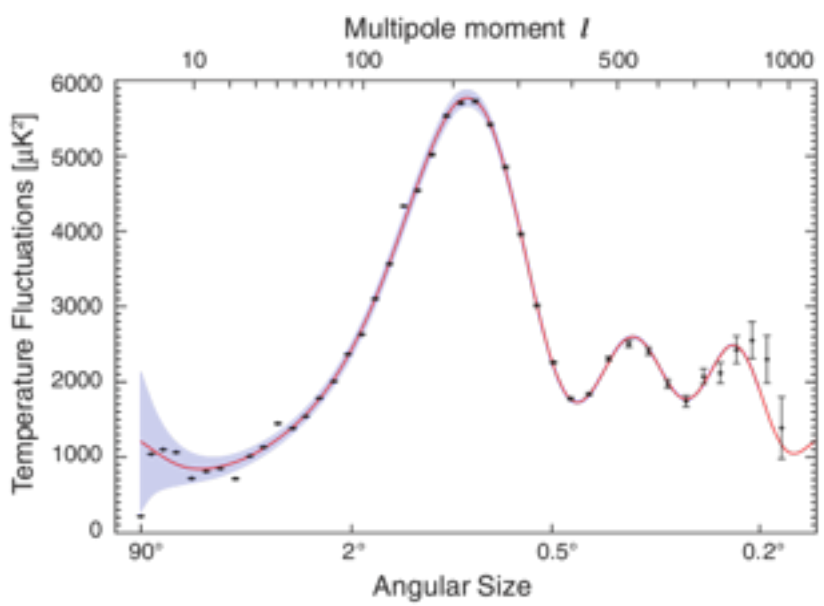
$$\theta = \frac{\delta T}{T} = \sum_{lm} a_{lm} Y_m^{l*}$$

- If CMB is Gaussian: 2-Point function contains all information: $C_l \equiv \langle a_{lm} a_{lm}^* \rangle = \frac{1}{(2l+1)} \sum_m a_{lm} a_{lm}^*$

- If non-Gaussian, we must calculate higher order n-point functions:

Examples: $\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle$ Bispectrum

$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} a_{l_4 m_4} \rangle$ Trispectrum



(credit : WMAP Team)

Example: Bispectrum

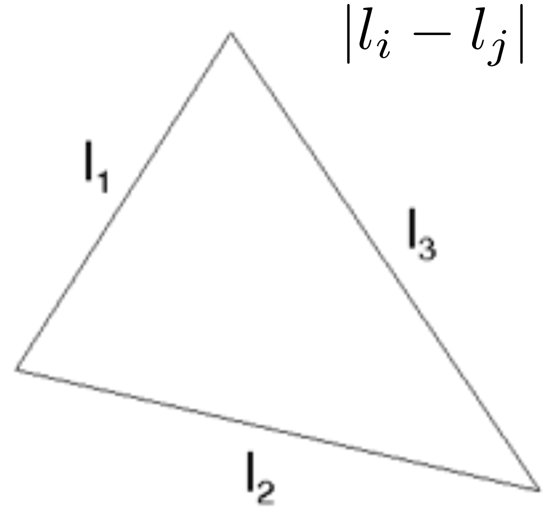
- We can measure non-Gaussianity looking at Bispectrum: $$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} \rangle = B_{l_1 l_2 l_3} \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$

  In Fourier space, modes form triangles.

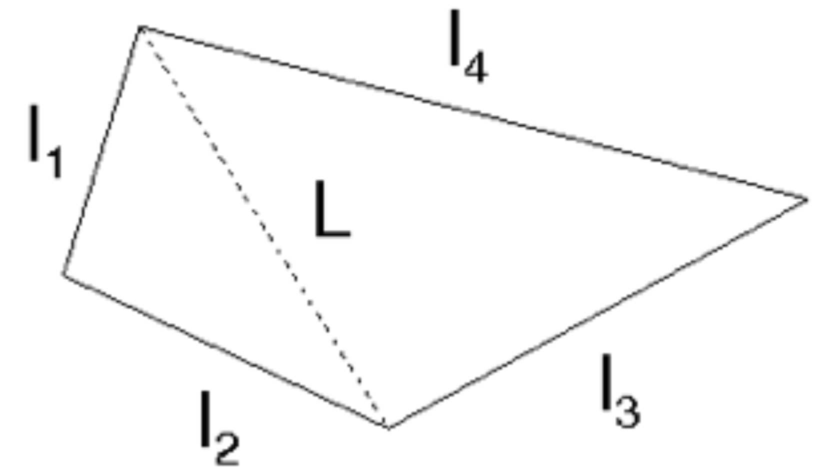
Flat sky.

$$|l_i - l_j| \le l_k \le l_i + l_j$$

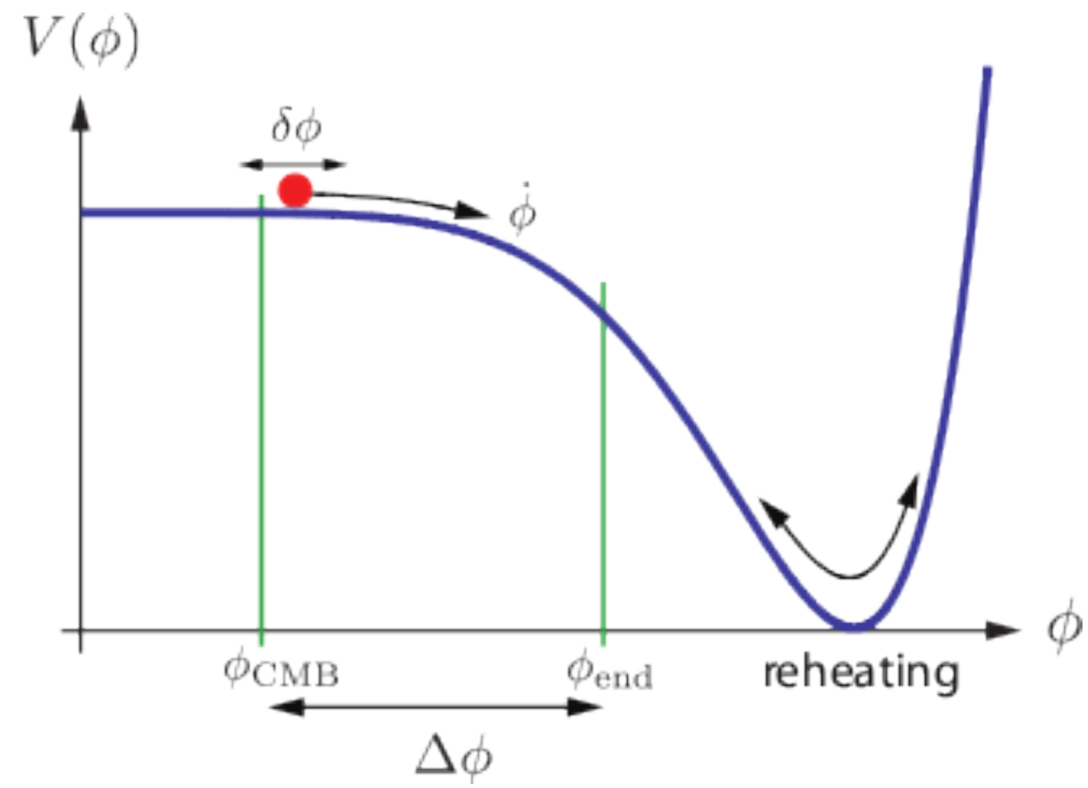
$$\langle \theta(\mathbf{l_1}) \theta(\mathbf{l_2}) \theta(\mathbf{l_3}) \rangle = (2\pi)^2 \delta(\mathbf{l_1} + \mathbf{l_2} + \mathbf{l_3}) B_{(\mathbf{l_1} \mathbf{l_2} \mathbf{l_3})}$$



Similarly, for trispectrum: (quads)



Inflation and NG

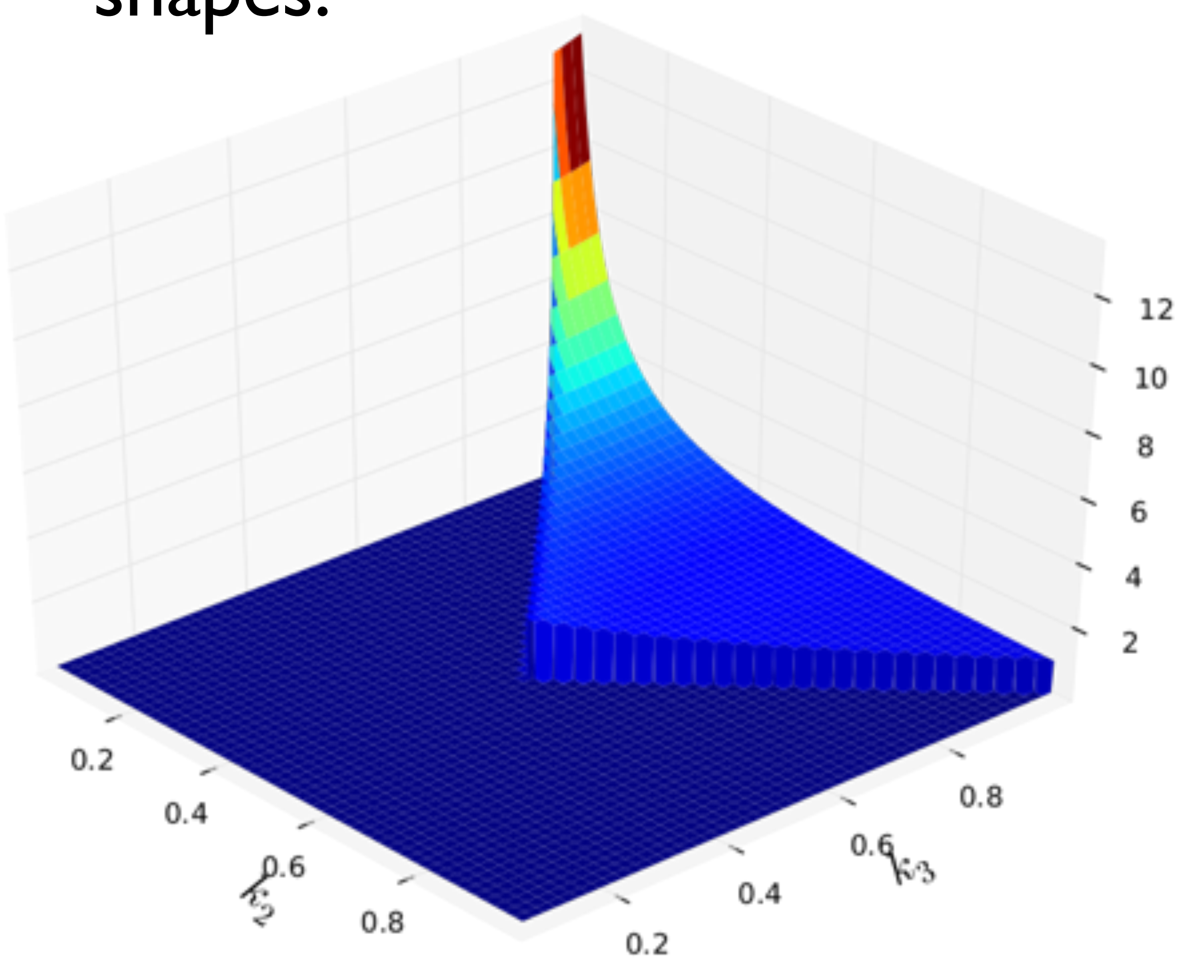


- Simplest Inflation models don't produce detectable non-Gausianity in curvature perturbations.
- Furthermore, the a_{lm} are sourced by curvature perturbations:

$$a_{lm} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3} \Phi(k) g_{Tl} Y^*_{lm}$$

Models And Shapes

- Different inflationary models predict maximal signal for different triangle configurations.
- Example: Some Multifield models peak for squeezed shapes.

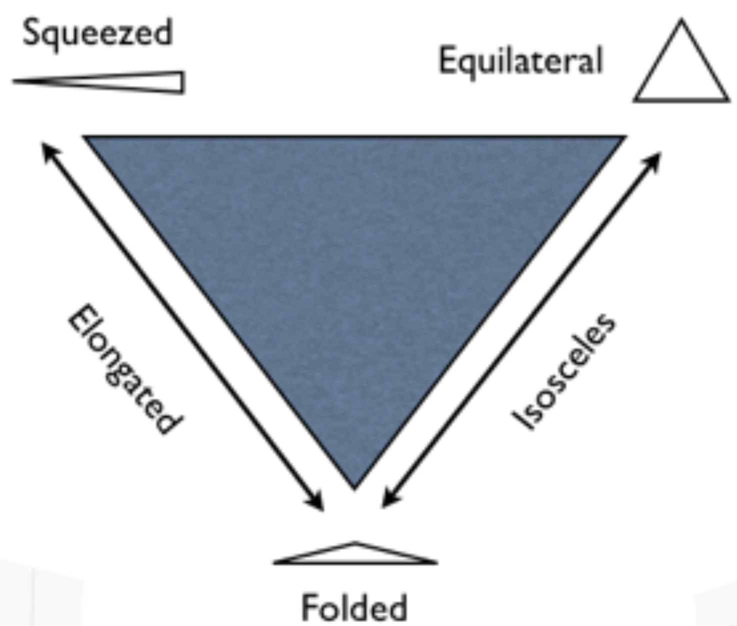


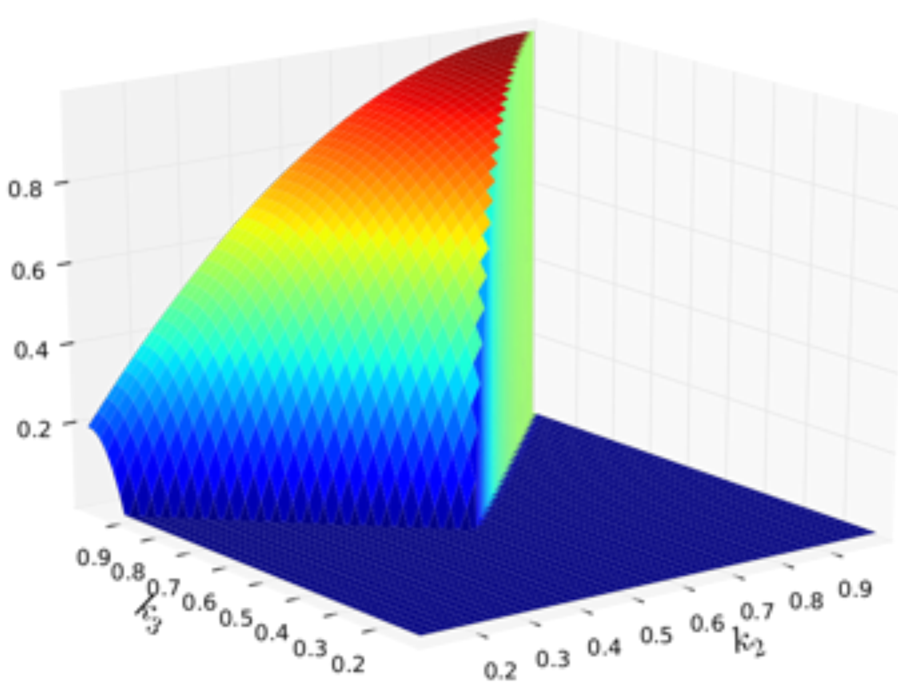
In plot, $k_1 = 1$

$$(k_1 \sim k_3 \gg k_2)$$

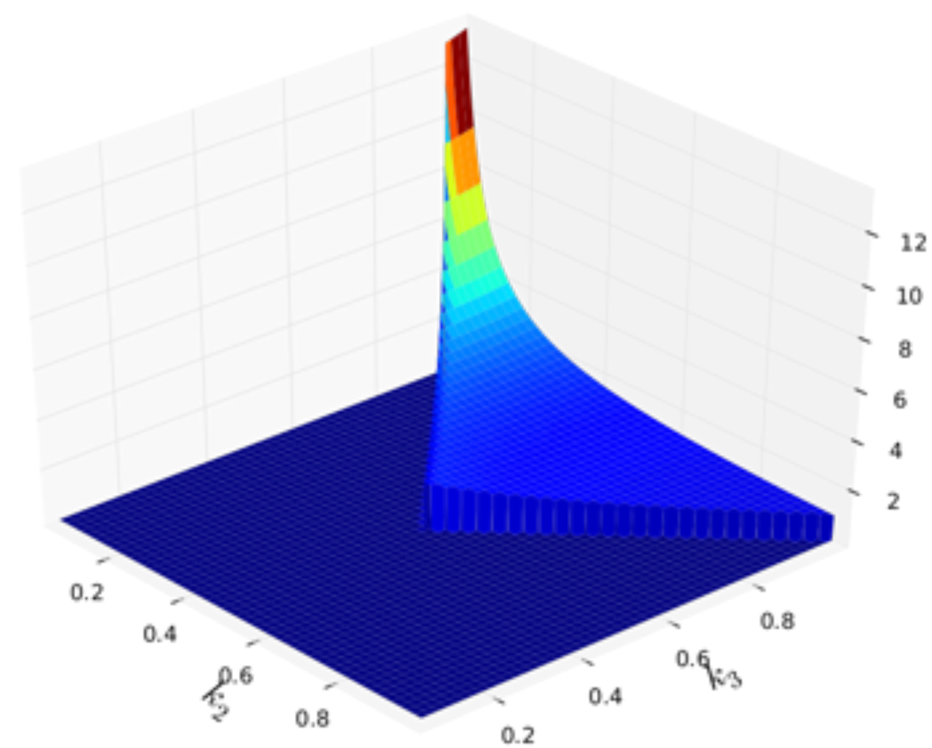
Shapes of non-Gaussianity

- Common shapes: Equilateral, Squeezed, etc...





Equilateral



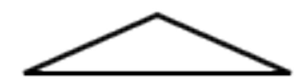
Squeezed

Why Shapes Are Important.

- 3-Point Shapes. (Well understood.)
- Multi-Field: squeezed.

- Non-Canonical Kinetic Terms: equilateral.

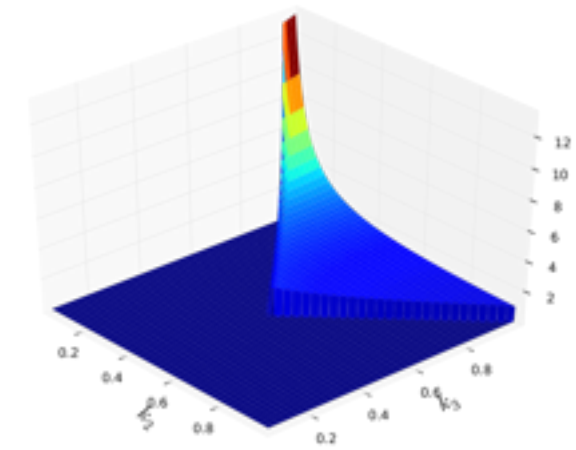
- Non-Adiabatic Vacuum: flattened.



- Possible to have linear combinations.

- 4-point shapes: (From Trispectrum)
- Example: Self interactions produces different squeezed shaped quadrilaterals.

The Local Model



- It's convenient to parameterize non-Gaussianity to second order as:

$$\Phi = \Phi_g + f_{\rm NL}\left[\Phi_g^2 - \langle\Phi_g\rangle^2\right] + g_{\rm NL}\Phi_g^3$$

- This is called the local model.
  (Dominant For Squeezed Shapes)

$f_{\rm NL}\ (l_1 \sim l_2 \gg l_3)$

$\tau_{\rm NL}\ (l_1 \sim l_2 \gg L)$

$g_{\rm NL}\ (l_1 \sim L \gg l_2)$

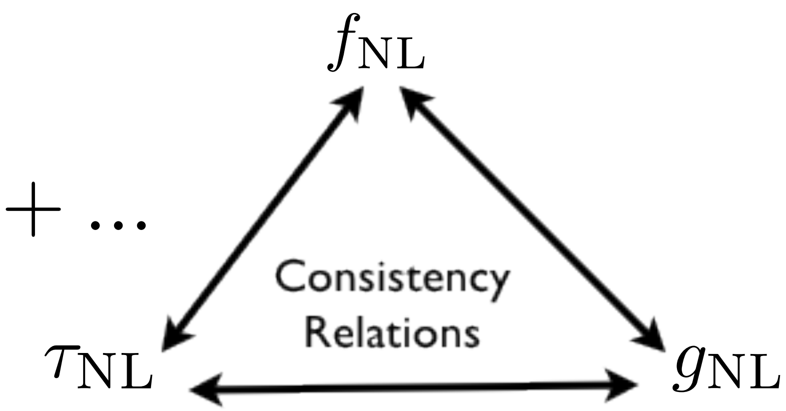
- We introduce: $A_{\rm NL} \equiv \dfrac{\tau_{\rm NL}}{(6f_{\rm NL}/5)^2}$

  (You'll see why in next slide.)

Single Field Inflation Is Special

- For single field: (N = e-folds)

$$\frac{5}{3}\Phi \equiv \delta N = N'\delta\phi + \frac{1}{2}N''\delta\phi^2 + \frac{1}{6}N'''\delta\phi^3 + \ldots$$



- One can show:

$$f_{\rm NL} = \frac{5}{6}\frac{N''}{(N')^2} \quad \tau_{\rm NL} = \frac{(N'')^2}{(N')^4} \quad g_{\rm NL} = \frac{25}{54}\frac{N'''}{(N')^3} \quad \tau_{\rm NL} = (5/6 f_{\rm NL})^2 \quad => A_{\rm NL} = 1$$

- $A_{\rm NL}$ not equal to unity rules out single field inflation.

- Robust, even for curvaton models, DBI, etc...

New Estimators And Measurements

- Relevant Papers:
  [Smidt et al. (2009) PRD. 80, 123005]

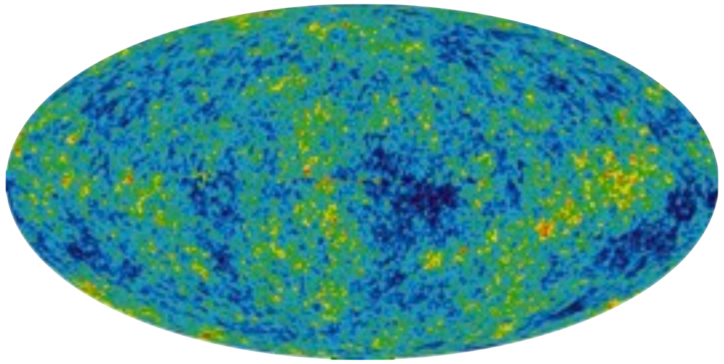
  [Calabrese, Smidt et al. (2010) PRD. 81,043529]

  [Smidt et al. (2009) Arxiv:1001.5026]

  [Smidt et al. (2010) PRD 123007]

- Collaborators:
  Alexandre Amblard, Christian Byrnes, Erminia Calabrese,
  Peter Coles, Asantha Cooray, Alan Heavens, Alessandro Melchiorri,
  Dipak Munshi and Paolo Serra

Extracting Non-Gaussianity :



(credit : WMAP Team)

- Reminder, we can write out curvature perturbations:

$$\Phi = \Phi_g + f_{\rm NL}\left[\Phi_g^2 - \langle\Phi_g\rangle^2\right] + g_{\rm NL}\Phi_g^3$$

- We can extract this information from CMB since:

$$a_{lm} = 4\pi(-i)^l \int \frac{d^3k}{(2\pi)^3}\Phi(k) g_{Tl} Y^*_{lm}$$

- Examples:

$$\langle a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}\rangle = B_{l_1 l_2 l_3}\begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & m_3 \end{pmatrix}$$

$$< a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3} a_{l_4 m_4} >_c = \sum_{LM} T^{l_3 l_4}_{l_1 l_2}(L)\begin{pmatrix} l_1 & l_2 & L \\ m_1 & m_2 & M \end{pmatrix}\begin{pmatrix} l_3 & l_4 & L \\ m_1 & m_2 & -M \end{pmatrix}$$

Our Recipe For Analysis

- Calculate Bispectrum/Trispectrum estimators theoretically. (Codes like CAMB, CMBFast...)
- Extract these same quantities in the Data.
- Compare these two to get a measure for the amount of non-Gaussianity.
- Use Gaussian simulations to calculate error bars.

The Theoretical Bispectrum

- For the local model:

$$B^{prim}_{l_1 l_2 l_3} \equiv 2 I_{l_1 l_2 l_3} \int r^2 dr \left[\beta_{l_1}(r)\beta_{l_2}(r)\alpha_{l_3}(r) + \beta_{l_3}(r)\beta_{l_1}(r)\alpha_{l_2}(r) + \beta_{l_2}(r)\beta_{l_3}(r)\alpha_{l_1}(r) \right]$$

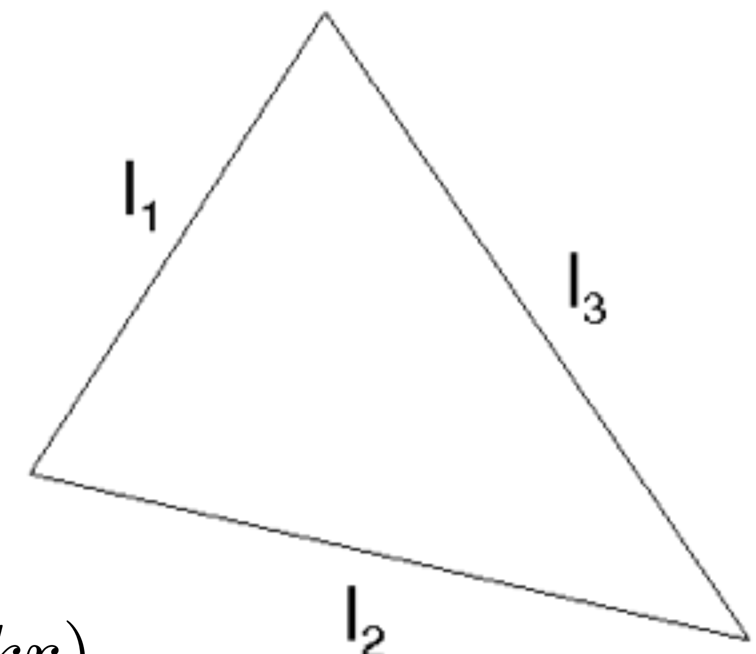
(for f_{nl}=1)

where :

$$I_{l_1 l_2 l_3} \equiv \sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}} \begin{pmatrix} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\alpha_l(r) \equiv \frac{2}{\pi} \int k^2 dk\, g_{Tl}(k) j_l(kr)$$

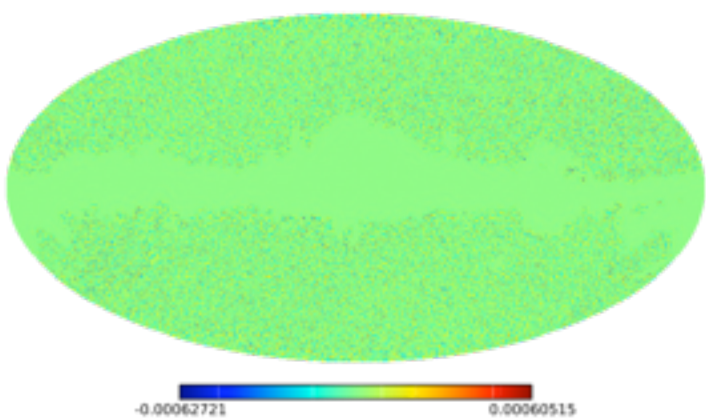
$$\beta_l(r) \equiv \frac{2}{\pi} \int k^2 dk\, P_\Phi(k) g_{Tl}(k) j_l(kr)$$



The WMAP Estimator: Skewness

- Common way to compute non-Gaussianity: the "skewness" estimator

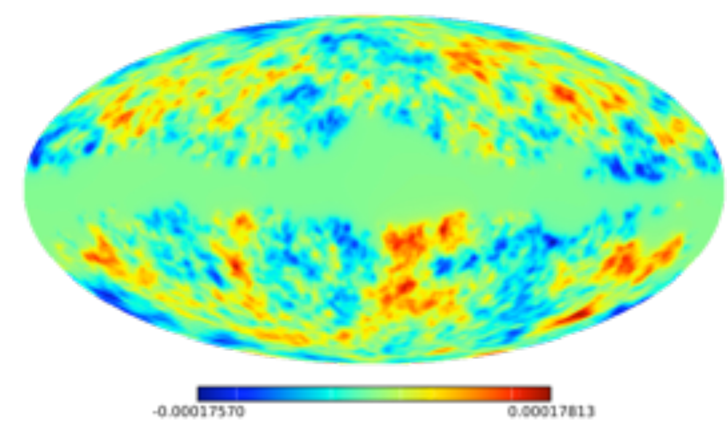
$$S_3 = f_{\rm NL} \sum_{l_1} \sum_{l_2} \sum_{l_3} \left\{ \frac{B_{l_1 l_2 l_3} B_{l_1 l_2 l_3}}{C_{l_1} C_{l_2} C_{l_3}} \right\}$$



A Map

It turns out:

$$S_3 \equiv \int r^2 dr \int d\hat{\Omega} A(r,\hat{\Omega}) B^2(r,\hat{\Omega})$$



B Map

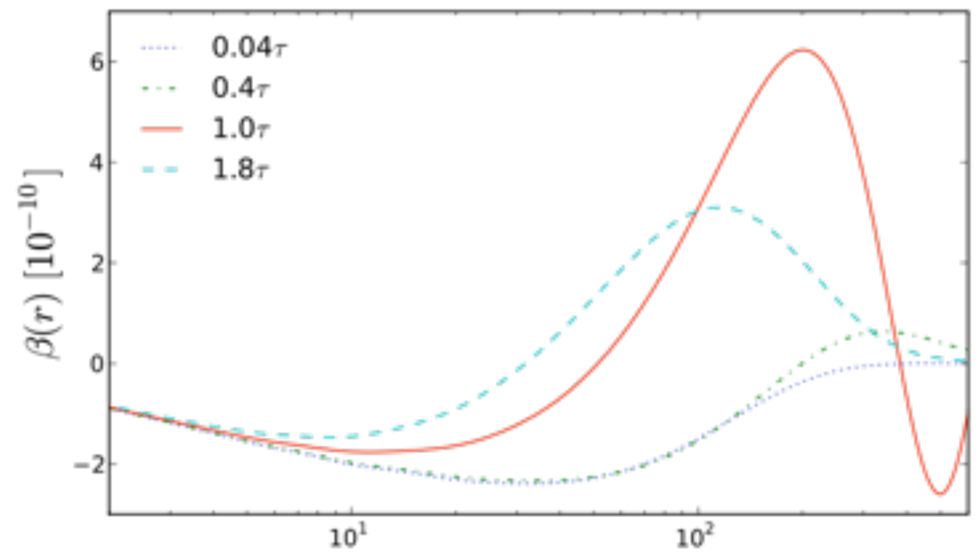
$$A(r,\hat{\Omega}) \equiv \sum_{lm} Y_{lm}(\hat{\Omega}) A_{lm}(r); \qquad A_{lm}(r) \equiv \frac{\alpha_l(r)}{C_l} b_l a_{lm}$$

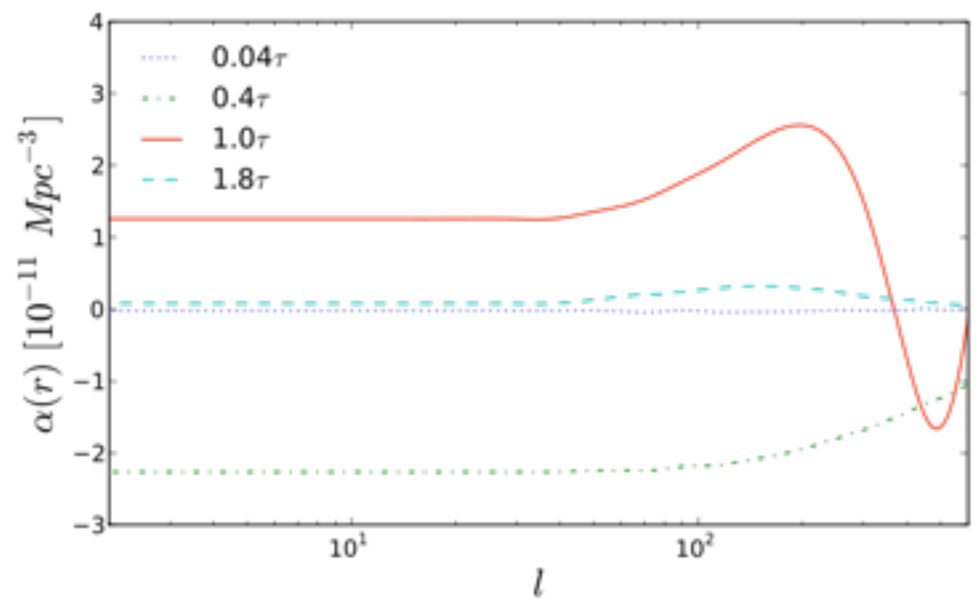
$$B(r,\hat{\Omega}) \equiv \sum_{lm} Y_{lm}(\hat{\Omega}) B_{lm}(r); \qquad B_{lm}(r) \equiv \frac{\beta_l(r)}{C_l} b_l a_{lm}$$

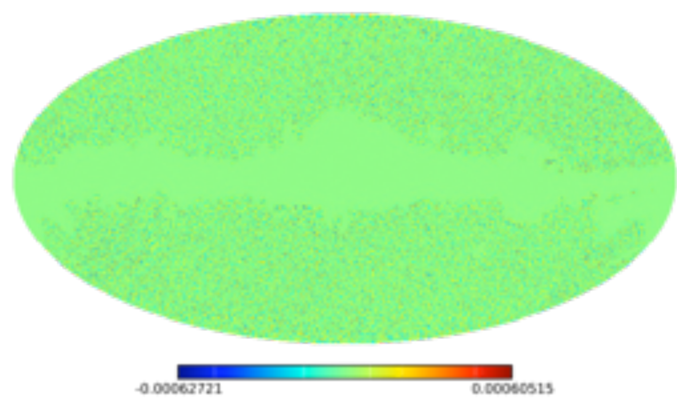
Komatsu et al. (2005) ApJ, 634, 14

Quantities Versus Look-back Time

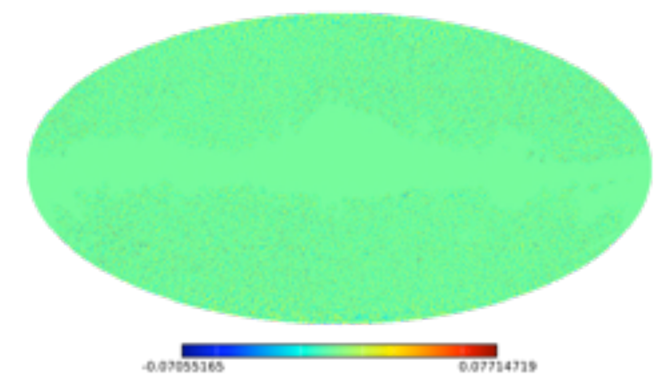
τ is the look-back time to end of decoupling



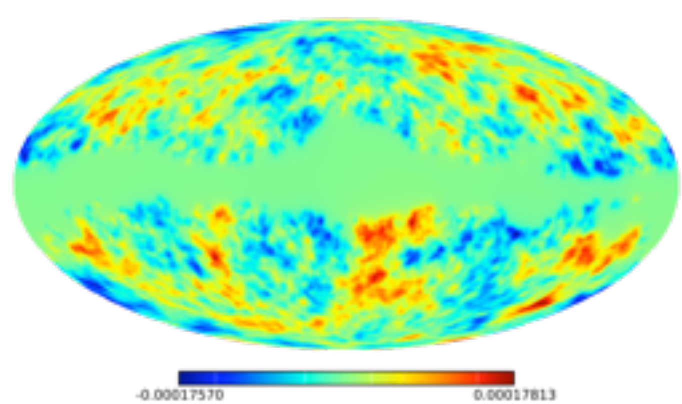




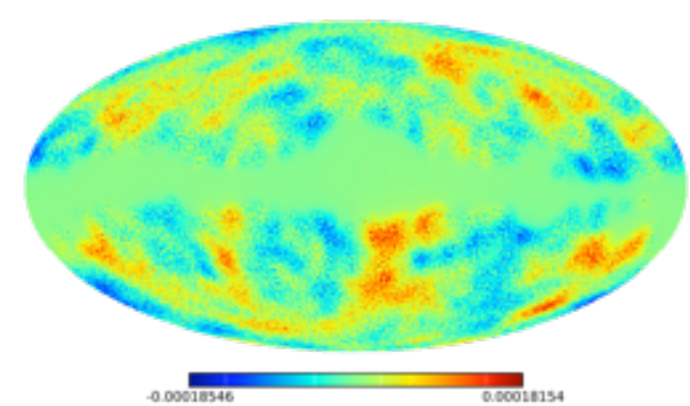
A Map 0.04τ



A Map 1.0τ



B Map 0.04τ



B Map 1.0τ

Decompressing the skewness estimator :

- We used optimized skewness power spectrum estimators $C_l^{2,1}$

  Cooray (2001)PRD, 64, 043516
  Munshi et al. (2010) MNRAS, 401, 2406
  Smidt et al. (2009) PRD 80, 123005

- Advantage over skewness estimator: scale dependance.

$$C_l^{A,B^2} \equiv \frac{1}{2l+1} \int r^2 dr \sum_m A_{lm}^*(r) B_{lm}^{(2)}(r); \qquad C_l^{AB,A} \equiv \frac{1}{2l+1} \int r^2 dr \sum_m (AB)_{lm}^*(r) B_{lm}(r)$$

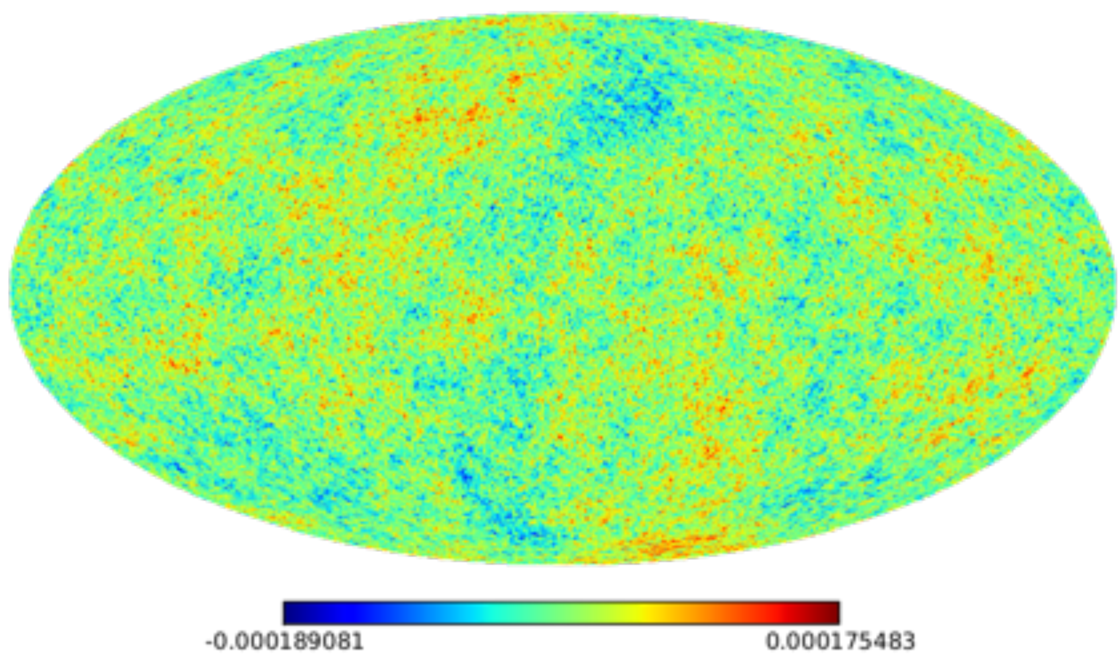
The skewness power spectrum :

$$C_l^{2,1} \equiv \left(C_l^{A,B^2} + 2C_l^{AB,B} \right) = \frac{f_{\rm NL}}{(2l+1)} \sum_{l_2} \sum_{l_3} \left\{ \frac{B_{ll_2l_3} B_{ll_2l_3}}{C_l C_{l_2} C_{l_3}} \right\}$$

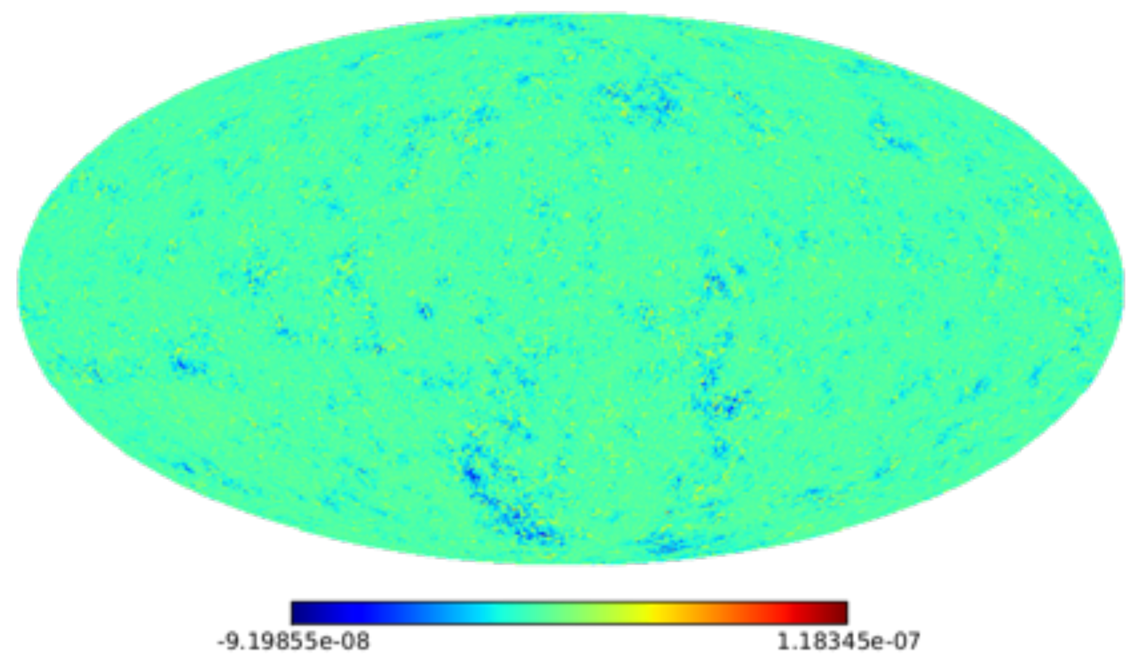
Testing Estimator

- Franz Elsner and Benjamin D. Wandelt have released non-Gaussian simulations.
- We can make maps for any f_{nl} using:

$$a_{lm} = a_{lm}^{G} + f_{\rm NL} a_{lm}^{NG}$$



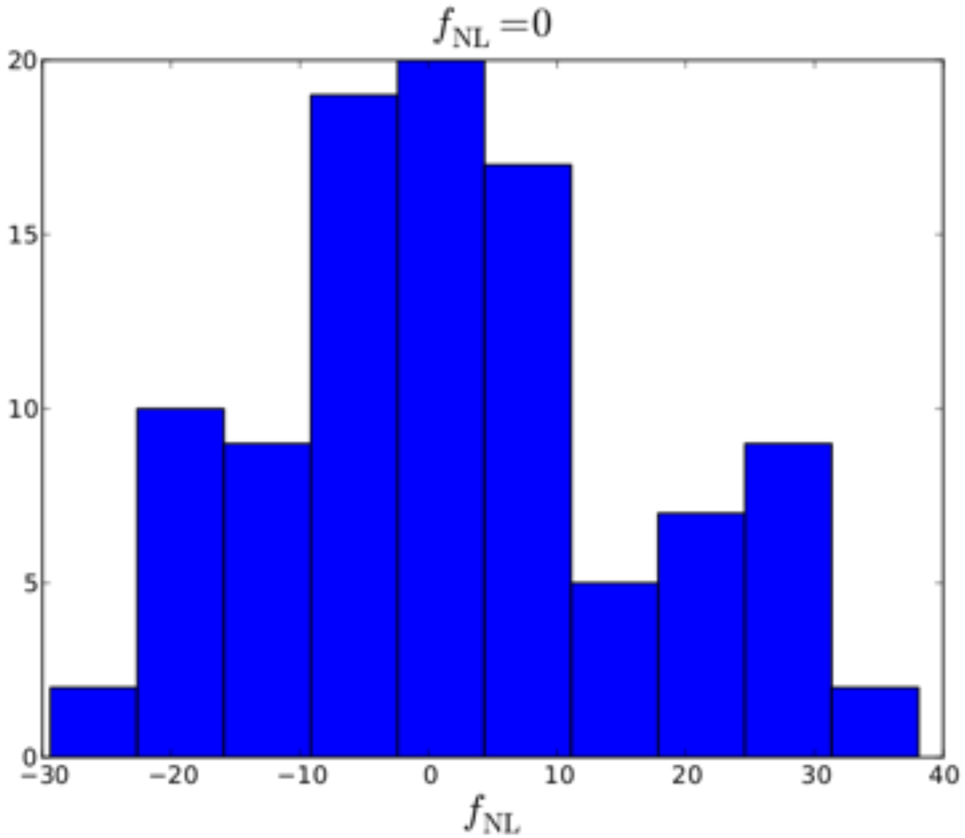
Gaussian Map

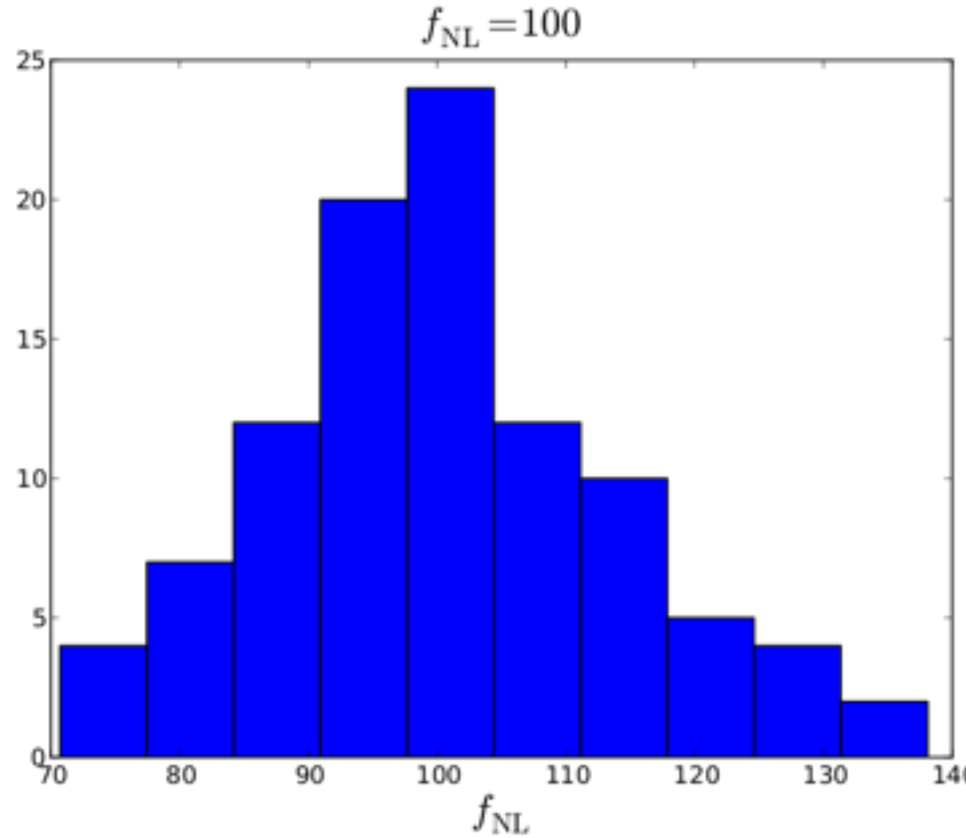


non-Gaussian Map

Results of Simulations

- Run our estimator on 100 of them gives:
- Good as fisher error estimate is +/- 13





f_{nl} = 2 +/- 14

f_{nl} = 101 +/- 14

Wait, Primordial Signal Could Be Contaminated!

Estimator optimized for point sources :

$$E(\hat{\mathbf{n}}) \equiv \sum_{lm} Y_{lm}(\hat{\mathbf{n}}) E_{lm}(r) \qquad E_{lm}(r) \equiv \frac{b_l}{\mathcal{C}_l} a_{lm}$$

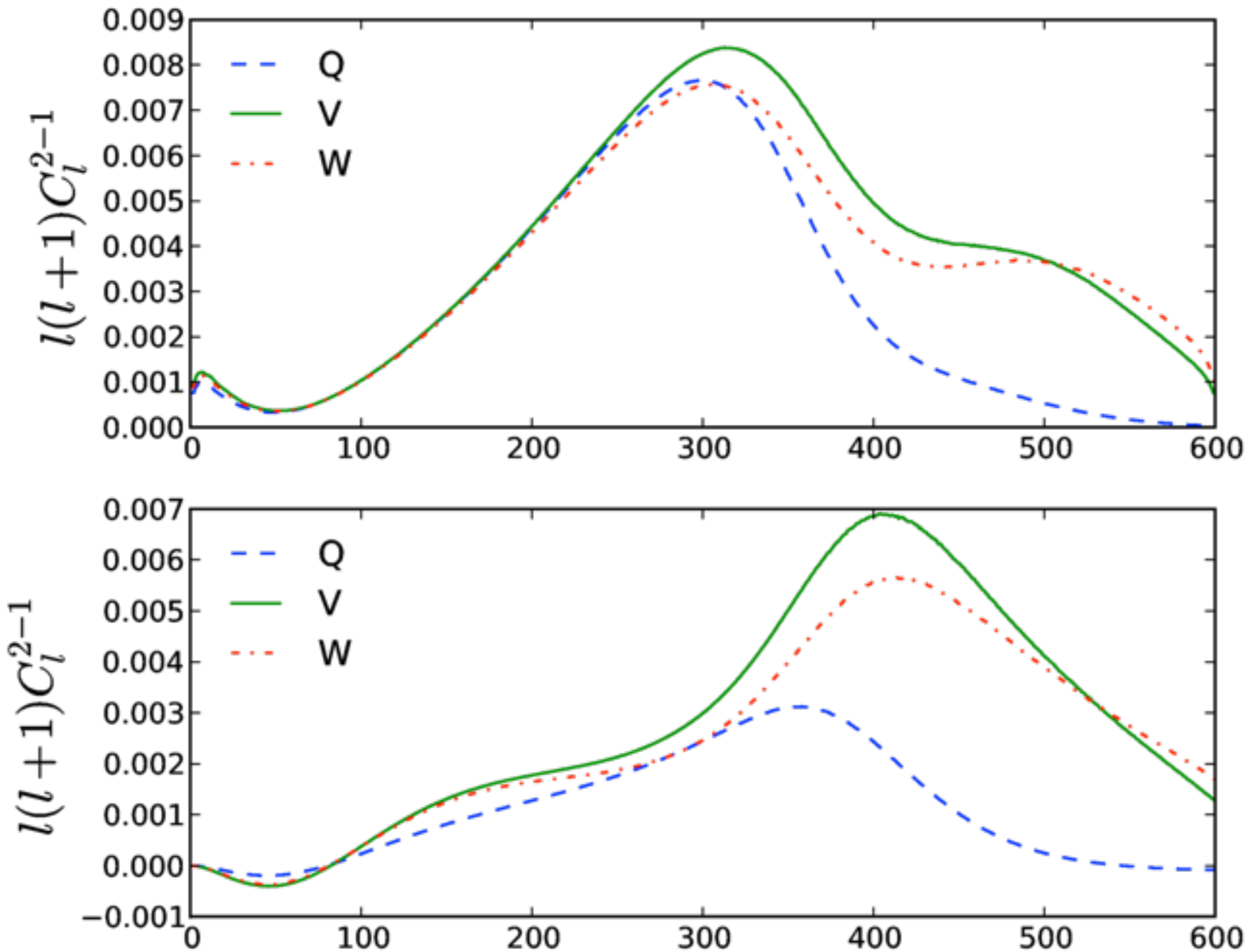
$$E_l^{2-1} = C_l^{E,E^2} \equiv \frac{1}{2l+1} \left[\sum_m \mathrm{Real} \left\{ E_{lm} \left(E^2 \right)_{lm} \right\} \right]$$

$$E_l^{2-1} = \frac{1}{(2l+1)} \left[\sum_{l'l''} \left\{ \frac{B_{ll'l''}^{PS,b_{\mathrm{ps}}=1} \hat{B}'_{ll'l''} b_l b'_l b''_l}{\mathcal{C}_l \mathcal{C}_{l'} \mathcal{C}_{l''}} \right\} \right]$$

Separating the sources in C_l^{2-1}:

Primordial NG
f_{nl}=1

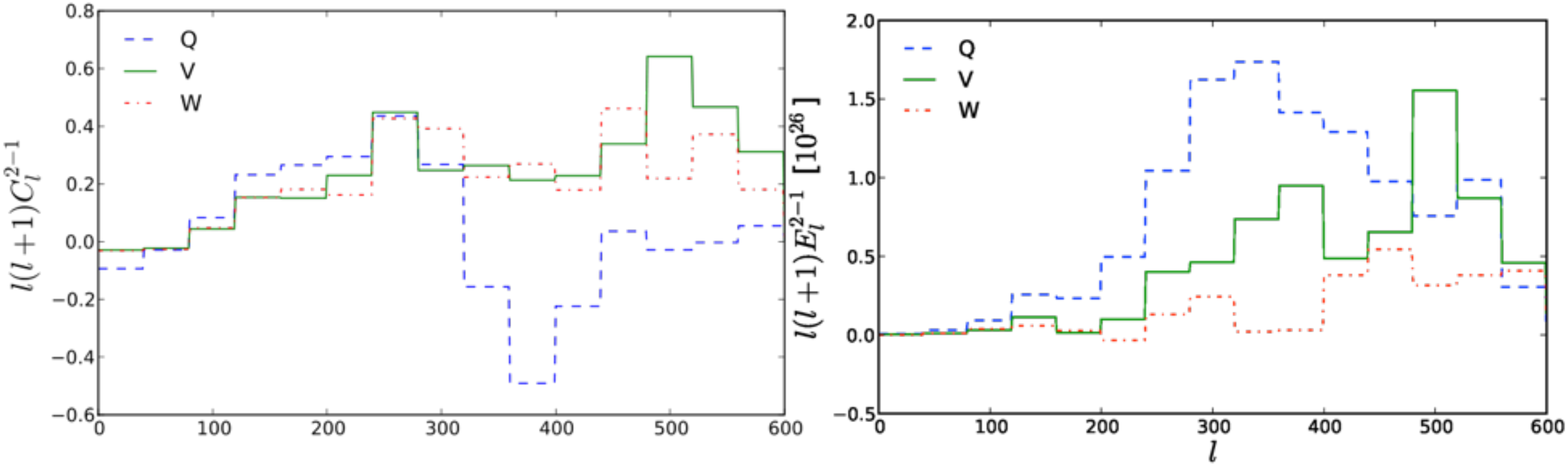
Point Sources
b_{ps}=1



Estimating $C_l^{(2,1)}$ and $E_l^{(2,1)}$ on WMAP5 data

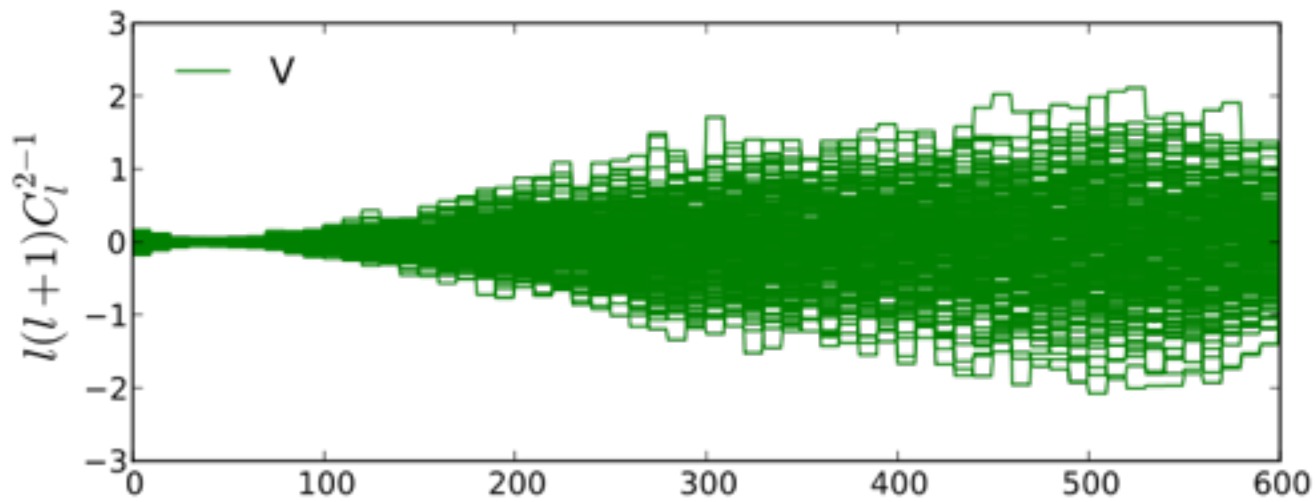
Smidt et al. (2009) PRD 80, 123005

- Analyzed three frequencies bands: Q, V and W (40, 60 and 90 GHz respectively)
- 250 Simulations of CMB and WMAP5 noise

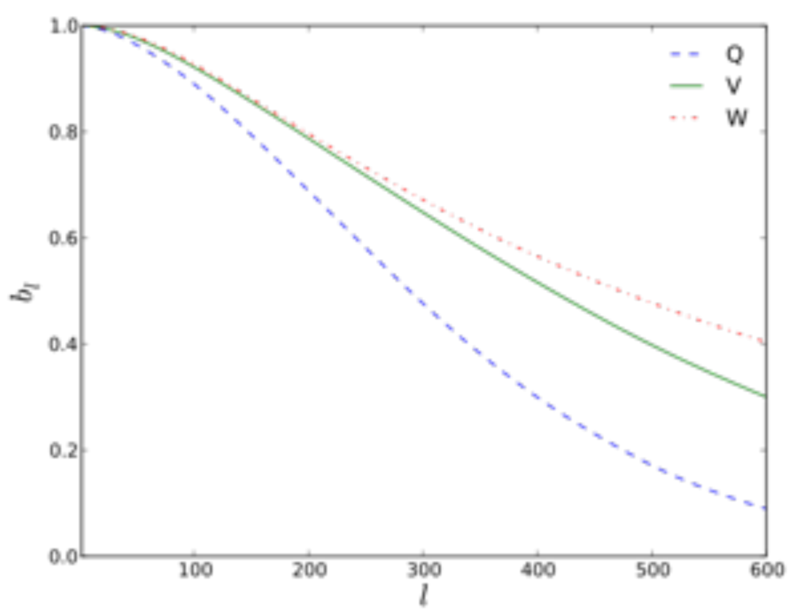


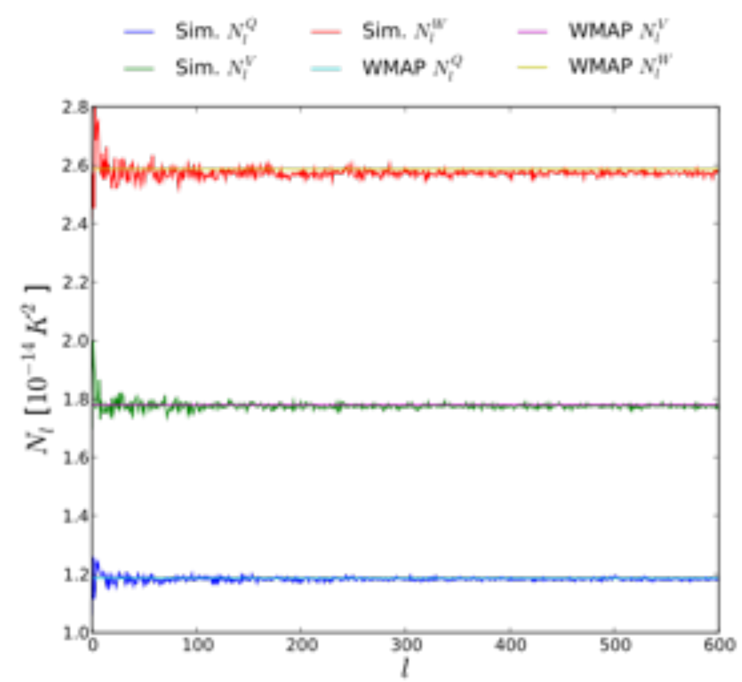
J. Smidt et al. (2009) PRD, 80, 123005

Simulations



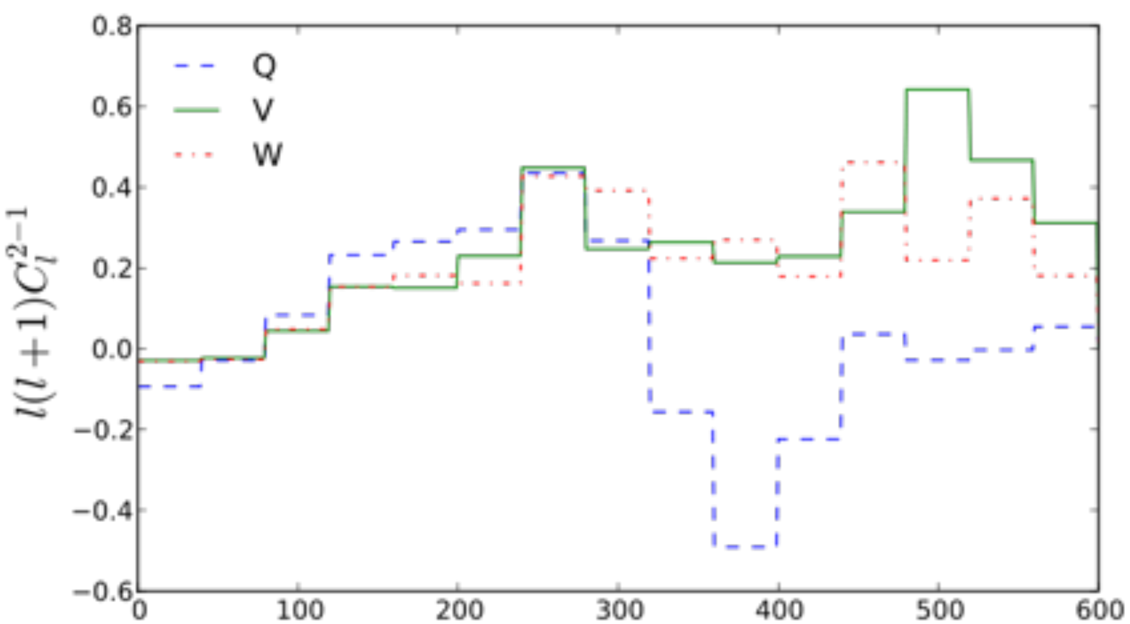
- To determine error we Created 250 masked Gaussian maps with proper noise.
- For simulated maps: $a_{lm}^S b_l + n_{lm} = a_{lm}^D$
- Gives covariance matrix



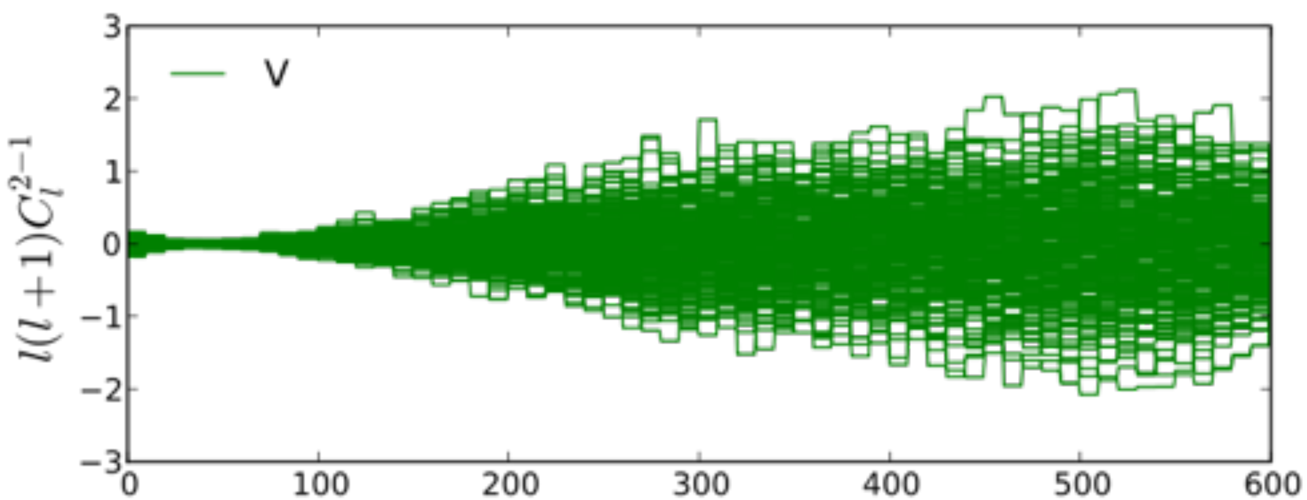


Side By Side

May help to see figures side by side.



Data



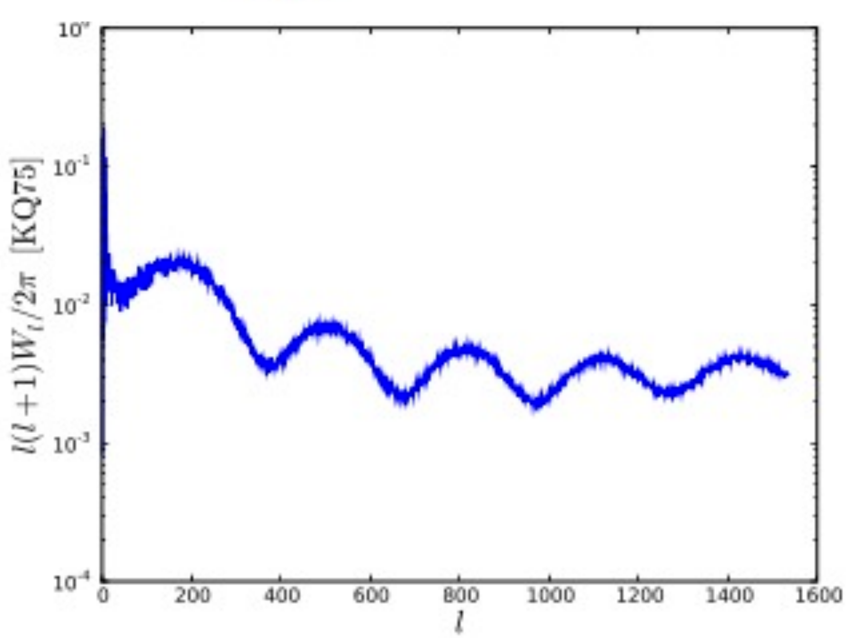
Simulations

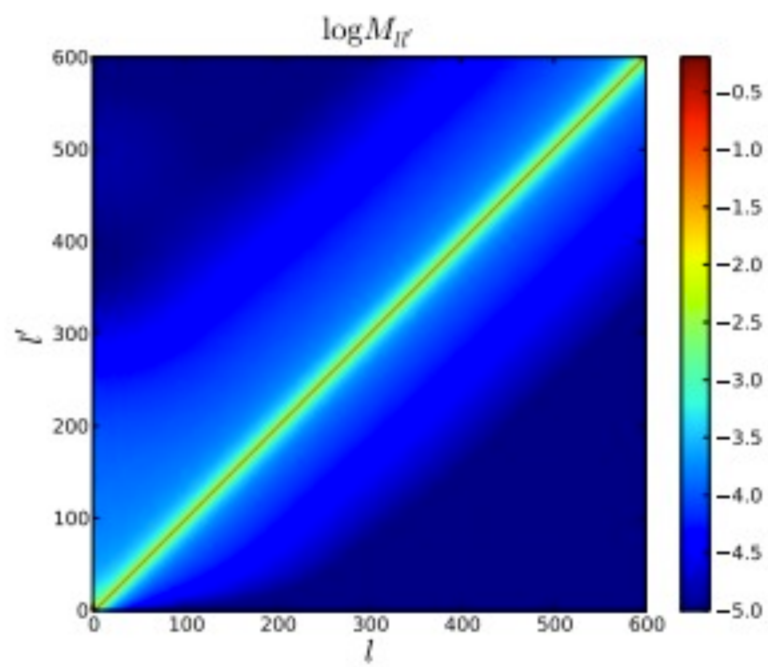
Removing Masking Effects

- To correct for cut sky we use a method devoloped by Hivon et al. 2001.

$$\begin{aligned}\tilde{a}_{lm} &= \int d\hat{\mathbf{n}} M(\hat{\mathbf{n}}) W(\mathbf{n}) Y_l^{m*}(\hat{\mathbf{n}}), \\ &= \sum_{l'm'} a_{l'm'} \int d\hat{\mathbf{n}} Y_{l'}^{m'}(\hat{\mathbf{n}}) W(\hat{\mathbf{n}}) Y_l^{m*}(\hat{\mathbf{n}}), \\ &= \sum_{l'm'} a_{l'm'} K_{lml'm'}[W],\end{aligned}$$

$$\tilde{C}_l = \sum_{l'} M_{ll'} C_{l'},$$





Parameter Estimation Explain How To get C matrix and show it

- Calculate Cov. Matrix from 250 binned simulations:

$$C_{ij} = \langle X_i X_j \rangle - \langle X_i \rangle \langle X_j \rangle$$

- To determine best fit values and errors we need to minimize

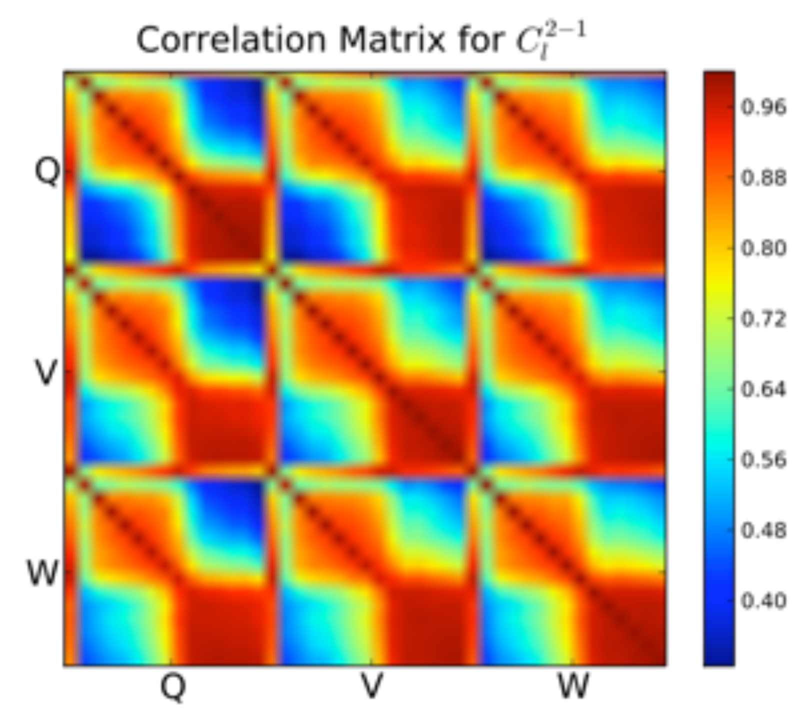
$$\chi^2 = (y - M \cdot p)^T C^{-1} (y - M \cdot p)$$

- We set the above's derivative to zero and solve for our parameters

$$p = (M^T C^{-1} M)^{-1} M^T C^{-1} \cdot y$$

- Our error for each parameter is

$$\Delta p = (M^T C^{-1} M)^{-1}$$



Fit for f_{nl} with the skewness power spectra :

Smidt et al. (2009) PRD 80, 123005

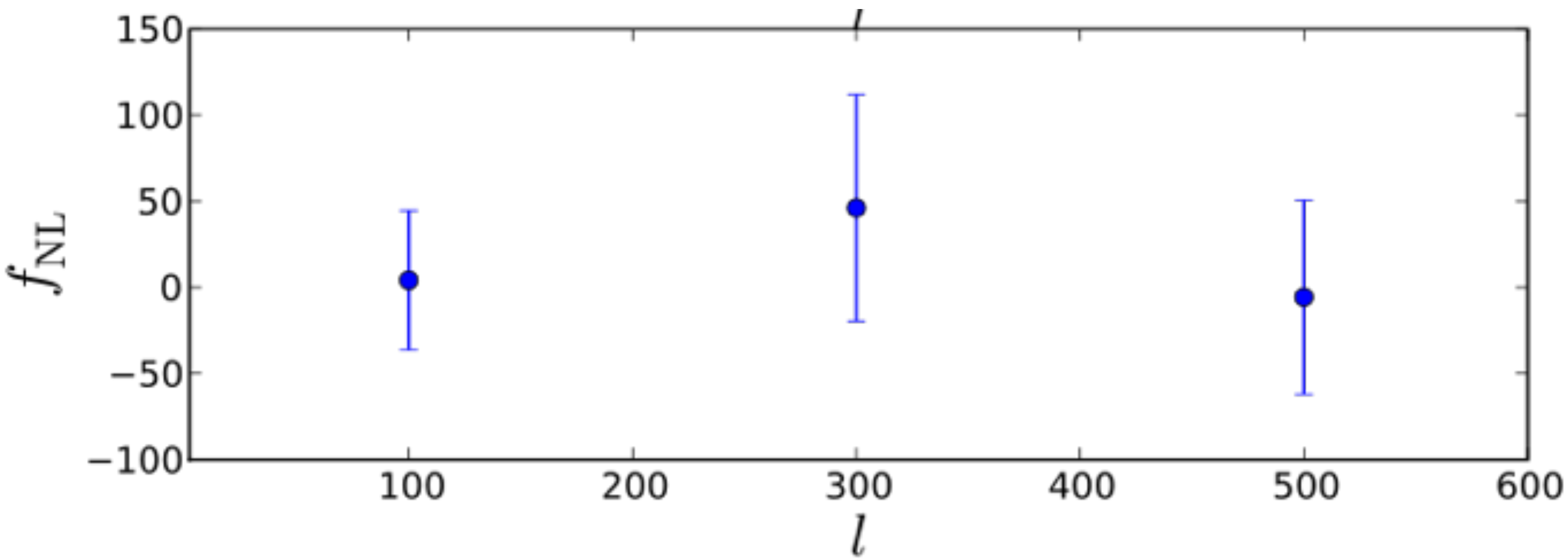
- f_{nl} from $C_l^{(2,1)}$ estimator :

  V: 15.7 ± 38.9, W: -13.5 ± 39.8, V+W: 14.3 ± 37.6

- f_{nl} from $C_l^{(2,1)}$ and $E_l^{(2,1)}$ estimator :

  V: 16.7 ± 27.1, W: 18.7 ± 27.2, V+W: 11.0 ± 24

- f_{nl} variation with scales :



f_{nl} comparison :

WMAP 5-Year, Skewness :	51 ± 30 (E. Komatsu et al. 2009, ApJS, 180,330)
WMAP 5-Year, Minkowski Functions :	-57 ± 61 (E. Komatsu et al. 2009, ApJS, 180,330)
WMAP 5-year, Wavelets :	31 ± 25 (A. Curto et al. 2009, ApJ, 706, 399)
WMAP 5-year, Needlets :	84 ± 40 (O. Rudjord et al. 2009, ApJ, 701, 369)
WMAP 5-year, N-point PDF :	30 ± 62 (P. Vielva et al. 2009, MNRAS, 397, 837)
WMAP 5-Year, Optimal Estimator :	32 ± 21 (K. Smith et al. 2009, JCAP, 909,6)
WMAP 5-year, Skew-power spectrum	11.0 ± 24 (Smidt et al. 2009, PRD, 80, 123005)

The error bars of the skewness power spectrum is comparable to best estimates but it does not find any hint of non-zero f_{nl}

Scale Dependance of f_{nl}

- The literature also describes another parameter, n_{fNL},that measures the scale dependance of f_{nl}:

$$n_{f_{\rm NL}}(k) \equiv \frac{d \ln |f_{\rm NL}(k)|}{d \ln k}$$

- We may approximate a constraint on this parameter as follows:

$$f_{\rm NL}(l) = f_{{\rm NL}_{200}} \left(\frac{l}{l_{200}}\right)^{n_{f_{\rm NL}}(l)}$$

- From data we constrain $n_{fNL(l)}$ = -0.1 ± 1.2
- Consistent with no scale dependance.

[Smidt et al. (2010) PRD 123007]

Let's Complicate It: Trispectrum

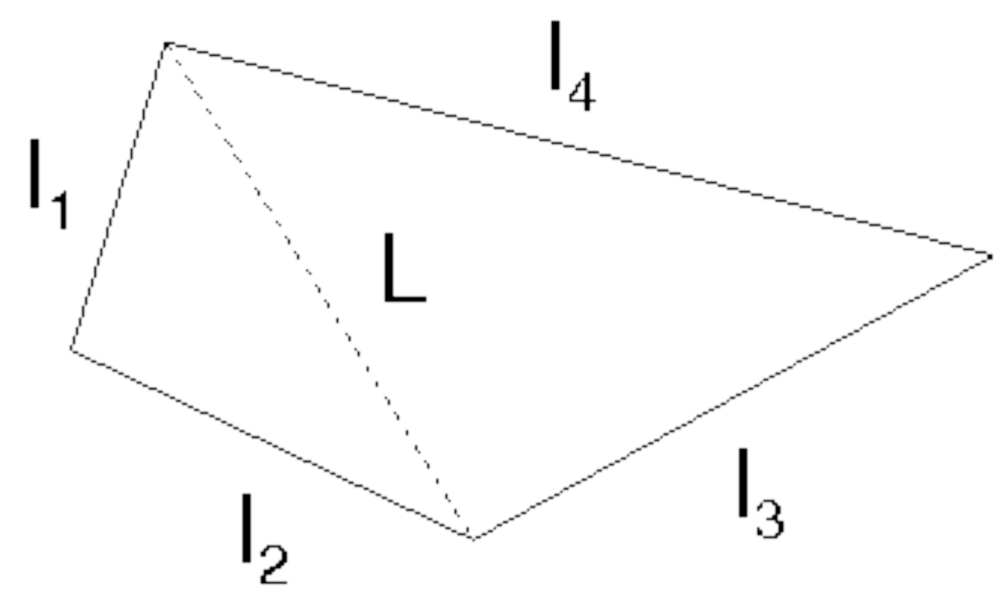
- Remember, We can look to the four point function for non-Gaussianity:

$$< a_{l_1m_1}a_{l_2m_2}a_{l_3m_3}a_{l_4m_4} >_c = \sum_{LM} T^{l_3l_4}_{l_1l_2}(L) \left(\begin{array}{ccc} l_1 & l_2 & L \\ m_1 & m_2 & M \end{array} \right) \left(\begin{array}{ccc} l_3 & l_4 & L \\ m_1 & m_2 & -M \end{array} \right)$$

$$T^{l_1l_2}_{l_3l_4}(L) = 4f_{\rm NL}^2 h_{l_1l_2L}h_{l_3l_4L}\int r_1^2 dr_1 \int r_2^2 dr_2 F_L(r_1,r_2)\alpha_{l_1}(r_1)\beta_{l_2}(r_1)\alpha_{l_3}(r_2)\beta_{l_4}(r_2)$$
$$+g_{\rm NL}h_{l_1l_2L}h_{l_3l_4L}\int r^2 dr \beta_{l_2}\beta_{l_4}(\alpha_{l_1}(r)\beta_{l_3}(r)+\alpha_{l_3}(r)\beta_{l_1}(r))$$

$$h_{l_1l_2l_3} = \sqrt{\frac{(2l_1+1)(2l_2+1)(2l_3+1)}{4\pi}} \left(\begin{array}{ccc} l_1 & l_2 & l_3 \\ 0 & 0 & 0 \end{array} \right)$$

$$F_L(r_1,r_2) = \frac{2}{\pi}\int k^2 dk P_\Phi j_L(kr_1)j_L(kr_2)$$



Kurtosis Power Spectrum For Trispectrum

- New estimators for the trispectrum that constrain τ_{nl} (a function of f_{nl}^2 and g_{nl}: 2 parameters are 2nd order

Theory:
$$K_l^{(2,2)} = \sum_{l_i} \frac{T^{l_3 l_4}_{l_1 l_2}(l) T^{l_3 l_4}_{l_1 l_2}(l)}{C_{l_1} C_{l_2} C_{l_3} C_{l_4}} \qquad K_l^{(3,1)} = \sum_{l_i L} \frac{T^{l_1 l_2}_{l_3 l}(L) T^{l_1 l_2}_{l_3 l}(L)}{C_{l_1} C_{l_2} C_{l_3} C_l}$$

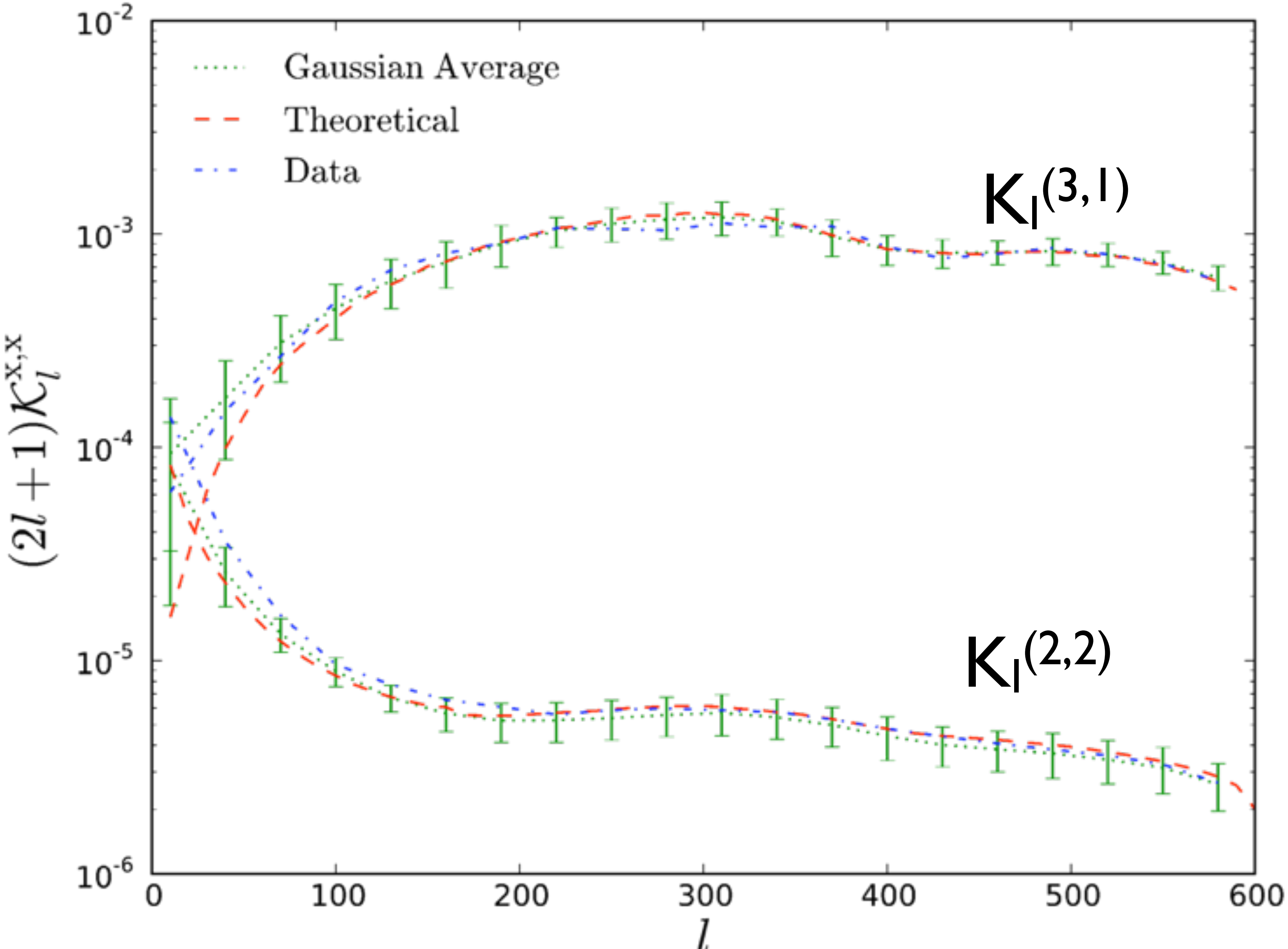
From Maps:
$$\begin{aligned} \mathcal{K}_l^{(2,2)}|_c &= \tau_{\rm NL} \mathcal{A}_l^{(2,2)} + g_{\rm NL} \mathcal{B}_l^{(2,2)} \\ \mathcal{K}_l^{(3,1)}|_c &= \tau_{\rm NL} \mathcal{A}_l^{(3,1)} + g_{\rm NL} \mathcal{B}_l^{(3,1)} \end{aligned}$$

with :
$$\mathcal{A}_l^{(3,1)} = 4 \int r_1^2 dr_1 \int r_2^2 dr_2 \mathcal{J}_l^{AB^2,A}(r_1, r_2) \qquad \mathcal{B}_l^{(3,1)} = 2 \int r^2 dr \mathcal{L}_l^{AB^2,M}(r)$$

$$\mathcal{A}_l^{(2,2)} = 4 \int r_1^2 dr_1 \int r_2^2 dr_2 \mathcal{J}_l^{AB,AB}(r_1, r_2) \qquad \mathcal{B}_l^{(2,2)} = 2 \int r^2 dr \mathcal{L}_l^{AB,BM}(r)$$

[Smidt et al. (2010) PRD 123007]

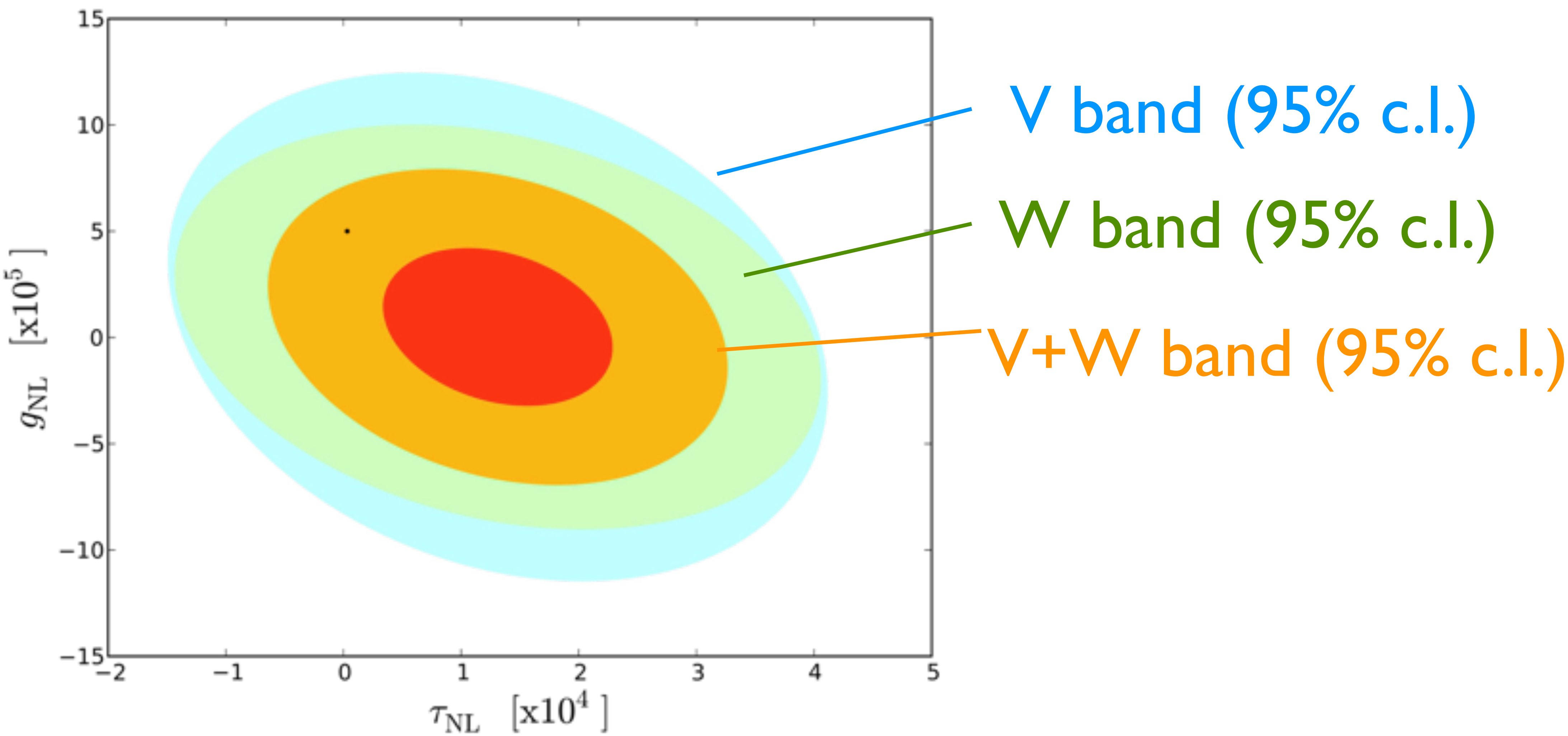
Comparing data and simulations



[Smidt et al. (2010) PRD 123007]

Results: (First Measurement.)

Smidt et al.(2010) Arxiv:1001.5026



	V	W	V+W
g_{nl}	$4.2 \pm 53 \times 10^4$	$4.1 \pm 59 \times 10^4$	$4.2 \pm 39 \times 10^4$
τ_{nl}	$1.32 \pm 1.27 \times 10^4$	$1.39 \pm 1.31 \times 10^4$	$1.35 \pm 0.98 \times 10^4$

previous constraint on τ_{nl} from COBE was $<10^8$

Summary of Results

- The skewness power spectrum gives $f_{nl} = 11.0 \pm 27$, consistent with 0.

[Smidt et al. (2009) PRD. 80, 123005]

- The trispectrum estimate gives $g_{nl} < 8.2 \times 10^5$ (95% c.l.).

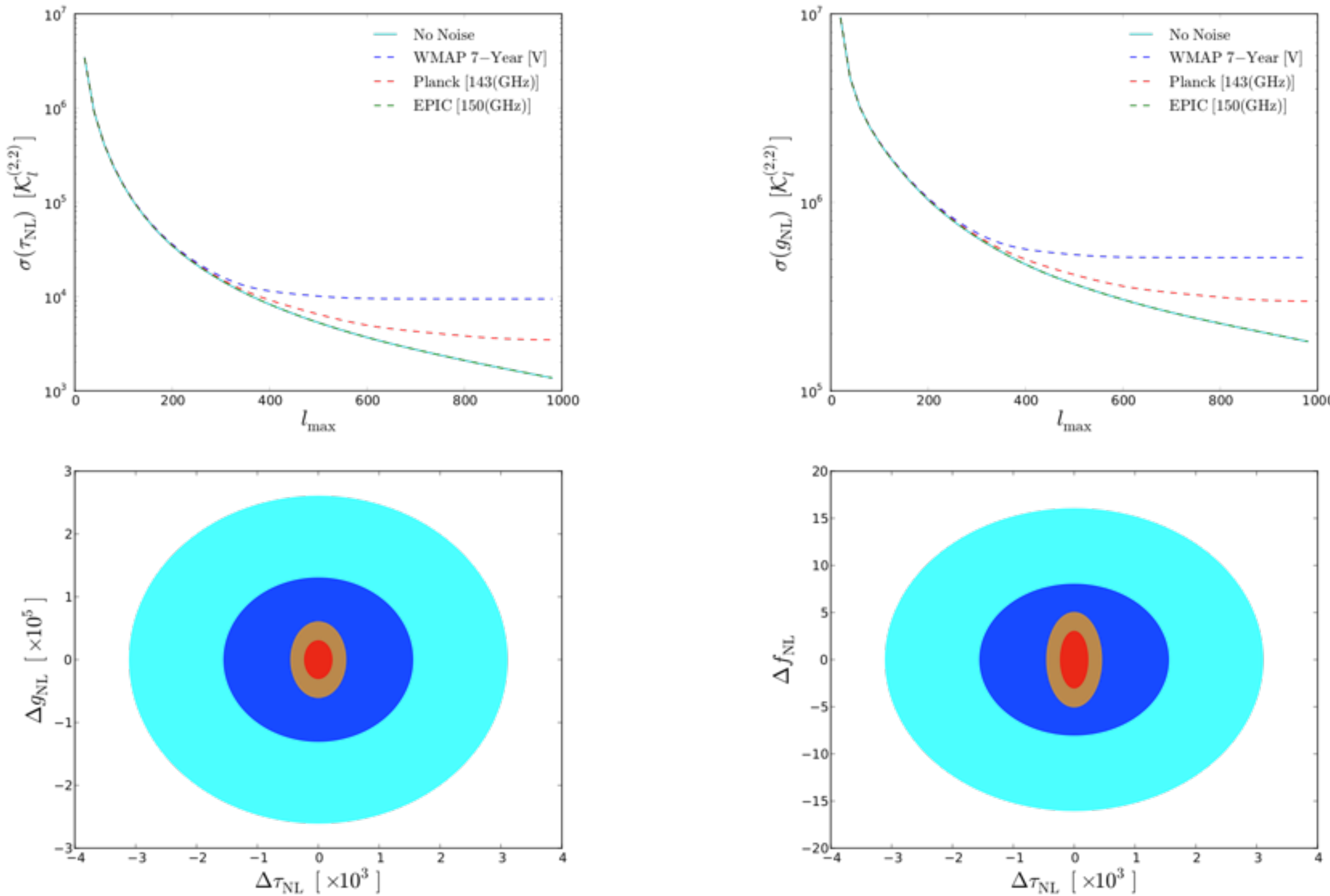
- The trispectrum estimate gives $\tau_{nl} < 3.3 \times 10^4$ (95% c.l.).

- Constraint on single field consistency relation is: $A_{NL} = 9.2 \pm 6.1$.

- Scale dependance is constrained to: $n_{fNL(l)} = -0.1 \pm 1.2$

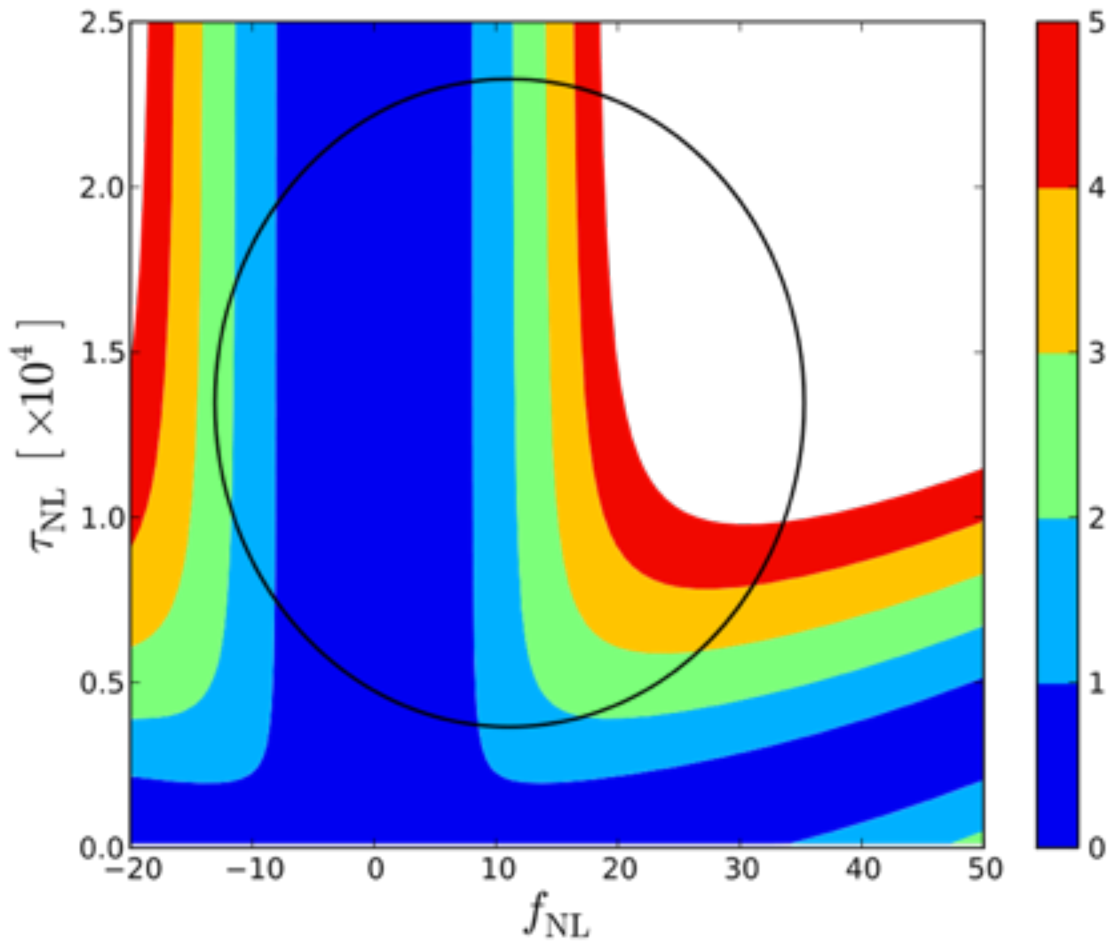
[Smidt et al. (2010) PRD 123007]

Expectations for Planck and EPIC

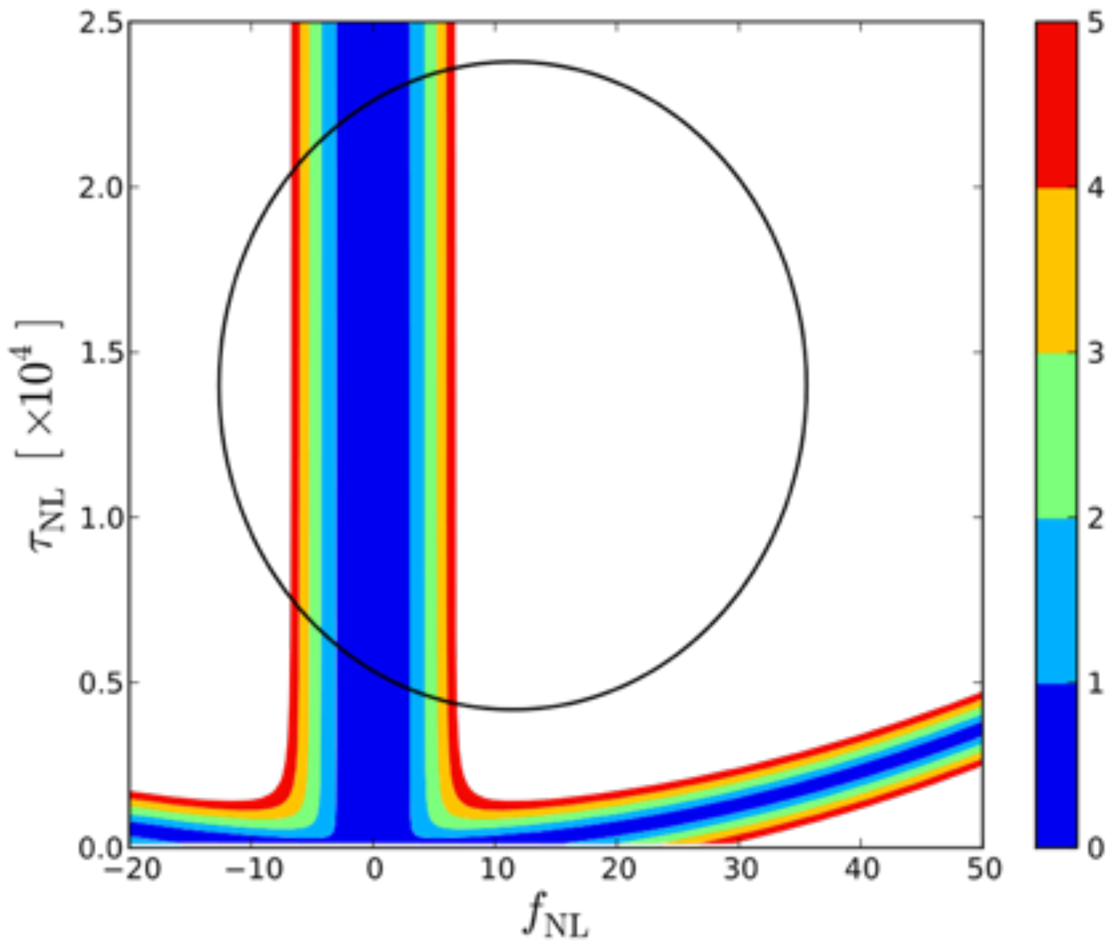


Ability To Rule Out Single Field Inflation

- Anywhere in white region, A_{NL} equal to unity is ruled out by > 5 σ.
- Circle is current 68% confidence region.

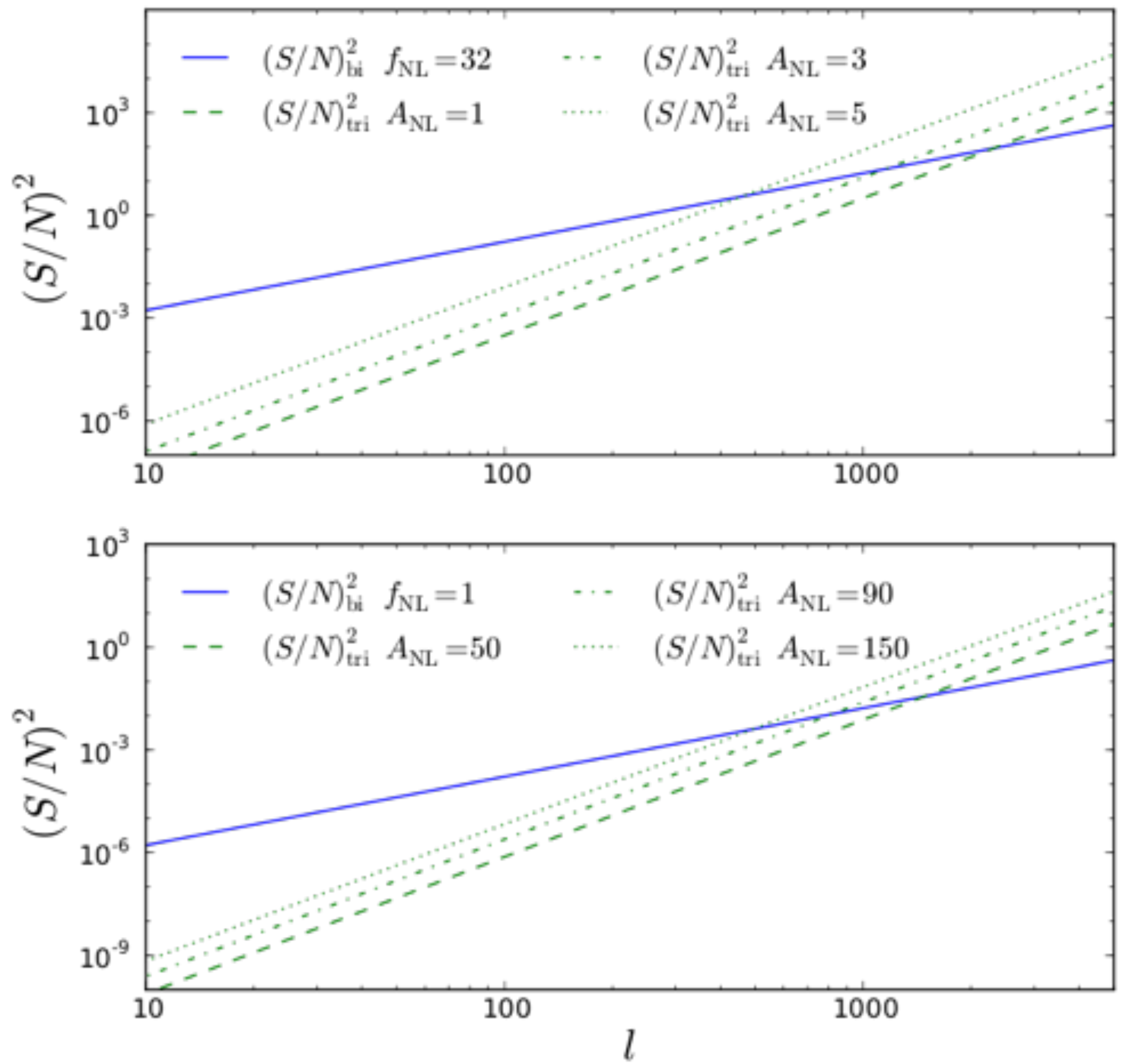


[Planck]



[EPIC]

Finally, for Planck trispectrum may be more important then bispectrum.



Measuring $C_l^{\Phi\Phi}$ directly from the trispectrum of the CMB.

[Smidt et al. 2010. arxiv:1012:1600]
(Accepted by ApJ Letters!)

A CONSTRAINT ON THE INTEGRATED MASS POWER SPECTRUM OUT TO $z = 1100$ FROM LENSING OF THE COSMIC MICROWAVE BACKGROUND

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(Dated: December 7, 2010)
Draft version December 7, 2010

ABSTRACT

The temperature fluctuations and polarization of the Cosmic Microwave Background (CMB) are now a well-known probe of the Universe at an infant age of 400,000 years. During the transit to us from the surface of last scattering, the CMB photons are expected to undergo modifications induced by the intervening large-scale structure. Among the expected secondary effects is the weak gravitational lensing of the CMB by the foreground dark matter distribution. We derive a quadratic estimator that uses the non-Gaussianities generated by the lensing effect at the four-point function level to extract the power spectrum of lensing potential fluctuations integrated out to $z \sim 1100$ with peak contributions from potential fluctuations at z of 2 to 3. Using WMAP 7-year temperature maps, we report the first direct constraints of this lensing potential power spectrum and find that it has an amplitude of $A_L = 0.96 \pm 0.60$, 1.06 ± 0.69 and 0.97 ± 0.47 using the W, V and W+V bands, respectively.

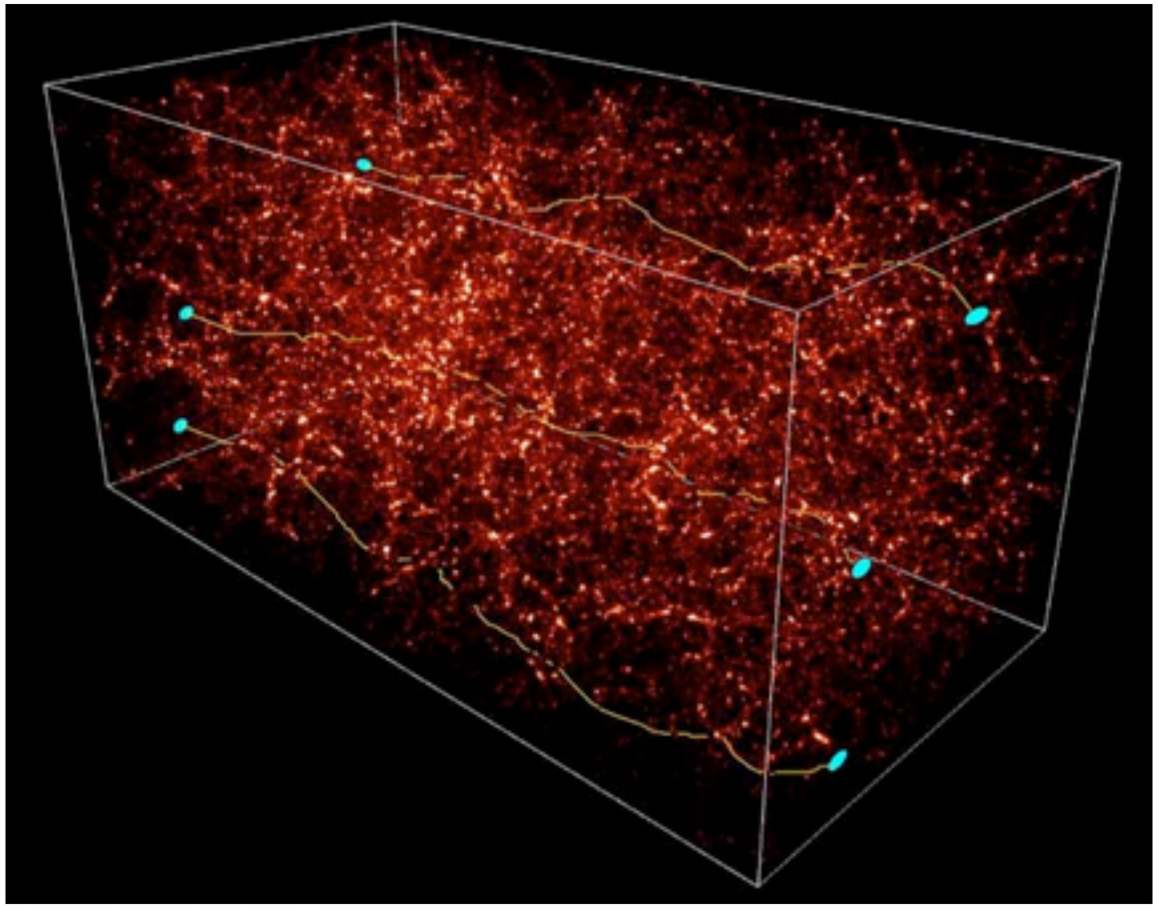
Subject headings: cosmology: cosmic microwave background — cosmology: observations — cosmology: theory — gravitational lensing

Weak Lensing

- Photons leave surface of last scattering.
- Deflected by large scale structure.

$$\text{Gravitational Potential } \phi$$

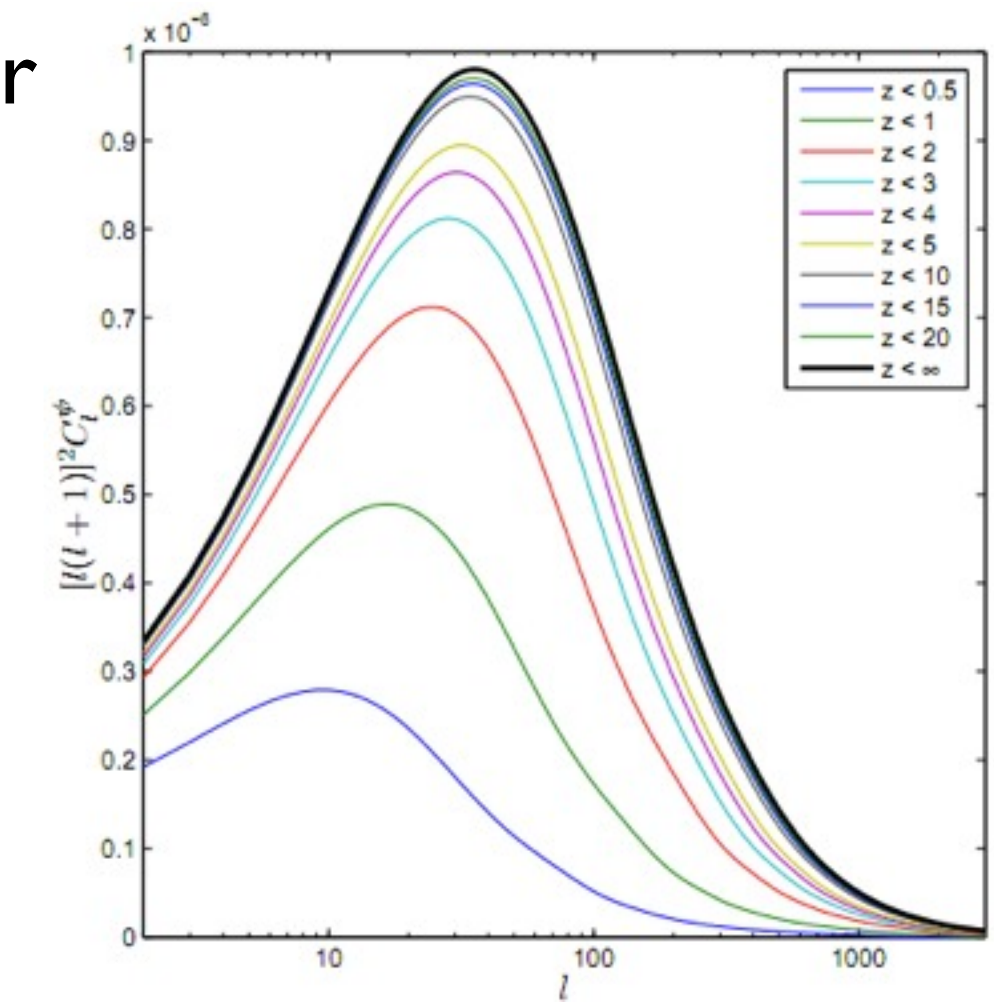
$$\text{Deflection Angle } \alpha = \nabla\phi$$



(Credit: S. Colombi (IAP), CFHT Team)

Previous Detections Of Lensing.

- Using Galaxy Shapes (Whittman et al. 2000, COSMOS)
  Out to z = 2.
- Redshifts above 2 effect the power ~30%. CMB z = 1100.
- Evidence from cross-correlations:
  Hirata et al. 2008 (2.5 σ),
  Smith et al. 2007 (3.4 σ)



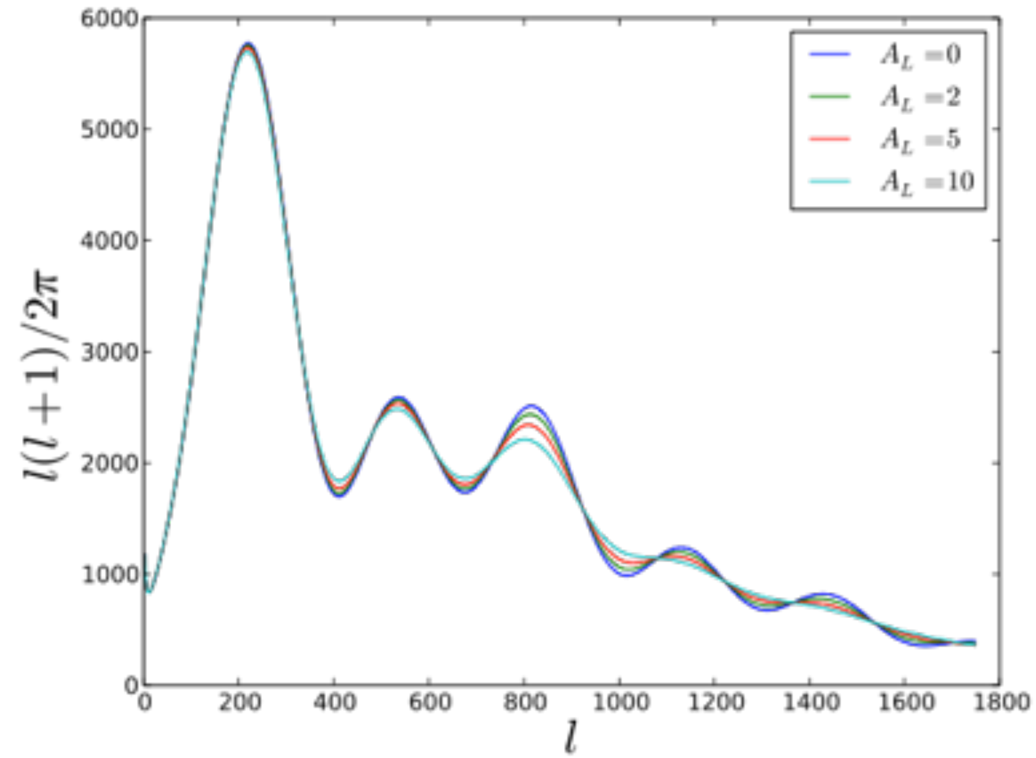
Lewis & Challinor 2006

Extracting lensing

$$\Theta(\mathbf{n}) = \left(\frac{\Delta T}{T}\right)(\mathbf{n}) = \sum_{lm} \Theta_{lm} Y^*_{lm}(\mathbf{n})$$

$$\Theta(\mathbf{n}) = \Theta(\mathbf{n} + \nabla\phi)$$

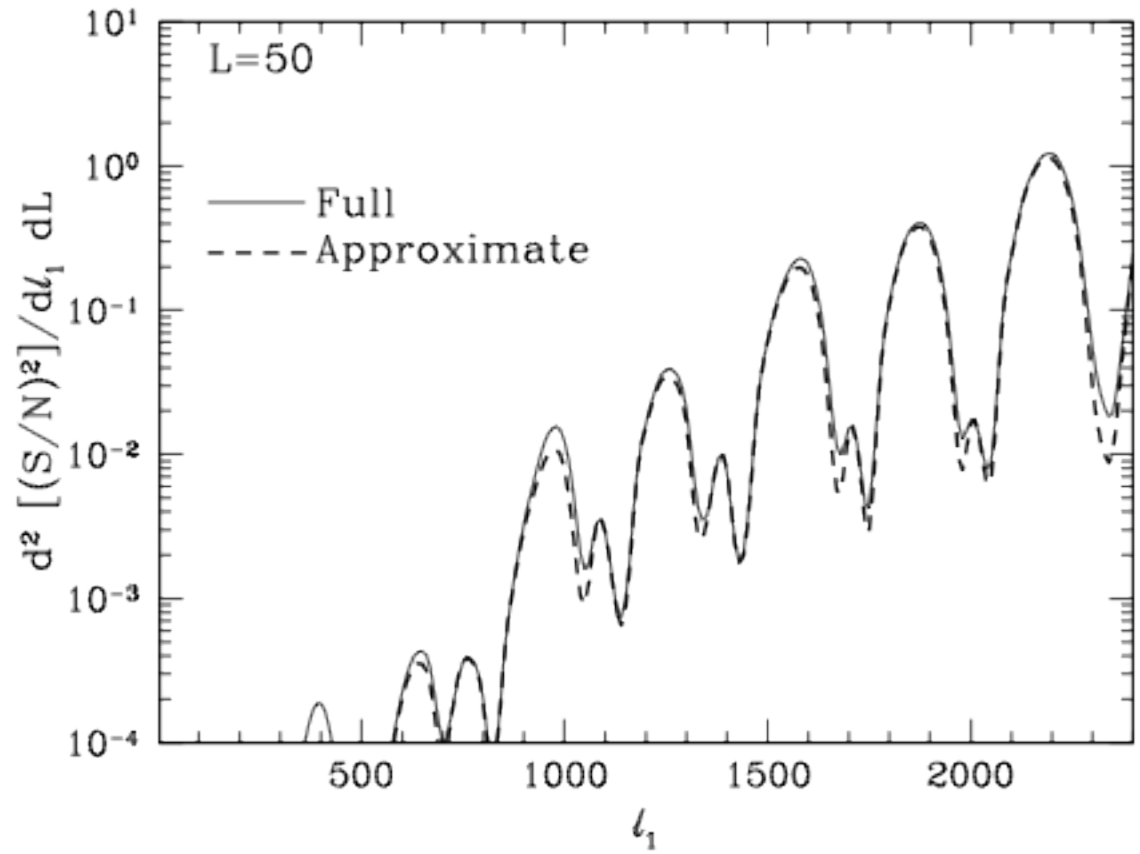
$$\sim \tilde{\Theta}(\mathbf{n}) + \nabla_i \phi(\mathbf{n}) \nabla^i \tilde{\Theta}(\mathbf{n})$$



$$\delta\Theta_{lm} = \Theta_{l'm'} + \sum_{LM} \sum_{l'm'} \phi_{LM} \tilde{\Theta}_{l'm'} (-1)^m \begin{pmatrix} l & l' & L \\ m & m' & -M \end{pmatrix} F_{ll'L}$$

$$F_{ll'L} = \sqrt{\frac{(2l+1)(2l'+1)(2L+1)}{4\pi}} \begin{pmatrix} l & l' & L \\ 0 & 0 & 0 \end{pmatrix} \frac{1}{2}[L(L+1) + l'(l'+1) - l(l+1)]$$

The Trispectrum



(Hu (2001) Phys.Rev.D64:083005)

$$\langle \Theta_{l_1 m_1} \Theta_{l_2 m_2} \Theta_{l_3 m_3} \Theta_{l_4 m_4} \rangle_c \sim$$

$$\sum_{LM} (-1)^M T^{l_1 l_2}_{l_3 l_4}(L) \begin{pmatrix} l_1 & l_2 & l_3 \\ m_1 & m_2 & -M \end{pmatrix} \begin{pmatrix} l_3 & l_4 & L \\ m_3 & m_4 & M \end{pmatrix}$$

$$T^{l_1 l_2}_{l_3 l_4}(L) = \underline{C^{\phi\phi}_L} \left(\tilde{C}_{l_2} F_{l_1 l_2 L} + \tilde{C}_{l_1} F_{l_2 l_1 L} \right) \left(\tilde{C}_{l_4} F_{l_3 l_4 L} + \tilde{C}_{l_3} F_{l_4 l_3 L} \right)$$

Lensing

- Reminder: For local non-Gaussianity, trispectrum was of the form: $$T^{l_1 l_2}_{l_3 l_4}(L) = \tau_{\rm NL} h_{l_1 l_2 L} h_{l_3 l_4 L} F(l_1, l_2, l_3, l_4, L)$$

  where F is a separable function for each l.
- For weak-lensing the trispectrum is:

$$T^{l_1 l_2}_{l_3 l_4}(L) = C^{\phi\phi}_L h_{l_1 l_2 L} h_{l_3 l_4 L} \left(C_{l_2} C_{l_4} I_{l_1 l_2 L} I_{l_1 l_2 L} + (3 \text{ perm.})\right)$$

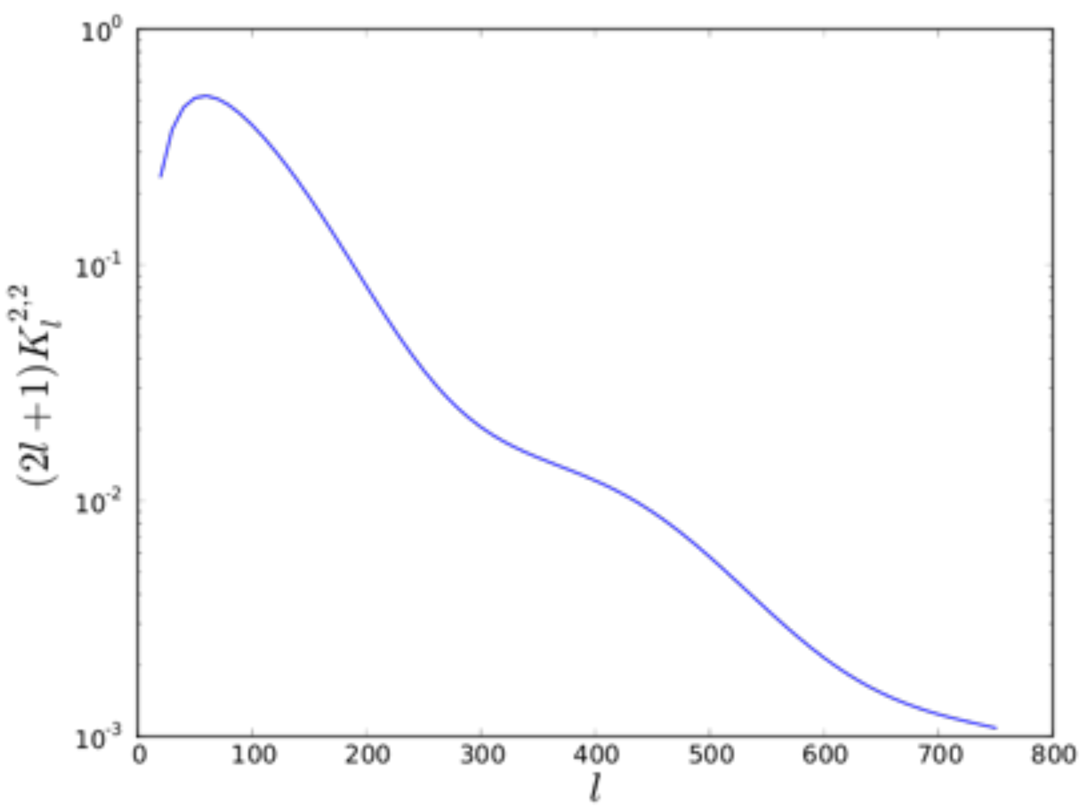
$$I_{l_1 l_2 l_3} = [l_3(l_3+1) + l_2(l_2+1) - l_1(l_1+1)]$$

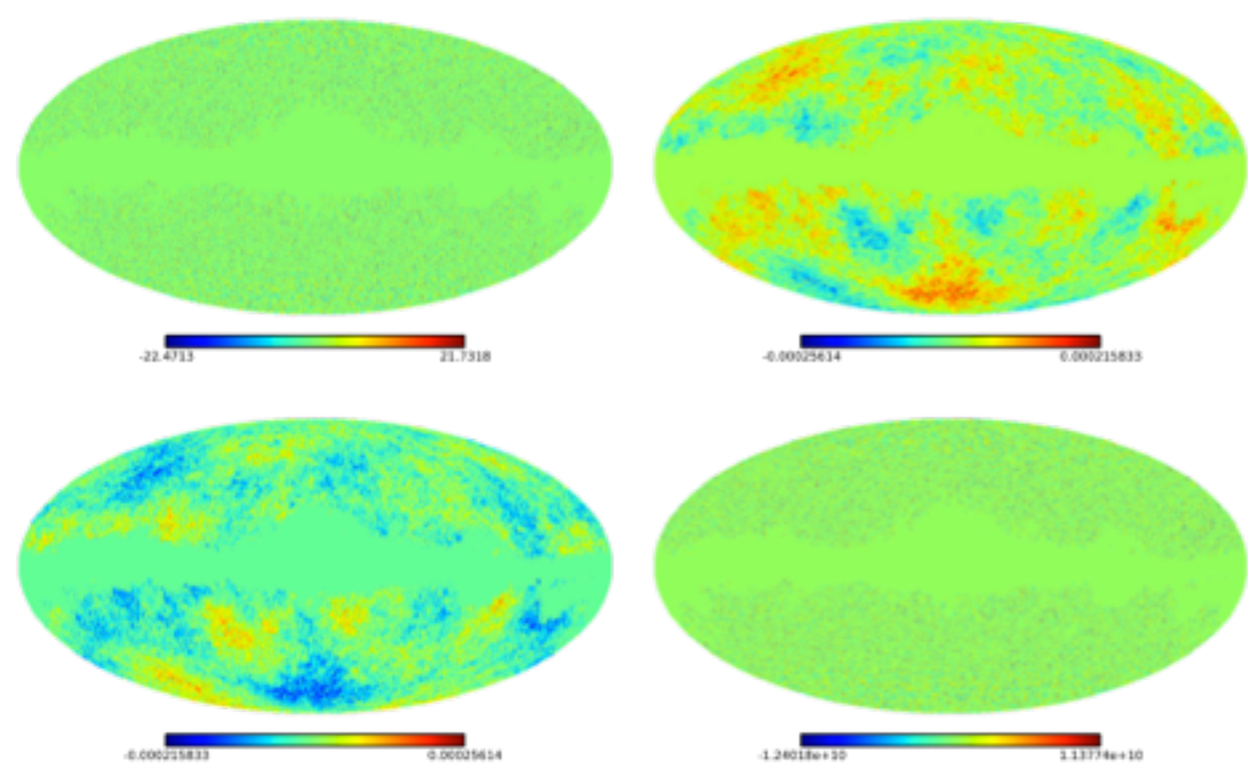
- This means we can measure $C_l^{\Phi\Phi}$ similar to how we measured $\tau_{nl.}$ (changing F appropriately)
- This decomposes into 36 pieces giving 36 weightings for CMB maps.

So We Play The Same Game

- We weight maps.
- We compare to theoretical 2-2 estimator

$$\mathcal{K}^{(2,2)}_{l(\mathrm{Lens})} = \frac{C_l^{\phi\phi}}{(2l+1)} \sum_{l_i} \frac{1}{(2l+1)} \frac{T^{l_3 l_4}_{l_1 l_2}(l) \hat{T}^{l_1 l_2}_{l_3 l_4}(l)}{\mathcal{C}_{l_1}\mathcal{C}_{l_2}\mathcal{C}_{l_3}\mathcal{C}_{l_4}};$$

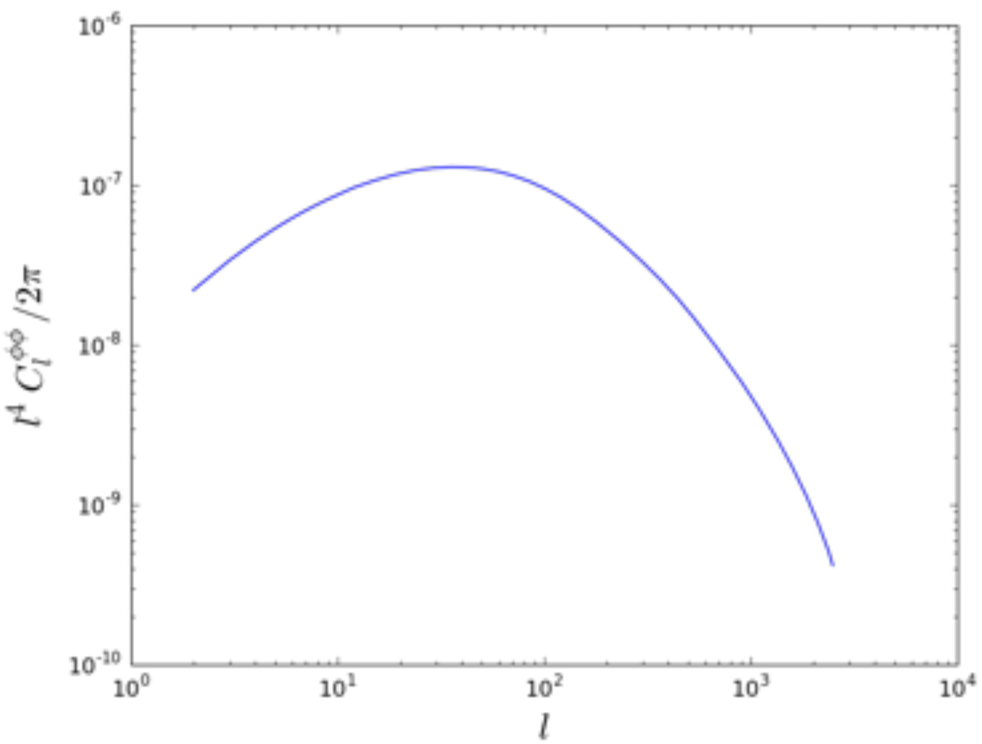


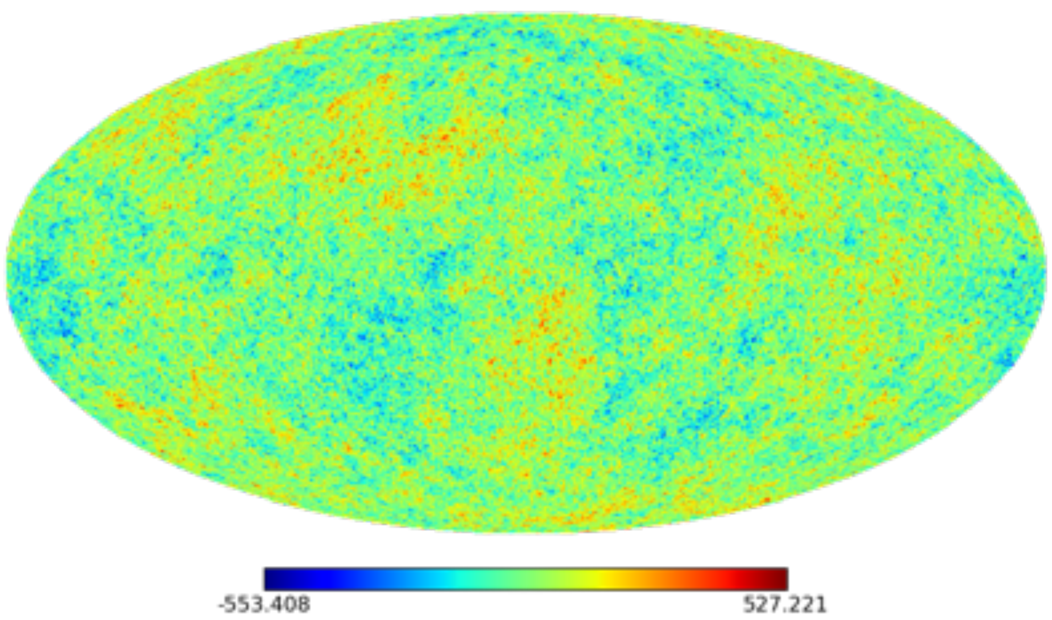


Example Weighted Maps

Test With Simulations

- Make 400 Gaussian Maps
- Used Lenspix to seed 400 Gaussian maps with arbitrary $C_l^{\Phi\Phi}$.
- Use Gaussian Maps to subtract off Gaussian piece.



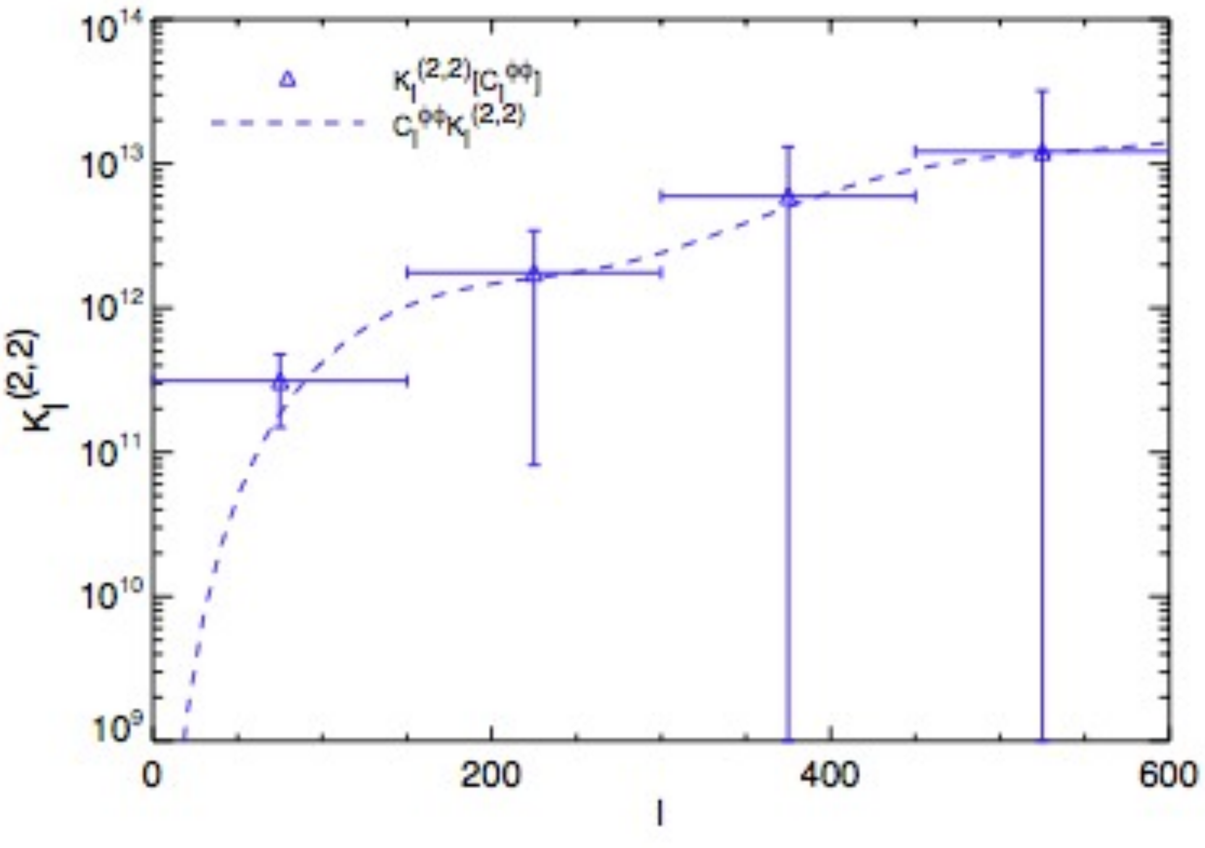


Results From Simulations

- Remember: $\mathcal{K}^{(2,2)}_{l(\mathrm{Lens})}|_{\mathrm{Data}} = C_l^{\phi\phi}\mathcal{K}^{(2,2)}_{l(\mathrm{Lens})}|_{\mathrm{Theory}}$

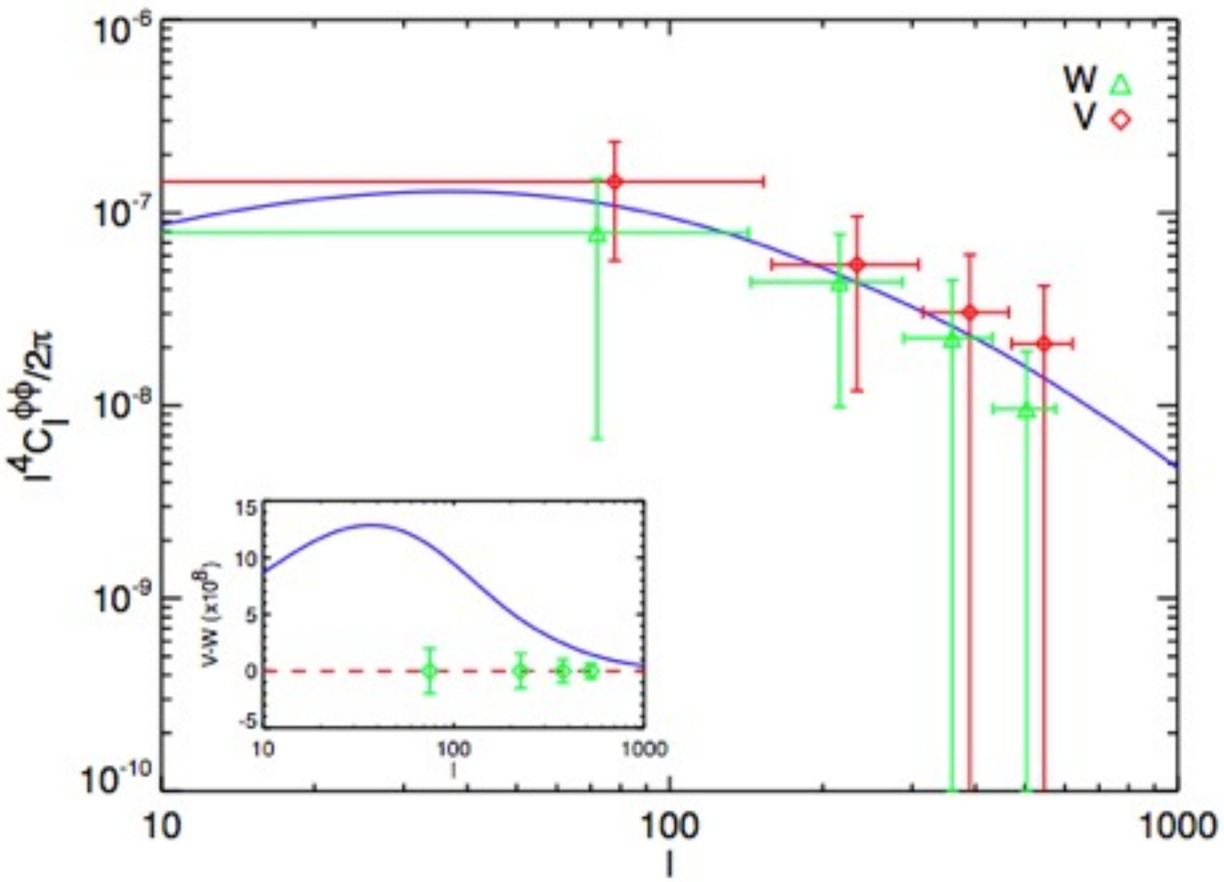
- We can constrain $C_l^{\Phi\Phi}$ directly:

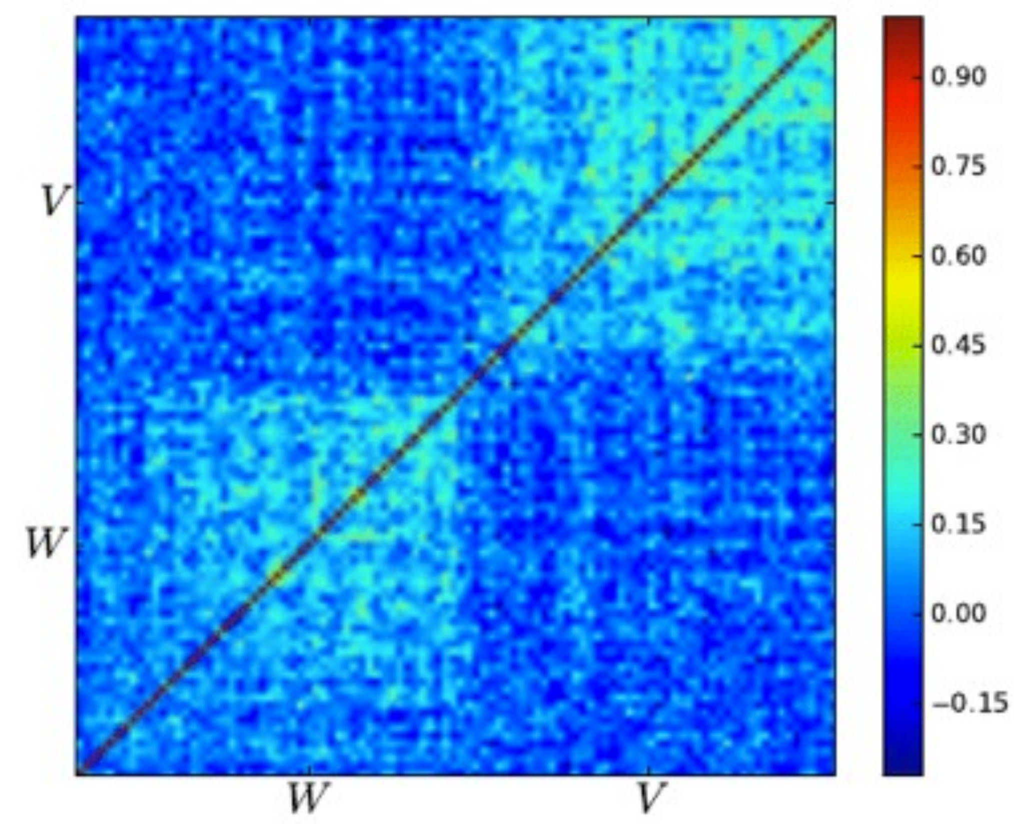
$$C_l^{\phi\phi} = \frac{\mathcal{K}^{(2,2)}_{l(\mathrm{Lens})}|_{\mathrm{Data}}}{\mathcal{K}^{(2,2)}_{l(\mathrm{Lens})}|_{\mathrm{Theory}}}$$



Results From WMAP Data

- Results from V and W band WMAP 7 data.
- Also a null test V - W.
- Use χ^2 and the covariance matrix as before.





A 2σ Excess For Lensing.

- To measure the lensing amplitude we constrain $A_l\ C_l^{\Phi\Phi}$
- $A_l = 0$ is for unlensed sky. $A_l = 1$ is fiducial.
- Use CosmoMC. (2 sigma measurement of lensing amplitude.)

Params.	WMAP7	WMAP7+A_L	WMAP7+A_L+$C_l^{\phi\phi}$
$10^3\Omega_b h^2$	22.51 ± 0.62	22.59 ± 0.63	22.60 ± 0.58
$10^2\Omega_{DM} h^2$	11.08 ± 0.57	11.04 ± 0.54	11.09 ± 0.54
τ	0.089 ± 0.016	0.090 ± 0.015	0.089 ± 0.015
n_s	0.967 ± 0.015	0.968 ± 0.014	0.968 ± 0.014
Ω_Λ	0.734 ± 0.031	0.737 ± 0.028	0.735 ± 0.027
Age/Gyr	13.8 ± 0.14	13.7 ± 0.14	13.7 ± 0.13
H_0^1	71.0 ± 2.7	71.3 ± 2.5	71.1 ± 2.4
$\mathbf{A_L}$	**1.0**	$\mathbf{0.87 \pm 1.05}$	$\mathbf{0.97 \pm 0.47}$

In Conclusion

- These new estimators are powerful.
- Can constrain many non-Gaussianity parameters directly with scale dependance.
- May also be used to constrain things in addition to NG such as lensing.
- Thanks for listening.