MODULI STABILIZATION

AND BRANE INFLATION

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Search for an Inflation Model in String Theory, with all moduli stabilized, is still on!

Recall Moduli Stabilzation of KKLT

Step-I: Starts with the assumption that, in a flux compactification of Type IIB on Calabi-Yau threefold, all moduli are fixed except for one Kahler modulus ρ which survives the compactification.

Leading to a constant superpotential W_0 (GVW flux superpotential) and the Kahler potential

$$\mathcal{K}(\rho, \bar{\rho}) \;=\; -3\ln(\rho + \bar{\rho})$$

Step-II: The volume modulus is then fixed through a non-perturbative correction to W_0, of the form:

$$W(\rho) \;=\; W_0 + A\, e^{-b\rho}$$

A is a constant, b is instanton charge for Euclidean D3- brane or $2\pi/n$ for a stack of n D7-branes wrapping a four-cycle.

KKLT studied the potential:

$$V_F \;=\; e^{\mathcal{K}} \; [G^{\bar{i}j}\overline{D_i W}\, D_j W \;-\; 3|W|^2]$$

Minimization of the potential leads to moduli stabilization.

The minimum of the potential turns out to be -ve (AdS)

Step-III: Uplifting:

Adding an anti-D3 brane to the system which sits at the tip of the throat. The warped tension of the anti-D3 branes lifts the AdS minimum to a local metastable dS vacuum.

Brane-anti-brane Inflation :

KKLMMT proposal starts with the one-modulus model of KKLT. Brane dynamics is added by including a mobile D3-brane which is drawn down the throat by it attraction towards the anti-D3 brane.

A D3-brane added to a Type IIB vacuum backreacts on the metric and changes the Kahler potential of the low energy four dimensional supergravity.

KKLMMT considered the modifications to Kahler potential, which depends upon the position, (z^i), of the D3-brane:

$$\mathcal{K}(\rho, \bar{\rho}) \;=\; -3\ln[\rho + \bar{\rho} - K(z, \bar{z})]$$

The non-perturbative potential was kept as in the KKLT model.

The potential $V = V_f + V_D$, where V_D includes the D3-brane interaction with anti-D3-brane (in the warped geometry) was calculated taking $K(z, \bar{z}) \equiv K(\phi, \bar{\phi}) = \phi\bar{\phi}$. It was assumed that such a potential has a dS minimum at some values of ρ and ϕ and the mass of ϕ (D3-brane moduli/ inflaton) was computed in an expansion about this minimum and it turned out that such a mass of the inflaton field lead to a slow-roll parameter, in the inflation model,

$$\eta \;=\; 2/3,$$

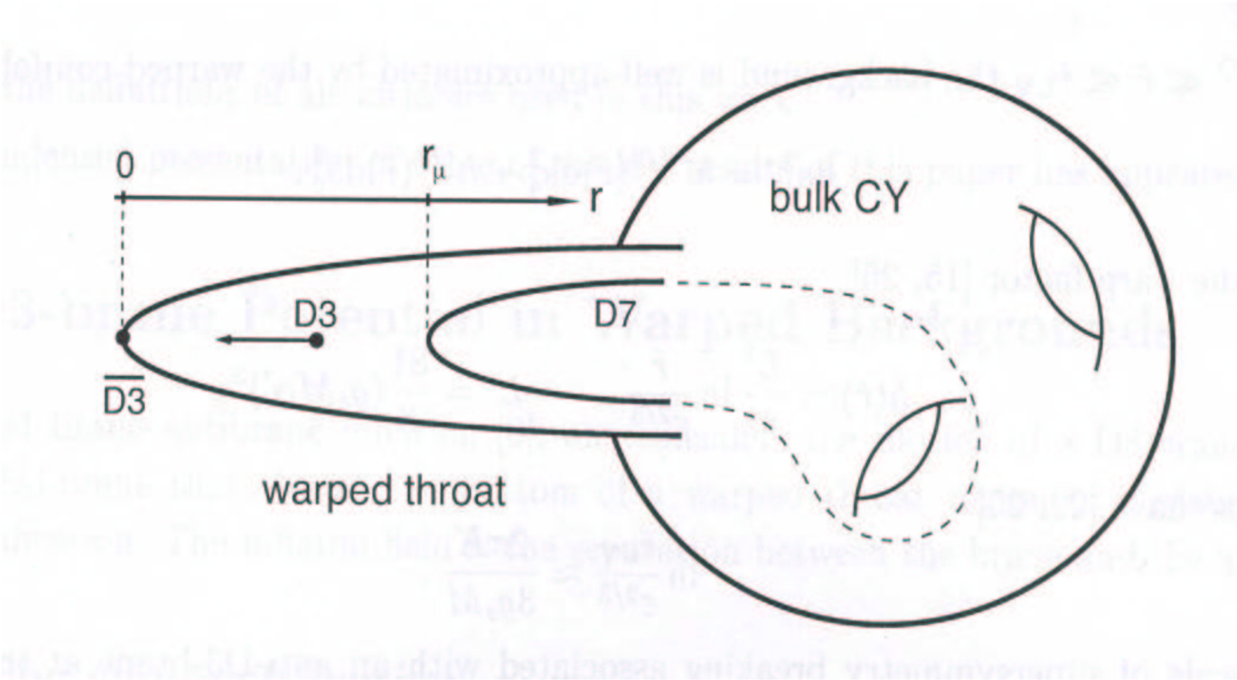
Incompatible with sustained slow-roll inflation.

Presence of D3-brane also modifies the non-perturbative superpotential Giddings and Maharana, hep-th/0507158

Does it solve the η problem?

This correction, for the warped compactification, has been computed

Bauman, Dymarsky, Klebanov, Maldacena, Mc Allister and Murugan hep-th/0607050


0
r_μ
r
bulk CY
D3
D7
$\overline{D3}$
warped throat

Result: $W = W_0 + W_{np}$

$$W_{np} = A(z_\alpha)\, e^{-b\rho}$$

$A(z_\alpha)$ depends upon the embedding of D7-branes wrapping the four-cycle and preserving SUSY, specified by $f(z_\alpha) = 0$.

$$A(z_\alpha) = A_0 \left(\frac{f(z_\alpha)}{f(0)} \right)^{1/n}$$

Application to Brane Inflation: BDKMS **Delicate Universe** 0705.3837 (hep-th), BDKM 0706.0360 (hep-th)

Kuperstein embedding: $f(z_1) = \mu - z_1$

$$K = \frac{3}{2} \left(\sum_{i=1}^{4} |z_i|^2 \right)^{2/3} = \frac{3}{2} r^2$$

$$\mathcal{K}(\rho, \bar{\rho}, z_\alpha, \bar{z}_\alpha) = -3M_{\rm pl}^2 \ln[\rho + \bar{\rho} - \gamma K]$$

Taking $z_1 = -r^{3/2}/\sqrt{2}$, σ = Re (ρ) and canonical normalized field $\phi = \sqrt{3T_3/2}r$ with $r_\mu^3 \equiv 2\mu^2$ i.e. $\phi_\mu^2 = 3/2T_3(2\mu^2)^{2/3}$ leads to the **full two-field potential:**

$$V(\phi,\sigma) = \frac{b|A_0|^2}{3M_{\rm pl}^2}\frac{e^{-2b\sigma}}{U^2(\phi,\sigma)}g^{2/n}(\phi)\Bigg\{2b\sigma + 6 - 6e^{b\sigma}\frac{|W_0|}{|A_0|}\frac{1}{g^{1/n}(\phi)} + \frac{3}{n}\left[c\frac{\phi}{\phi_\mu} - \left(\frac{\phi}{\phi_\mu}\right)^{3/2} - \left(\frac{\phi}{\phi_\mu}\right)^3\right]\frac{1}{g^2(\phi)}\Bigg\} + \frac{D(\phi)}{U^2(\phi,\sigma)}\,,$$

where

$$U(\phi,\sigma) = 2\sigma - \frac{\gamma}{T_3}\phi^2\,, g(\phi) = 1 + \left(\frac{\phi}{\phi_\mu}\right)^{3/2}$$

$$D(\phi) = D_0\left(1 - \frac{27D_0}{64\pi^2\phi^4}\right)\,, c = 1/(6\pi\gamma T_3\phi_\mu^2)$$

and $D_0 = 2h_0^{-1}T_3$ i.e. twice the warped tension at the tip.

An effective single field potential: $V(\phi) \equiv V(\sigma_*(\phi), \phi)$ where $\partial_\sigma V|_{\sigma_*(\phi)} = 0$ (instantaneous minimum)

Assumes σ is more massive than ϕ and evolves adiabatically while remaining in its instantaneous minimum

$$\sigma_* \approx \sigma_0 \left[1 + c_{3/2}\left(\frac{\phi}{\phi_\mu}\right)^{3/2}\right] .$$

σ_0 related to γ and $c_{3/2}$ is related to n and W_0/A_0 **Numerical simulation does not support the assumption for truely generic configuration of a D3-brane in a compact space**

Working with the approximated expression yields number of e-foldings to be less than 10 even for highly finetuned parameters

σ is not even canonical scalar field but χ is

$$\frac{\chi}{M_{\rm pl}} = \sqrt{\frac{3}{2}} \ln \sigma .$$

Best bet is to consider a two-field inflation with $V(\chi, \phi)$

For flat FRW metric with scale factor a, Eqns of motion:

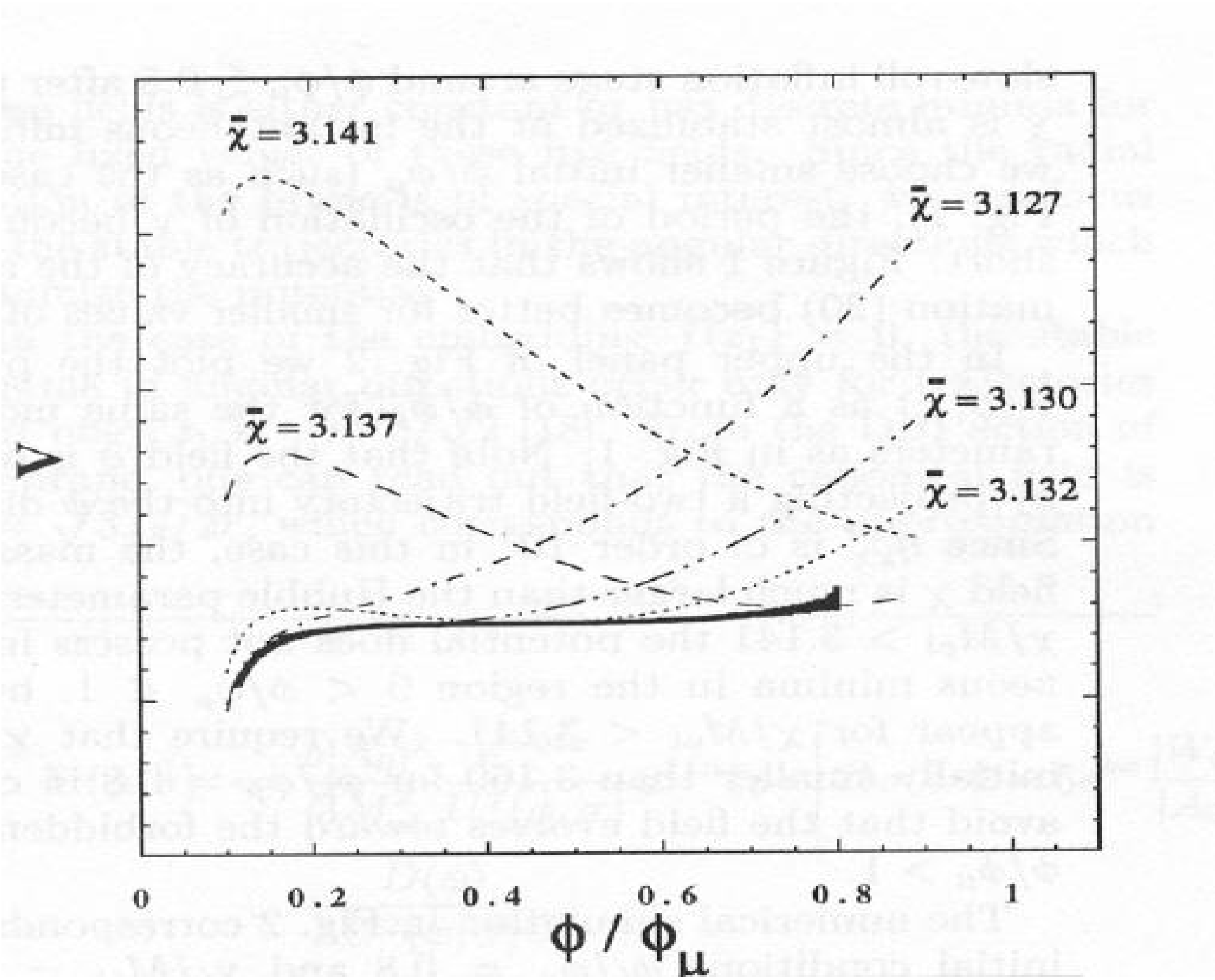
$$\dot{H} = -\frac{1}{2M_{\rm pl}^2}(\dot{\phi}^2 + \dot{\chi}^2)\,,$$
$$\ddot{\phi} + 3H\dot{\phi} + V_{,\phi} = 0\,,$$
$$\ddot{\chi} + 3H\dot{\chi} + V_{,\chi} = 0$$

and the constraint eqn:

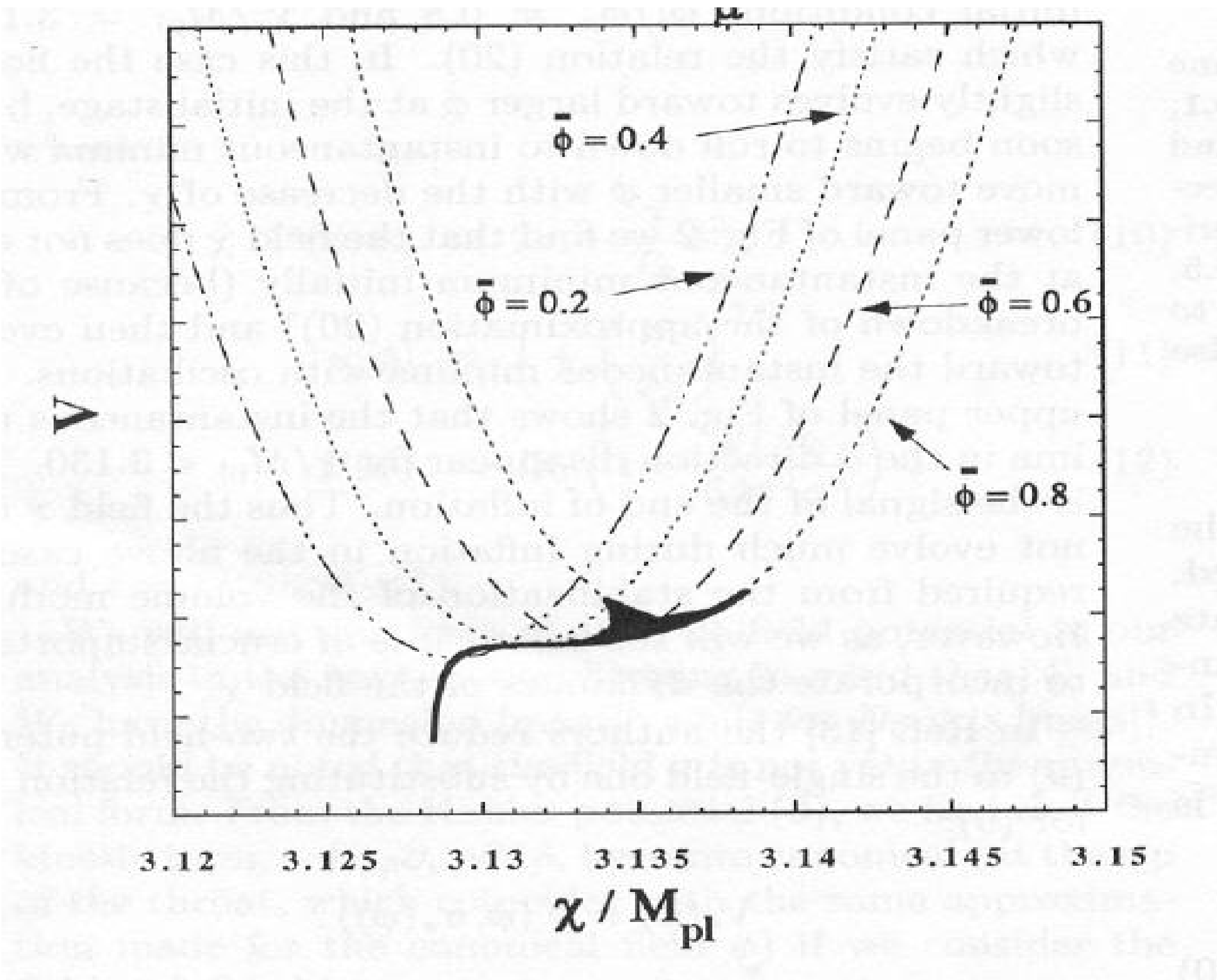
$$3H^2 = \frac{1}{M_{\rm pl}^2}\left[\frac{1}{2}\dot{\phi}^2 + \frac{1}{2}\dot{\chi}^2 + V(\phi,\chi)\right]$$

Slow roll parameters:

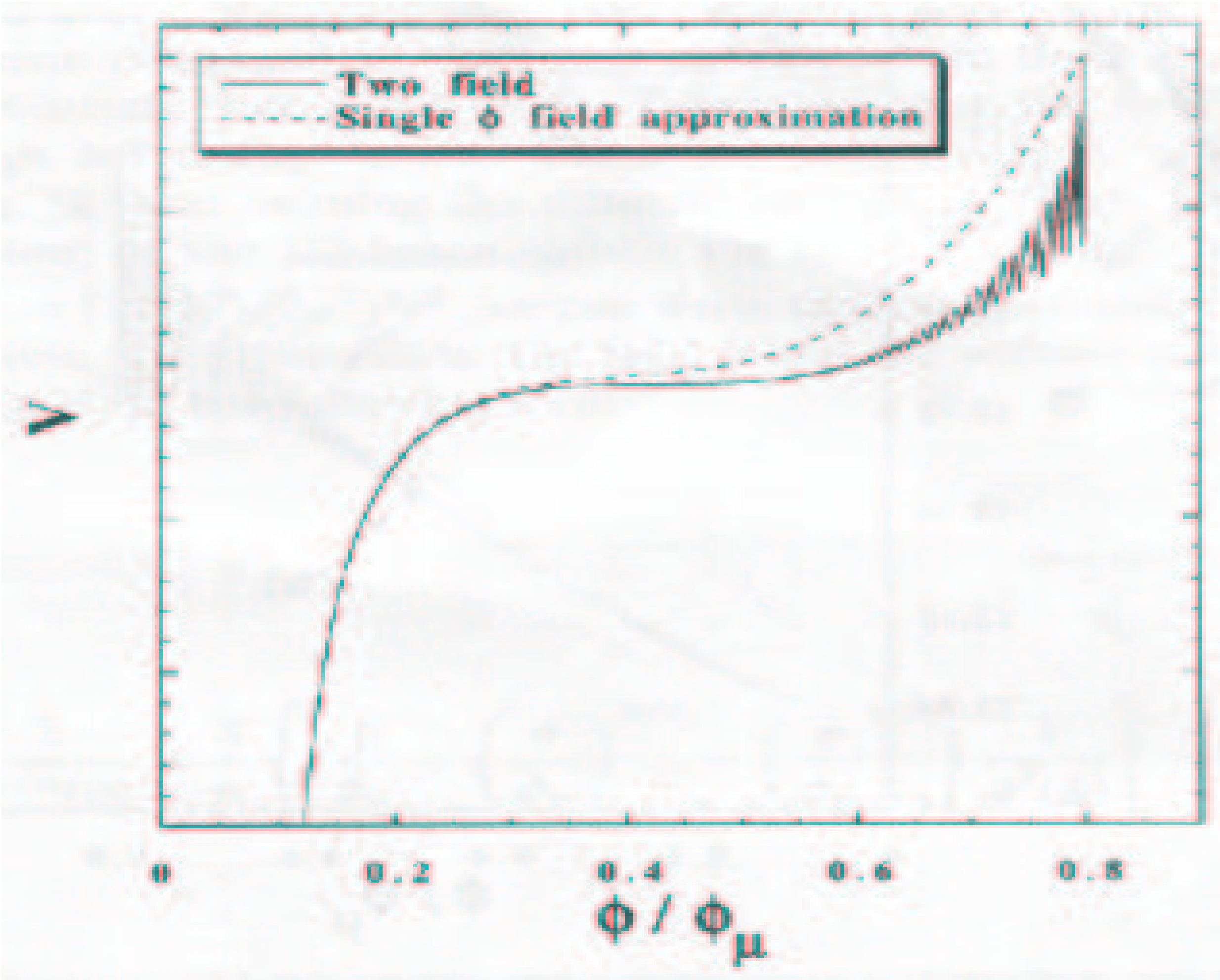
$$\epsilon_\phi = \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\phi}}{V}\right)^2\,, \quad \epsilon_\chi = \frac{M_{\rm pl}^2}{2}\left(\frac{V_{,\chi}}{V}\right)^2\,,$$
$$\eta_{\phi\phi} = M_{\rm pl}^2\frac{V_{,\phi\phi}}{V}\,, \quad \eta_{\chi\chi} = M_{\rm pl}^2\frac{V_{,\chi\chi}}{V}\,, \quad \eta_{\phi\chi} = M_{\rm pl}^2\frac{V_{,\phi\chi}}{V}\,.$$



Potential vs $\bar\phi$ for various fixed $\bar\chi$ for $n = 8, A_0 = 1, b\sigma_0 = 10.1, W_0 = 3.496 \times 10^{-4}, D_0 = 1.215 \times 10^{-8}, \phi_\mu = 0.25$ **Solid curve is obtained by solving the background eqn numerically for initial condition**$\phi/\phi_\mu = 0.8$ and $\bar\chi \equiv \chi/M_pl = 3.1385$



Potential vs $\bar{\chi}$ for fixed $\bar{\phi}$



Solid curve is for Potential obtained by numerically solving the background eqns in two-field model

Dotted curve is for $V(\phi, \sigma_*(\phi))$

Acceptable value of Number of e-foldings in two-field model

This discripancy reflects that the background trajectory is not given by the fieldϕ

but by the field ψ satisfying:

$$\dot{\psi} = (\cos\theta)\dot{\phi} + (\sin\theta)\dot{\chi} \ \ , \tan\theta = \dot{\chi}/\dot{\phi} \, .$$

$$\dot{s} \equiv -(\sin\theta)\dot{\phi} + (\cos\theta)\dot{\chi} = 0$$

fields donot move to the direction orthogonal to ψ.

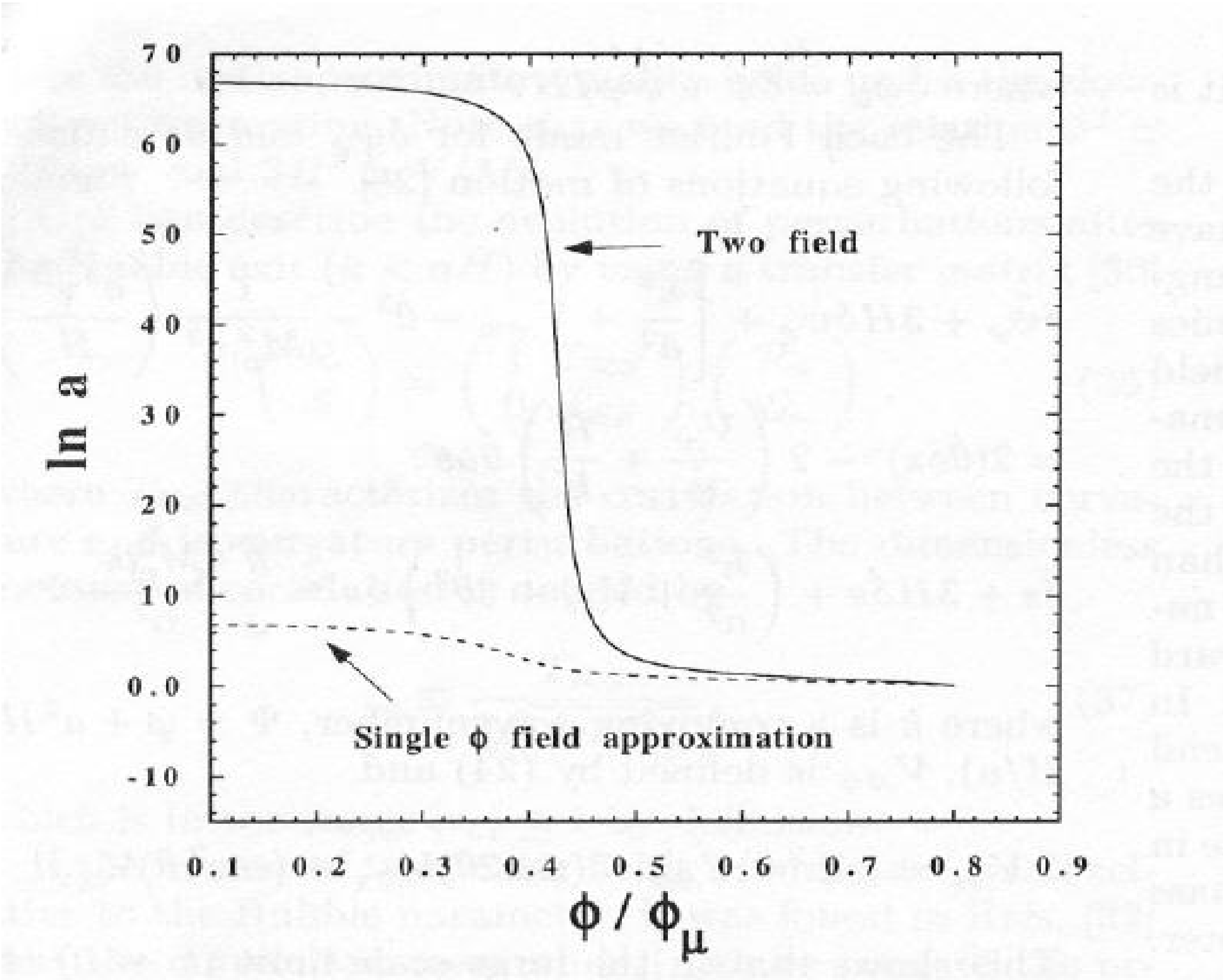
Correct single field description of inflation dynamics is in terms of ψ with mass sqared:

$$V_{,\psi\psi} = (\cos^2\theta)V_{,\phi\phi} + (\sin 2\theta)V_{,\phi\chi} + (\sin^2\theta)V_{,\chi\chi} \, .$$

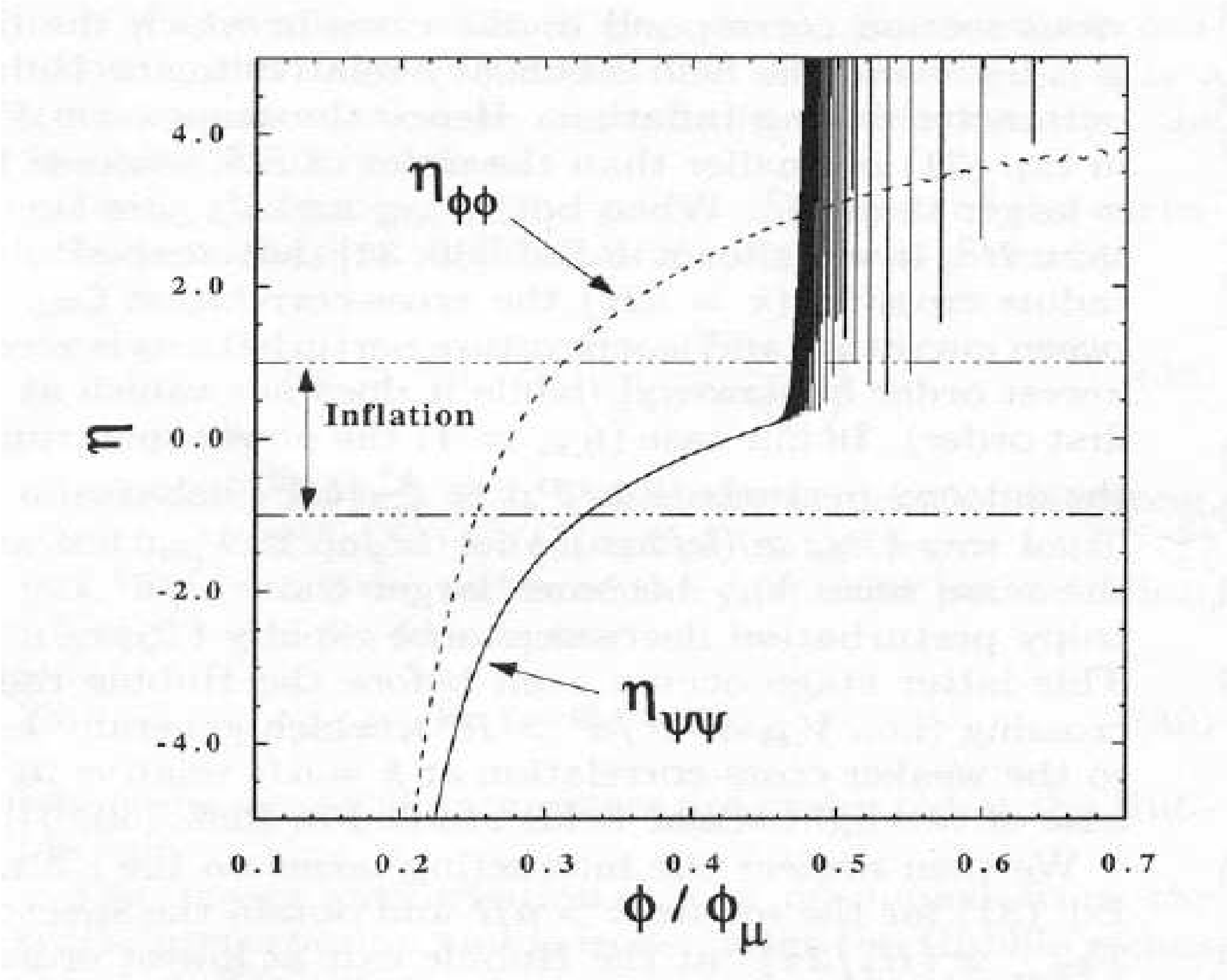
$$V_{,ss} = (\sin^2\theta)V_{,\phi\phi} - (\sin 2\theta)V_{,\phi\chi} + (\cos^2\theta)V_{,\chi\chi} .$$

Then the slow-roll parameter, $\eta_{\psi\psi} \equiv M_{\rm pl}^2 V_{,\psi\psi}/V$, is

$$\eta_{\psi\psi} = (\cos^2\theta)\eta_{\phi\phi} + (\sin 2\theta)\eta_{\phi\chi} + (\sin^2\theta)\eta_{\chi\chi} \, .$$



Period of inflation is given by $|\eta_{\psi\psi}| < 1$ **which is for** $0.3 \leq \phi/\phi_\mu \leq 0.5$.



$\eta_{\phi\phi}$ is larger than unity during this period

Possible to have larger number of e-foldings for $D_0 = 1.218 \times 10^{-8}$ instead of $D_0 = 1.215 \times 10^{-8}$, increases to 148.

The field stays for longer time at the instantaneous minima

If $D_0 = 1.210 \times 10^{-8}$, number of e-foldings decreases to 43

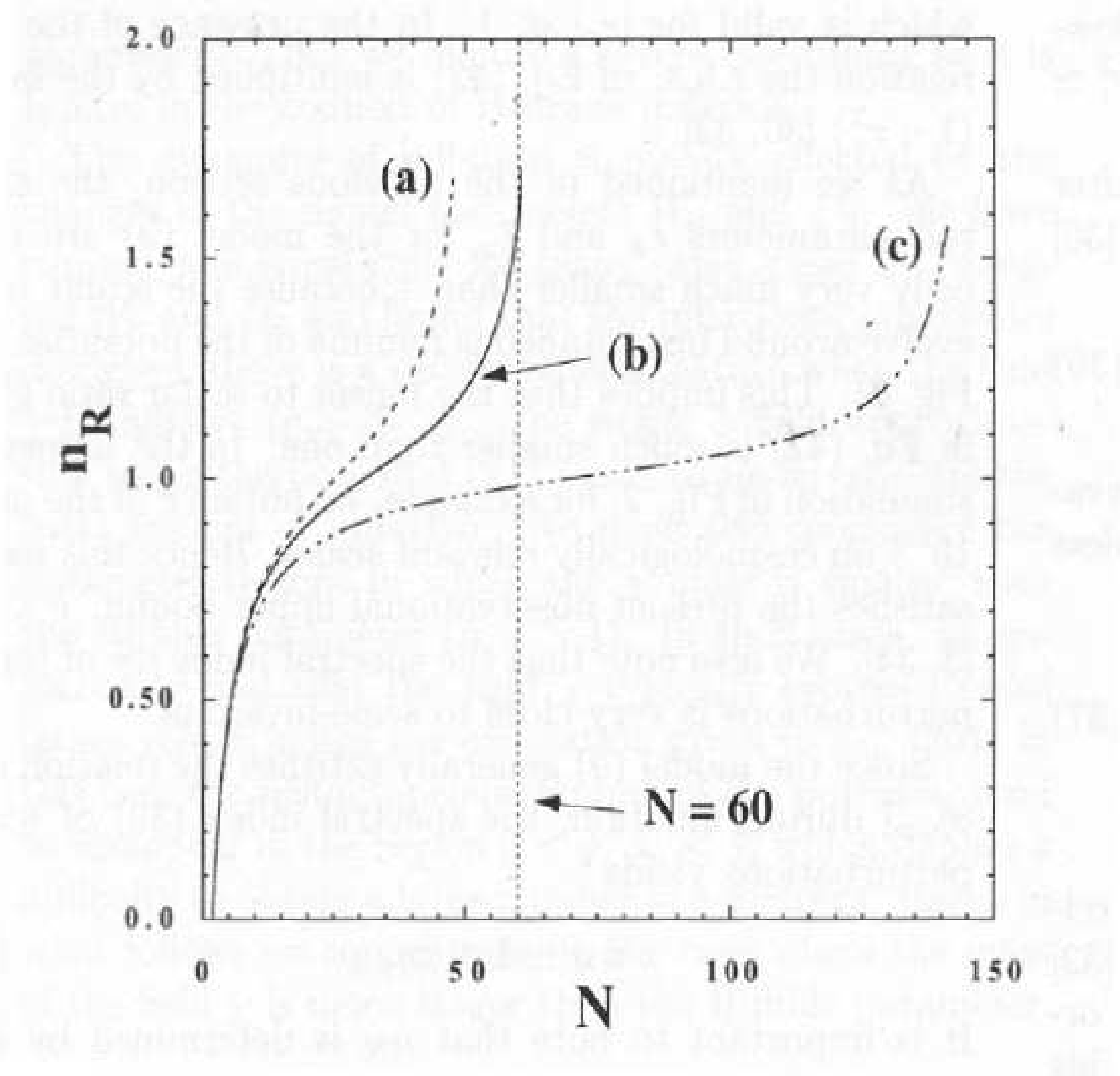
This shows how sensitive it is to model parameters and reflects severe fine tuning

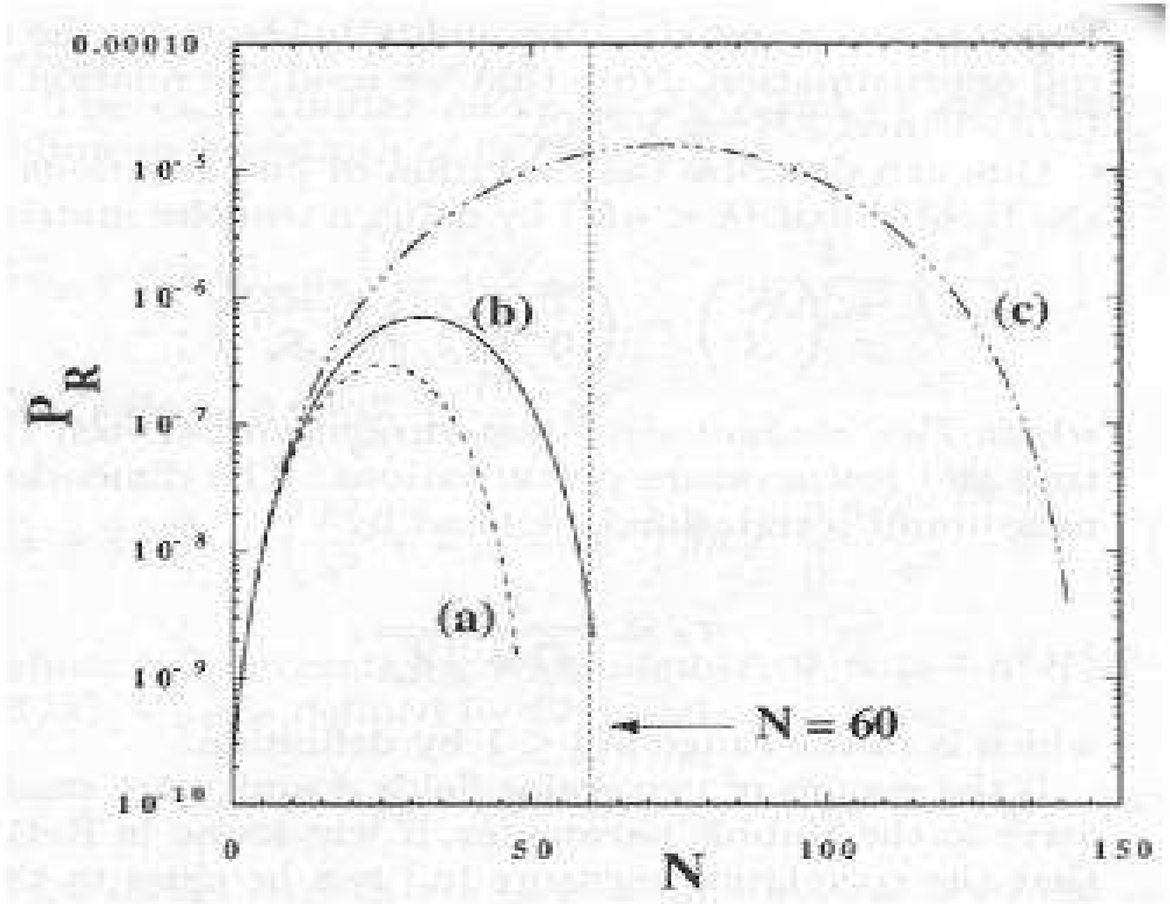
In two-field model, density perturbations are different from that of single-field model due to presence of isocurvature (entropy) perturbations.

denote field perturbations in ϕ and χ as $\delta\phi$ and $\delta\chi$ and define, with earlier defn of θ

$$\delta\psi \equiv (\cos\theta)\delta\phi + (\sin\theta)\delta\chi\,,$$
$$\delta s \equiv -(\sin\theta)\delta\phi + (\cos\theta)\delta\chi$$

Perturb the spacetime around FRW and go through the standard analysis:


2.0
1.5
1.0
0.50
0.0
n_R
(a)
(b)
(c)
N = 60
0
50
100
150
N


0.00010
10⁻⁵
10⁻⁶
10⁻⁷
10⁻⁸
10⁻⁹
10⁻¹⁰
P_R
(a)
(b)
(c)
N = 60
0
50
100
150
N

Results

(1) Tensor to scalar ratio is found to be 10^{-5}, consistent with observational bound < 0.3.

(2) The spectral index of scalar perturbation turns out to be $n_{\mathcal{R}} \simeq 1 + 2\eta_{\psi\psi}$ i.e determined by $\eta_{\psi\psi}$ instead of $\eta_{\phi\phi}$, consistent with our observation that background trajectory is along ψ direction.

(3) However, we found that when the spectrum approaches the scale scale invarint value $n_{\mathcal{R}} = 1$, the amplitude $\mathcal{P}_{\mathcal{R}}$ tends to be larger than the COBE normalized value (2.4×10^{-9}) by about three order of maginude.

Thus the correction to non-perturbative superpotential, found as yet, is not adequate!