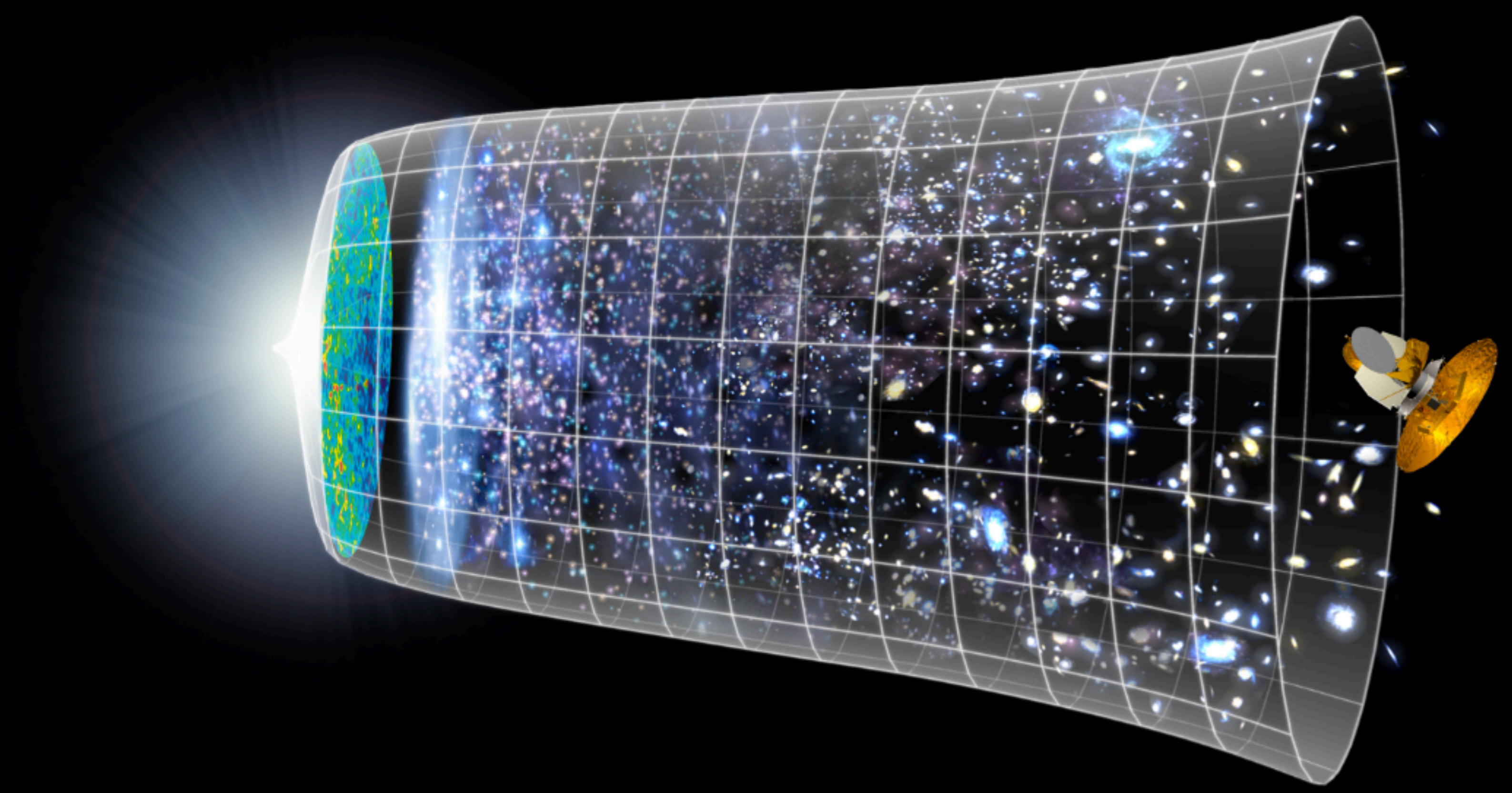
Primordial Non-Gaussianity in the CMB



Amit Yadav
Institute for Advanced Study, Princeton

PFNG-2010, **Harish-Chandra research Institute, Allahabad**

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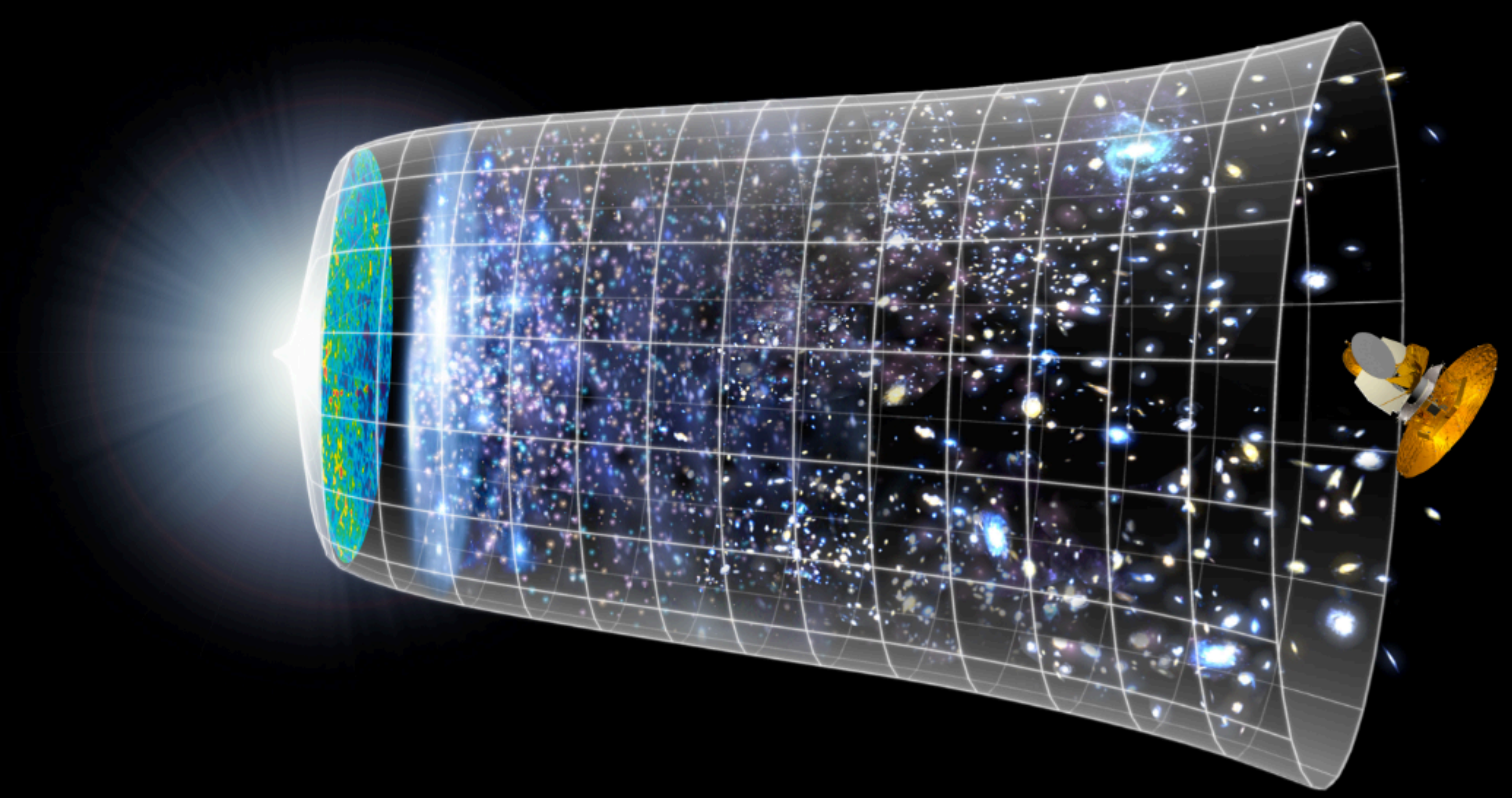

It is well established that the observed structure originated from seed perturbations imprinted in the early universe

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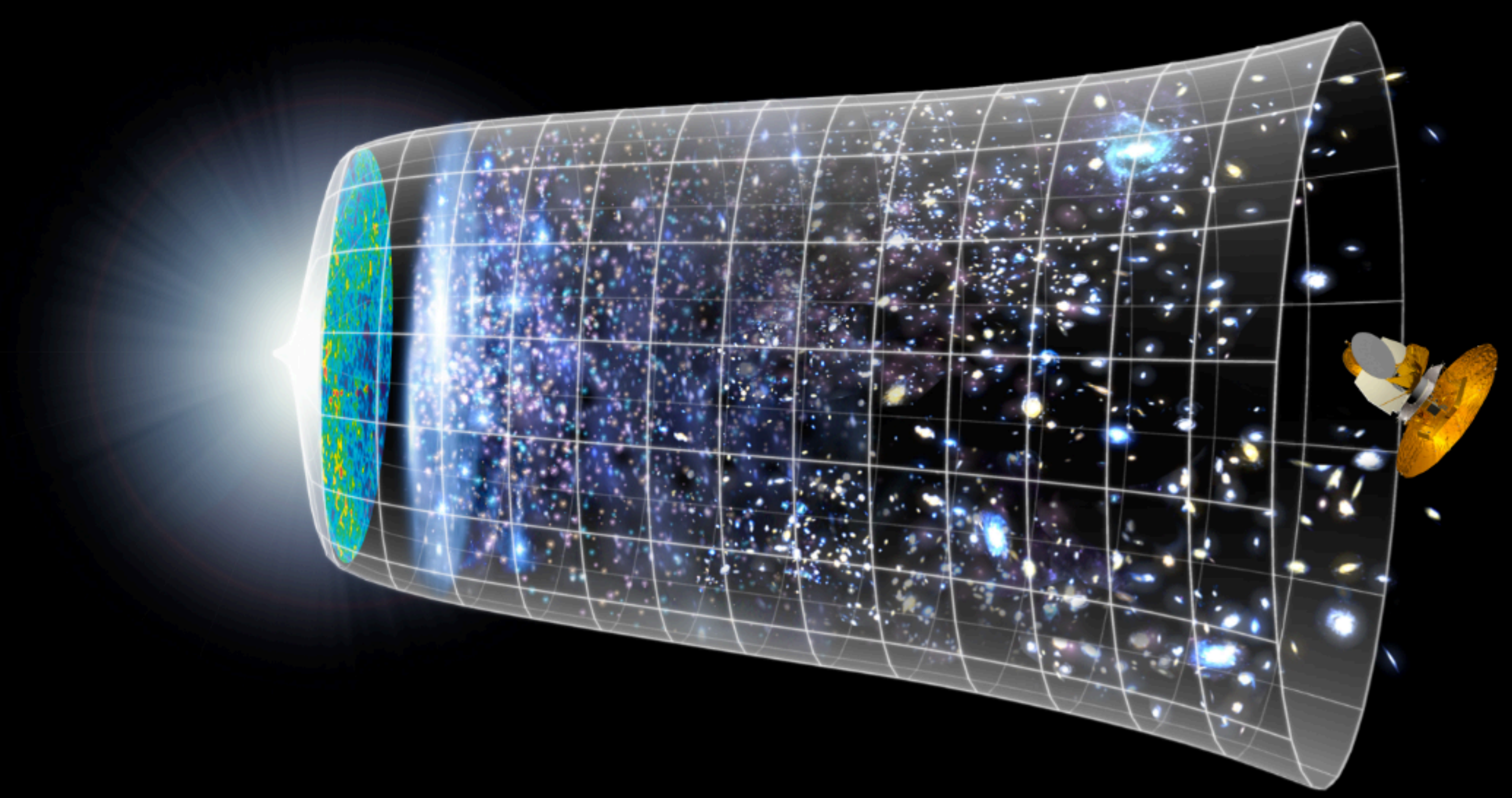

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The physics responsible for generating the seed perturbations is largely unknown.

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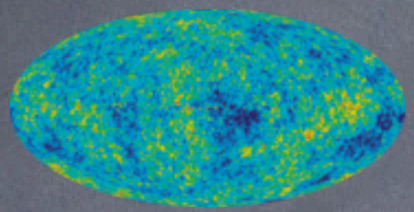
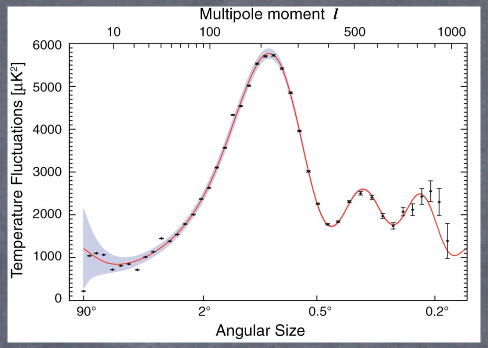
Inflation (Exponentially Expanding Phase)



1) Scalar Fluctuations

- Adiabatic — 5 %
- Nearly Gaussian — 0.2 %
- Nearly Scale Invariant — 5%



2) Tensor Fluctuations (gravitational waves)

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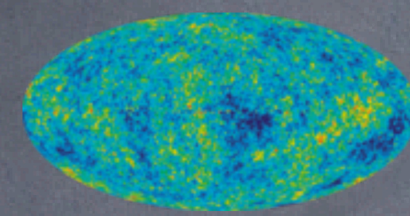

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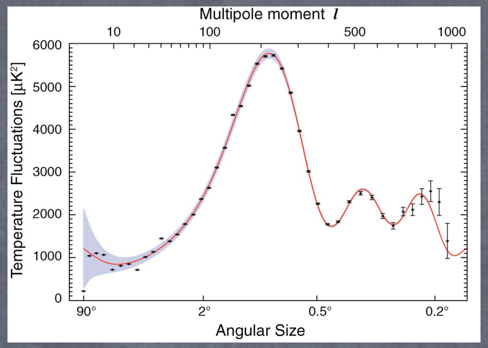

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Problem

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(2) Alternatives to inflation, cyclic model is also consistent with data

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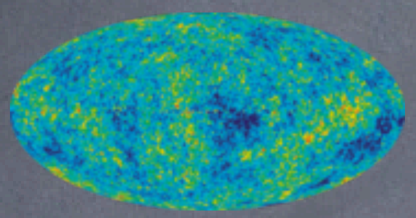

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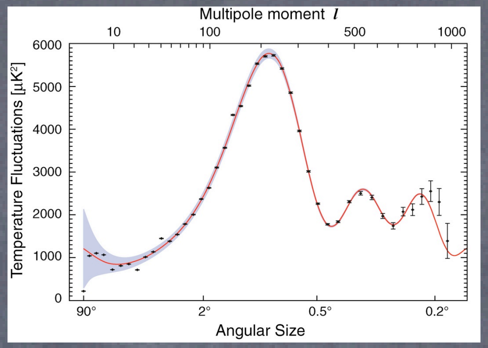

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- kinetic and potential terms
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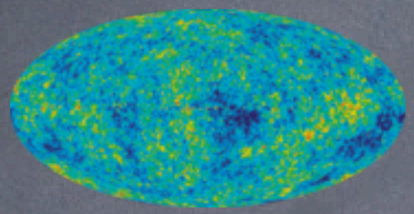

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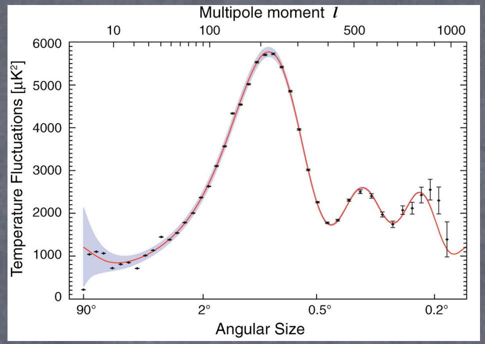

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- Non-Gaussianity in the CMB
- Gravitational waves (B-modes of CMB)
- Isocurvature modes

Non-Gaussianity= deviation from Gaussianity

For a Gaussian random field $\Phi(x)$

the statistical properties are completely characterized by its two point correlation function (or its power spectrum): $\langle\Phi(\mathbf{k_1})\Phi(\mathbf{k_2})\rangle = \delta(k_1+k_2)P_\Phi(k_1)$

All higher order connected correlation functions are vanishing

3-point/bispectrum $\langle\Phi(\mathbf{k_1})\Phi(\mathbf{k_2})\Phi(\mathbf{k_3})\rangle = 0$

4-point/trispectrum $\langle\Phi(\mathbf{k_1})\Phi(\mathbf{k_2})\Phi(\mathbf{k_3})\Phi(\mathbf{k_4})\rangle_c = 0$

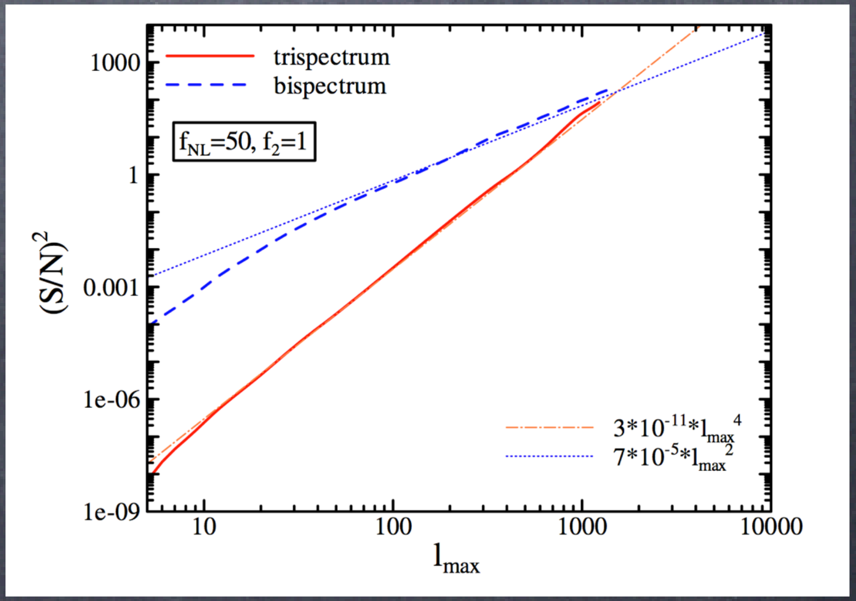
Minkowski Functionals

Wavelets

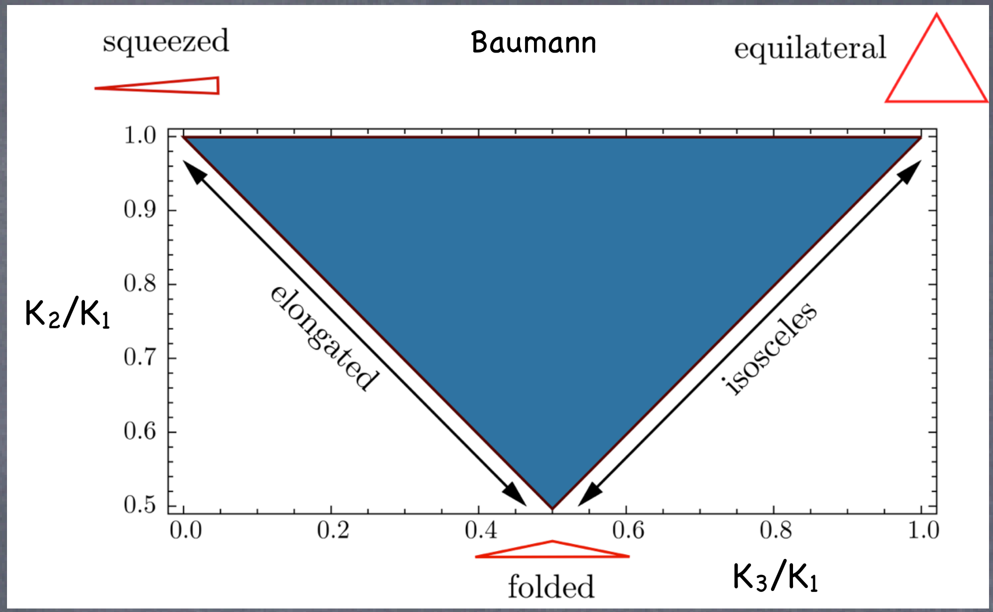
Bispectrum

- Bispectrum contains nearly all the information for local-Gaussianity
  eg. Okamoto & Hu (2002), Babich (2005), Kogo & Komatsu (2006), Creminelli et at. (2007)
- For two scalar fields, it is possible to have large trispectrum

e.g. Byrnes et al. (2006), Engel et al. (2009)



What does inflation predict?



$$B_\Phi(\mathbf{k_1},\mathbf{k_2},\mathbf{k_3}) = \langle \Phi(\mathbf{k_1})\Phi(\mathbf{k_2})\Phi(\mathbf{k_3})\rangle$$

$$= f_{NL}\delta(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3})F(k_1,k_2,k_3)$$

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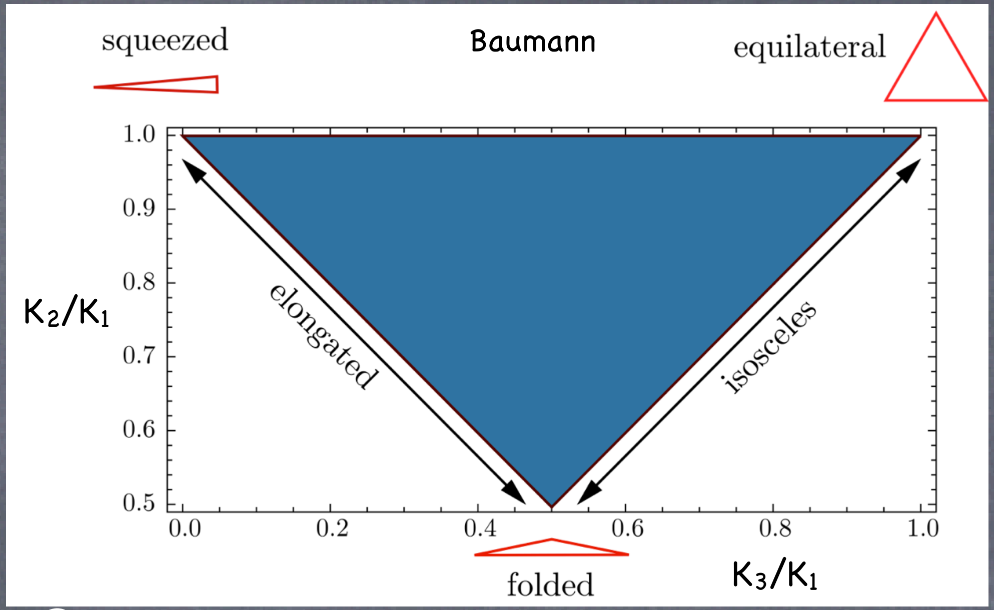

Maldacena (2003)

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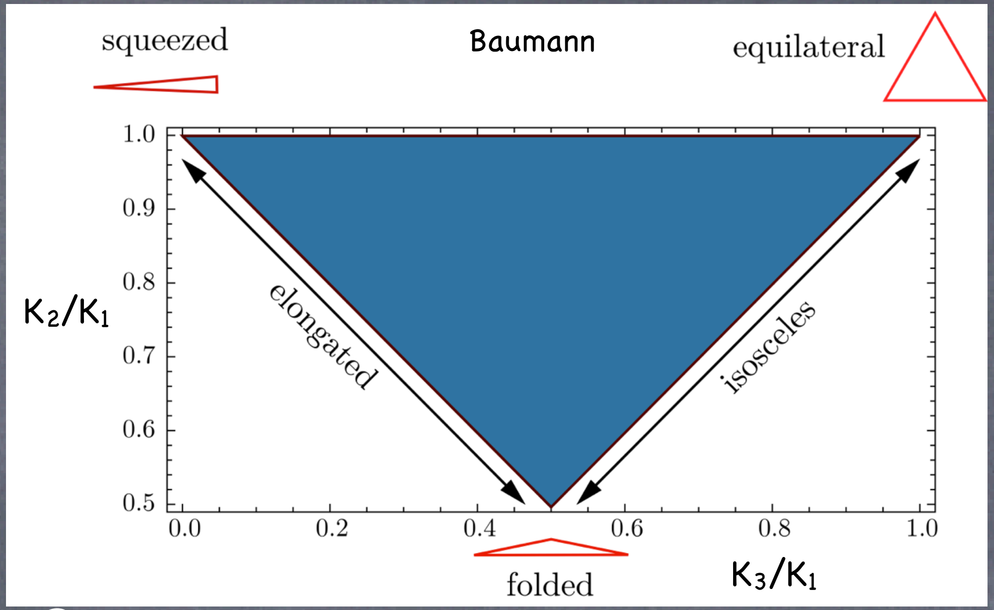

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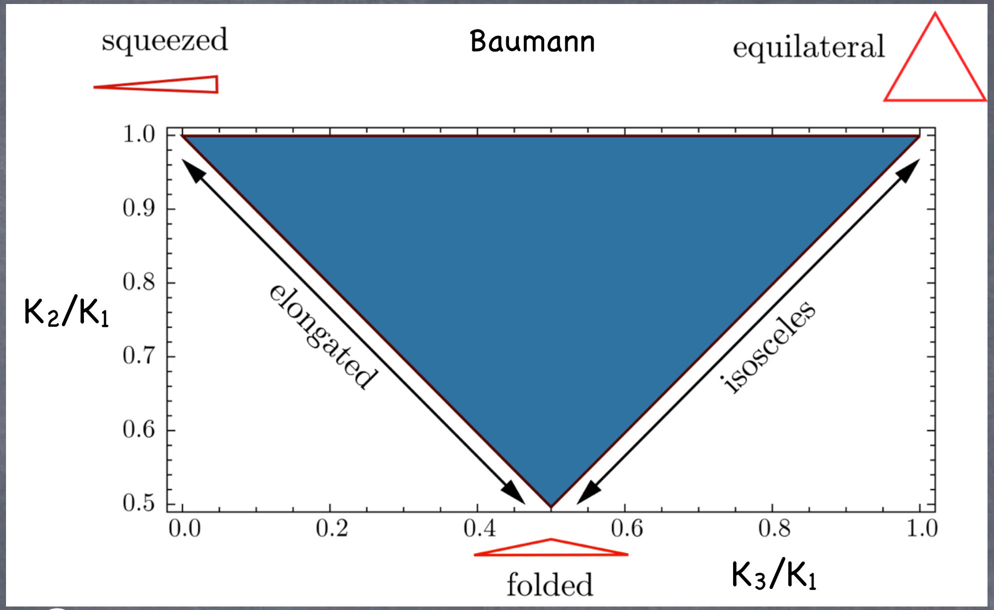

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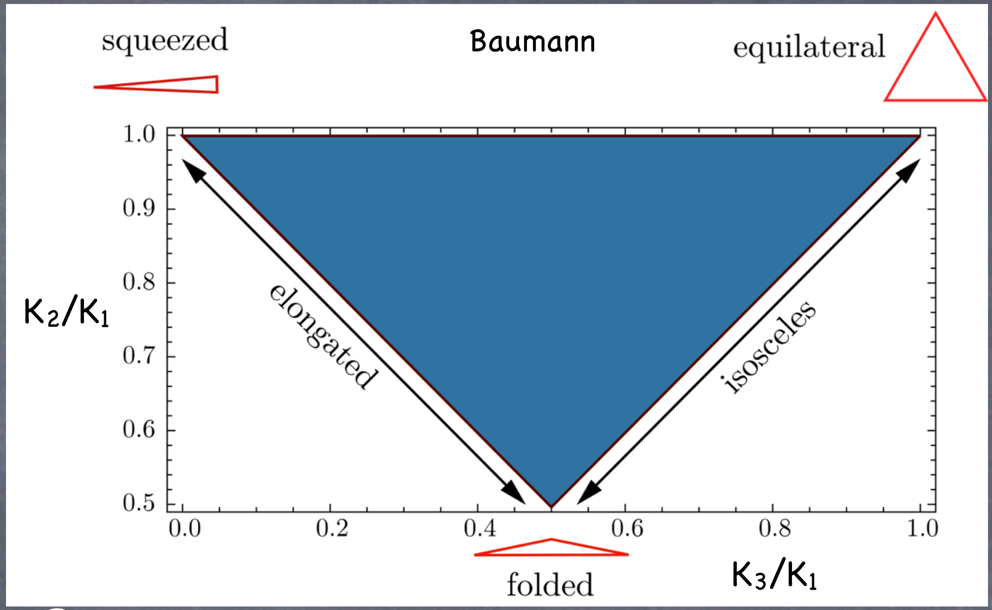

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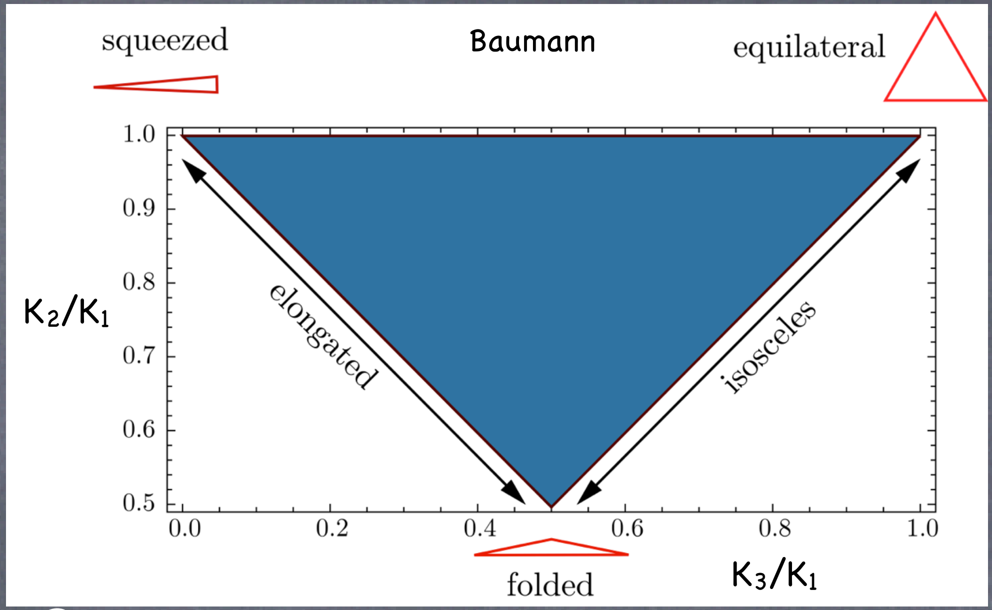

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f_{NL} ~ 0.05 canonical inflation (single field, Maldacena 2003, Acquaviva et al. 2003)

f_{NL} ~ 100 DBI inflation (Alishahiha et al. 2004) — Equilateral

f_{NL} >10 curvaton models (Lyth et al. 2003) — Local

f_{NL} ~ 100 ghost inflation (Arkani-Hamed et al. 2004) — Equilateral

f_{NL} ~ 20-100 Cyclic models (Creminelli et al. 07, Buchbinder et. al 07, Koyama et. al 07)

How is Cosmic Microwave Background Linked to the Primordial Non-Gaussianity?

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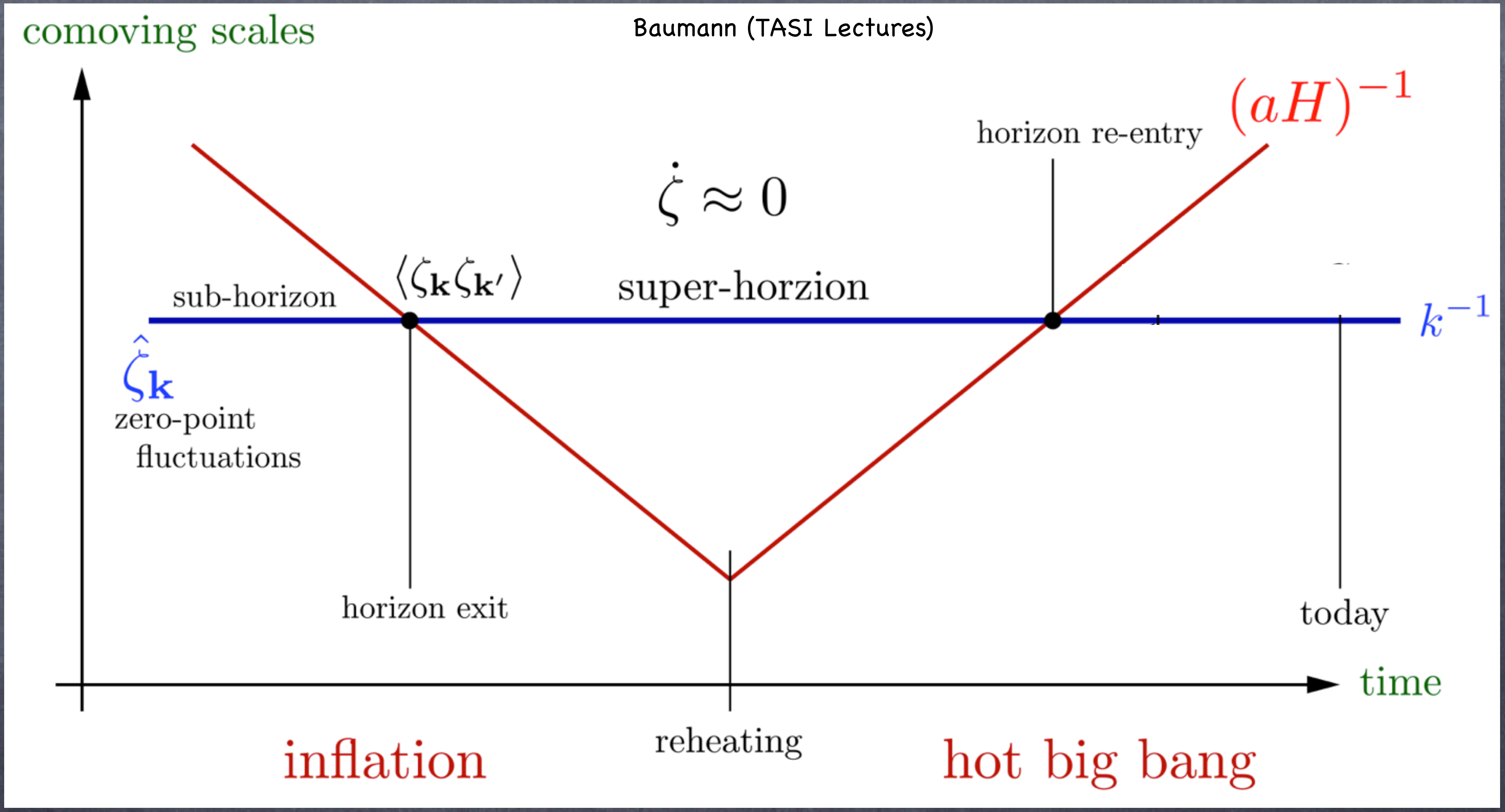

ζ

- Gauge invariant
- Conserve outside the horizon

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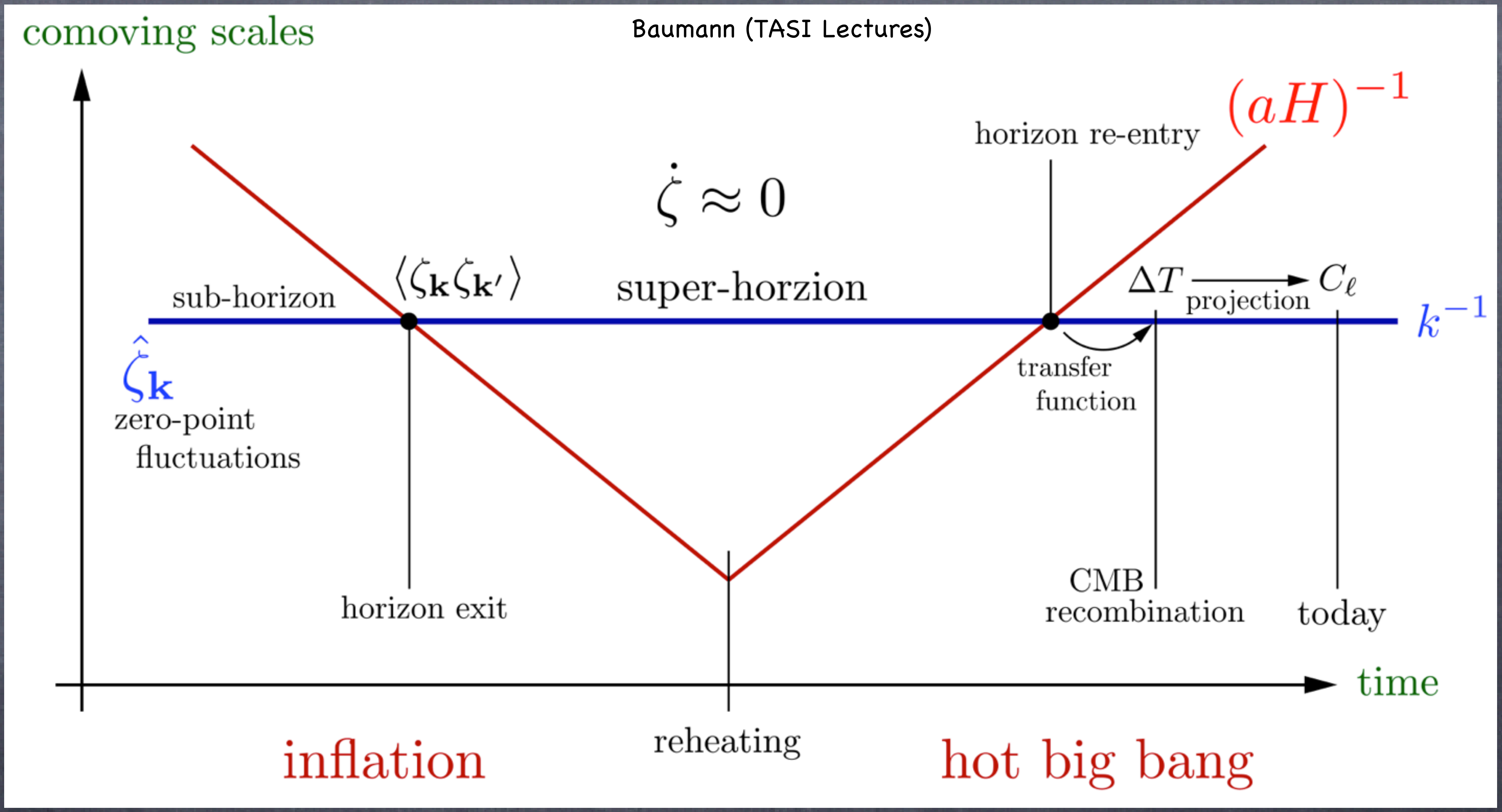

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CMB Bispectrum

Math people this slide
Picture people next slide

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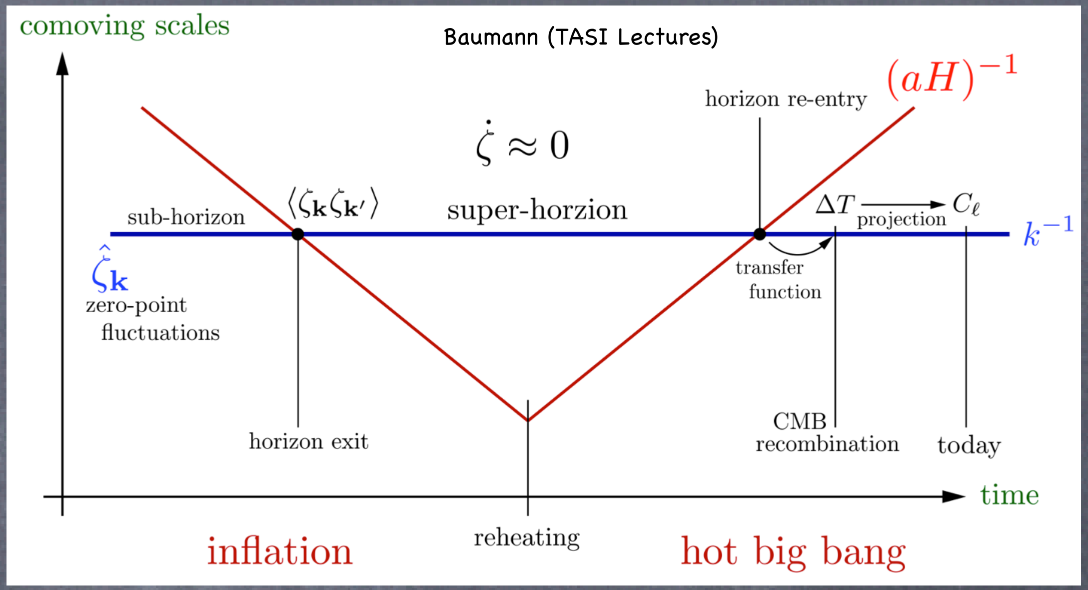

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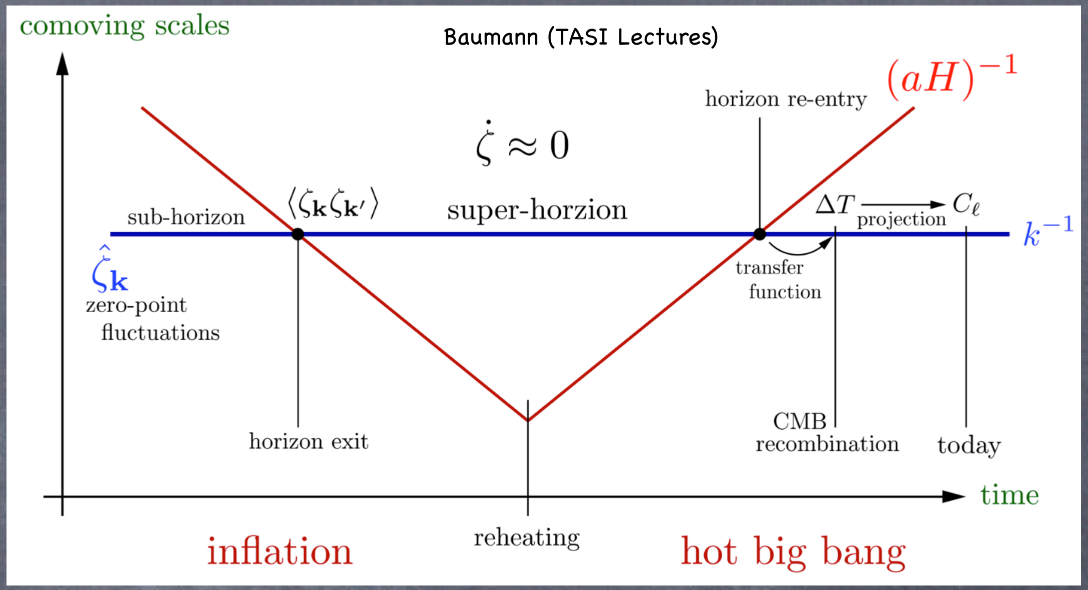
Transfer function
CMBFast, CAMB



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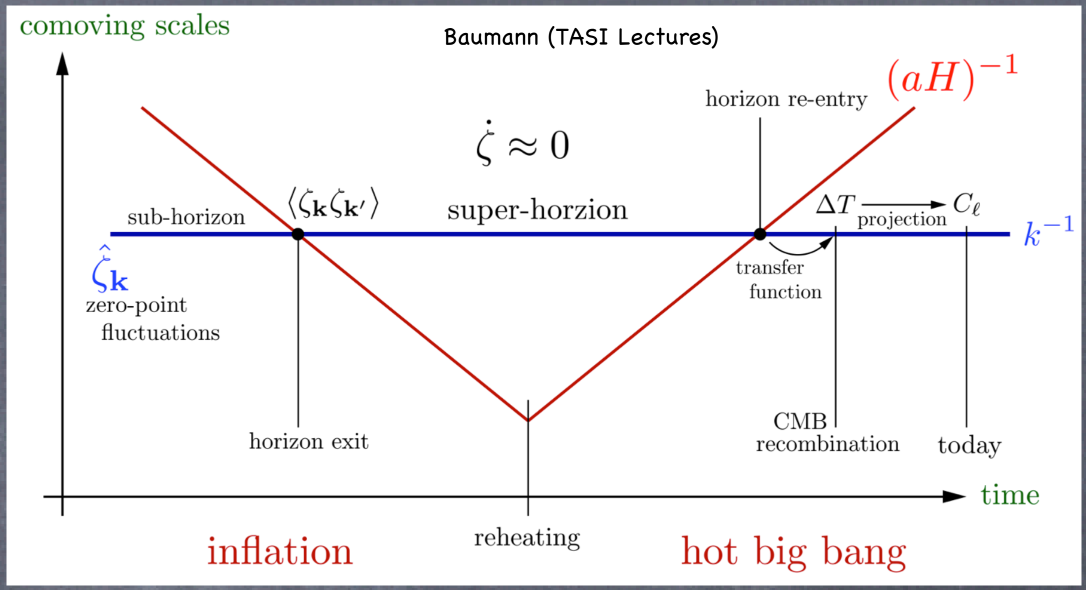

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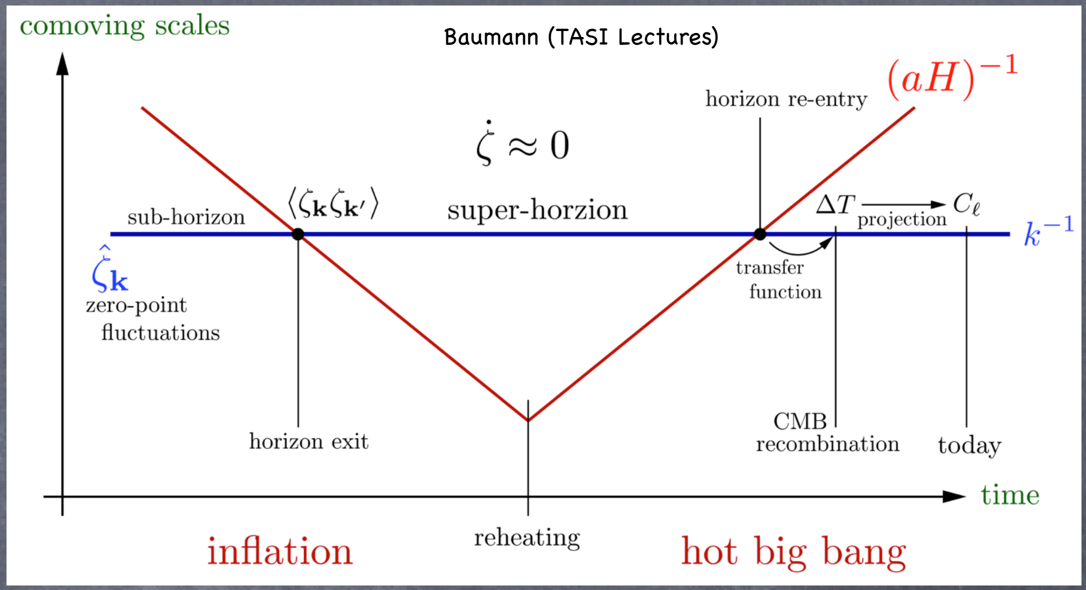

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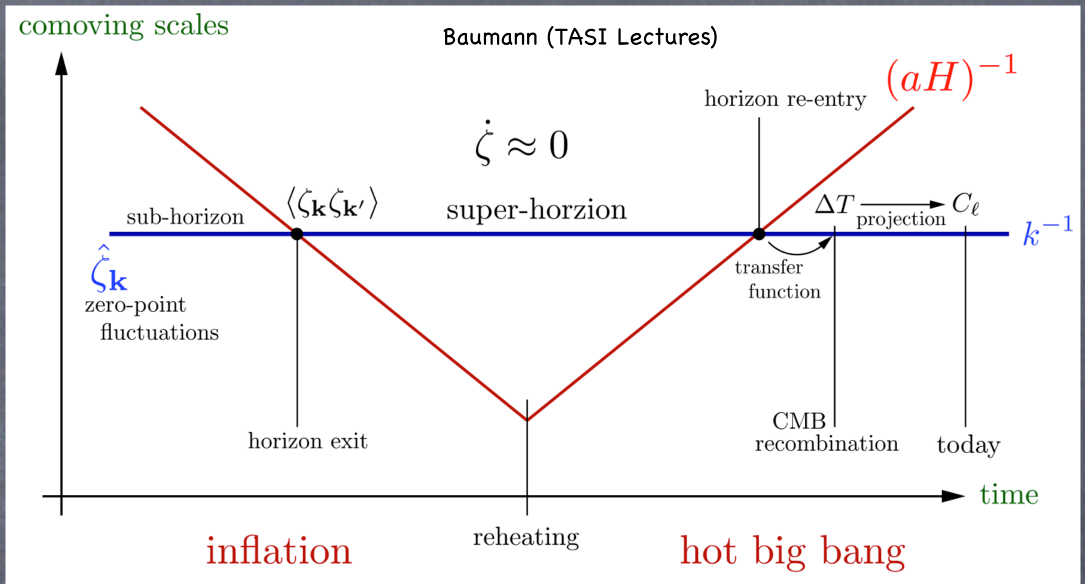

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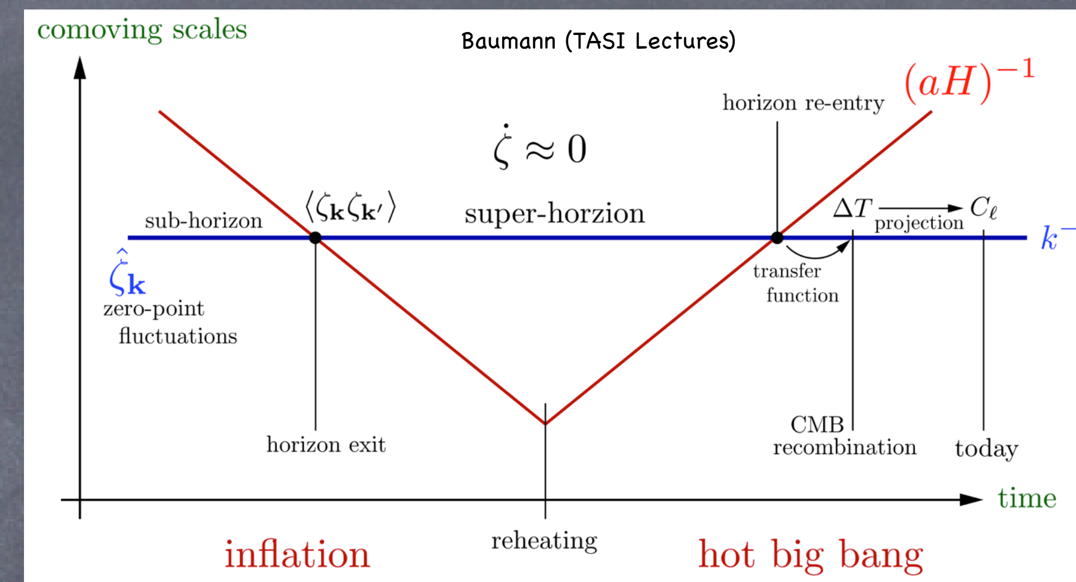

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$$N_{pix}^{5/2}$$

Primordial
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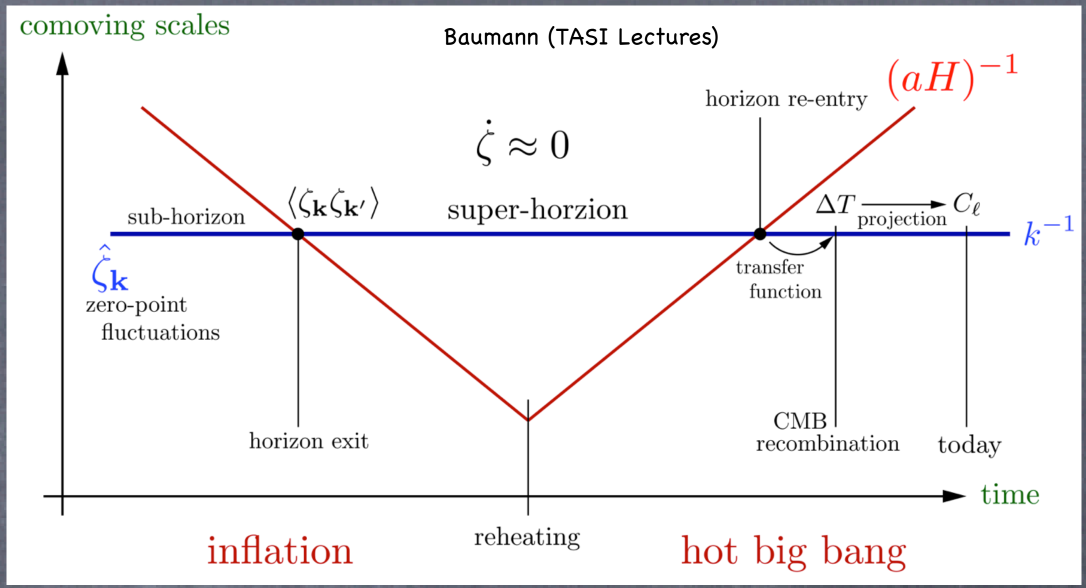

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$$N_{pix}^{5/2}$$

Primordial
Bispectrum

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CMB Bispectrum

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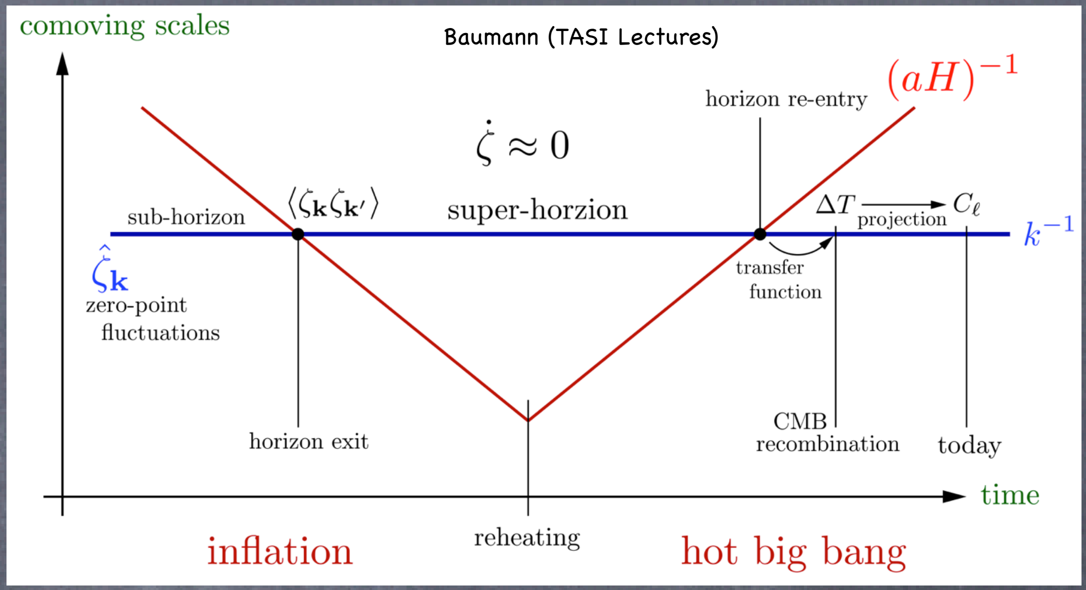

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Primordial Bispectrum

$$F(k_1,k_2,k_3) = f_1(k_1) f_2(k_2) f_3(k_3)$$

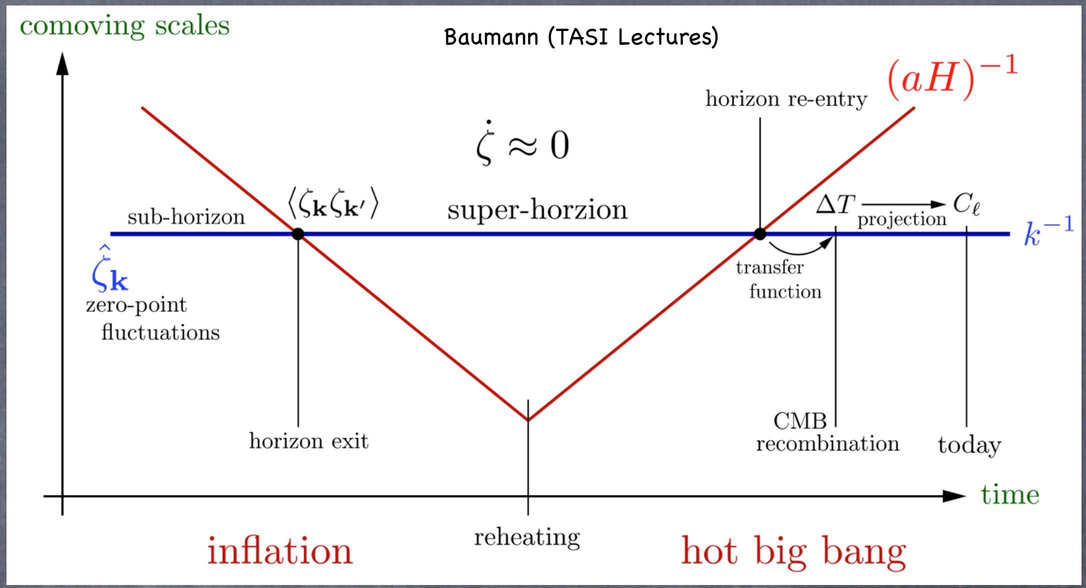
$$\hat{f}_{NL} = \frac{1}{N}\cdot\int d^2\hat{n}\int_0^\infty r^2 dr \prod_{i=1}^3 \sum_{l_i m_i}\int \frac{2k^2 dk}{\pi} j_{l_i}(kr) f_i(k)\Delta^T_{l_i}(k) C^{-1}_{l_i} a_{l_i m_i} Y_{l_i m_i}(\hat{n})$$

CMB Bispectrum

Math people this slide
Picture people next slide

$$a_{\ell m} = 4\pi(-i)^{\ell} \int \frac{d^3k}{(2\pi)^3} \Phi(\mathbf{k})\, g_{\ell}(k) Y^{\star}_{\ell m}(\hat{k})$$

Transfer function
CMBFast, CAMB



$$\hat{f}_{NL} = \frac{1}{N} \sum_{\ell_i m_i} \begin{pmatrix} \ell_1 & \ell_2 & \ell_3 \\ m_1 & m_2 & m_3 \end{pmatrix} \frac{B_{\ell_1 \ell_2 \ell_3}}{C_{\ell_1} C_{\ell_2} C_{\ell_3}}\, a_{\ell_1 m_1} a_{\ell_2 m_2} a_{\ell_3 m_3}$$

$$\hat{f}_{NL} = \frac{1}{N} \cdot \sum_{l_i m_i} \int d^2\hat{n}\, Y_{l_1 m_1}(\hat{n}) Y_{l_2 m_2}(\hat{n}) Y_{l_3 m_3}(\hat{n}) \int_0^{\infty} r^2 dr\; j_{l_1}(k_1 r) j_{l_2}(k_2 r) j_{l_3}(k_3 r)\; C^{-1}_{l_1} C^{-1}_{l_2} C^{-1}_{l_3}$$

$$\int \frac{2k_1^2 dk_1}{\pi} \frac{2k_2^2 dk_2}{\pi} \frac{2k_3^2 dk_3}{\pi} F(k_1, k_2, k_3) \Delta^T_{l_1}(k_1) \Delta^T_{l_2}(k_2) \Delta^T_{l_3}(k_3)\; a_{l_1 m_1} a_{l_2 m_2} a_{l_3 m_3}\,,$$

$N_{pix}^{5/2}$

Primordial Bispectrum

$$F(k_1, k_2, k_3) = f_1(k_1) f_2(k_2) f_3(k_3)$$

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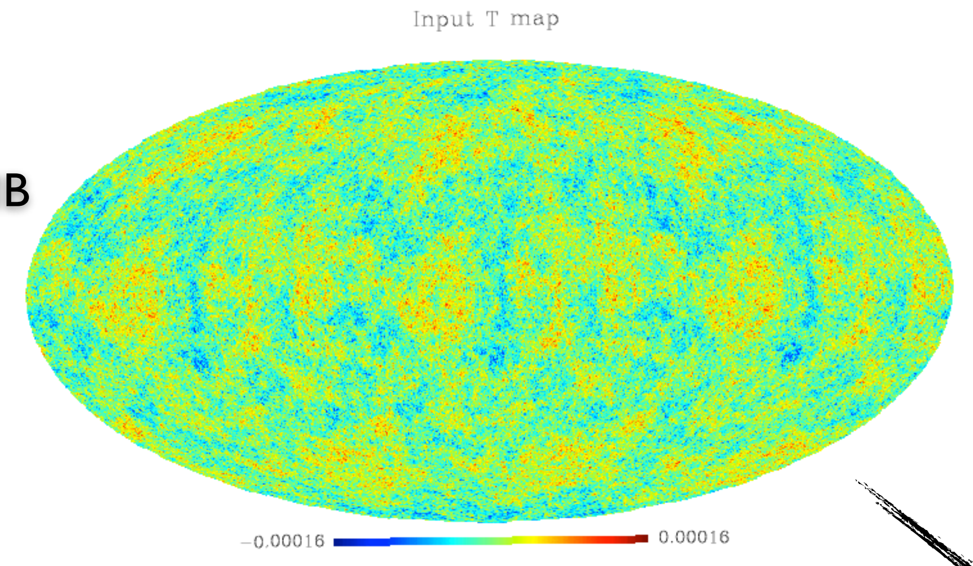
$N_{pix}^{3/2}$

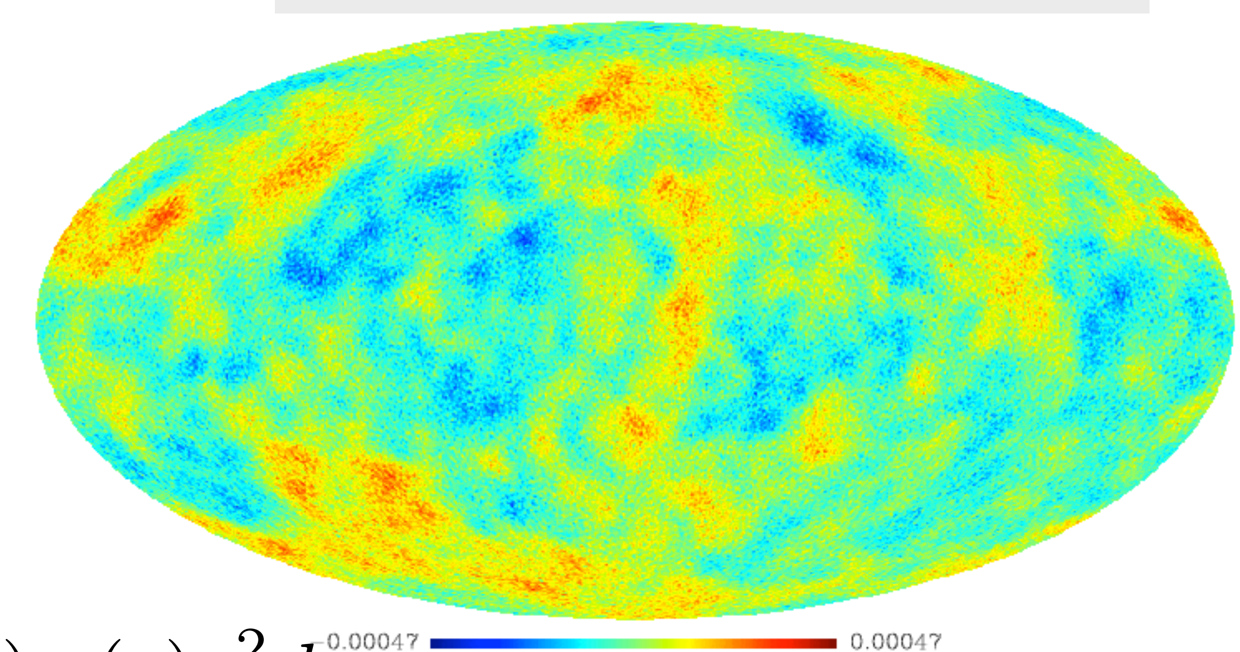
Primordial non-Gaussianity from CMB

CMB



On large scales $\Delta T = -\frac{1}{3}\Phi$

Curvature Perturbations $\Phi(r)$

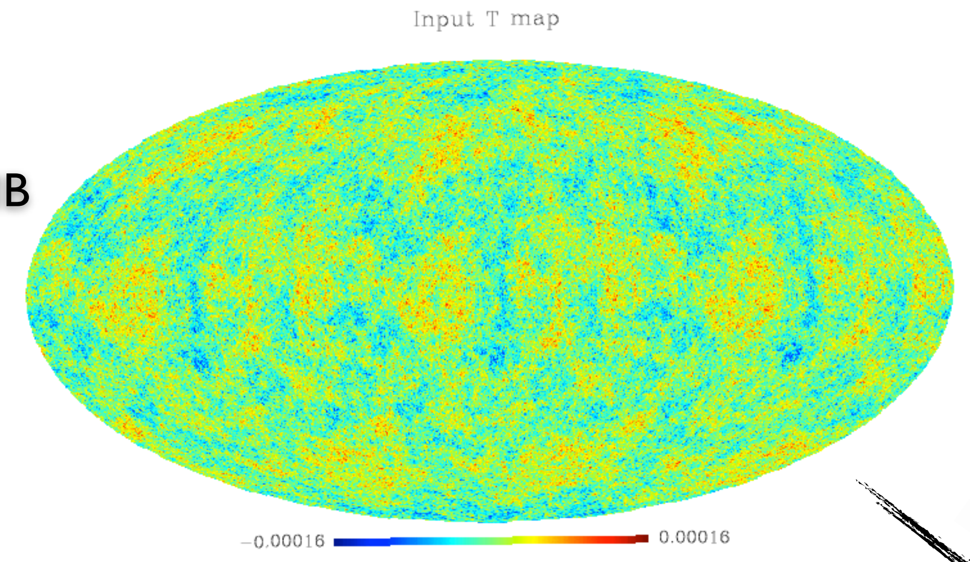


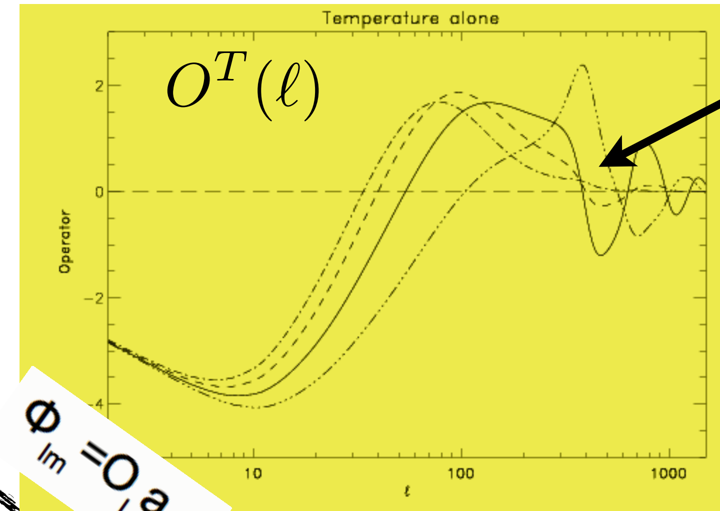
$$\hat{f}_{NL} = \frac{1}{N}\int \Phi^2(r)w(r)r^2dr$$

Komatsu, Spergel, Wandelt 05; Yadav and Wandelt 05

Primordial non-Gaussianity from CMB

CMB



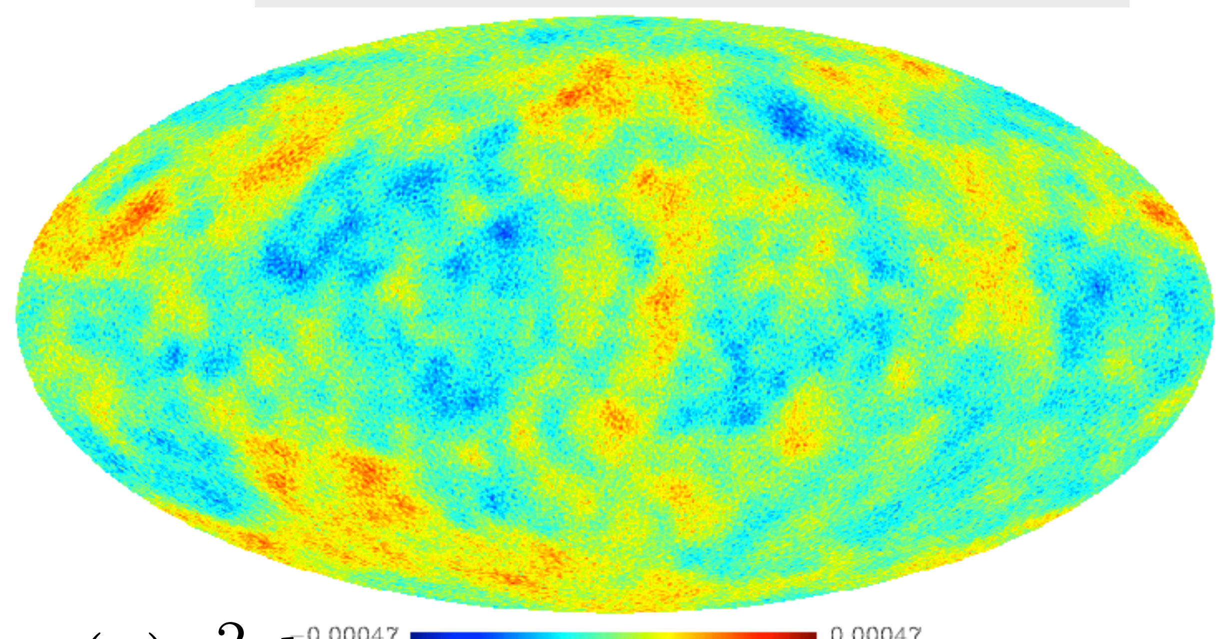


Filter goes to zero (reconstruction fails)

$\Phi_{lm} = O_l a_{lm}$

On large scales $\Delta T = -\frac{1}{3}\Phi$

Curvature Perturbations $\Phi(r)$



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Primordial non-Gaussianity from CMB

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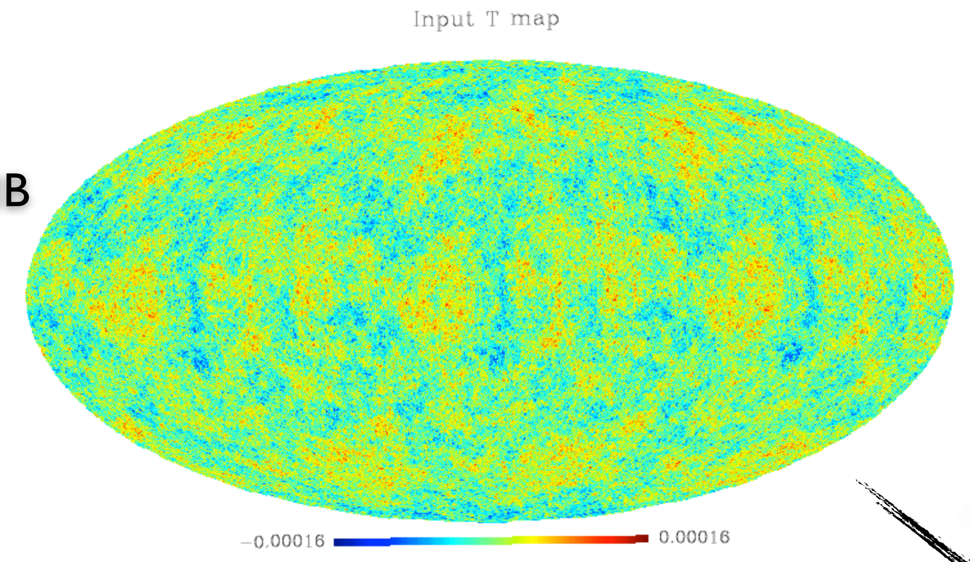

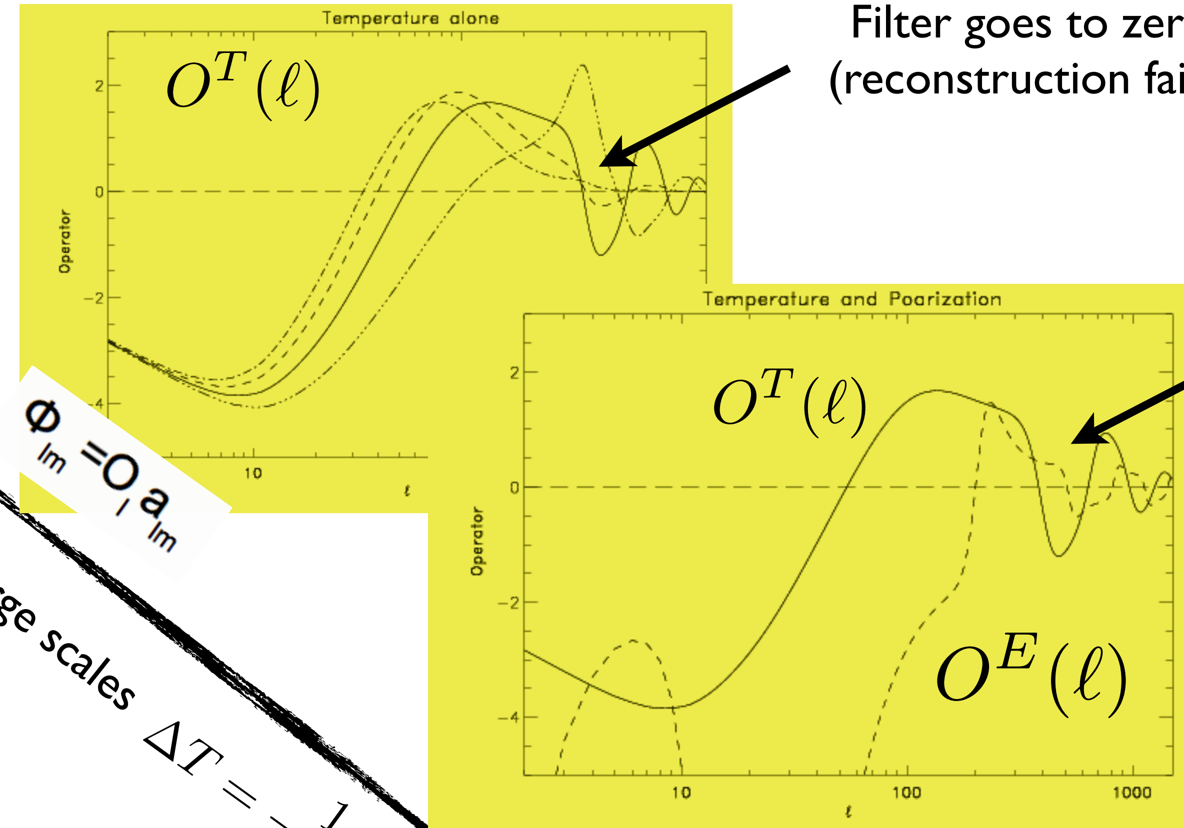

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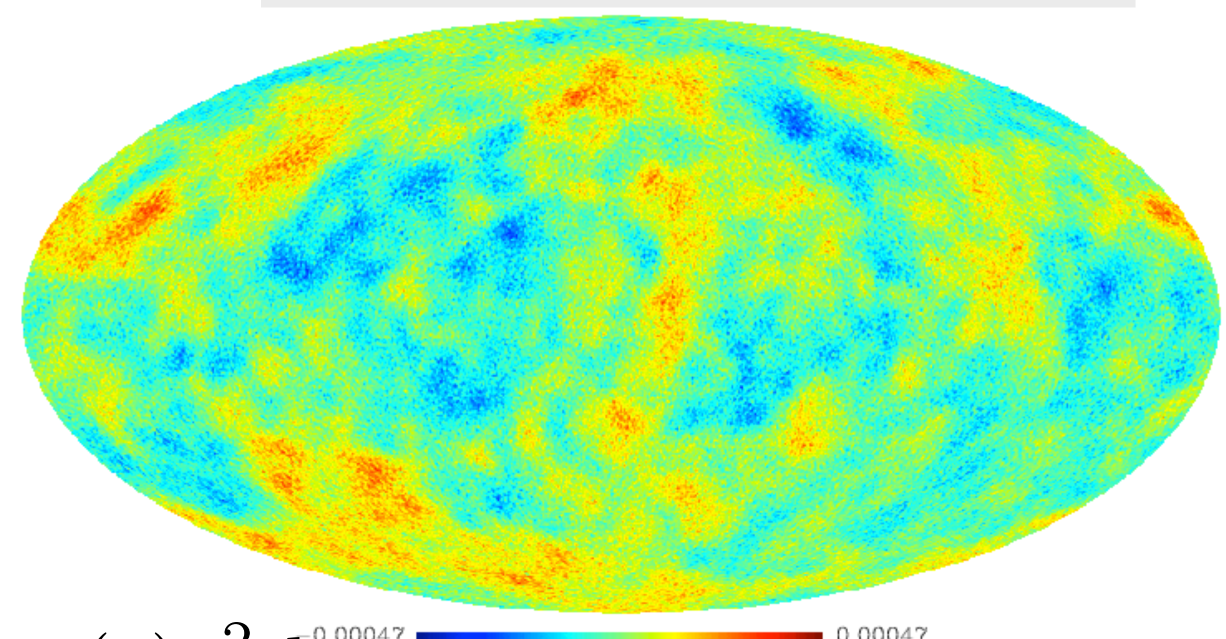

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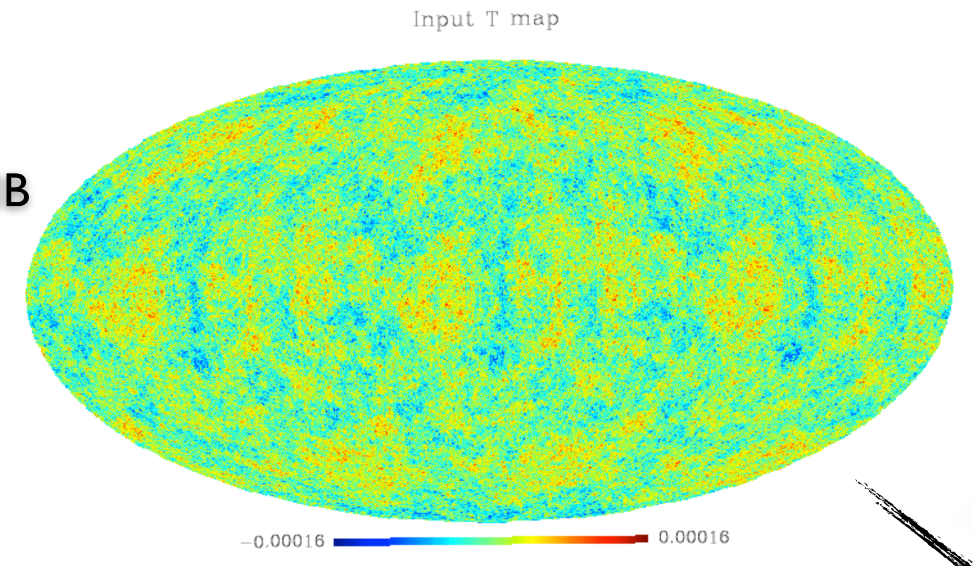
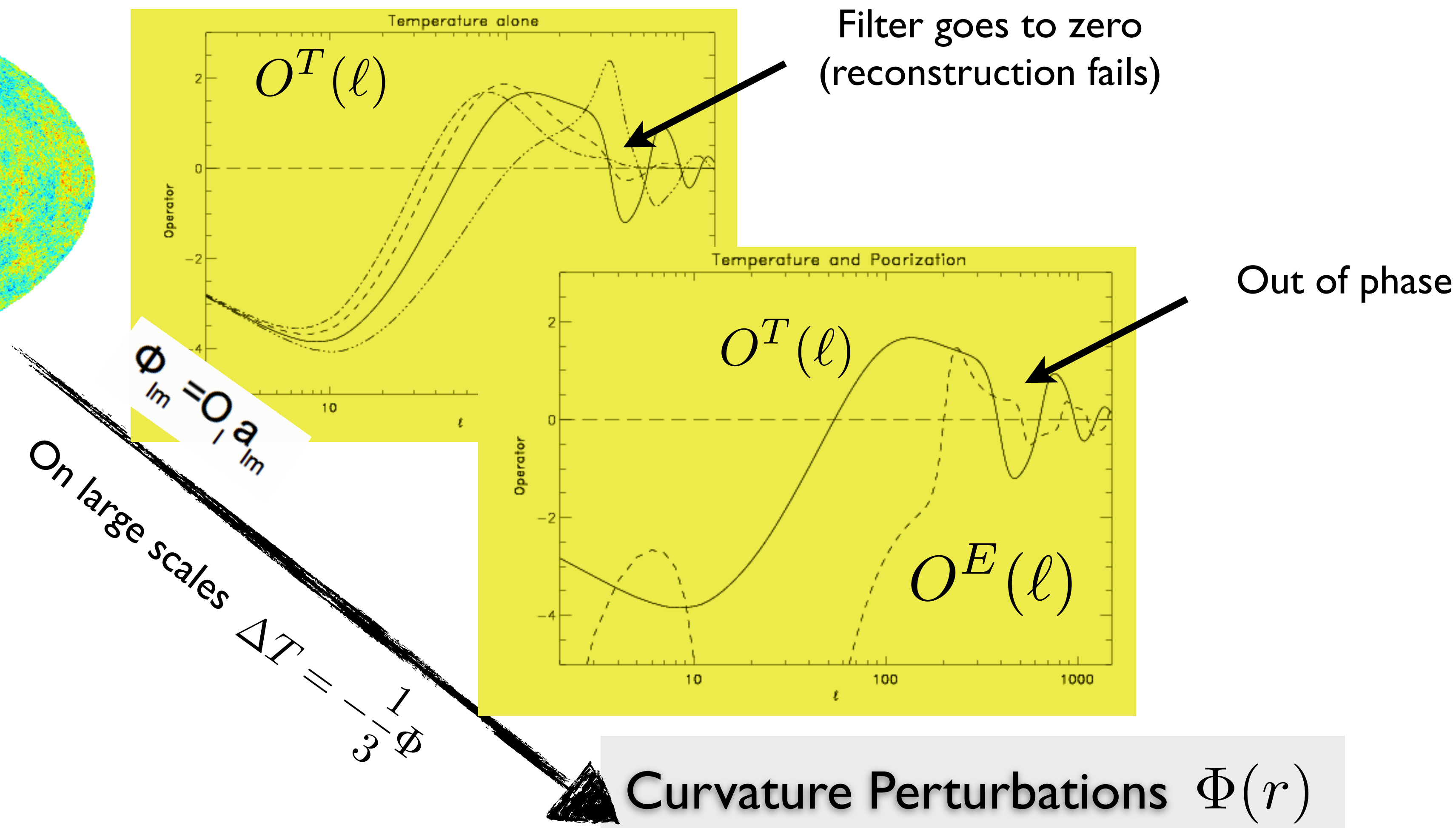
Komatsu, Spergel, Wandelt 05; Yadav and Wandelt 05

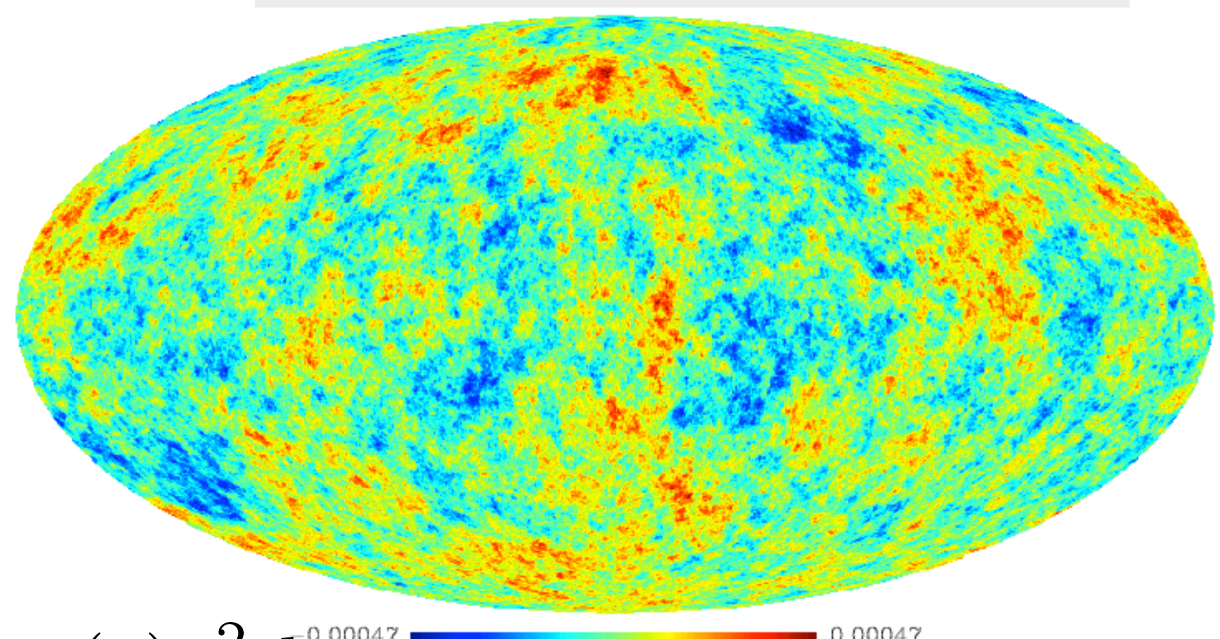
Primordial non-Gaussianity from CMB

CMB





Curvature Perturbations $\Phi(r)$



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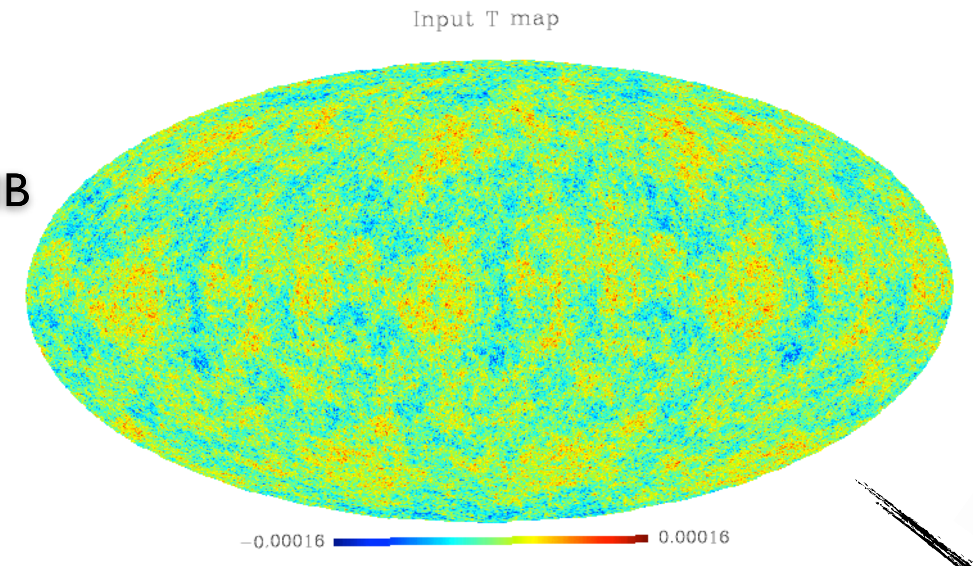
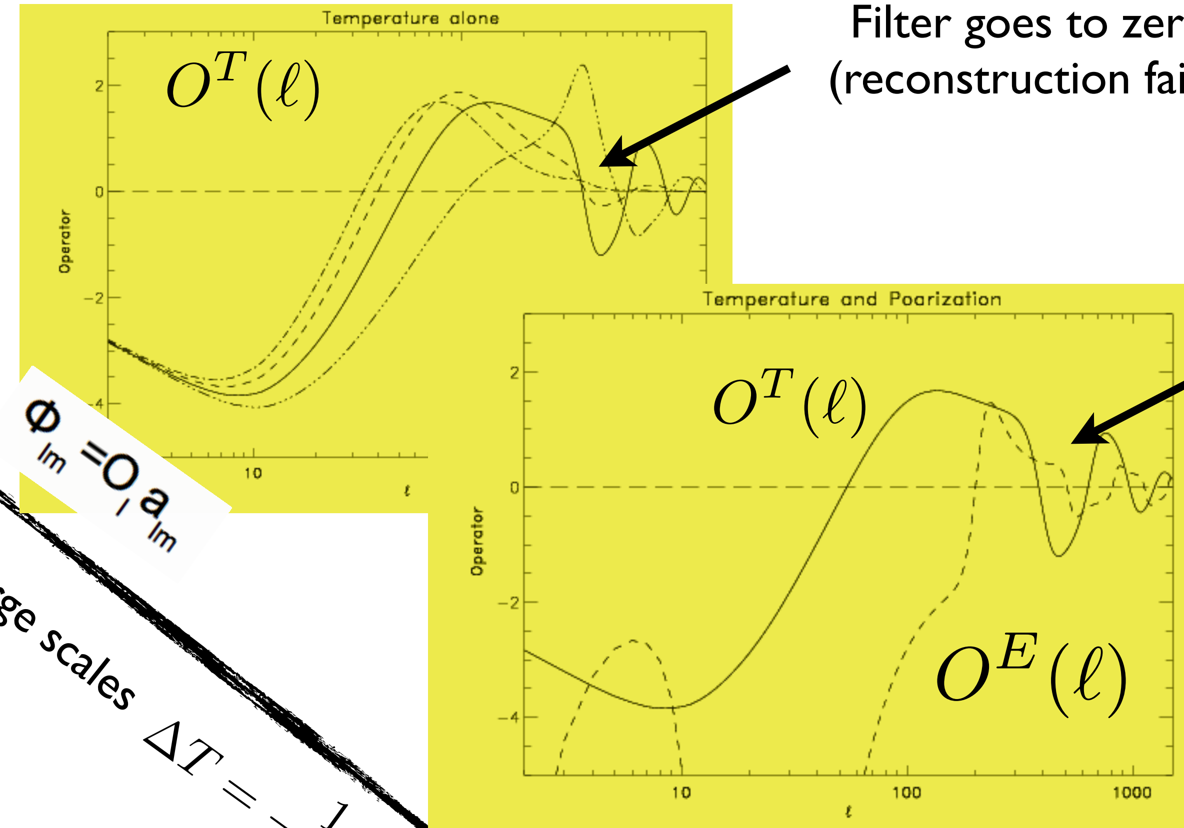
Komatsu, Spergel, Wandelt 05; Yadav and Wandelt 05

Primordial non-Gaussianity from CMB

CMB





Filter goes to zero (reconstruction fails)

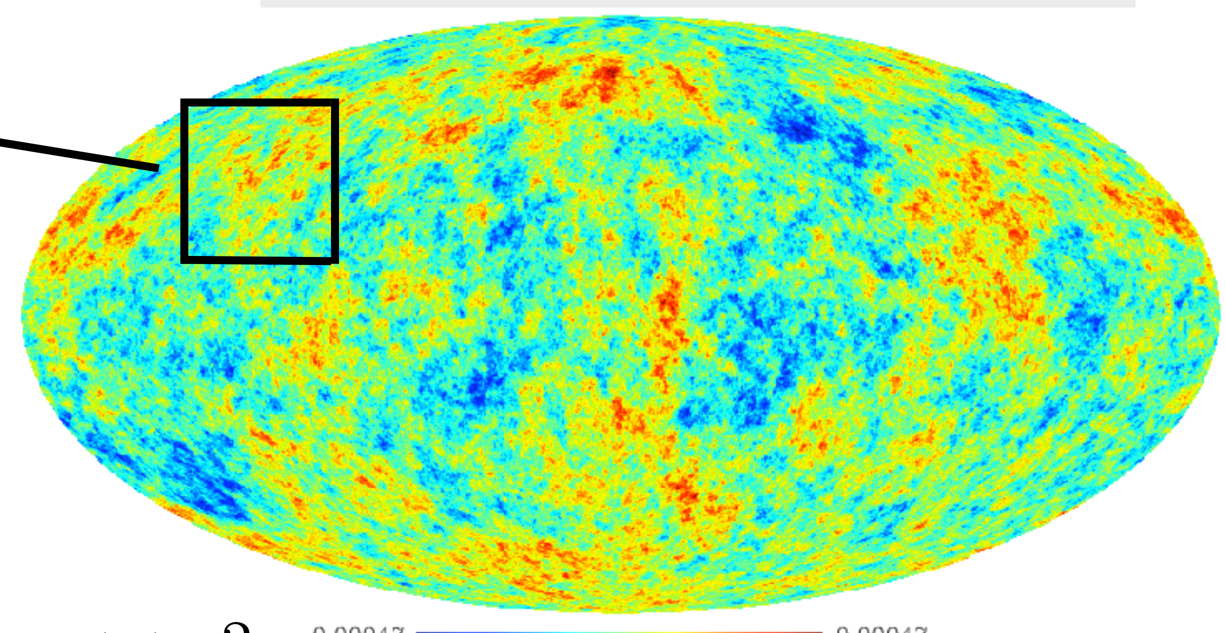
Out of phase

$\Phi_{lm} = O_l a_{lm}$

Curvature Perturbations Movie $\Phi(r)$

On large scales $\Delta T = -\frac{1}{3}\Phi$

Curvature Perturbations $\Phi(r)$



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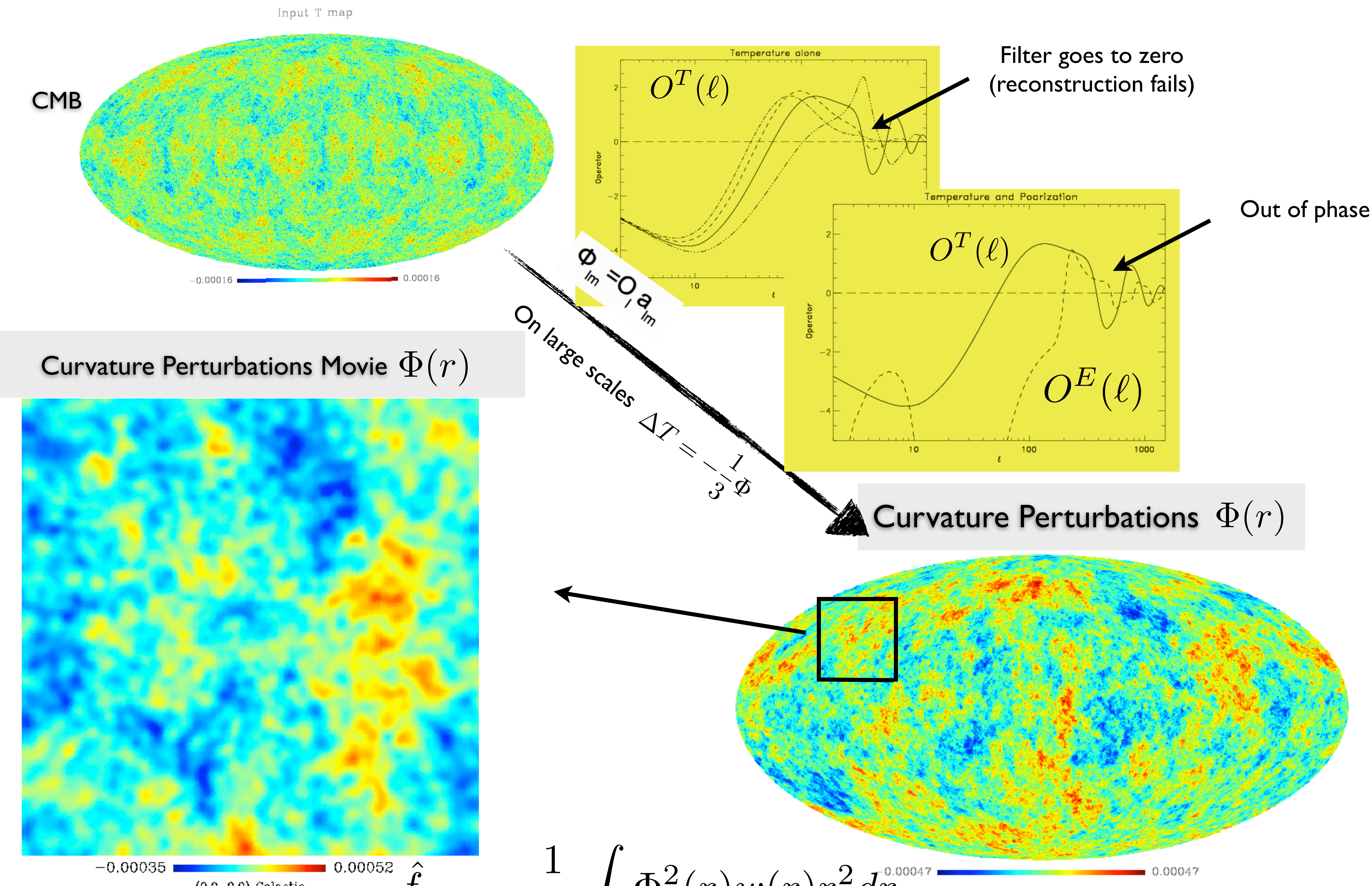
Komatsu, Spergel, Wandelt 05; Yadav and Wandelt 05

Primordial non-Gaussianity from CMB



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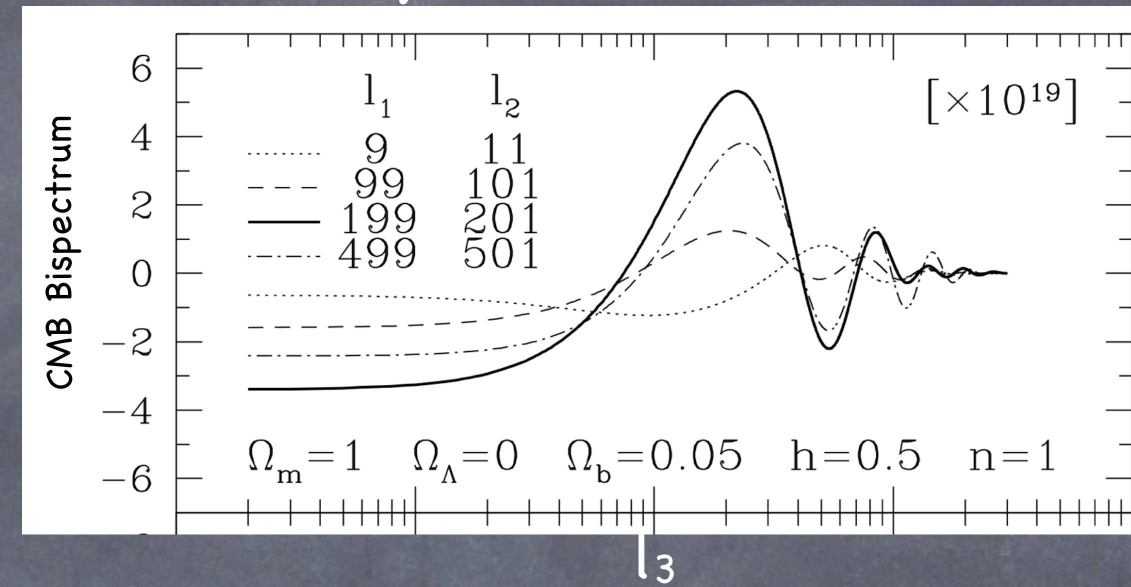
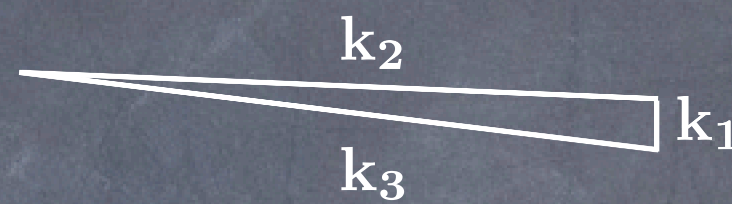
Komatsu, Spergel, Wandelt 05; Yadav and Wandelt 05

Squeezed Bispectrum/Local Non-Gaussianity

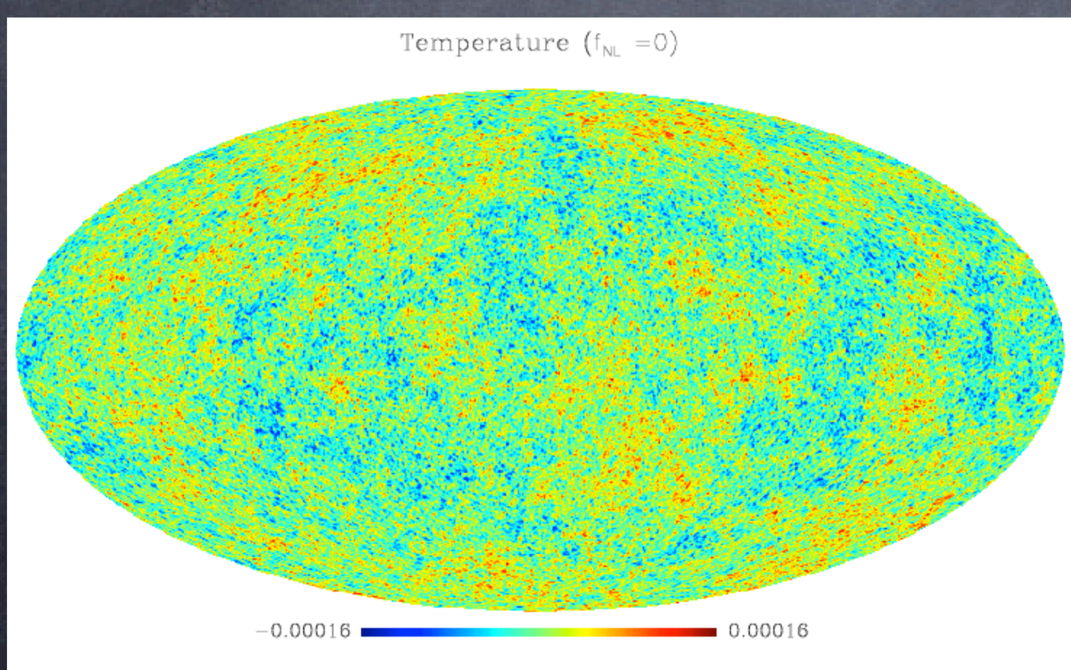
$$\Phi(x) = \Phi_G(x) + f_{NL}\Phi_G^2(x)$$

$$F(k_1,k_2,k_3) = \Delta_\Phi^2 \cdot \left(\frac{1}{k_1^3 k_2^3} + \frac{1}{k_1^3 k_3^3} + \frac{1}{k_2^3 k_3^3} \right)$$

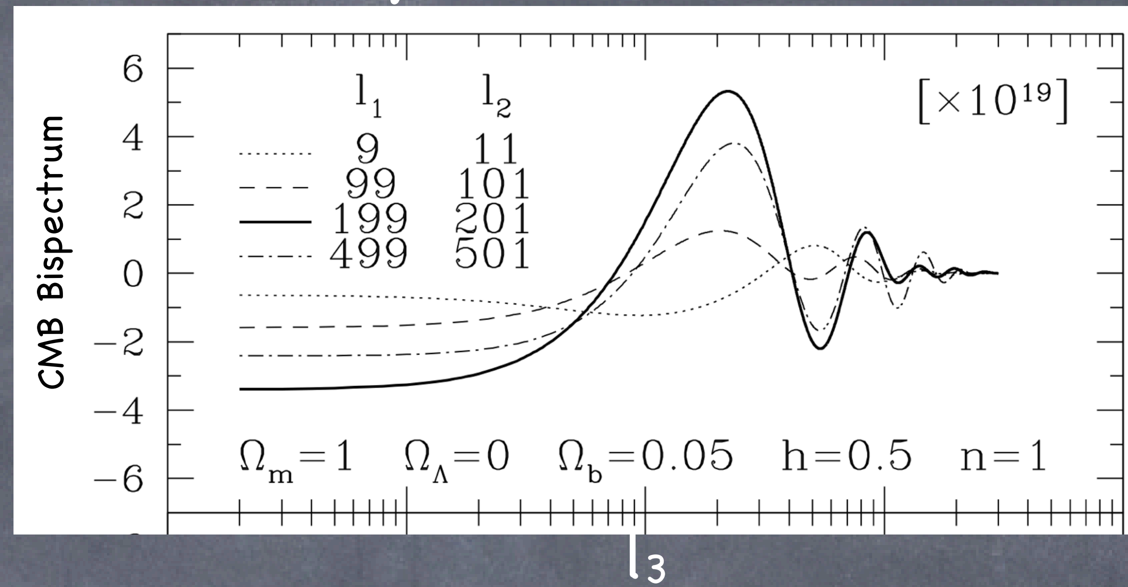




- Large values for squeezed triangle configuratio
- Represents inflationary models where perturb generated outside the horizon
  - Multifield inflation, Curvaton
  - Inhomogeneous reheating...



Squeezed Bispectrum/Local Non-Gaussianity



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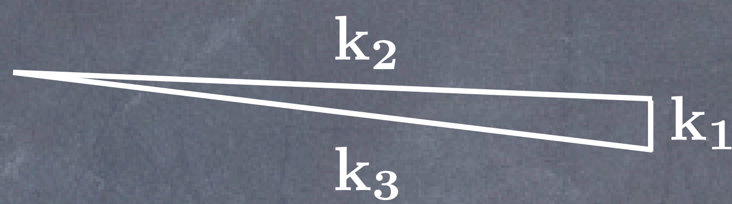

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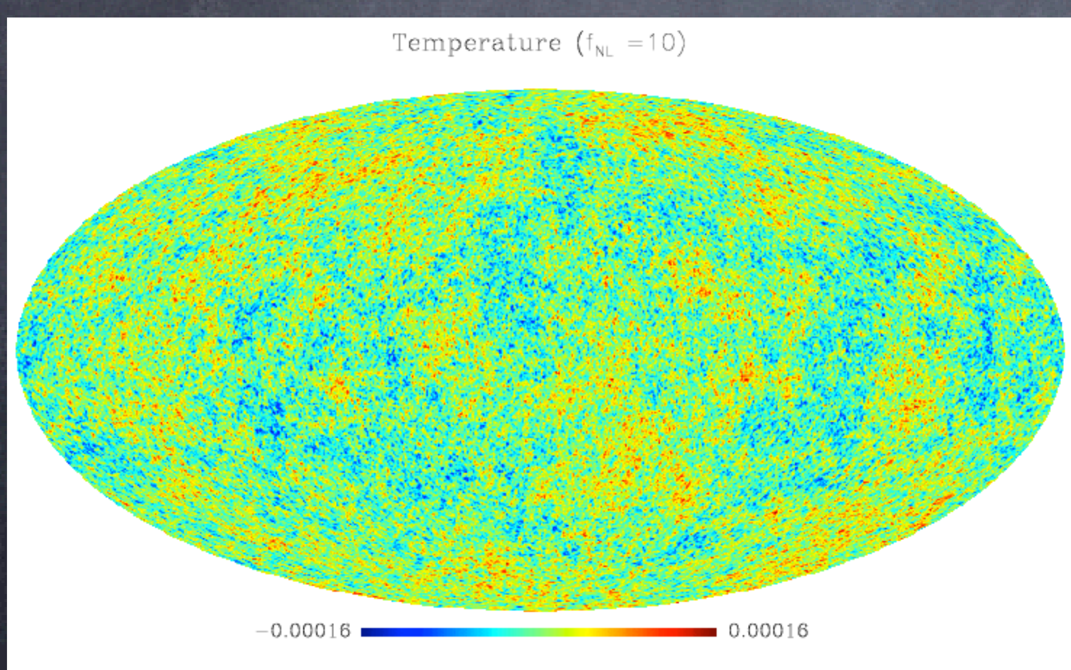

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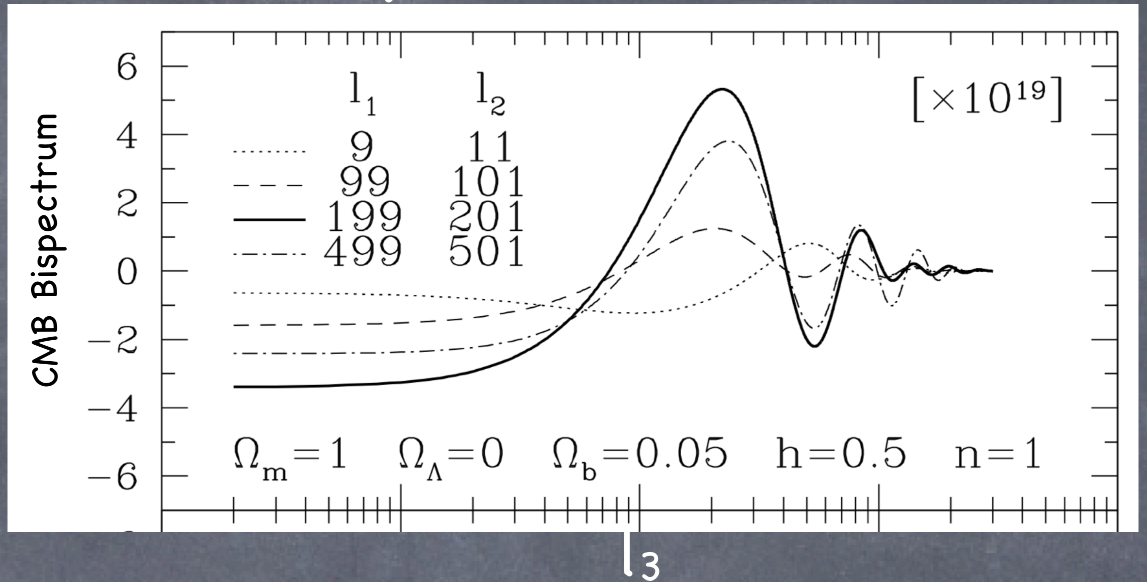

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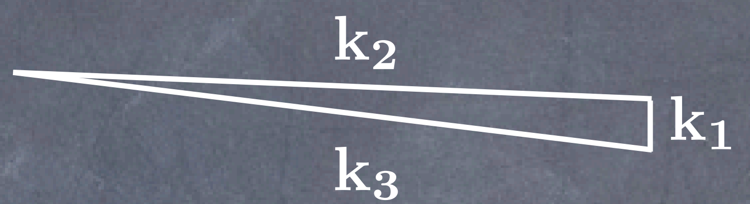

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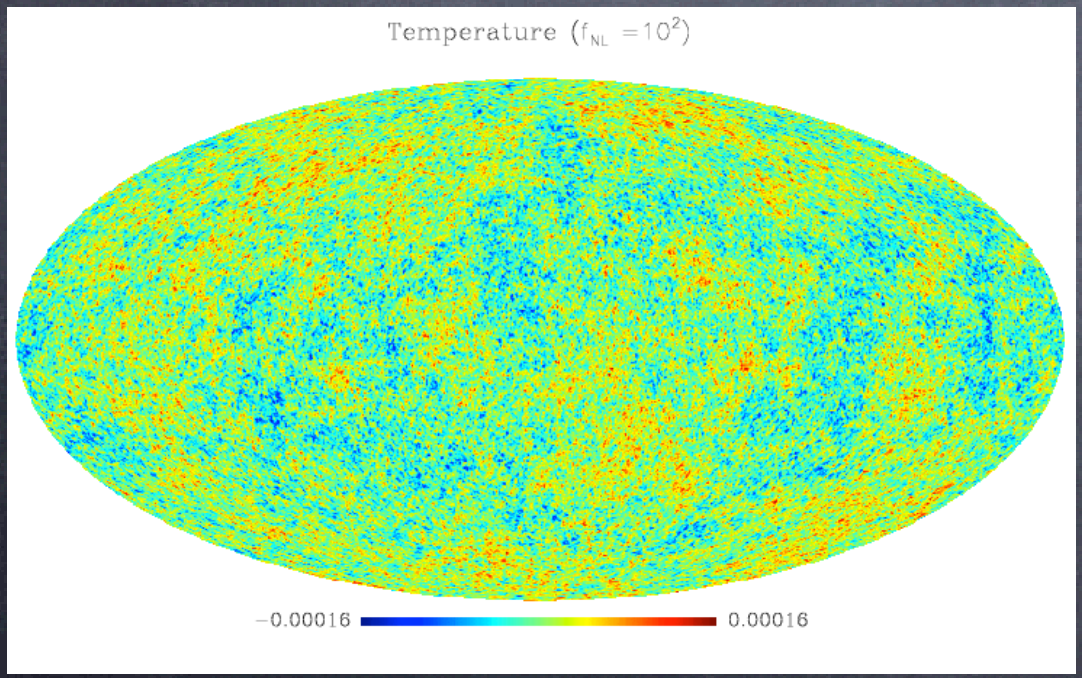

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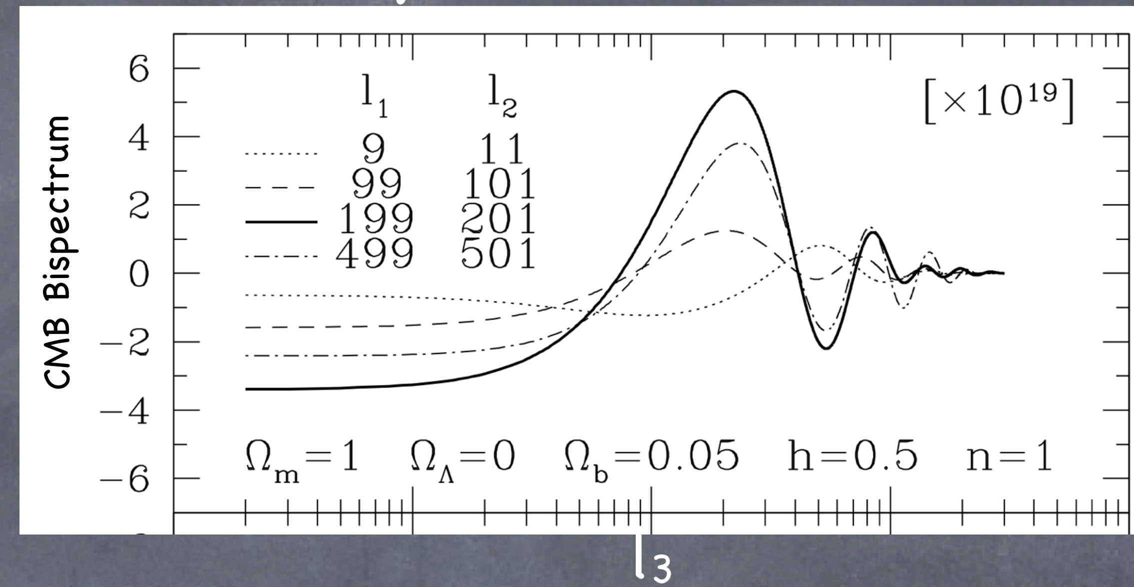

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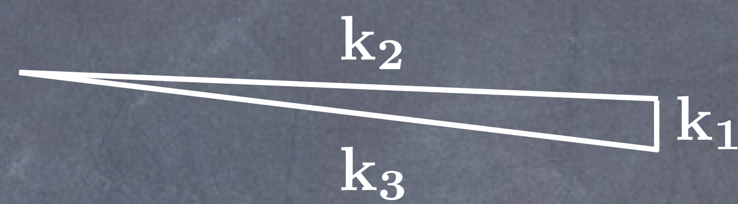

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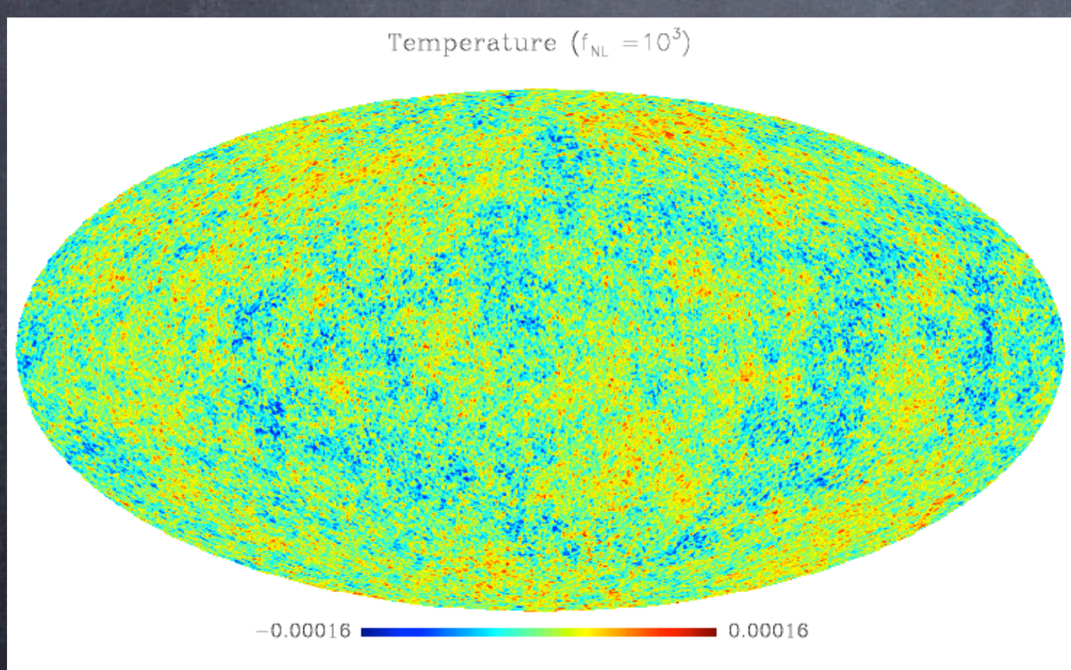
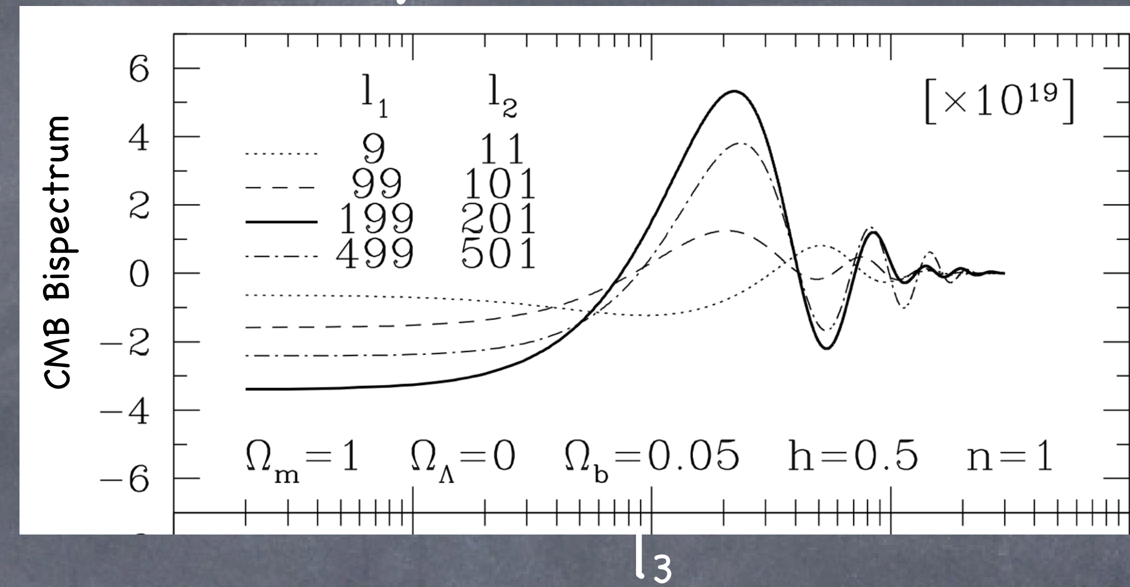


Squeezed Bispectrum/Local Non-Gaussianity



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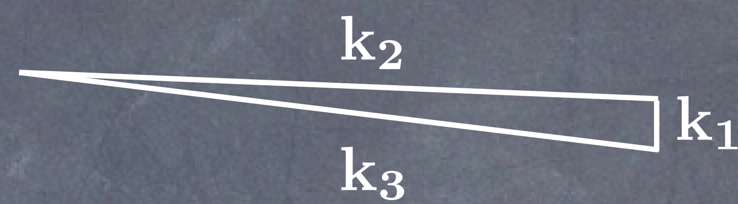

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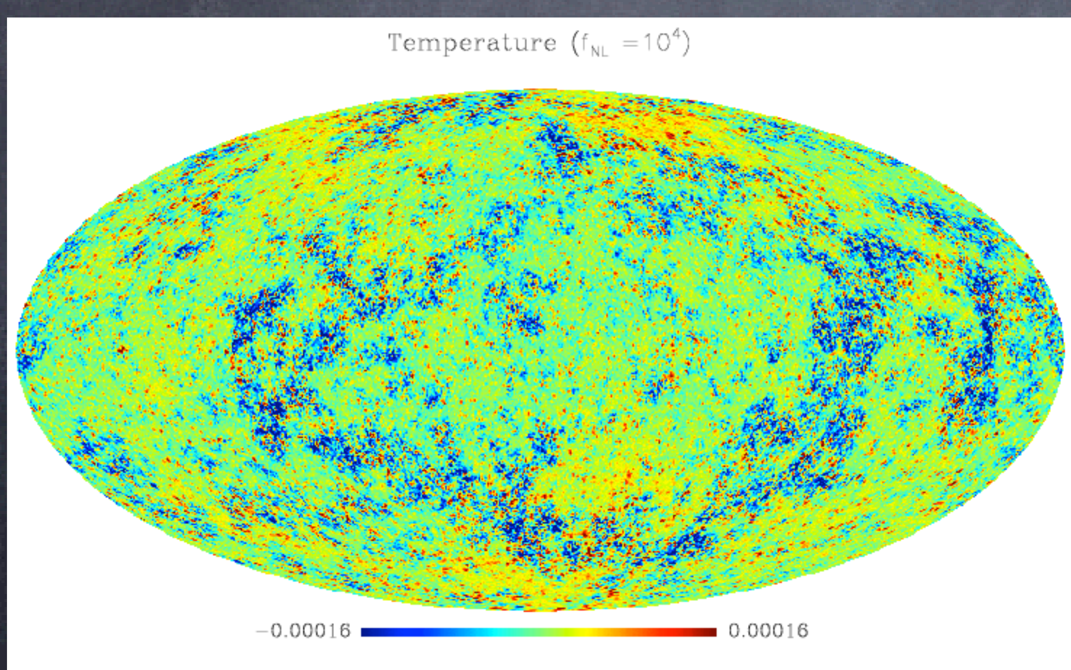

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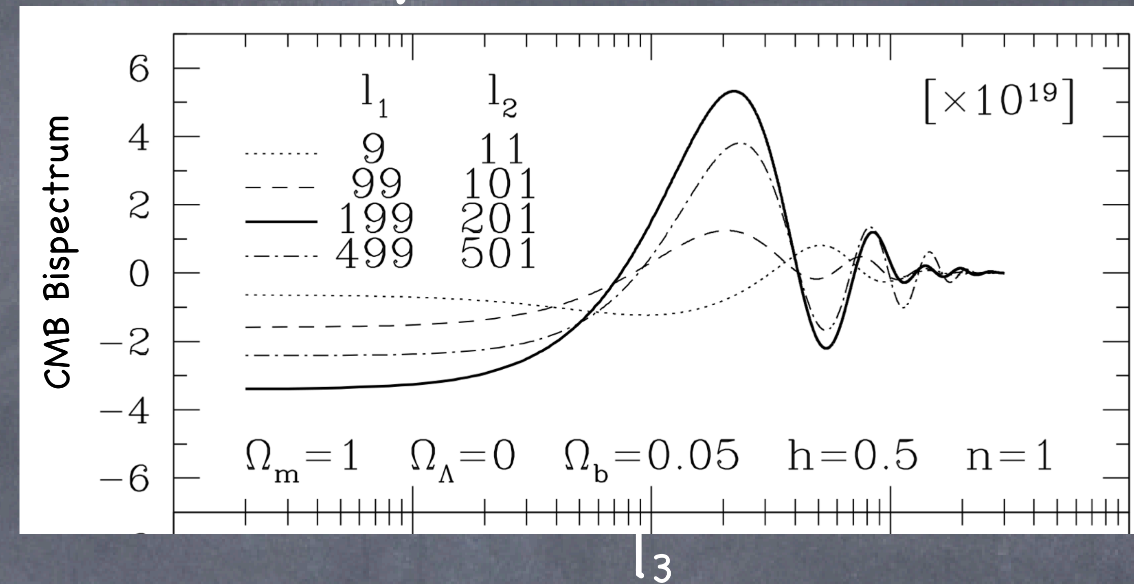

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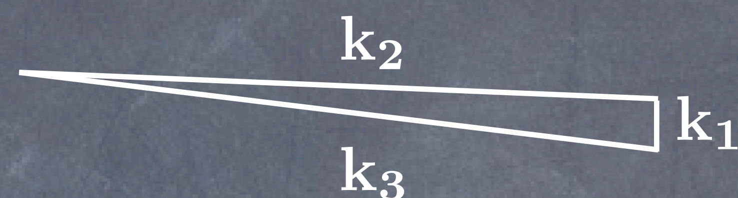

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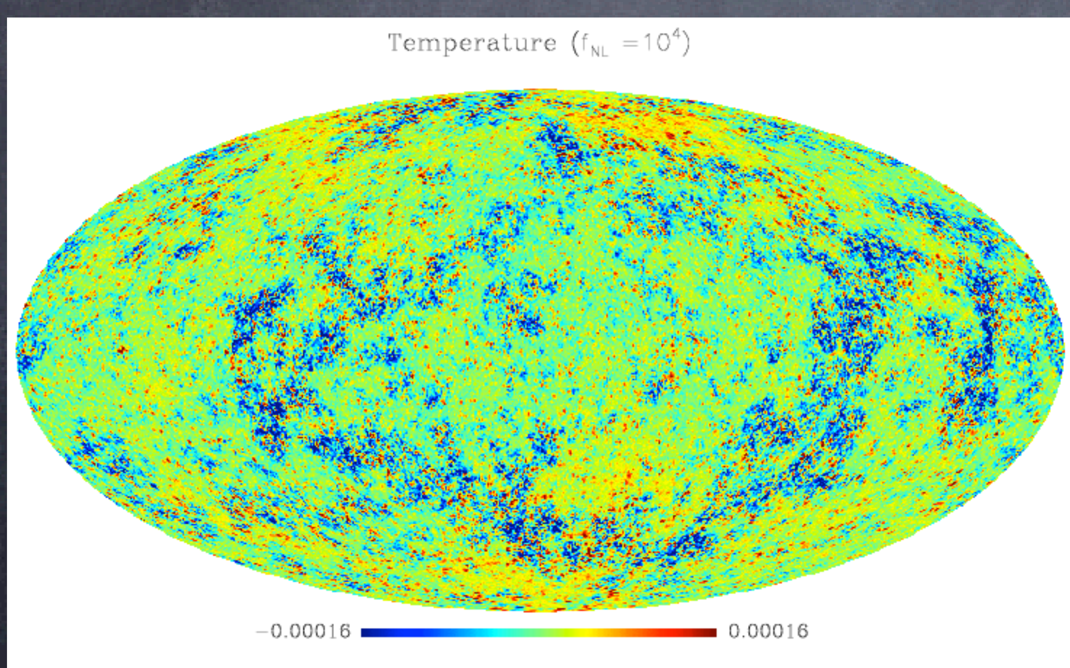
data	Method	$f_{NL}^{local} \pm 2\sigma$ error	
COBE	Bispectrum non-optimal	$\lvert f_{NL}\rvert < 1500$	Komatsu et al. (2002)
WMAP 1-year	Bispectrum non-optimal	39.5 ± 97.5	E. Komatsu et al. (2003)
WMAP 1-year	Bispectrum near-optimal-v1	47 ± 74	Creminelli et al. (2005)
WMAP 3-year	Bispectrum non-optimal	30 ± 84	Spergel et al. (2006)
WMAP 3-year	Bispectrum near-optimal-v1	32 ± 68	Creminelli et al. (2006)
WMAP 3-year	Bispectrum near-optimal	87 ± 62	Yadav and Wandelt (2008)
WMAP 3-year	Bispectrum near-optimal	69 ± 60	Smith et al. (2009)
WMAP 3-year	Bispectrum optimal	58 ± 46	Smith et al. (2009)
WMAP 5-year	Bispectrum near-optimal	51 ± 60	Komatsu et al. (2008)
WMAP 5-year	Bispectrum optimal	38 ± 42	Smith et al. (2009)
WMAP 7-year	Bispectrum optimal	32 ± 42	Komatsu et al. (2010)

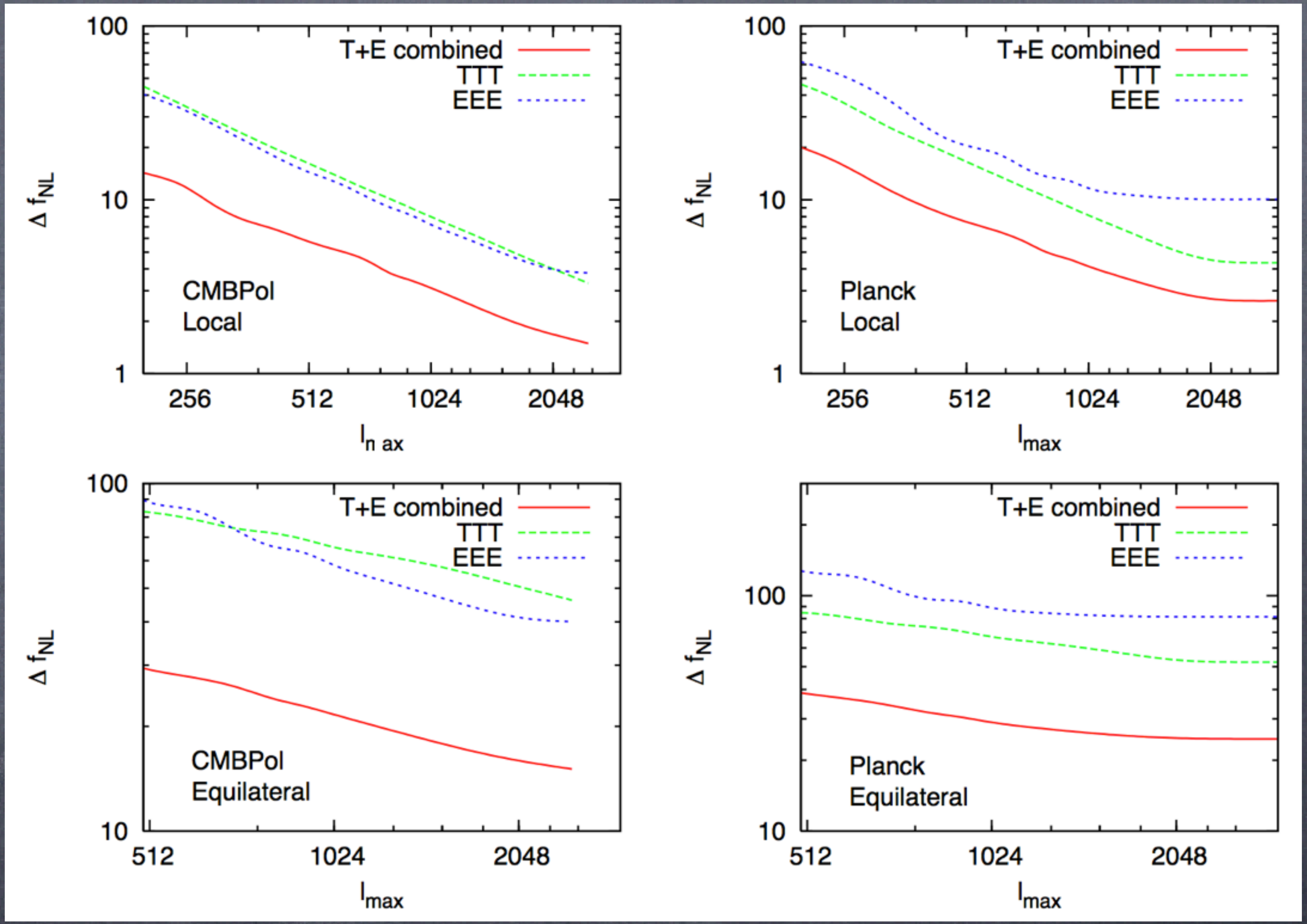


$$f_{NL}^{\rm equil} = 26 \pm 140 \ (68\%\ {\rm CL}).$$

$$f_{NL}^{\rm orthog} = -202 \pm 104 \ (68\%\ {\rm CL}).$$

Temperature + E-Polarization

1. Cross Check: independent analysis for E and T
2. Combined T+E analysis



Concerns:

secondary anisotropies

ISW-lensing f_{NL}~ 10 (Smith & Zaldarriaga 06)

Recombination f_{NL}~ few (Pitrou 2010)

Instrumental systematics

Babich & Zaldarriaga (2004); Yadav, Komatsu, Wandelt (2007)
Yadav & Wandelt (2005)

Beyond f_{NL}: the case for running non-Gaussianity

- fNL has been shown to be scale-dependent in several models of with a variable speed of sound, such as DBI inflation.

- Non-Gaussianity can be larger (or smaller) at different scales and for different observables

$$f_{NL} \rightarrow f_{NL}\left(\frac{K}{k_\star}\right)^{n_{NG}}$$

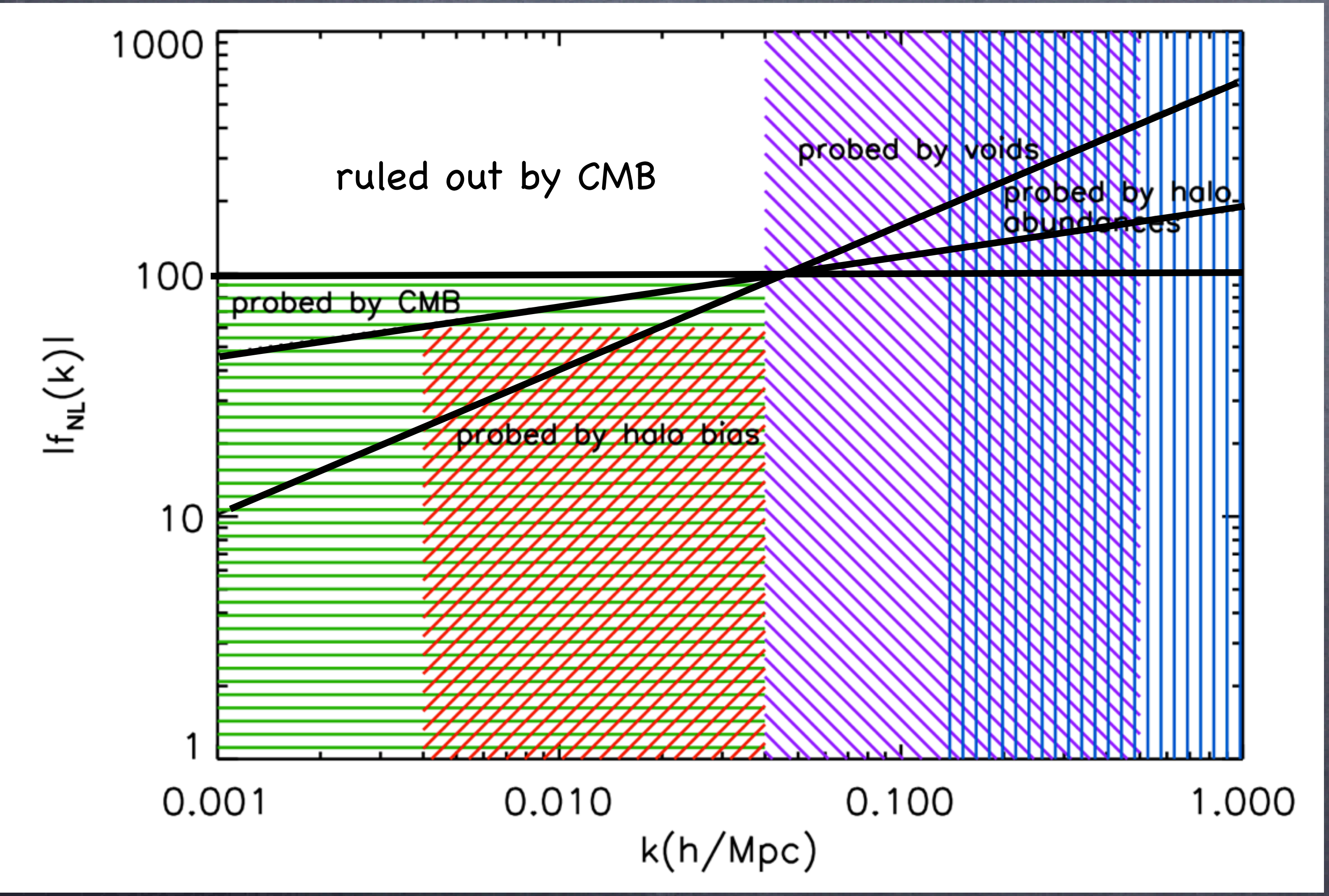


Figure stolen from, Emiliano Sefusatti's talk

n_{NG}=0
n_{NG}=0.2
n_{NG}=0.6

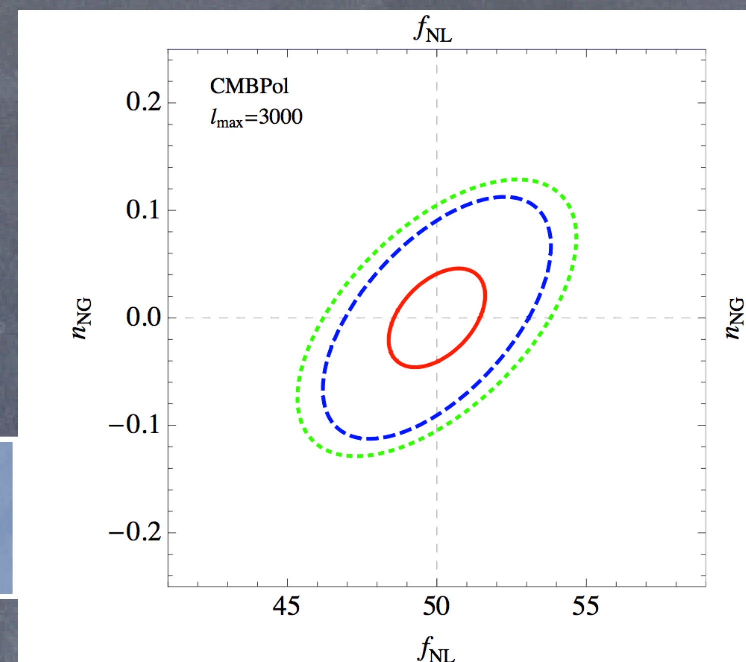
LoVerde, Miller, Shandera, Verde (2008)
Sefusatti, Liguori, Yadav, Jackson, Pajer (2009)

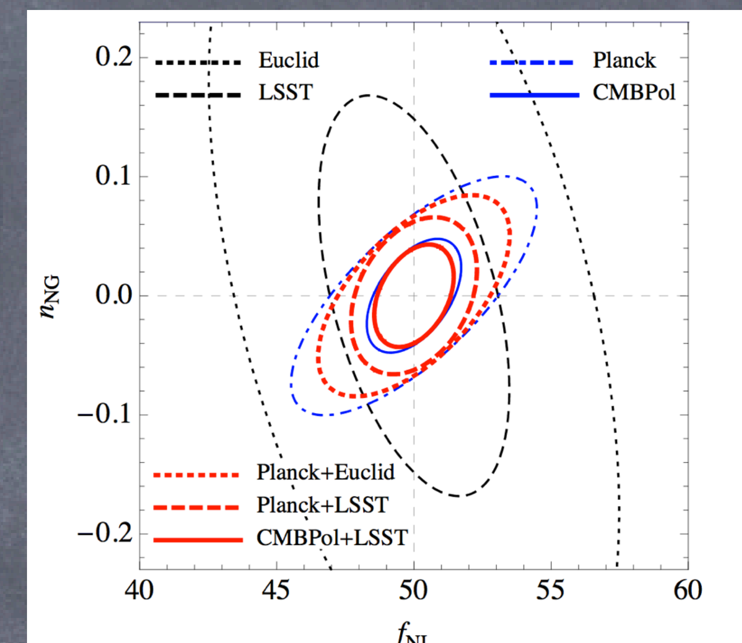
Beyond f_{NL}: the case for running non-Gaussianity

- f_{NL} has been shown to be scale-dependent in several models of with a variable speed of sound, such as DBI inflation.
- Current observations only constrain the magnitude of f_{NL}. However if f_{NL} is large enough, it may be also possible in the near future to constrain its possible dependence on scale.

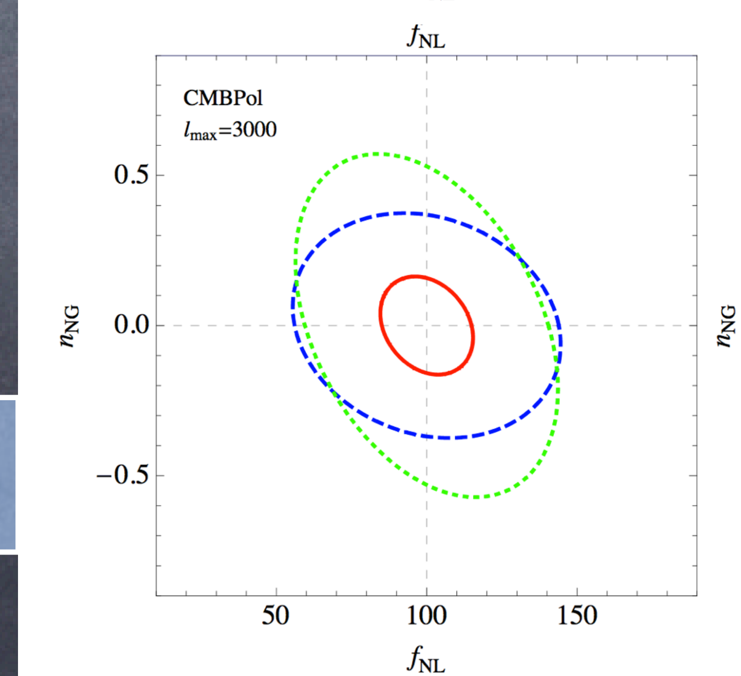
$$f_{NL} \rightarrow f_{NL}\left(\frac{K}{k_\star}\right)^{n_{NG}}$$

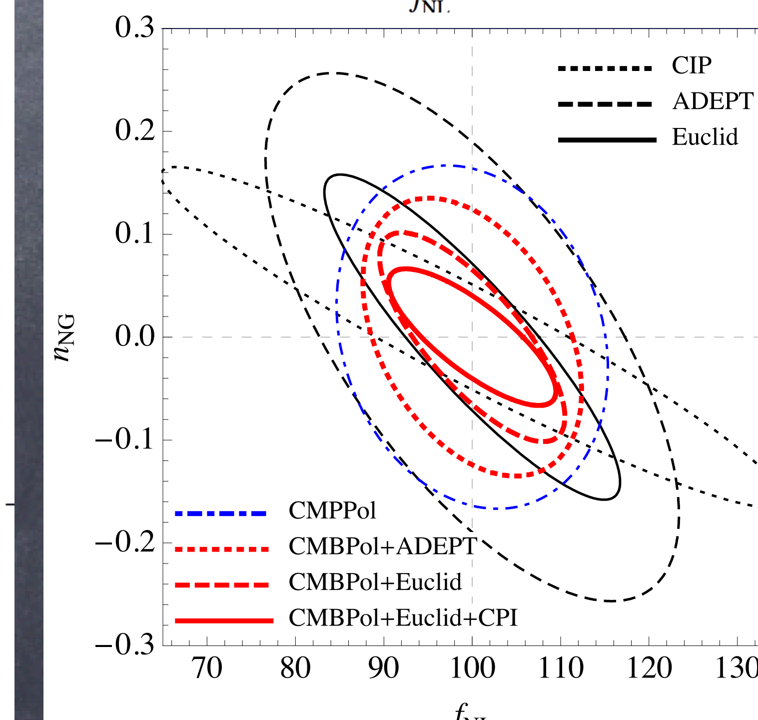
Local





Equilateral



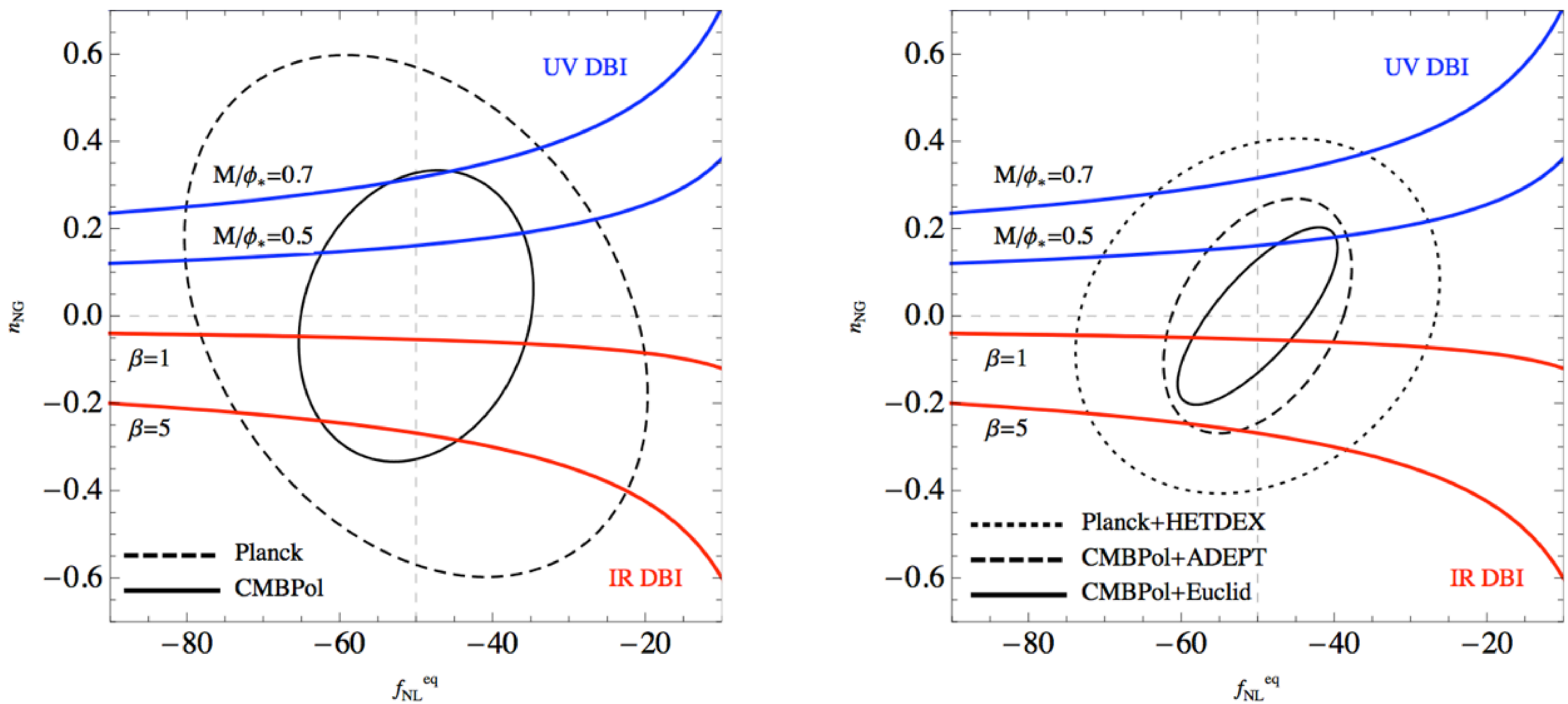


Sefusatti, Liguori, Yadav, Jackson, and Pajer (2009)

Beyond f_{NL}: the case for running non-Gaussianity

An example: DBI Inflation

$$\mathcal{L}(\phi) = -\frac{\phi^4}{\lambda}\sqrt{1-\lambda\frac{\dot{\phi}^2}{\phi^4}} + \frac{\phi^4}{\lambda} - V(\phi), \qquad V(\phi) = \begin{cases} \frac{1}{2}m\phi^2 & UV \\ V_0 - \frac{1}{2}m\phi^2 & IR \end{cases} \quad (m^2 \equiv \beta H^2)$$



Limits on $n_{\rm NG}$ can provide stronger constraints on the parameters of the models

Sefusatti, Liguori, Yadav, Jackson, and Pajer (2009)

Instrumental effects on CMB Bispectrum

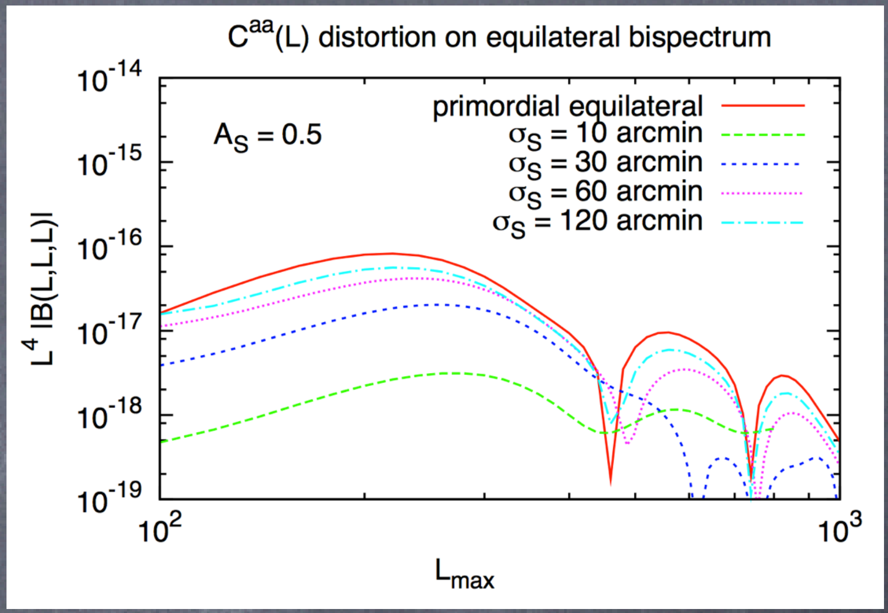
Su, Yadav et al. (2010)

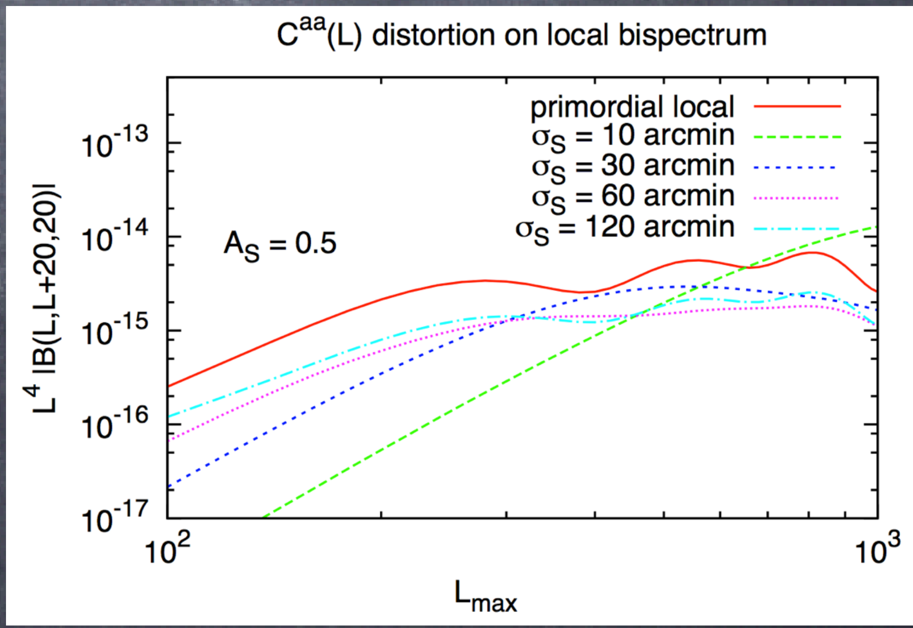
Detector gain: $\delta T^{out}(n) = (1 + a(n))\delta T^{in}(n)$

$$C_\ell^{aa} \propto \exp(-\ell(\ell+1)\sigma_S^2)$$

$$\delta B^{TTT}_{(\mathbf{l}_1,\mathbf{l}_2,\mathbf{l}_3)} = \int \frac{d^2\mathbf{l}'}{(2\pi)^2} C^{aa}_{\mathbf{l}'} \left\{ B^{TTT}_{(\mathbf{l}_1,\mathbf{l}_2-\mathbf{l}',\mathbf{l}_3+\mathbf{l}')} + B^{TTT}_{(\mathbf{l}_1-\mathbf{l}',\mathbf{l}_2+\mathbf{l}',\mathbf{l}_3)} + B^{TTT}_{(\mathbf{l}_1+\mathbf{l}',\mathbf{l}_2,\mathbf{l}_3-\mathbf{l}')} \right\}$$

(1) Linear systematics can only distorts primordial bispectrum





Instrumental effects on CMB Bispectrum

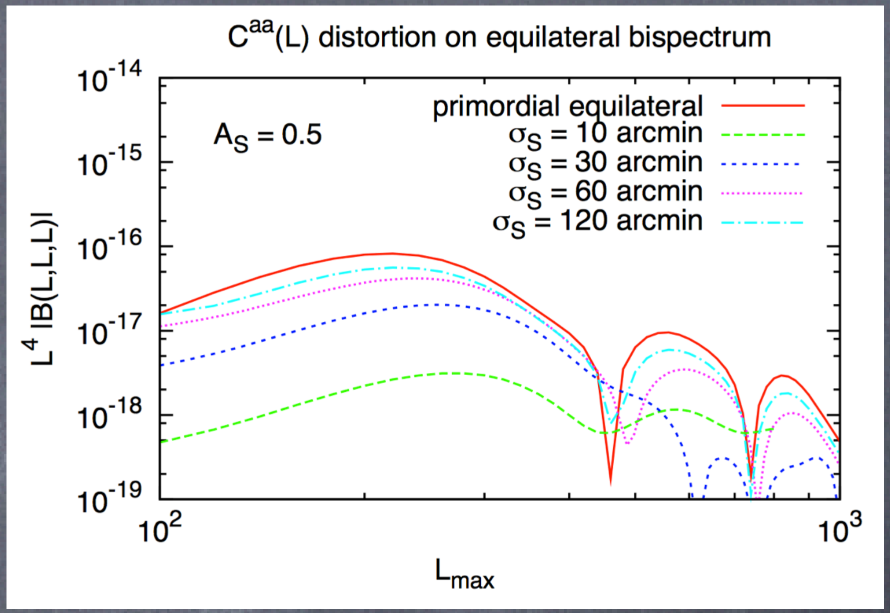
Su, Yadav et al. (2010)

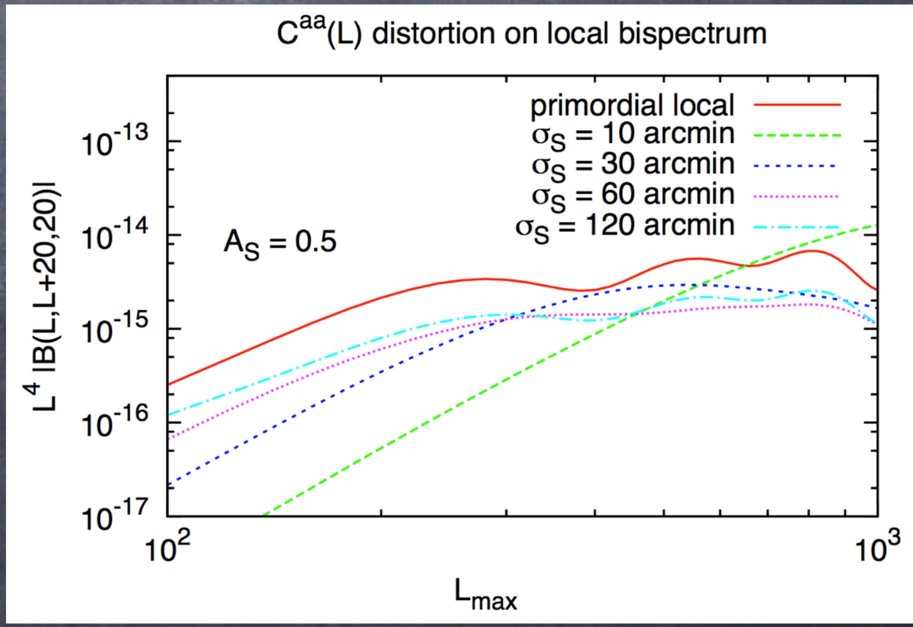
Detector gain: $\delta T^{out}(n) = (1 + a(n))\delta T^{in}(n) + b\left(\delta T^{in}\right)^2$

$$C_\ell^{aa} \propto \exp(-\ell(\ell+1)\sigma_S^2)$$

$$\delta B^{TTT}_{(\mathbf{l}_1,\mathbf{l}_2,\mathbf{l}_3)} = \int \frac{d^2\mathbf{l}'}{(2\pi)^2} C^{aa}_{\mathbf{l}'} \left\{ B^{TTT}_{(\mathbf{l}_1,\mathbf{l}_2-\mathbf{l}',\mathbf{l}_3+\mathbf{l}')} + B^{TTT}_{(\mathbf{l}_1-\mathbf{l}',\mathbf{l}_2+\mathbf{l}',\mathbf{l}_3)} + B^{TTT}_{(\mathbf{l}_1+\mathbf{l}',\mathbf{l}_2,\mathbf{l}_3-\mathbf{l}')} \right\}$$

(1) Linear systematics can only distorts primordial bispectrum





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Su, Yadav et al. (2010)

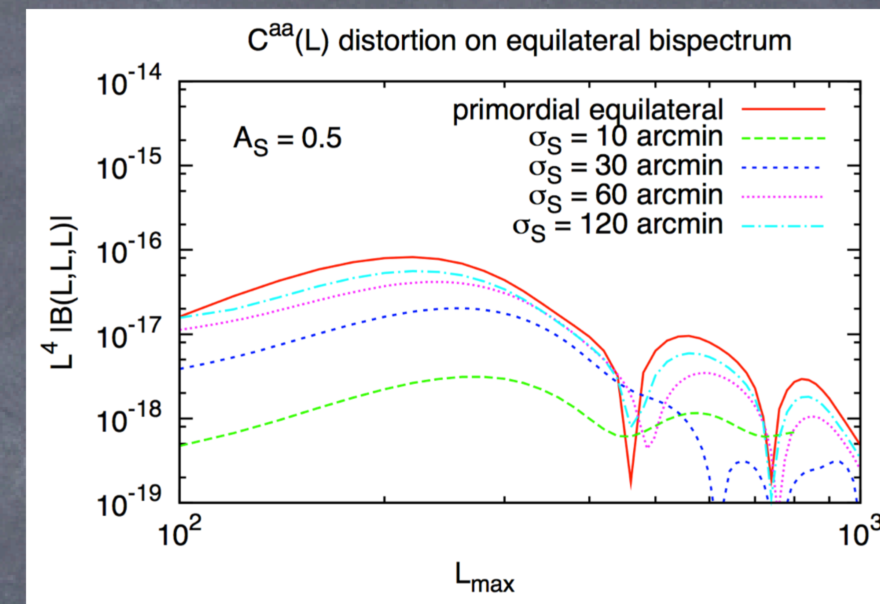
Detector gain: $\delta T^{out}(n) = (1 + a(n))\delta T^{in}(n) + b\left(\delta T^{in}\right)^2$

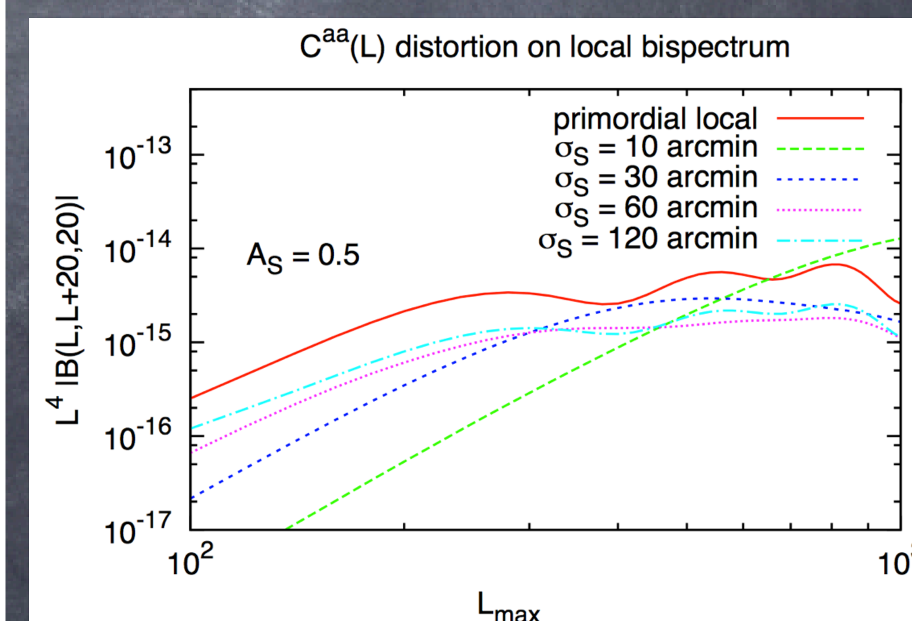
$$\delta B^{TTT}_{(l_1,l_2,l_3)} = \int \frac{d^2 l'}{(2\pi)^2} C^{aa}_{l'} \left\{ B^{TTT}_{(l_1,l_2-l',l_3+l')} + B^{TTT}_{(l_1-l',l_2+l',l_3)} + B^{TTT}_{(l_1+l',l_2,l_3-l')} \right\}$$

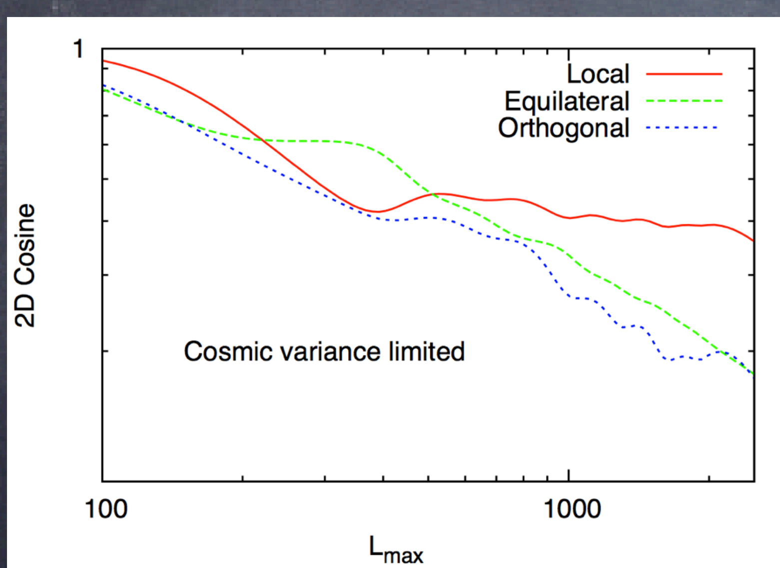
$$C^{aa}_\ell \propto \exp(-\ell(\ell+1)\sigma_S^2)$$

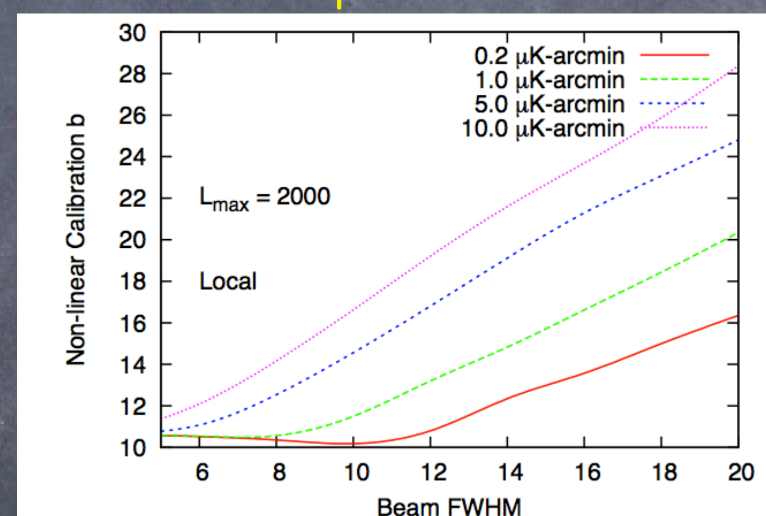
(1) Linear systematics can only distorts primordial bispectrum

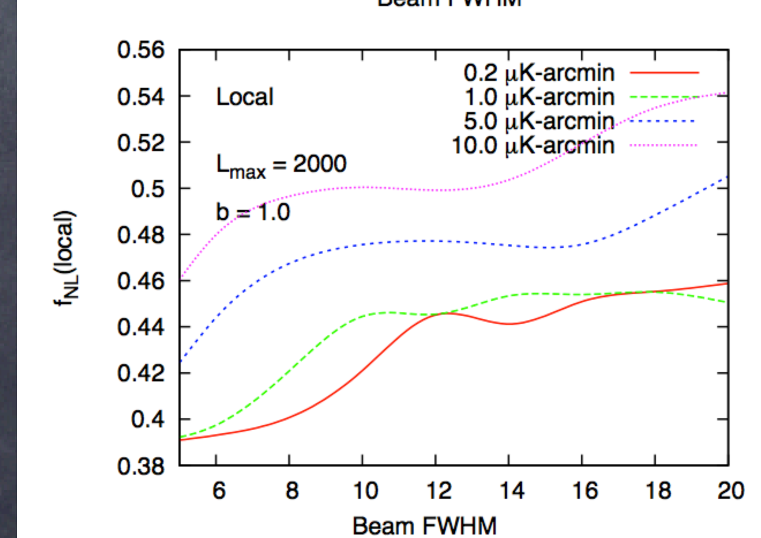
(2) Instrumental nonlinearities can generate spurious bispectrum and if these non-linearities are not controlled by dedicated calibration, they can produce $f_{NL} \sim O(10)$ before their effect is visible in CMB dipole.











Instrumental effects on CMB polarization Bispectrum

Su, Yadav et al. (2010)

Differential Gain $\delta[Q+iU](n) = [\gamma_1 + i\gamma_2](n)T(n)$

Differential Pointing $\delta[Q+iU](n) = \sigma[d_1 + id_2](n)[\partial_1 + \partial_2]T(n)$

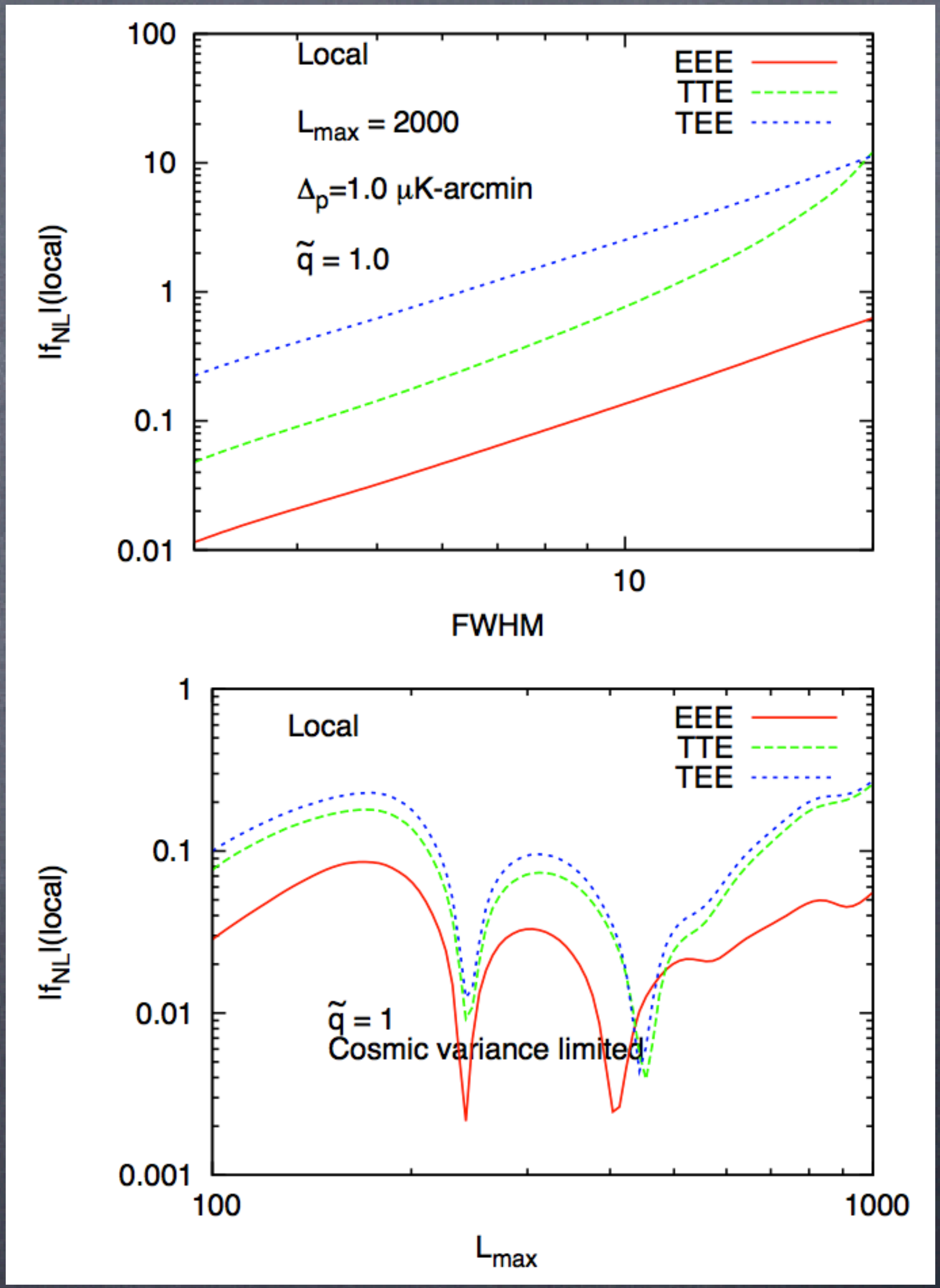
Differential Ellipticity $\delta[Q+iU](n) = \sigma^2 q(n)[\partial_1 + \partial_2]^2 T(n)$

$$B^{EEE,\tilde{S},bias}_{(\mathbf{l}_1,\mathbf{l}_2,\mathbf{l}_3)} = \tilde{S}(0)C^{TE}_{\ell_1}C^{TE}_{\ell_2}W^{\tilde{S}}_E(\mathbf{l}_3,-\mathbf{l}_1,-\mathbf{l}_2) + \text{perm.}$$

$$B^{TTE,\tilde{S},bias}_{(\mathbf{l}_1,\mathbf{l}_2,\mathbf{l}_3)} = \tilde{S}(0)C^{TT}_{\ell_1}C^{TT}_{\ell_2}W^{\tilde{S}}_E(\mathbf{l}_3,-\mathbf{l}_1,-\mathbf{l}_2) + \text{perm.}$$

$$B^{TEE,\tilde{S},bias}_{(\mathbf{l}_1,\mathbf{l}_2,\mathbf{l}_3)} = \tilde{S}(0)C^{TT}_{\ell_1}C^{TE}_{\ell_2}W^{\tilde{S}}_E(\mathbf{l}_3,-\mathbf{l}_1,-\mathbf{l}_2) + \tilde{S}(0)C^{TT}_{\ell_1}C^{TE}_{\ell_3}W^{\tilde{S}}_E(\mathbf{l}_2,-\mathbf{l}_1,-\mathbf{l}_3) + \text{perm.}$$

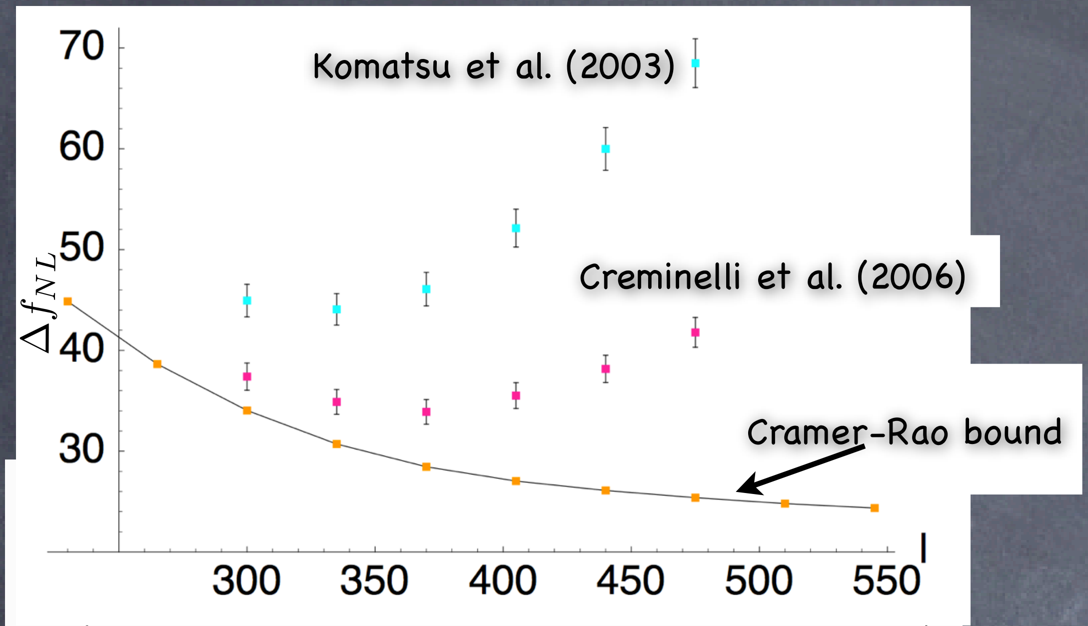
Non-linear Systematics $\tilde{S}$	$W^{\tilde{S}}_E(\mathbf{l},\mathbf{l}',\mathbf{l}'')$
Monopole leakage $\tilde{\gamma}_a$	$2\cos[2(\varphi_{\mathbf{l}-\mathbf{l}'-\mathbf{l}''} - \varphi_{\mathbf{l}})]$
Monopole leakage $\tilde{\gamma}_b$	$-2\sin[2(\varphi_{\mathbf{l}-\mathbf{l}'-\mathbf{l}''} - \varphi_{\mathbf{l}})]$
Dipole leakage $\tilde{d}_a$	$\sigma[\mathbf{l}'\sin(\varphi_{\mathbf{l}'} + \varphi_{\mathbf{l}-\mathbf{l}'-\mathbf{l}''} - 2\varphi_{\mathbf{l}}) + \mathbf{l}''\sin(\varphi_{\mathbf{l}''} + \varphi_{\mathbf{l}-\mathbf{l}'-\mathbf{l}''} - 2\varphi_{\mathbf{l}})]$
Dipole leakage $\tilde{d}_b$	$\sigma[\mathbf{l}'\cos(\varphi_{\mathbf{l}'} + \varphi_{\mathbf{l}-\mathbf{l}'-\mathbf{l}''} - 2\varphi_{\mathbf{l}}) + \mathbf{l}''\cos(\varphi_{\mathbf{l}''} + \varphi_{\mathbf{l}-\mathbf{l}'-\mathbf{l}''} - 2\varphi_{\mathbf{l}})]$
Quadrupole leakage $\tilde{q}$	$-2\sigma^2\{\mathbf{l}'^2\cos[2(\varphi_{\mathbf{l}'} - \varphi_{\mathbf{l}})] + \mathbf{l}''^2\cos[2(\varphi_{\mathbf{l}''} - \varphi_{\mathbf{l}})]\}$

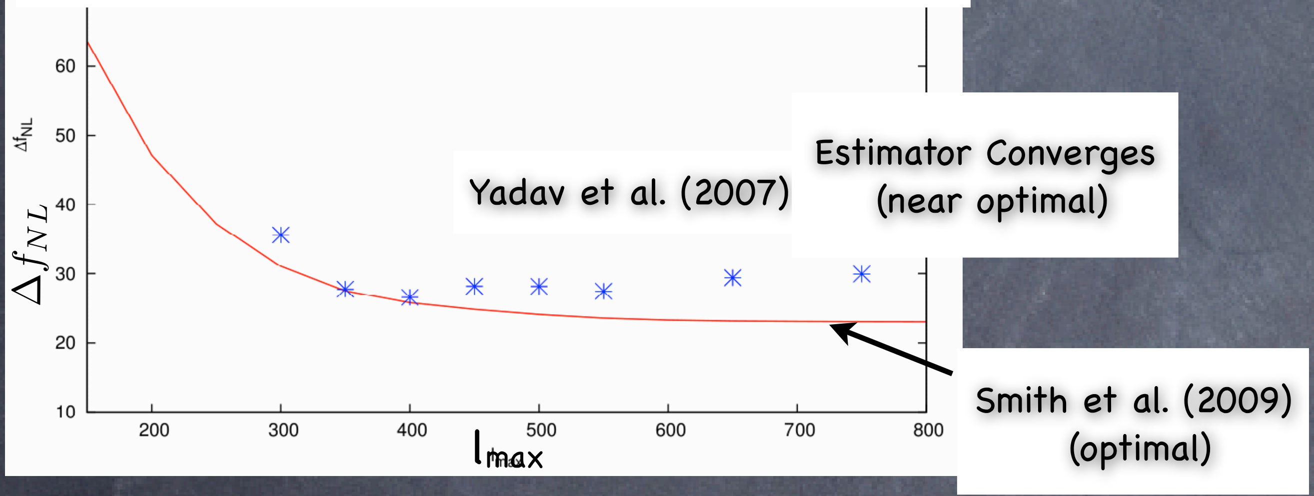


Executive Summary

- Characterization of non-Gaussianity is a powerful probe of the the early universe. Non-Gaussianity measures the strength of interaction and field content during inflation.
- Current status with CMB: $\Delta f_{NL} \sim 20$, $f_{NL} \sim 40$.
- Using combined T+E data of Planck: $\Delta f_{NL} \sim 4$ (in few years)
- If nonG is indeed large, running can also be constrained.
- If instrumental non-linearities are not controlled by dedicated calibration, it can generate $f_{NL} \sim O(10)$ before showing up in CMB dipole.

Constraints on local f_{NL}





data	Method	$f_{NL}^{local} \pm 2\sigma$ error	
COBE	Bispectrum non-optimal	$\|f_{NL}\| < 1500$	Komatsu et al. (2002)
WMAP 1-year	Bispectrum non-optimal	39.5 ± 97.5	E. Komatsu et al. (2003)
WMAP 1-year	Bispectrum near-optimal-v1	47 ± 74	Creminelli et al. (2005)
WMAP 3-year	Bispectrum non-optimal	30 ± 84	Spergel et al. (2006)
WMAP 3-year	Bispectrum near-optimal-v1	32 ± 68	Creminelli et al. (2006)
WMAP 3-year	Bispectrum near-optimal	87 ± 62	Yadav and Wandelt (2008)
WMAP 3-year	Bispectrum near-optimal	69 ± 60	Smith et al. (2009)
WMAP 3-year	Bispectrum optimal	58 ± 46	Smith et al. (2009)
WMAP 5-year	Bispectrum near-optimal	51 ± 60	Komatsu et al. (2008)
WMAP 5-year	Bispectrum optimal	38 ± 42	Smith et al. (2009)
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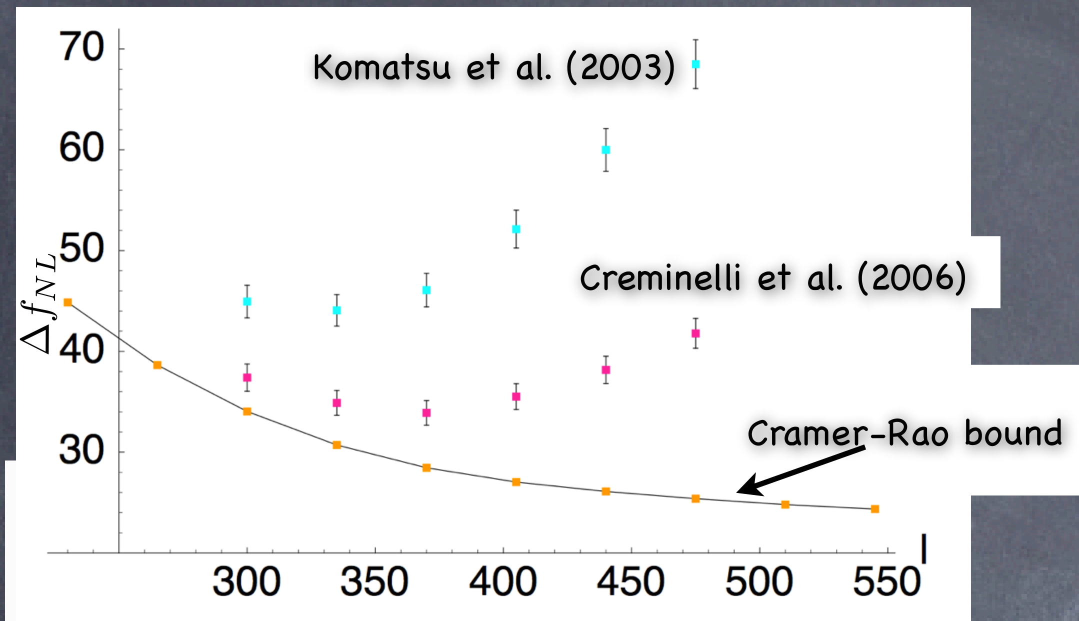

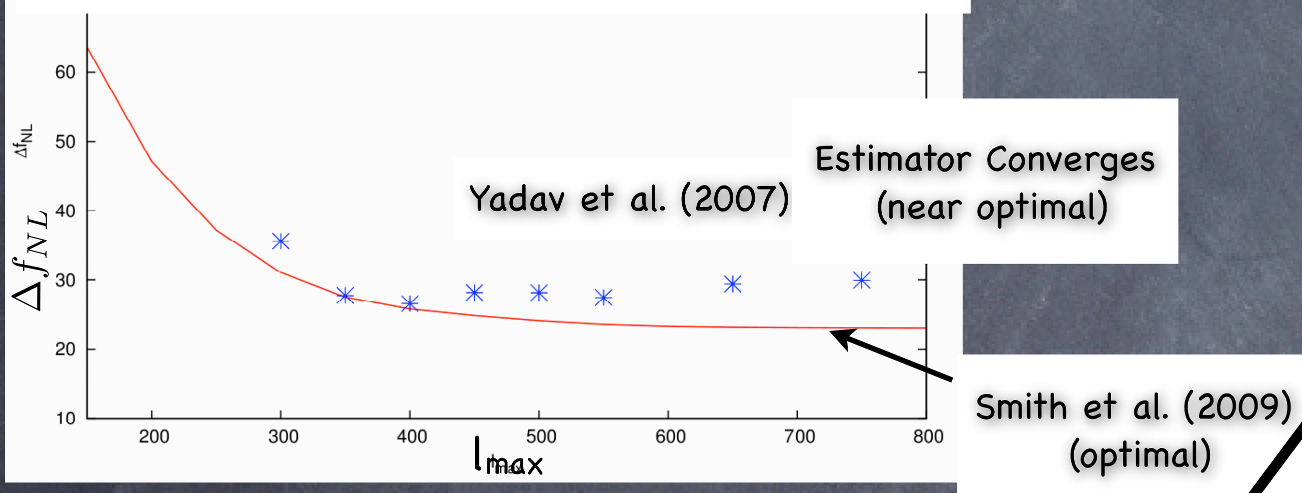

Hint/Evidence of non-Gaussianity

~2.5σ deviation from Gaussianity, in wmap3 year data

Yadav & Wandelt (2008); Smith, Senatore and Zaldarriaga 2009)

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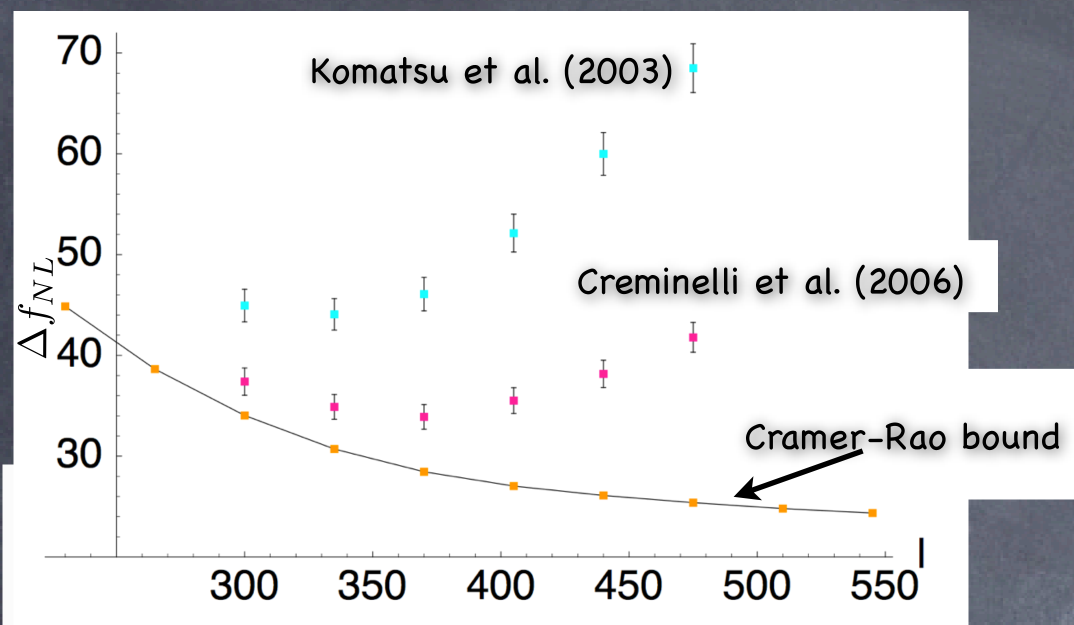

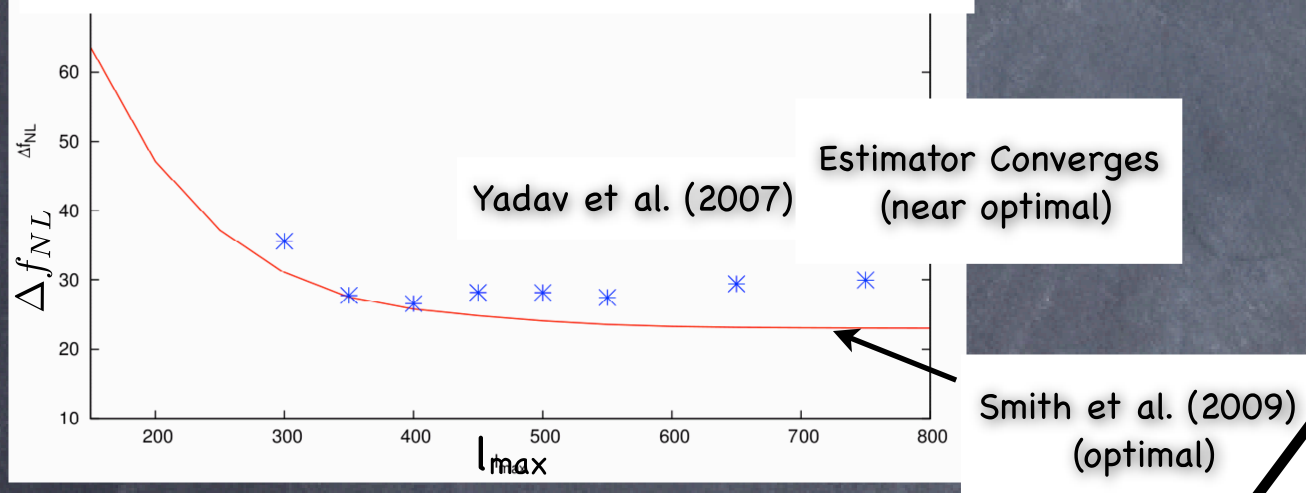

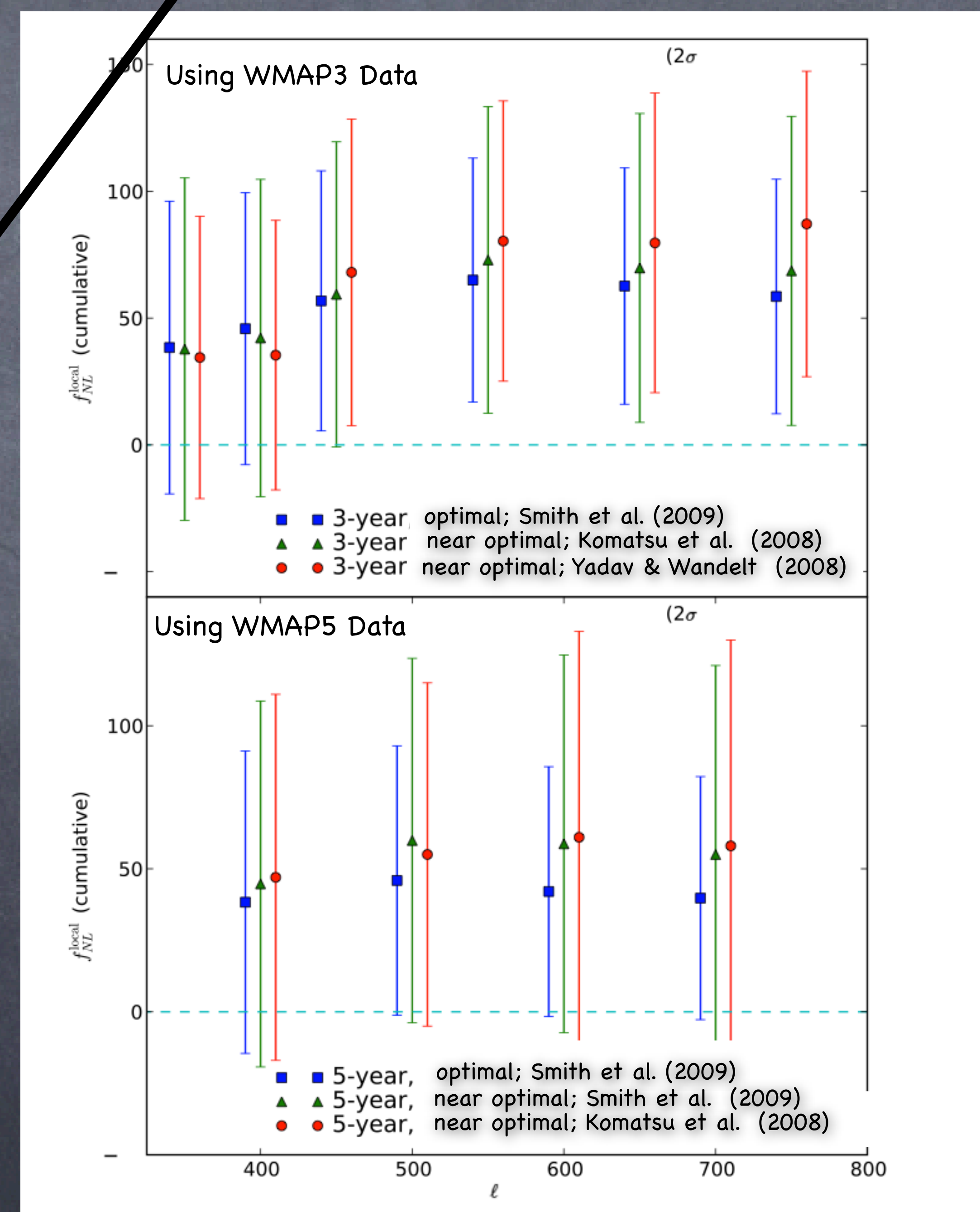

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